Dynamical Symmetry breaking in supersymmetric extension of Nambu–Jona-Lasinio model

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Outline

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Introduction to NJL model

- Dynamical mass generation and symmetry breaking is a very interesting theoretical topic with important phenomenological applications.
- Nambu adopted the idea of Cooper pairing to construct a classic model of dynamical mass generation and symmetry breaking. This is the Nambu–Jona-Lasinio (NJL) model with a strong attractive four-fermi interaction.
- The nonperturbative gap equation shows that the interaction induces a bi-fermion vacuum condensate which serves as the source for the fermion Dirac mass.
- For the SM this phenomenon was studies by W. A. Bardeen, C. T. Hill and M. Lindner.
- The idea of constructing a supersymmetric version of the NJL model (SNJL) was introduced in 1982 by W. Buchmúller and S. T. Love, and continued in 1984 W. Buchmúller and U. Ellwanger.
- As a direct supersymmetrization (SNJL model), two chiral superfields are used to construct a dimension-six operator that has a four-fermion interaction in its D term.
- It was realized that nontrivial vacuum is not possible due to supersymmetric cancelation, unless soft supersymmetry (SUSY) breaking is incorporated.
- The approach leads to the MSSM as the low energy effective field theory, with interesting relations among some of the model parameters.

- Fitting the current top mass from the SNJL model pushes $\tan\beta$ value into a narrow window between 0.5 to 1.5 which is essentially excluded by the LEP result.
- A supersymmetric extension with a holomorphic dimension five operator was introduced recently by Dong Won Jung, Jae Sik Lee and Otto Kong.
- The idea is to formulate a model that gives rise to the Minimal Supersymmetric Standard Model as the low energy effective theory with both Higgs superfields as composites.
- In addition, to fully accommodate the masses and mixing of the quarks in the MSSM, dimension-five four-superfield interactions cannot be avoided.
- A renormalization group analysis is performed to establish the phenomenological viability of the scenario, with admissible background scale that could go down to the TeV scale.
- Within the model, the fine-tuning problem on the four-fermion coupling while allowing the lower top mass can be eliminated. Moreover, it is shown that the bottom quark Yukawa coupling plays a more important role than the top Yukawa, and the composite scale could be very low.
- The holomorphic SNJL model provides an interesting alternative for dynamical symmetry breaking and its phenomenology is worth a more serious investigation at the LHC and ILC (Christophe Grojean talk: Composite Higgs physics at a linear collider).

(1)
$$\begin{aligned}\mathcal{L} &= i\partial_m\psi_+\sigma^m\bar{\psi}_+ + i\partial_m\psi_-\sigma^m\bar{\psi}_- \\ &+ g^2\psi_+\psi_-\bar{\psi}_+\bar{\psi}_-\end{aligned}$$

where ψ_+, ψ_- are two-component Weyl spinors and the coupling g has mass dimension -1 which shows that the model is non-renormalizable and has to be provided with a cut-off Λ. The Lagrangian $\mathcal{L}$ is invariant under the chiral $U(1)$ transformations:

(2)
$$\begin{aligned}U(1)_V &: \quad \psi_\pm \rightarrow e^{\pm i\alpha}\psi_\pm \\ U(1)_A &: \quad \psi_\pm \rightarrow e^{i\beta}\psi_\pm\end{aligned}$$

We can rewrite $\mathcal{L}$ as

(3)
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

where

(4)
$$\mathcal{L}_0 = i\partial_m\psi_+\sigma^m\bar{\psi}_+ + i\partial_m\psi_-\sigma^m\bar{\psi}_- - m\psi_-\psi_+$$

(5) $$\mathcal{L}_I = g^2 \psi_+ \psi_- \bar{\psi}_+ \bar{\psi}_- + \; m\psi_- \psi_+$$

Figure 1: Diagrams for proper self-energy $\Gamma^2(p)$.

The mass m is defined by

(6) $$\Gamma^2(p)_{\gamma p=-m} = 0$$

where $\Gamma^2(p)$ is the proper two- point function. This leads to the gap equation:

(7) $$m = mg^2 \frac{\Lambda^2}{8\pi^2} \left[1 - \frac{|m|^2}{\Lambda^2} \, \ln \frac{\Lambda^2}{|m|^2} + O(1/\Lambda^4) \right] \; .$$

which has nontrivial solution i.e. $m \neq 0$ for the coupling constant satisfying the inequality

(8) $$g^2 > \frac{8\pi^2}{N_C \Lambda^2} \ ,$$

which indicates that a vacuum expectation value of the operator $\psi_+\psi_-$ is formed and the $U(1)_A$ symmetry is dynamically broken.

- The phenomenon of dynamical symmetry breaking can be also studied from the effective potential of an auxilary filed ϕ and one can rewrite the Lagrangian in Eq.(1)as

(9) $$\begin{aligned} \mathcal{L} &= i\partial_m \psi_+ \sigma^m \bar{\psi}_+ + i\partial_m \psi_- \sigma^m \bar{\psi}_- \\ -\phi^* \phi &+ g\phi\psi_+\psi_- + g\phi^* \bar{\psi}_+ \bar{\psi}_- \end{aligned}$$

Equation of motion for ϕ gives $\phi = g\bar{\psi}_+\bar{\psi}_-$ and one can construct tadpole graphs from the interaction terms $g\phi\psi_+\psi_-, g\phi^*\bar{\psi}_+\bar{\psi}_-$ which gives the first derivative of the effective potential of ϕ that after equating it with zero we get the gap equation obtained before. for coupling g^2 satisfy the inequality (8), ϕ acquires VEV $\neq 0$ and a dirac mass is generated that breaks the chiral $U(1)_A$ symmetry.

Supersymmetric extensions of NJL model

We start with the following Lagrangian density for what one wants to have as a Dirac pair of chiral ('quark') superfields, $\Phi_\pm(y,\theta) = A_\pm(y) + \sqrt{2}\theta\psi_\pm(y) + \theta^2 F_\pm(y)$:

(10) $$\mathcal{L} = \int d^4\theta \left[\left(\Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_- \right)(1-\Delta) + \left(\mathcal{M}\, \Phi_+ \Phi_-\, \delta^2(\bar\theta) + h.c. \right) \right] \; + \; \mathcal{L}_I \; .$$

Here, $\Delta = \tilde{m}^2\theta^2\bar\theta^2$ characterizes a soft supersymmetry breaking mass-squared $\tilde{m}^2$ for the scalar fields $A_\pm$ and $\mathcal{M}$ a superfield Dirac mass parameter. An important point to note is that $\mathcal{M}$ should be considered like a constant superfield with a supersymmetric as well as a supersymmetry breaking part. We write

(11) $$\mathcal{M} = m - \theta^2\eta \; ,$$

where m is the usual (supersymmetric) Dirac mass and η its supersymmetry breaking counterpart.

- m corresponds to Dirac mass for the fermion pair $\psi_\pm$ and $|m|^2$ contributions to both $A_\pm$ mass-squared.
- η gives (so-called left-right) mass mixing between the $A_\pm$ pair, and does not correspond to a mass eigenvalue.

- The crucial step is to write the effective action as $\Gamma \equiv \Gamma(\Phi_+, \Phi_-, \Phi_+^\dagger, \Phi_-^\dagger, \theta, \bar{\theta})$ accordingly, where the explicit dependence on θ and $\bar{\theta}$ allows supersymmetry breaking parts to be included. We are interested in the quadratic part $\Gamma_{+-}^{(2)}(p, \theta)$ in Γ, with

(12) $$\Gamma = \int \frac{d^4 p}{2\pi^4} \int d^2\theta \, \Phi_+(-p, \theta) \, \Gamma_{+-}^{(2)}(p, \theta^2) \, \Phi_-(p, \theta) + h.c. + \cdots \ ,$$

where

(13) $$\Phi_\pm(p, \theta) = \int d^4x \, e^{-ip.x} \Phi_\pm(x, \theta) \ .$$

- As a self-consistent Hartree approximation for the dynamically generated nonzero $\mathcal{M}$, the interaction Lagrangian density $\mathcal{L}_I$ is taken to contain a $-\left[\mathcal{M} \, \Phi_+ \Phi_- \delta^2(\bar{\theta}) + h.c.\right]$ term and at least an extra true interaction term which is to be the true origin of the nonzero Dirac mass parameter.

One looks for nontrivial solution for $\mathcal{M}$ from the equation

(14) $$\Gamma^{(2)}_{+-}(p,\theta^2)\Big|_{\text{on-shell}} = 0$$

which is equivalent to the vanishing of the proper self-energy

(15) $$\Sigma_{+-}(p,\theta^2)\big|_{\text{on-shell}} = 0$$

from diagrams produced by $\mathcal{L}_I$. With the $\mathcal{M}$ term in $\mathcal{L}_I$, we have

(16) $$-\mathcal{M} = \Sigma^{(loop)}_{+-}(p,\theta^2)\Big|_{\text{on-shell}} \;,$$

where $\Sigma^{(loop)}_{+-}$ denotes the lowest order contributions to the proper self-energy from loop diagrams involving the true interaction, *i.e.* either one of the four-superfield interactions we will consider.

The superfield propagators in the generic framework. We obtained, within the formulation of Grisaru, Siegel and Roček ,

$$\langle T(\Phi_\pm(1)\Phi^\dagger_\pm(2))\rangle = \frac{-i}{p^2+|m|^2}\delta^4_{12} - \frac{i}{[(p^2+|m|^2+\tilde{m}^2)^2-|\eta|^2]}\left(\eta\bar{m}\theta_1^2+\bar{\eta}m\bar{\theta}_1^{\,2}\right)\delta^4_{12}$$
$$+\frac{i\,[\tilde{m}^2(p^2+|m|^2+\tilde{m}^2)-|\eta|^2]}{(p^2+|m|^2)[(p^2+|m|^2+\tilde{m}^2)^2-|\eta|^2]}\left[|m|^2\theta_1^2\bar{\theta}_1^{\,2}+\frac{D_1^2\theta_1^2\bar{\theta}_1^{\,2}\overline{D}_1^2}{16}\right]\delta \tag{17}$$

$$\langle T(\Phi_+(1)\Phi_-(2))\rangle = \frac{i\,\bar{m}}{p^2(p^2+|m|^2)}\frac{D_1^2}{4}\delta^4_{12}$$
$$-\frac{i}{[(p^2+|m|^2+\tilde{m}^2)^2-|\eta|^2]}\left[\frac{\bar{\eta}\,D_1^2\bar{\theta}_1^{\,2}}{4}-\frac{\eta|m|^2\,D_1^2\theta_1^{\,2}}{4p^2}\right]\delta^4_{12}$$
$$+\frac{i\,\bar{m}\,[\tilde{m}^2(p^2+|m|^2+\tilde{m}^2)-|\eta|^2]}{(p^2+|m|^2)[(p^2+|m|^2+\tilde{m}^2)^2-|\eta|^2]}\left[\frac{D_1^2\theta_1^2\bar{\theta}_1^{\,2}}{4}+\frac{\bar{\theta}_1^{\,2}\theta_1^2D_1^2}{4}\right]\delta^4_{12}\ . \tag{18}$$

where $\delta^4_{12}=\delta^4(\theta_1-\theta_2)$.

Next, we introduce the interactions of interest that are expected to lead to nontrivial $\Sigma_{+-}^{(loop)}$.

- Consider the dimension six four-superfield interaction

(19)
$$g^2 \int d^4\theta \, \Phi_+^\dagger \Phi_-^\dagger \Phi_+ \Phi_- \, (1 - \tilde{m}_C^2 \theta^2 \bar{\theta}^2)$$

  coming with a supersymmetry breaking part. This interaction gives the SNJL model, here extended to include the supersymmetry breaking $\tilde{m}_C^2$ part.

- The dimension five four-superfield interaction is given by

(20)
$$-\frac{G}{2} \int d^4\theta \, \Phi_+ \Phi_- \Phi_+ \Phi_- \, (1 + B\theta^2) \, \delta^2(\bar{\theta}) \; .$$

  It is really a superpotential term, as indicated by the $\delta^2(\bar{\theta})$, hence holomorphic. This HSNJL model is proposed as an alternative supersymmetrization of the NJL model.

  With the above, we are ready to implement the supergraph evaluation of $\Sigma_{+-}^{(loop)}(p, \theta^2)$ at one-loop level.

We use a technique from Miller on one-loop tadpole, extending it to the proper self-energy diagram. The technique can be considered as re-writing the effective action as

(21) $$\Gamma = \int \frac{d^4p}{2\pi^4} \int d^2\theta \, \Phi_+(-p,\theta_1) \left[\int d^2\bar{\theta} \, \Gamma^{(2)}_{+-}(p,\theta^2) \, \delta^2(\bar{\theta}) \right] \Phi_-(p,\theta_2) \Big|_{\theta_1=\theta_2=\theta} + h.c. + ..$$

splitting the vertex with θ_1 and θ_2 distinct from θ in the evaluation of the diagrams before finally enforcing the equal limit. The proper self-energy diagram is to be taken as an integrand over $d^4\theta$, with amplitude at the $\theta_1 = \theta_2 = \theta$ limit contributing to $\Sigma^{(1loop)}_{+-}(p,\theta^2)$ as given schematically by

(22) $$\int d^2\bar{\theta} \left[\text{Amplitude} \right] \Big|_{\theta_1=\theta_2=\theta} \longrightarrow \int d^2\bar{\theta} \, \Sigma^{(1loop)}_{+-}(p,\theta^2) \, \delta^2(\bar{\theta}) \; .$$

Note that at the one-loop level, $\Sigma^{(1loop)}_{+-}$ is actually independent of the external momentum p, with a loop momentum integral to be evaluated with a cut-off Λ.

Dimension six operator case(SNJL)

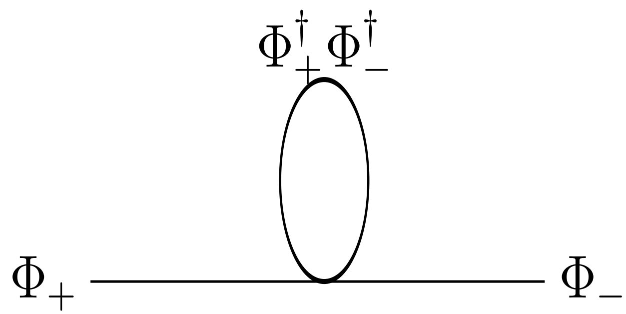

Figure 2: Superfield diagram for proper self-energy $\Sigma_{+-}(p, \theta^2)$ with the dimension six four-superfield interaction.

The propagator as from (the conjugate of) Eq.(18) has three terms, the last two each has two parts (from the two fractions inside the big bracket). We present the contribution from each part separately schematically as

$$\Sigma^{(fig1)} = \Sigma_1^{(fig1)} + \Sigma_{2a}^{(fig1)} + \Sigma_{2b}^{(fig1)} + \Sigma_{3a}^{(fig1)} + \Sigma_{3b}^{(fig1)} ,$$

One may read off directly from the results the contributions to the the supersymmetric (m) and supersymmetry breaking (η) parts of gap equation,

$$-\mathcal{M} = \Sigma^{(fig1)}$$

Through the term by term supergraph calculations of the partial amplitudes, we obtain

$$\int d^2\bar\theta\, \Sigma_{2a}^{(fig1)}\, \delta^2(\bar\theta) = \int d^2\bar\theta\, (-\eta g^2)(1 - \tilde m_C^2 \theta^2 \bar\theta^2)\, I_2(|m|^2, \tilde m^2, |\eta|, \Lambda^2)\ ,$$

$$\int d^2\bar\theta\, \Sigma_{3a}^{(fig1)}\, \delta^2(\bar\theta) = \int d^2\bar\theta\, m g^2 (1 - \tilde m_C^2 \theta^2 \bar\theta^2)\, I_1(|m|^2, \tilde m^2, |\eta|, \Lambda^2)\, \bar\theta^2\ ,$$

(23)
$$\int d^2\bar\theta\, \Sigma_{3b}^{(fig1)}\, \delta^2(\bar\theta) = \int d^2\bar\theta\, m g^2 (1 - \tilde m_C^2 \theta^2 \bar\theta^2)\, I_1(|m|^2, \tilde m^2, |\eta|, \Lambda^2)\, \bar\theta^2\ ,$$

where

$$I_1(|m|^2, \tilde m^2, |\eta|, \Lambda^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{[\tilde m^2 (k^2 + |m|^2 + \tilde m^2) - |\eta|^2]}{(k^2 + |m|^2)[(k^2 + |m|^2 + \tilde m^2)^2 - |\eta|^2]}$$

$$= \frac{1}{16\pi^2} \left[\frac{1}{2} (|m|^2 + \tilde m^2) \ln \frac{(|m|^2 + \tilde m^2 + \Lambda^2)^2 - |\eta|^2}{(|m|^2 + \tilde m^2)^2 - |\eta|^2} - |m|^2 \ln \frac{(|m|^2 + \Lambda^2}{|m|^2} \right.$$

(24)
$$\left. + |\eta| \left(\tanh^{-1} \frac{|m|^2 + \tilde m^2 + \Lambda^2}{|\eta|} - \tanh^{-1} \frac{|m|^2 + \tilde m^2}{|\eta|} \right) \right]\ ,$$

$$I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + |m|^2 + \tilde{m}^2)^2 - |\eta|^2}$$

$$= \frac{1}{16\pi^2} \left[\frac{1}{2} \ln \frac{(|m|^2 + \tilde{m}^2 + \Lambda^2)^2 - |\eta|^2}{(|m|^2 + \tilde{m}^2)^2 - |\eta|^2} \right.$$

$$\left. + \frac{|m|^2 + \tilde{m}^2}{|\eta|} \left(\tanh^{-1} \frac{|m|^2 + \tilde{m}^2 + \Lambda^2}{|\eta|} - \tanh^{-1} \frac{|m|^2 + \tilde{m}^2}{|\eta|} \right) \right] . \tag{25}$$

For the gap equation (16) with $\Sigma^{(fig1)}$, we hence obtain

$$m = 2mg^2 I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \; ,$$

$$\eta = -\eta \, g^2 \tilde{m}_C^2 \, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \; . \tag{26}$$

A couple of remarks are in order:

- $\tilde{\mathbf{m}}_{\mathbf{C}}^{\mathbf{2}} = \mathbf{0}$, we get $\eta = 0$ and hence the gap equation for m reduces to

$$m = 2m \, g^2 I(|m|^2, \tilde{m}^2, \Lambda^2) \; , \tag{27}$$

where

$$I(|m|^2, \tilde{m}^2, \Lambda^2) = \frac{1}{16\pi^2}\left[|m|^2 \ln\frac{|m|^2(|m|^2+\tilde{m}^2+\Lambda^2)}{(|m|^2+\tilde{m}^2)(|m|^2+\Lambda^2)} + \tilde{m}^2 \ln\left(1+\frac{\Lambda^2}{|m|^2+\tilde{m}^2}\right)\right] .$$

(28)

- $\tilde{m} \to \infty$ where the scalar particles of $\Phi_\pm$ become heavy and decoupled, m becomes the simple Dirac fermion/quark mass which then satisfies the equation

  (29) $$m = \frac{mg^2}{8\pi^2}\left[\Lambda^2 + |m|^2 \ln|m|^2 - |m|^2 \ln(\Lambda^2+|m|^2)\right] ,$$

  giving

  (30) $$(g^2)^{-1} = \frac{\Lambda^2}{8\pi^2}\left[1 - \frac{|m|^2}{\Lambda^2}\ \ln\frac{\Lambda^2}{|m|^2} + O(1/\Lambda^4)\right] .$$

  This is the standard NJL result for the pure fermionic model usually given with and extra N_c factor for the $\psi_\pm$ fermions as colored quarks.

- $\tilde{m}^2 = 0$, the gap equation will have a trivial solution $m = 0$ which corresponds to the SNJL model with an exactly supersymmetric Lagrangian.

- $\tilde{m}^2 \neq 0$, the gap equation will have a non trivial solution for the coupling constant satisfying the inequality

(31)
$$g^2 > \frac{8\pi^2}{\tilde{m}^2 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}^2}\right)} \; ,$$

generating a mass for the Dirac fermion pair.

- To further illustrate the power of our general gap equation results, we report also result for a scenario on the other extreme where $m = 0$ but $\eta \neq 0$ solution for Eq.(26). Naively, one enforces zero m in the the I_2 integral of the equation for η. Nontrivial solution for the latter exists under the condition

(32)
$$\frac{1}{16\pi^2}\left[\ln\left(1 + \frac{\Lambda^2}{\tilde{m}^2}\right) - \frac{\Lambda^2}{\Lambda^2 + \tilde{m}^2}\right] \leq \frac{1}{-g^2 \tilde{m}_C^2} < \frac{1}{16\pi^2}\ln\left(1 + \frac{\Lambda^2}{2\tilde{m}^2}\right) \; ,$$

Dimension five operator case (HNJL)

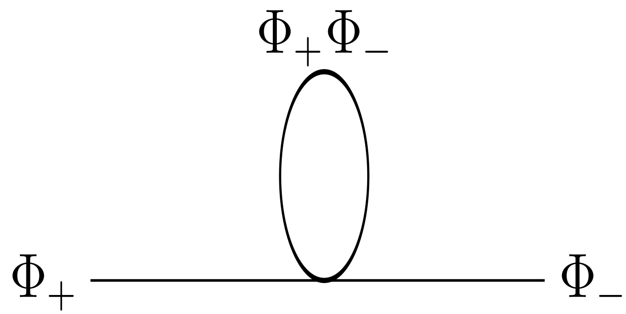

Figure 3: Superfield diagram for proper self-energy $\Sigma_{+-}(p,\theta^2)$ with the dimension five four-superfield interaction.

Again, we write schematically

$$\Sigma^{(fig2)} = \Sigma_1^{(fig2)} + \Sigma_{2a}^{(fig2)} + \Sigma_{2b}^{(fig2)} + \Sigma_{3a}^{(fig2)} + \Sigma_{3b}^{(fig2)} \ .$$

The non vanishing partial amplitude are

$$\begin{aligned}
\int d^2\bar\theta\, \Sigma_{2a}^{(fig2)}\, \delta^2(\bar\theta) &= \int d^2\bar\theta\ \frac{\bar\eta G}{2}(1+B\theta^2)\, I_2(|m|^2,\tilde m^2,|\eta|,\Lambda^2)\,\delta^2(\bar\theta)\ , \\
\int d^2\bar\theta\, \Sigma_{3a}^{(fig2)}\, \delta^2(\bar\theta) &= \int d^2\bar\theta\ \frac{-\bar m G}{2}(1+B\theta^2)\, I_1(|m|^2,\tilde m^2,|\eta|,\Lambda^2)\,\theta^2\,\delta^2(\bar\theta)\ , \\
\int d^2\bar\theta\, \Sigma_{3b}^{(fig2)}\, \delta^2(\bar\theta) &= \int d^2\bar\theta\ \frac{-\bar m G}{2}(1+B\theta^2)\, I_1(|m|^2,\tilde m^2,|\eta|,\Lambda^2)\,\theta^2\,\delta^2(\bar\theta)\ . \qquad (33)
\end{aligned}$$

where $I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$ and $I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$ are the same loop integrals as in Eq.(25). That gives the gap equation with $\Sigma^{(fig2)}$ as

$$\begin{aligned} m &= \frac{\bar{\eta}G}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)\ , \\ \eta &= \bar{m}G\, I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) - \frac{\bar{\eta}GB}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)\ . \end{aligned} \tag{34}$$

- As can be seen, the gap equation result is the important fact that the equations for m and η are completely coupled. If $\eta = 0$ then $m = 0$ and thus no dynamical mass generation.
- Considering only the case of real values for m and η under the assumption of a real and small B value, we find that nontrivial solution exists for large enough G (taken as real and positive here by convention) satisfying

$$G > \sqrt{G_0^2 + b^2} + b \sim G_0 + b\ , \tag{35}$$

where

(36) $$G_0^2 = \frac{512\pi^2}{\tilde{m}^2 \ln\left(1+\frac{\Lambda^2}{\tilde{m}^2}\right)\left[\ln\left(1+\frac{\Lambda^2}{\tilde{m}^2}\right) - \frac{\Lambda^2}{\tilde{m}^2+\Lambda^2}\right]}$$

gives the critical G^2 for $B=0$, and

(37) $$b = B\,\frac{8\pi^2}{\tilde{m}^2 \ln\left(1+\frac{\Lambda^2}{\tilde{m}^2}\right)}\ .$$

- Solution condition for more general cases is to be further investigated.
- To give an explicit symmetry breaking picture for the simplest version of the HSNJL model (with singlet superfields), one can consider a Z_4 symmetry under which both the Φ_+ and Φ_- superfields have a basic charge $e^{i\pi/2}$.
- The dimension five interaction obviously respects the symmetry while the $\Phi_+\Phi_-$ Dirac mass term is not allowed, that is till the Z_4 symmetry is dynamically broken by the $\Phi_+\Phi_-$ vacuum condensate. The condensate leaves a Z_2 symmetry that survives.

Summary

- We presented above the NJL model for dynamical symmetry breaking and its supersymmetrc extensions a dimension six or dimension five four-superfield interactions.
- They could each be used as a mechanism for dynamical electroweak symmetry breaking. The two kinds of models (SNJL and HSNJL models) have otherwise very different theoretical mass generation features, with phenomenological implications.
- The explicit symmetry breaking picture of the simplest HSNJL model maybe considered as $Z_4 \rightarrow Z_2$.
- While the particular symmetry breaking may not be of much interest, a version of the HSNJL with the basics superfields being (gauge) multiplets or a version with more than two basic superfield multiplets can achieve a rich spectrum of dynamical symmetry breaking: the original target for the idea of the HSNJL model is the one for electroweak symmetry breaking.
- studying phenomenology of the HSNGL at LHC and ILC is in progress.