Improving Photon Position Resolution Using Shower Fitting

A Status Report

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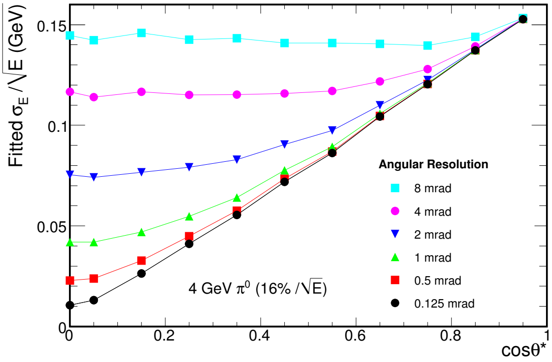
July 20, 2011

Outline

Motivation

The improvement in π^0 energy resolution from mass-constrained fitting of $\pi^0 \to \gamma\gamma$ decay depends crucially on the determination of the opening angle, ψ_{12}, between the two photons: $m^2 = 4E_1E_2\sin^2(\psi_{12}/2)$. This is a function of the reconstructed position vectors of the two photons in the ECAL, and so depends on the measurement of $(\phi, \cos\theta)$ for each photon.



Electromagnetic Shower Fitting

- Electromagnetic showers are well-behaved and governed by well understood stochastic processes.
- Sophisticated parametrizations of both shower shapes and fluctuations and their correlations are used in "fast shower simulation".
- In particular, Grindhammer and Peters (hep-ex/0001020) have a well-tuned model which reproduces full GEANT. This is used in "GFLASH".
- Plan: Use these parametrizations to improve the reconstruction of photon 4-vectors by fitting the observed cell energies to the model.
- Will exploit the very narrow core of EM showers near the start of the shower to improve position resolution. (I demonstrated this in an old study with 1mm^2 cells.)

Grindhammer-Peters Model

Describes the spatial energy distribution of electromagnetic showers in terms of the energy deposition in a 3-d volume element in the scaled longitudinal variable, t (in X_0), scaled radial variable, r (in R_M), and the azimuthal variable, ϕ.

$$dE(t, r, \phi) = E \, f_L(t) \, f_R(r) \, f(\phi) \, dt \, dr \, d\phi$$

The *average* longitudinal distribution uses the usual gamma distribution:

$$f_L(t; \alpha, \beta) = \beta \, (\beta \, t)^{\alpha-1} \exp{(-\beta \, t)}/\Gamma(\alpha)$$

where the shape parameter, α, and the scaling parameter, β, are related to the longitudinal center of gravity, $< t >$, and the shower maximum depth, T, by $< t >= \alpha/\beta$ and $T = (\alpha - 1)/\beta$.

Average Radial Profile vs Shower Depth

GP use two component ansatz with R_C, (R_T) being the median radial extent of the core (tail) and p giving the relative weight of the core.

$$f(r) = p f_C(r) + (1-p) f_T(r) = p \frac{2rR_C^2}{(r^2+R_C^2)^2} + (1-p)\frac{2rR_T^2}{(r^2+R_T^2)^2}$$

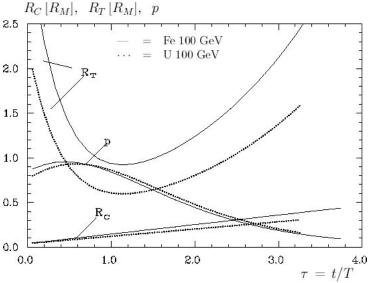


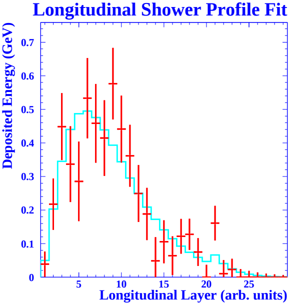
Figure 10:

The shower depth in units of the shower maximum, $\tau = t/T$ is used to parameterize the radial profile parameters, namely $R_C(\tau), R_T(\tau), p(\tau)$.

Longitudinal Shower Fit

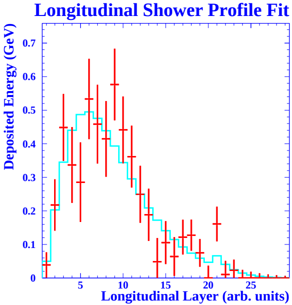
Example: 5 GeV photon, ILD00. Maximum likelihood fit to the measured energies per pseudo-layer for the 3 free parameters using a suitable choice of fit parameters: $\log E, \log(\alpha - 1), \log T$.

Use GSL simplex implementation for minimimization. For now, neglect angle of incidence issues; the β parameter accommodates this anyway.



Use Gamma distribution for sampling fluctuations with stochastic errors of $17\%/\sqrt{E}$ and $24\%/\sqrt{E}$. (GP eqns 12-15).

Longitudinal Shower Fit Remarks



These fits work remarkably well with typically 98% of fits leading to sensible answers even with all 3 parameters floating. Currently using default EM sampling fractions. The fitted energies are competitive (and highly correlated) with the measured energy. Can imagine allowing for t-dependent weights on a shower-by-shower basis and there may be some room to improve the energy resolution.

Using the Radial Profile Expectations

Simulating the radial profile fluctuations vs depth needs knowledge of the longitudinal fluctuations for the particular shower. So, the longitudinal fit is used to calculate $\tau_i = t/T_i$ where T_i is the fitted shower max.

So far, I took the evaluated R_C, R_T, p values for each pseudo-layer of an individual shower assuming $\cos\theta = 0$, and then used the median R of that radial profile at that fitted shower depth to calculate weight functions for position estimation.

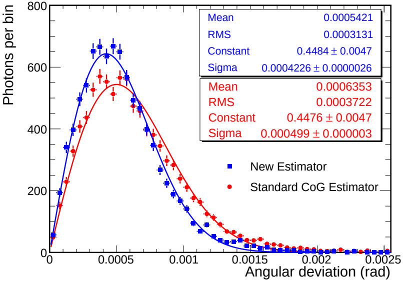
The default position estimator one usually uses for photon momentum reconstruction is the shower center-of-gravity

$\vec{r}_{CoG} = (\sum_{i=1}^{N} E_i \vec{r}_i)/(\sum_{i=1}^{N} E_i)$

We investigated $\vec{r}_{LW} = (\sum_{i=1}^{N} E_i w_i \vec{r}_i)/(\sum_{i=1}^{N} E_i w_i)$ with $w_i = R^{-\alpha}$. We also investigated using different values of α for determining ϕ and $\cos\theta$, denoting the parameter, α_ϕ and α_θ.

First Results

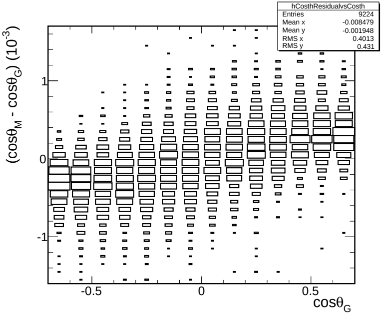
For 5 GeV photons found resolution optimized for $\alpha_\phi = 1.0$ and $\alpha_\theta = 0.5$ and angular resolution improved from 0.50 to 0.42 mrad.

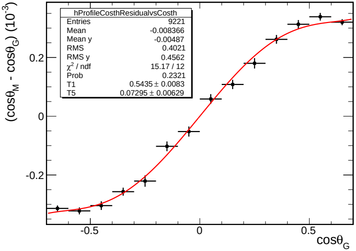


One way of quantifying the essential "2-d" nature of photon cluster reconstruction is to measure the distribution of separation angle between the reconstructed photon and the generator photon in space. This can be described by a Rayleigh distribution if the resolution contribution from both components is similar - which is a fair approximation.

Bias in Photon $\cos\theta$ Reconstruction

Example: 5 GeV with $\alpha_\theta = 0.5$. Residuals biased by up to $\approx 0.3 \times 10^{-3}$.
Better resolution in $\cos\theta$ for high $|\cos\theta|$.

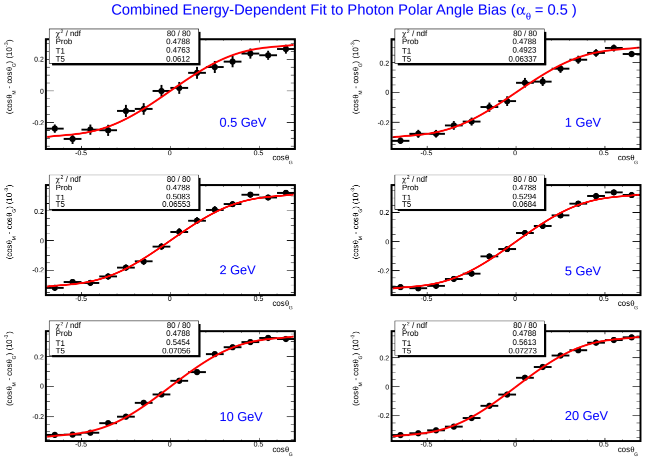




Should fit with a suitable odd function of $\cos\theta$. Chose to fit using a Chebyshev polynomial (1st kind). With the coefficients of T_1 and T_5 as the only free parameters can obtain reasonable fits.

In this case the fit function is basically $T_1 \cos\theta + T_5 \cos 5\theta$.

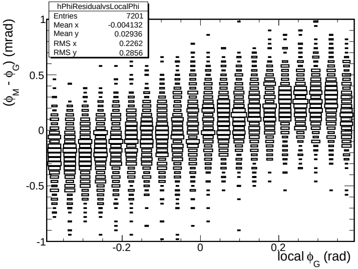
Energy Dependent Fit of $\cos\theta$ Bias

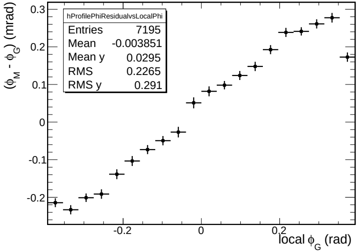


Fit using 4 free parameters: $(A_1 + B_1 \log E)\cos\theta + (A_5 + B_5 \log E)\cos 5\theta$

Bias in Photon ϕ Reconstruction wrt Octant Center

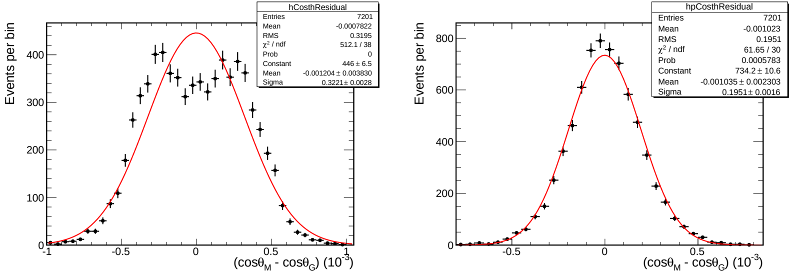
Example: 20 GeV with $\alpha_\phi = 1.0$. Residuals biased by up to ≈ 0.3 mrad too.





Mean bias is positive. Indication of B-field effect? associated with e^-/e^+ asymmetry in showers ? We do have + B along +z-axis ? Indication also of issues associated with octant module overlaps at octant boundary.

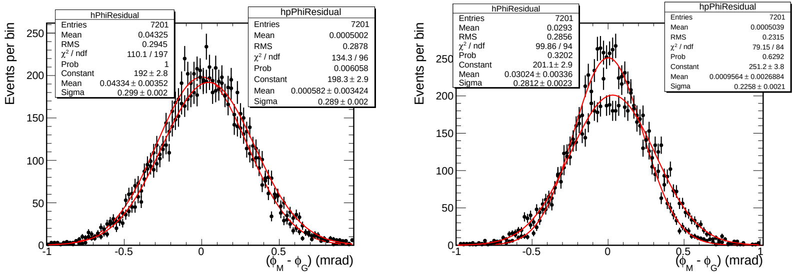
Snapshot of current improvements: 20 GeV photon



$\alpha_\theta = 0.5$. Left: before correcting $\cos\theta$ bias. Right: after correction.

Snapshot of current improvements: 20 GeV photon

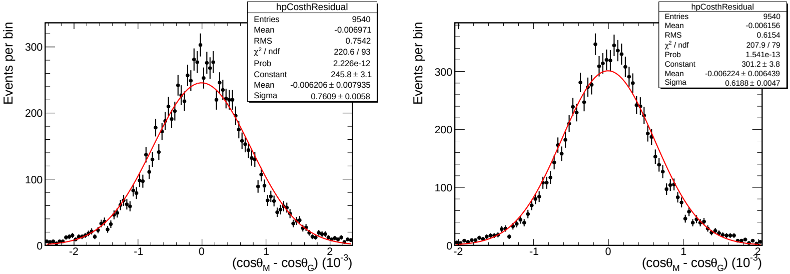
Before (hPhiResidual) and after (hpPhiResidual) correcting ϕ bias.



Left: $\alpha_\phi = 0.0$ (CoG). Right: $\alpha_\phi = 1.0$.

Snapshot of current improvements: 1 GeV photon

After correcting $\cos\theta$ bias.



Left: $\alpha_\theta = 0.0$ (CoG). Right: $\alpha_\theta = 0.5$.

Conclusions/Summary/Open Issues

- Shower fitting has potential to improve calorimeter measurements.
- First attempts at longitudinal fits appear very promising, and indicate that it is feasible to measure the 3 main longitudinal shower parameters.
- Longitudinal weighting of position estimates shows improvement over shower CoG.
- Need to take care of systematics in ϕ and $\cos\theta$ (which are there also for CoG ...) - in progress, and re-visit α optimization.
- Not sure how easy it will be to really adapt the method seamlessly to all incidence angles.
- Full 3-d fitting of all cells to a shower model looks to be worth pursuing for photons - particularly with regard to reducing finite cell-size type systematics.

Backup Slides

Position Resolution Algorithms

1. v01-09 PandoraPFANew - uses unweighted cluster centroid in first pseudo-layer (oops ...).
2. Shower center-of-gravity (was the default in MarlinReco / PandoraPFA).
3. Longitudinal weighting using shower fit (this talk)

I have a version of PfoCreationAlgorithm::CreateNeutralPfos() which implements 1 and 2 above and an energy-weighted version of 1. (CoG is the obvious default ...).

Chebyshev Polynomials of the First Kind

A particular orthogonal polynomial

$$T_1(x) = x$$
$$T_3(x) = 4x^3 - 3x$$
$$T_5(x) = 16x^5 - 20x^3 + 5x$$

In fits using just the T_1 and T_5 contributions, the T_1-T_5 correlation coefficient is small leading to robust fits.

Rayleigh Distribution

Can describe the magnitude of a vector, $\vec{r}$, whose two components, x and y are Gaussianly distributed, uncorrelated and with the same variance.

$$p(r;\sigma) = (r/\sigma^2)\exp(-r^2/(2\sigma^2))$$