Phenomenology of the Higgs Triplet Model

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Mariko Kikuchi (U of Toyama)

S. Kanemura, K. Yagyu, arXiv: 1201.6287 [hep-ph]
M. Aoki, S. Kanemura, M. Kikuchi. K. Yagyu, arXiv: 1204.1951 [hep-ph]

KILC12, Daegu, Korea, 25th April 2012

Introduction

- **The Higgs sector is unknown.**
  - Minimal? or Non-minimal?
  - The Higgs boson search is underway at the LHC.
    The Higgs boson mass is constrained to be
    115 GeV < mh < 127 GeV or mh > 600 GeV.
  - By the combination with electroweak precision data at the LEP,
    we may expect that a light Higgs boson exists.

- **There are phenomena which cannot be explained in the SM.**
  - Tiny neutrino masses
  - Existence of dark matter
  - Baryon asymmetry of the Universe

- **New physics may explain these phenomena above the TeV scale.**
  - Extended Higgs sectors are often introduced.

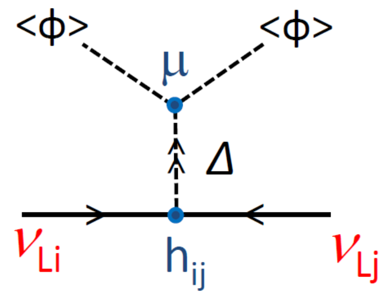
Explanation by extended Higgs sectors

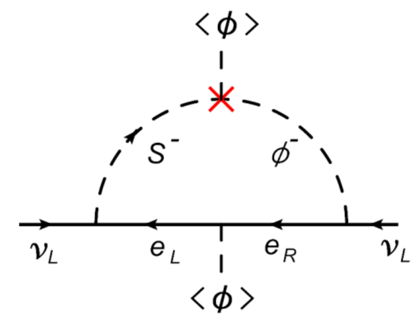
- **Tiny neutrino masses**

  - The type II seesaw model

  - Radiative seesaw models

  (e.g. Zee model)





- **Dark matter**

  - Higgs sector with an unbroken discrete symmetry

- **Baryon asymmetry of the Universe**

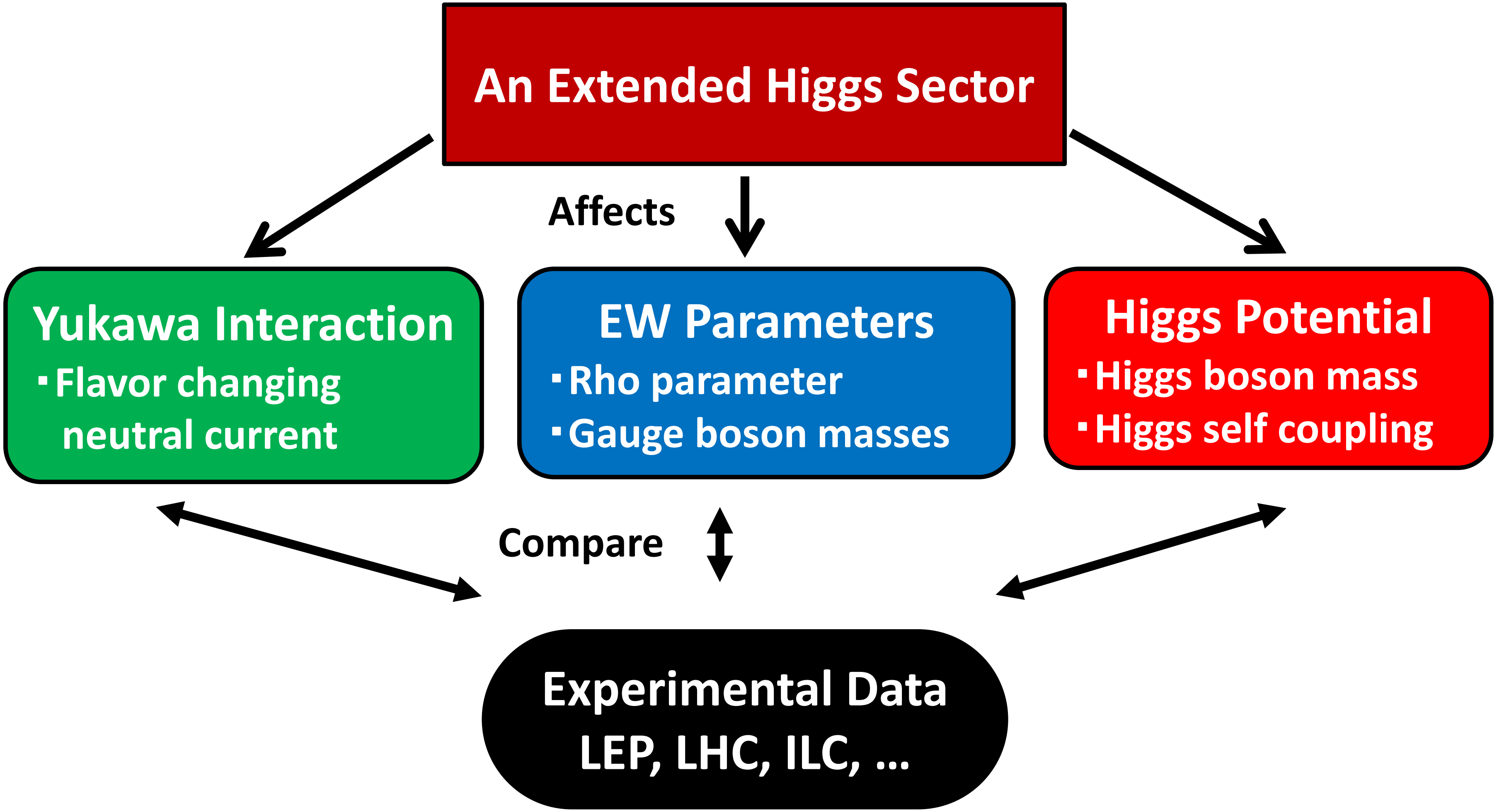
  - Electroweak baryogenesis

Introduced extended Higgs sectors

SU(2) doublet Higgs + Singlet [$U(1)_{B-L}$ model]
+ Doublet [Inert doublet model]
+ Triplet [Type II seesaw model], etc…

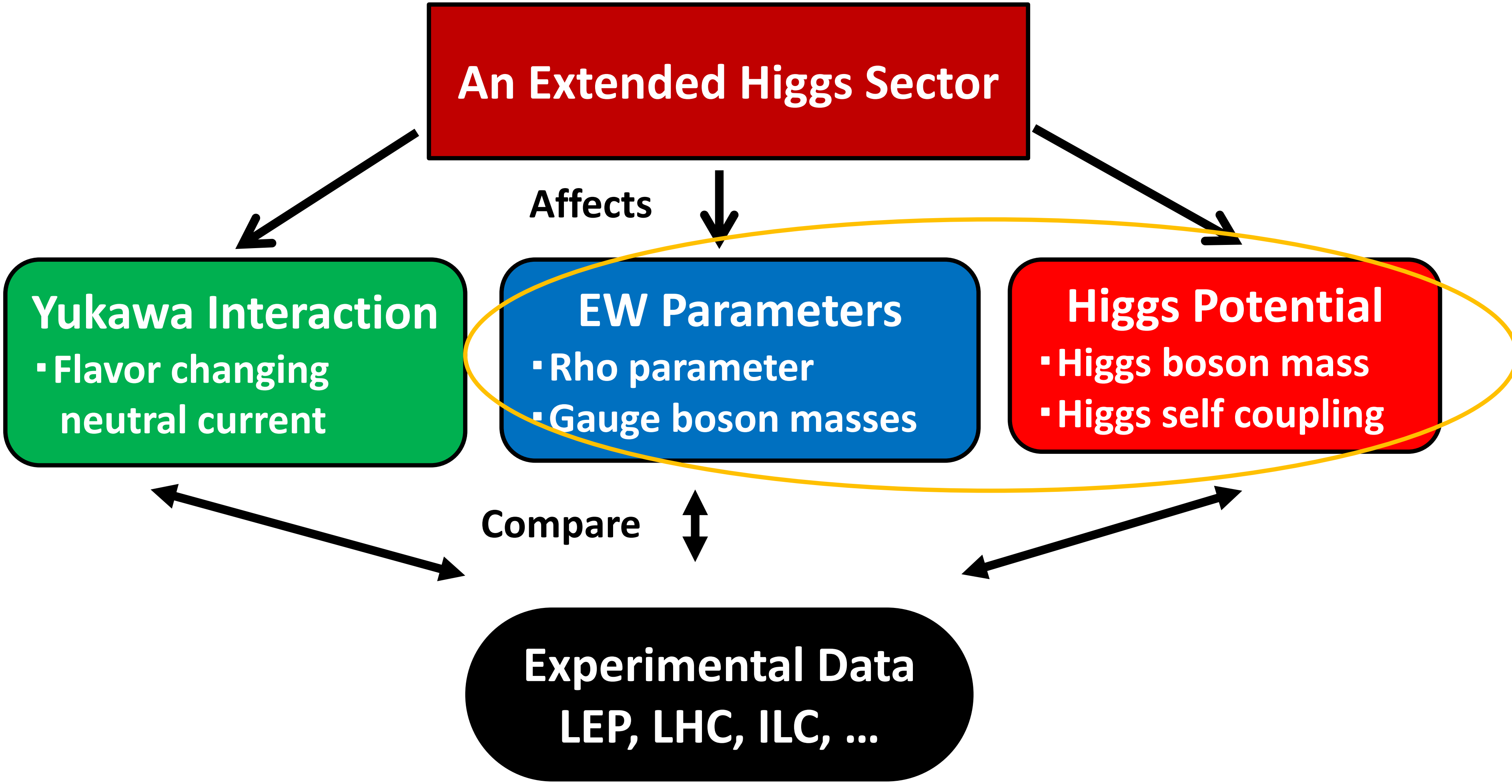
Studying **extended Higgs sectors** is important to understand the **phenomena beyond the SM.**

How we can constrain various Higgs sectors?



It is necessary to prepare precise calculations for observables in the Higgs sector in order to distinguish various Higgs sectors.

How we can constrain various Higgs sectors?



In this talk, we focus on the renormalization of electroweak parameters and the Higgs potential.

The electroweak rho parameter

★ The experimental value of the rho parameter is quite close to unity.

$\boxed{\rho_{\mathrm{exp}} \sim 1}$

Tree-level expression for the rho parameter （Kinetic term of Higgs fields）

$$\boxed{\rho_{\mathrm{tree}} = \frac{\sum_i \left[T_i(T_i+1) - Y_i^2\right] v_i^2}{\sum_i 2Y_i^2 v_i^2}}$$

Y_i : hypercharge
T_i : isospin
v_i : VEV

$\rho_{\mathrm{tree}} = 1$

- Standard Model
- Multi-doublet (with singlet) model

There is the custodial SU(2) sym. in the kinetic term

$\rho_{\mathrm{tree}} \neq 1$

- Higgs Triplet Model
- Model with larger isospin representaion fields.

The custodial SU(2) sym. is broken in the kinetic term.

These sector affects the rho parameter by the loop effects.

Higgs Potential

Yukawa interaction

Higgs Potential

Non-minimal Higgs sectors

Direct evidence → **Discovery of extra Higgs bosons; i.e., Charged Higgs bosons $H^{\pm}$, …**

★Measuring the mass spectrum, decay branching ratios of these extra Higgs bosons.

Indirect signature → **Deviation of (SM-like) Higgs couplings (hVV, hff, hhh) from those predicted values in the SM.**

★ Once "SM-like" Higgs boson (h) is discovered, precise measurements for the Higgs mass and the triple Higgs coupling turns to be very important.

$$\boxed{V_{\text{Higgs}} = \frac{1}{2}\underline{m_h^2}h^2 + \frac{1}{3!}\underline{\lambda_{hhh}}h^3 + \frac{1}{4!}\lambda_{hhhh}h^4 + \cdots}$$

Precise calculation of these physics quantities is very important to discriminate various Higgs sectors.

The Higgs Triplet Model

The Higgs triplet field Δ is added to the SM.

	SU(2)I	U(1)Y	U(1)L
Φ	2	1/2	0
Δ	3	1	-2

Cheng, Li (1980); Schechter, Valle, (1980); Magg, Wetterich, (1980); Mohapatra, Senjanovic, (1981).

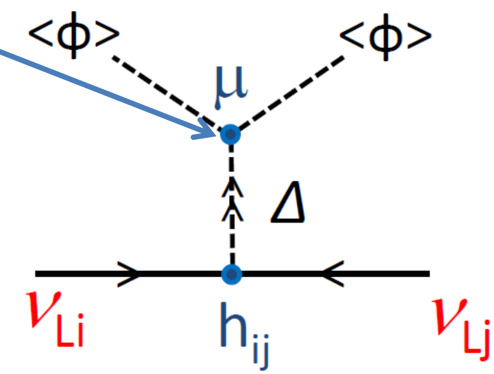
- **Neutrino Yukawa interaction:**

$$\mathcal{L}_Y = h_{ij}\overline{L_L^{ci}}\cdot\Delta L_L^j$$

- **Higgs Potential:**

Lepton number breaking parameter

$$\begin{aligned}V &= m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + [\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.}]\\&+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}[(\Delta^\dagger\Delta)^2]\\&+ \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi.\end{aligned}$$



- **Mass eigenstates:** (SM-like) h, (Triplet-like) $H^{\pm\pm}$, $H^\pm$, H, A

- **Neutrino mass matrix**

$$(m_\nu)_{ij} = h_{ij}\frac{\mu\langle\phi^0\rangle^2}{M_\Delta^2} = h_{ij}v_\Delta$$

M_Δ : Mass of triplet scalar boson.
v_Δ : VEV of the triplet Higgs

Important predictions (Tree-Level)

★ **Rho parameter deviates from unity.**

$$\rho_{\text{tree}} = \frac{1+\frac{2v_\Delta^2}{v_\Phi^2}}{1+\frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}$$

★ **Characteristic mass relation is predicted.**

$$m_{H^{++}}{}^2 - m_{H^+}{}^2 \simeq m_{H^+}{}^2 - m_A{}^2$$

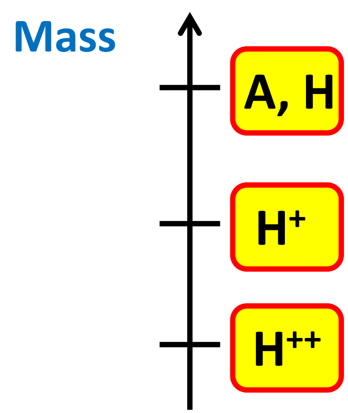
Under $v_\Delta \ll v_\Phi$ (From experimental data $\rho_{\text{exp}} \sim 1$)

$$\boxed{m_h^2 \simeq 2\lambda_1 v^2}$$

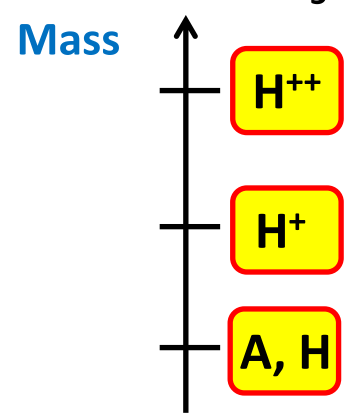
$$M_\Delta^2 \equiv \frac{v_\Phi^2 \mu}{\sqrt{2} v_\Delta}$$

$$m_{H^{++}}^2 \simeq M_\Delta^2 - \frac{v^2}{2}\lambda_5$$
$$m_{H^+}^2 \simeq M_\Delta^2 - \frac{v^2}{4}\lambda_5$$
$$m_A^2 \simeq m_H^2 = M_\Delta^2$$

Case I ($\lambda_5 > 0$)



Case II ($\lambda_5 < 0$)



How these predictions are modified by the radiative corrections?

Important predictions (Tree-Level)

★ **Rho parameter deviates from unity.**

$$\rho_{\text{tree}} = \frac{1+\frac{2v_\Delta^2}{v_\Phi^2}}{1+\frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}$$

Renormalization of the electroweak parameters

★ **Characteristic mass relation is predicted.**

$$m_{H^{++}}{}^2 - m_{H^+}{}^2 \simeq m_{H^+}{}^2 - m_A{}^2$$

Renormalization of the Higgs potential

Under $v_\Delta \ll v_\Phi$ (From experimental data $\rho_{exp} \sim 1$)

$$m_h^2 \simeq 2\lambda_1 v^2$$

$$M_\Delta^2 \equiv \frac{v_\Phi^2 \mu}{\sqrt{2} v_\Delta}$$

$$m_{H^{++}}^2 \simeq M_\Delta^2 - \frac{v^2}{2}\lambda_5$$

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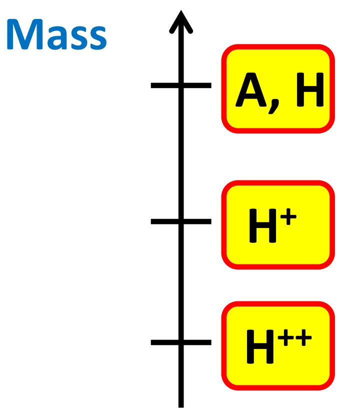
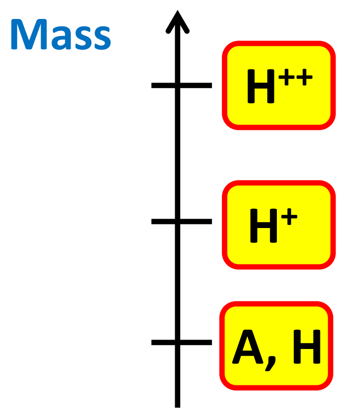
Case I ($\lambda_5 > 0$)



Case II ($\lambda_5 < 0$)



We first discuss renormalization of the EW parameters.

Model w/ ρ_{tree} = 1 and Model w/o ρ_{tree} = 1

Model w/ ρ_{tree} =1

EW parameters are described by **3 input parametes:** **α_{em}, G_F , m_Z** and the relation: **$c_W^2 = m_W^2/m_Z^2$.**

Counter term δs_W^2 is determined as

$$\frac{\delta s_W^2}{s_W^2} = \frac{s_W^2}{c_W^2}\left[\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right] = \boxed{\frac{s_W^2}{c_W^2}\left[\frac{\Pi^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi^{WW}(m_W^2)}{m_W^2}\right]} \sim \rho_{1\text{-loop}}$$

One-loop corrections to the rho parameter measures the ~~custodial~~ sym.

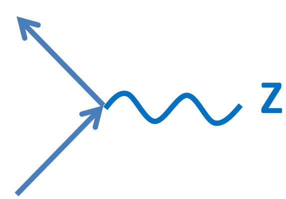
Model w/o ρ_{tree} =1

EW parameters are described by **4 input parameters**:
α_{em}, G_F , m_Z and $\hat{s}_W^2$.

Blank, Hollik (1997)

$\hat{s}_W^2$ is defined by the Ze^+e^- vertex:

$$\mathcal{L} = \bar{e}\frac{g}{2\hat{c}_W}(v_e\gamma_\mu - a_e\gamma_\mu\gamma_5)eZ^\mu$$



$$\boxed{1 - 4\hat{s}_W^2(m_Z) = \frac{\mathrm{Re}(v_e)}{\mathrm{Re}(a_e)}}$$

To determine the counter term $\delta\hat{s}_W^2$, additional renormalization condition is necessary.

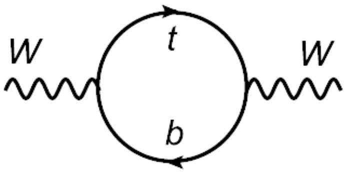
→Effects of the ~~custodial~~ sym. is absorbed by the renormalization of $\delta\hat{s}_W^2$.

One-loop corrections to the rho parameter **does not** measure the ~~custodial~~ sym.

Radiative corrections to the rho parameter

Model w/ ρ_{tree} =1

$$\delta\rho \simeq \frac{1}{16\pi^2}\frac{(m_t - m_b)^2}{m_W^2} + \cdots$$



Custodial sym. breaking in the Yukawa sector

Quadratic dependence of the mass splitting appears as the effect of the custodial symmetry breaking.

Peskin, Wells (2001);
Grimus, Lavoura, Ogreid, Osland (2008);
Kanemura, Okada, Taniguchi, Tsumura (2011).

Model w/o ρ_{tree} =1

$$\delta\rho \simeq \frac{1}{16\pi^2}\ln\frac{m_t}{m_b} + \cdots$$

Quadratic dependence of the mass splitting disappears by the renormalization, and only **logarithmic dependence** is remained.

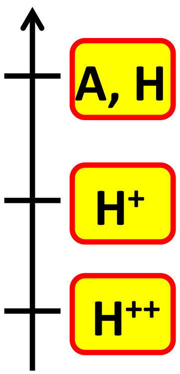
One-loop corrected rho parameter in the HTM

Kanemura, KY, arXiv: 1201.6287 [hep-ph]

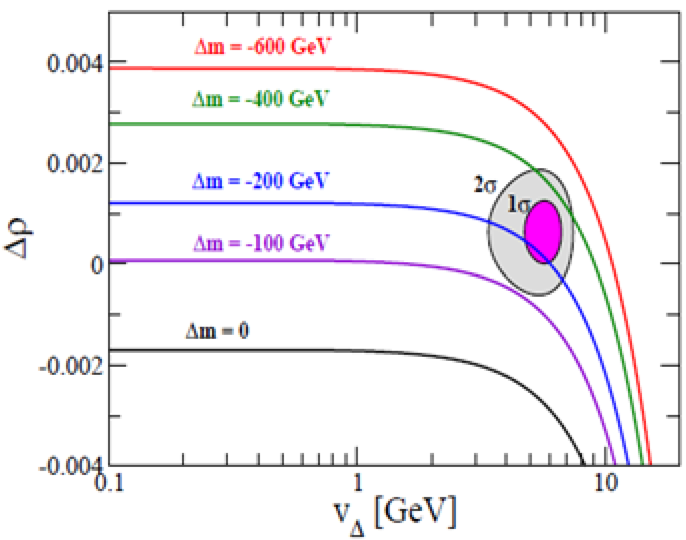
$$\Delta\rho \equiv \rho - \rho_{\rm SM}(m_h^{\rm ref} = 125\ {\rm GeV})$$

$$\Delta\rho^{\rm exp} = 0.000632 \pm 0.000621$$

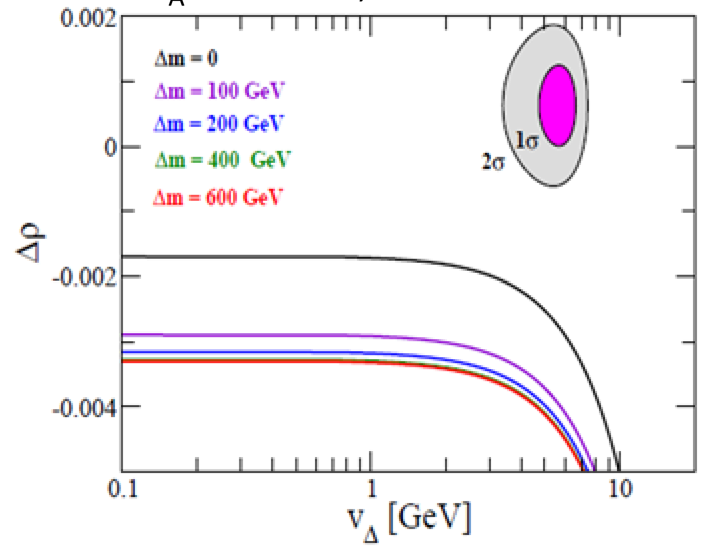
Case I



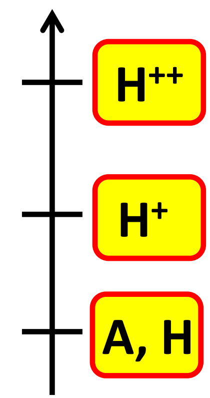
m_{H++} = 150 GeV, mh = 125 GeV



m_A= 150 GeV, mh = 125 GeV



Case II



$\Delta m = m_{H++} - m_{H+}$

v_Δ is calculated by the tree level formula: $$v_\Delta^2 = \frac{\hat{s}_W^2(1-\hat{s}_W^2)}{2\pi\alpha_{\rm em}} m_Z^2 - \frac{\sqrt{2}}{4G_F}$$

In Case I, **mH^{++} = 150 GeV, 100 GeV <|Δm| < 400 GeV , 3 GeV < vΔ < 8 GeV** is favored, while Case II is highly constrained by the data.

Renormalization of the Higgs potential

Aoki, Kanemura, Kikuchi, KY, arXiv: 1204.1951

8 parameters in the potential	→	8 physical parameters
$\mu, m, M, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$		$v, v_\Delta, m_{H++}, m_{H+}, m_A, m_h, m_H, \alpha$

Counter terms

$\delta v, \delta v_\Delta, \delta m_{H++}^2, \delta m_{H+}^2, \delta m_A^2, \delta m_h^2, \delta m_H^2, \delta\alpha$

Tadpole: $\delta T_\varphi, \delta T_\Delta,$

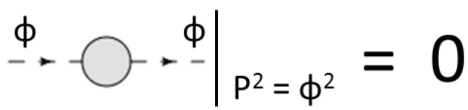
Wave function renormalization: $\delta Z_{H++}, \delta Z_{H+}, \delta Z_A, \delta Z_H, \delta Z_h, \ldots$

Renormalization of G_F and $\hat{s}_W^2$ → $\delta v, \delta v_\Delta$

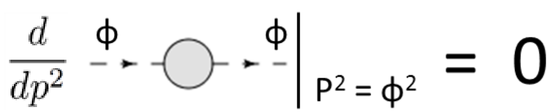
Vanishing 1-point function

→ $\delta T_\varphi, \delta T_\Delta$

On-shell condition

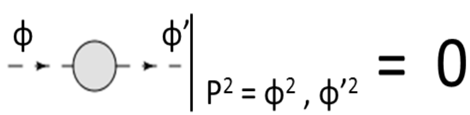


→ $\delta m_{H++}^2, \delta m_{H+}^2, \delta m_A^2, \delta m_h^2, \delta m_H^2,$



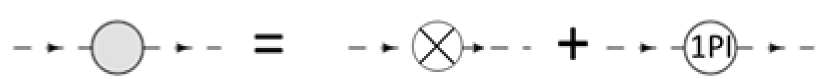
→ $\delta Z_{H++}, \delta Z_{H+}, \delta Z_A, \delta Z_H, \delta Z_h, \ldots$

No-mixing condition



→ $\delta\alpha, \ldots$

where 2-point function is defined by



Radiative corrections to the mass spectrum

Aoki, Kanemura, Kikuchi, KY, arXiv: 1204.1951

Ratio of the squared mass difference *R*

$$R \equiv \frac{m^2_{H++} - m^2_{H+}}{m^2_{H+} - m^2_A}$$

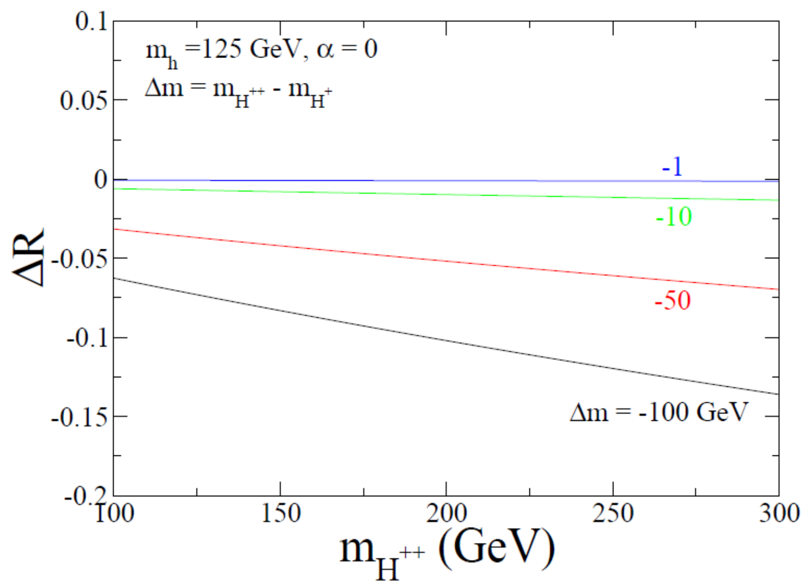
Tree level: $R^{\rm tree} = 1 + \underbrace{\left(\frac{v_\Delta^2}{v^2}\right)}_{\text{Less than } 10^{-3}} \simeq 1$

Loop level: $R^{\rm loop} = 1 + \underbrace{\Delta R}_{\text{Loop correction}} + \left(\frac{v_\Delta^2}{v^2}\right)$

$$\Delta R = \frac{\Pi^{\rm 1PI}_{H++H--}[m^2_{H++}] - 2\Pi^{\rm 1PI}_{H+H-}[m^2_{H+}] + \Pi^{\rm 1PI}_{AA}[(m^2_A)_{\rm tree}]}{m^2_{H++} - m^2_{H+}}$$

$(m_A^2)_{\rm tree}$ is determined by m_{H++}^2 and m_{H+}^2 : $(m^2_A)_{\rm tree} = 2m^2_{H+} - m^2_{H++}$

Case I

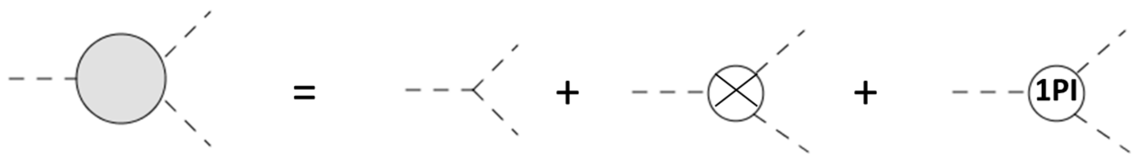


In favored parameter sets by EW precision data: m_{H++} = O(100)GeV, $|\Delta m| \sim$ 100GeV, ΔR can be as large as **O(10)%**.

One-loop corrected hhh coupling

Aoki, Kanemura, Kikuchi, KY, arXiv: 1204.1951

On-shell renormalization of the hhh coupling:



For $v_\Delta^2 << v^2$

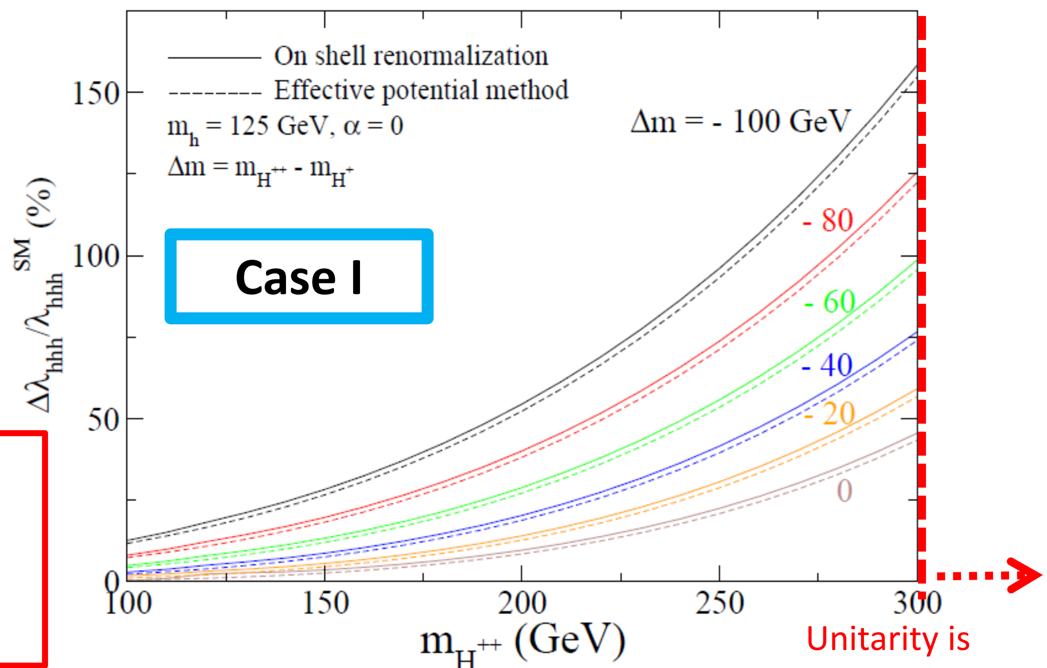
$$\frac{\lambda_{hhh}^{HTM}}{\lambda_{hhh}^{SM}} \simeq 1 + \frac{1}{12\pi^2 m_h^2 v^2}\left(2m_{H^{++}}^4 + 2m_{H^+}^4 + m_A^4 + m_H^4\right)$$

Quartic mass dependence of Δ-like Higgs bosons appears to the hhh coupling ⇒ Non-decoupling property of the Higgs sector.

Results for the renormalization of the EW parameter suggests m_{H++} = O(100)GeV, $|\Delta m|$ ∼100GeV.

In this set, deviation of hhh is predicted more than 25%

By measuring the **mass spectrum** as well as the **hhh coupling**, the HTM can be tested.

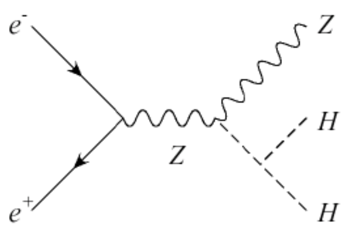


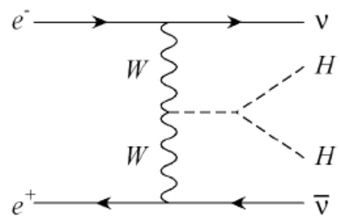
Unitarity is broken

Measuring hhh and the mass spectrum at the ILC

Measuring the hhh coupling

$e^+e^- \to Zhh$ **$e^+e^- \to hh\nu\nu$**

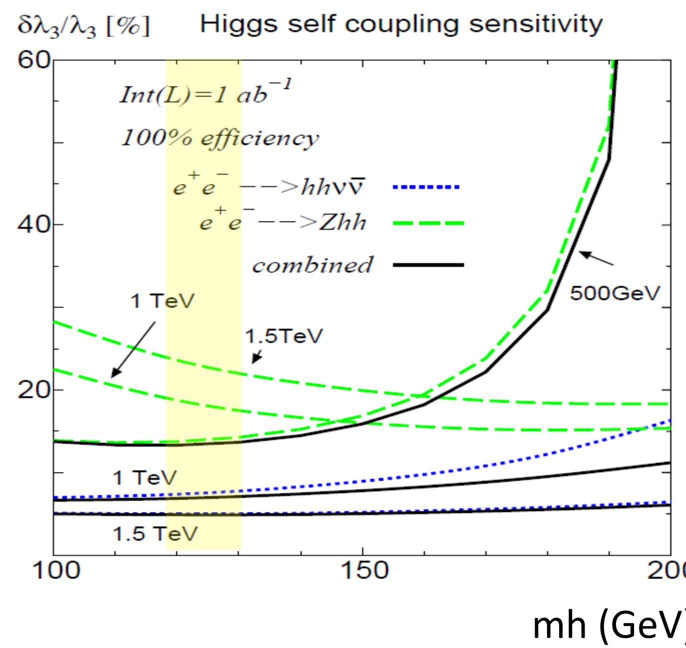




O(10)% precision may be expected.

Recent analysis was given by Suehara-san's talk.

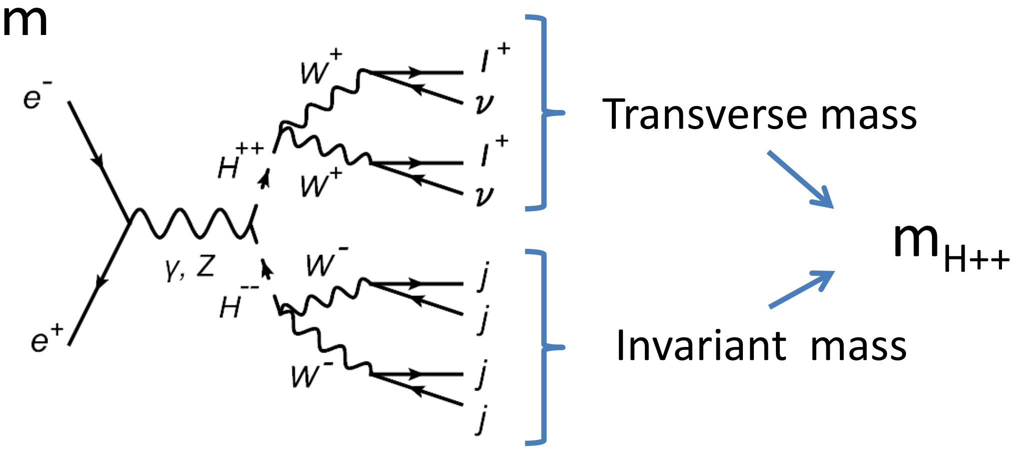
Yasui, Kanemura, Kiyoura, Odagiri, Okada, Senaha, Yamashita, hep-ph/0211047



Measuring the mass spectrum

Case I

Ex. $e^+e^- \to H^{++}H^{--} \to W^+W^+W^-W^-$
$\to l^+ l^+$ 4jet + missing



Summary

- Precise calculations of EW parameters as well as Higgs couplings (hhh, hVV, hff) are important to discriminate various Higgs sectors.
- The important predictions in the Higgs Triplet Model:

$$\boxed{\rho_{\text{tree}} = \frac{1+\frac{2v_\Delta^2}{v_\Phi^2}}{1+\frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}}$$

$$\boxed{m_{H++}{}^2 - m_{H+}{}^2 \simeq m_{H+}{}^2 - m_A{}^2}$$

- Renormalization of the EW parameters

  - 4 input parameters (not 3 as the SM) are necessary in to describe the EW parameters.

  ⇒ **mA > mH+ > mH++ with Δm=O(100) GeV** is favored.

- Renormalization of the Higgs potential

  - One-loop corrected mass spectrum**: ΔR = O(10)%**

  - One-loop corrected hhh coupling **: deviation from the SM prediction can be as large as O(100) %**

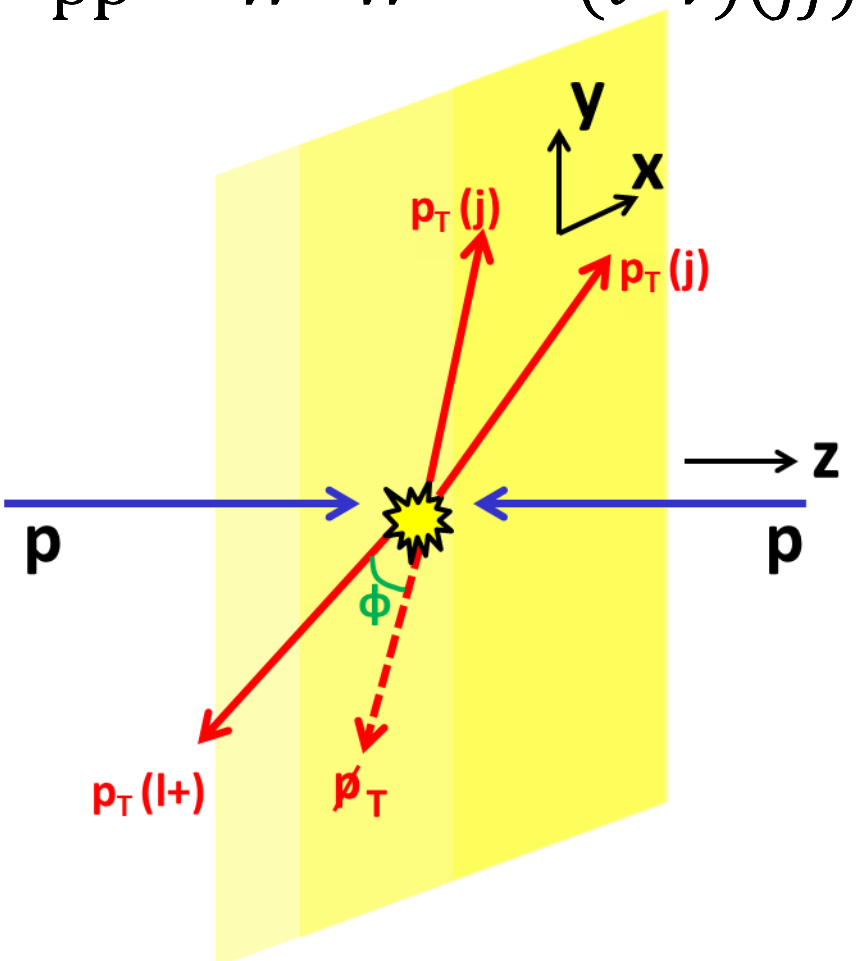
- These observables may be able to measured at the ILC.

Back up slides

Transverse mass distribution

Ex.) Measurement for W boson mass

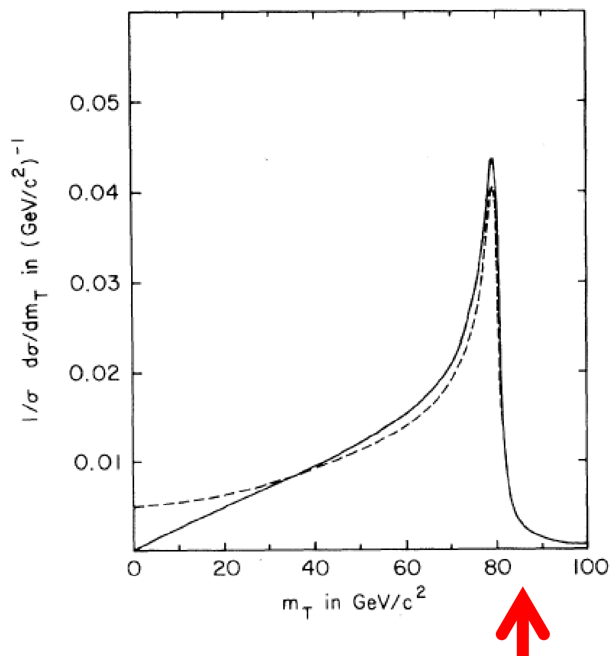
$$pp \to W^+W^- \to (l^+\nu)(jj)$$



Definition of the transverse mass

$$M_T(l^+\not{E}_T) = \sqrt{\left(P_T^{l+} + p_T\right)^2}$$

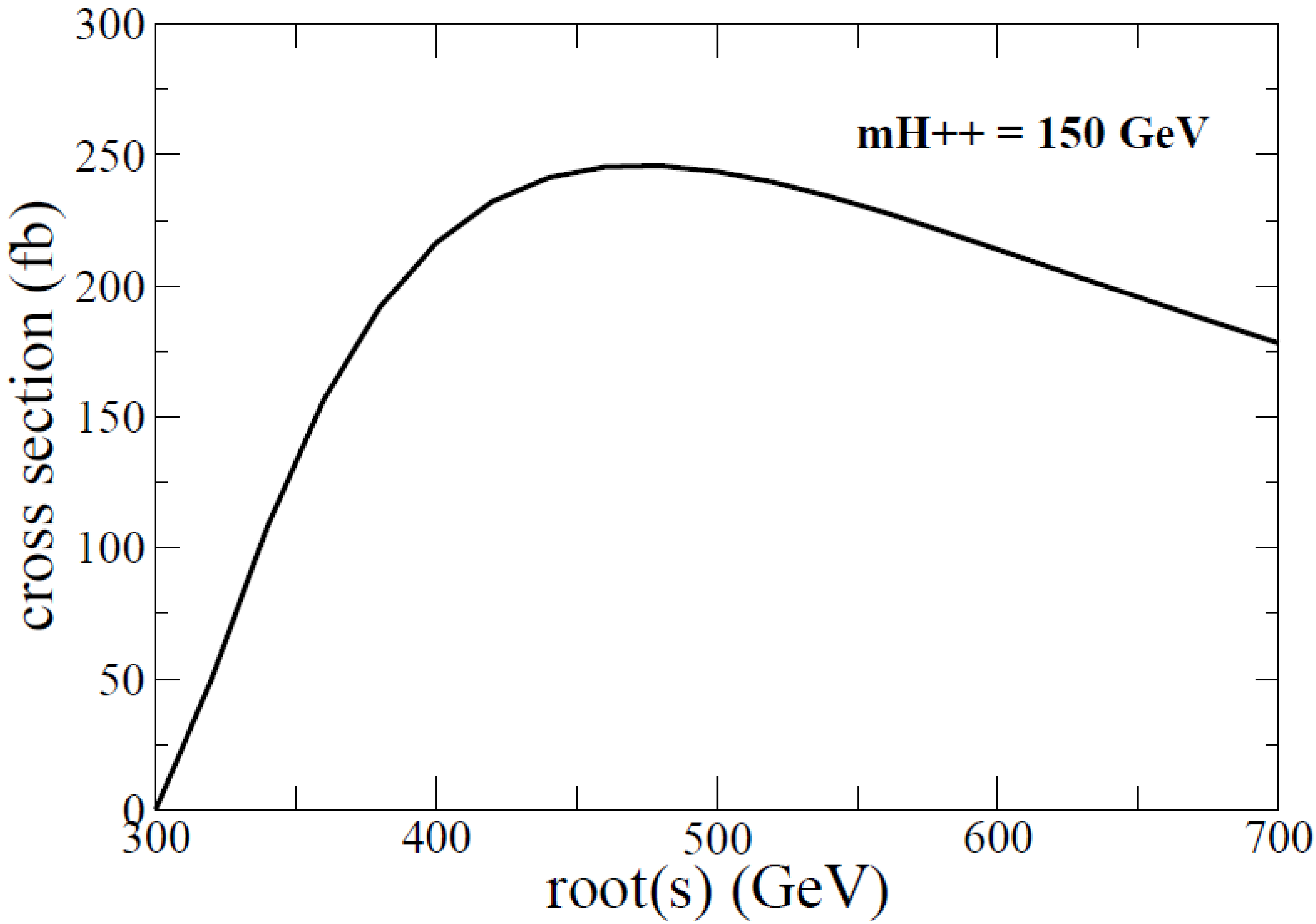
$$\simeq \sqrt{2P_T^{l+}\not{E}_T(1-\cos\phi)}$$



m_W

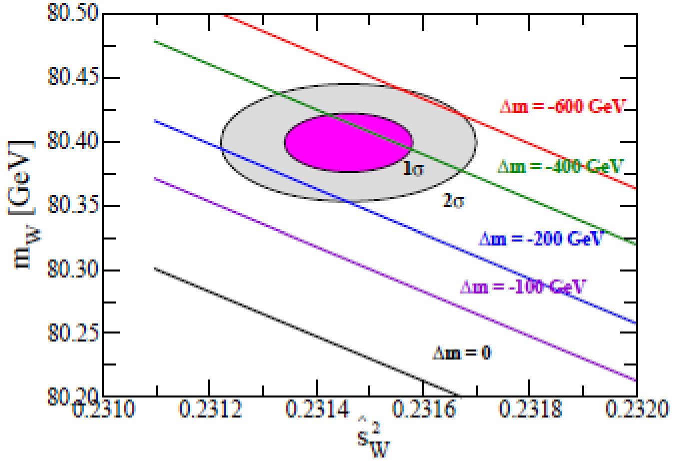
Smith,Neerven,Vermaseren,PRL(1983)

Cross section of $e^+e^- \rightarrow H^{++}H^{--}$

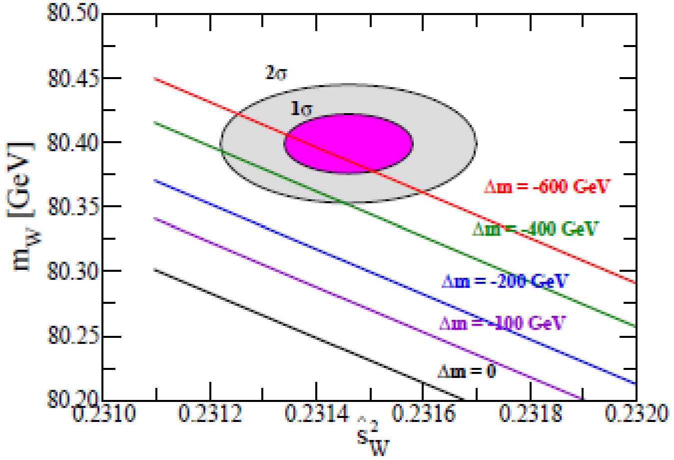


Large mh and mH^{++} case (Case I)

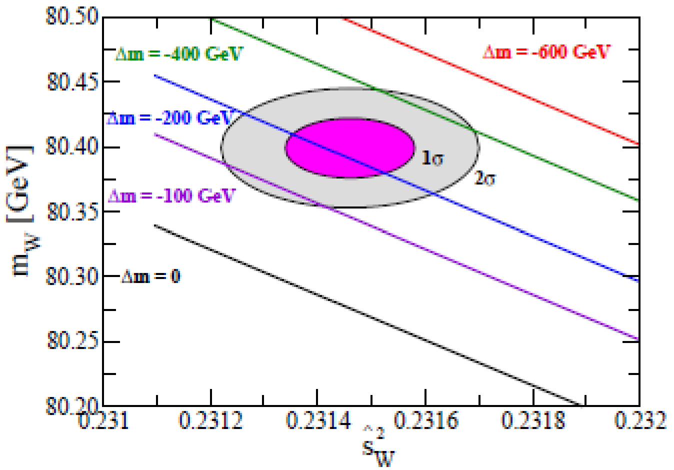
m_h = 125 GeV and mH^{++} = 150 GeV



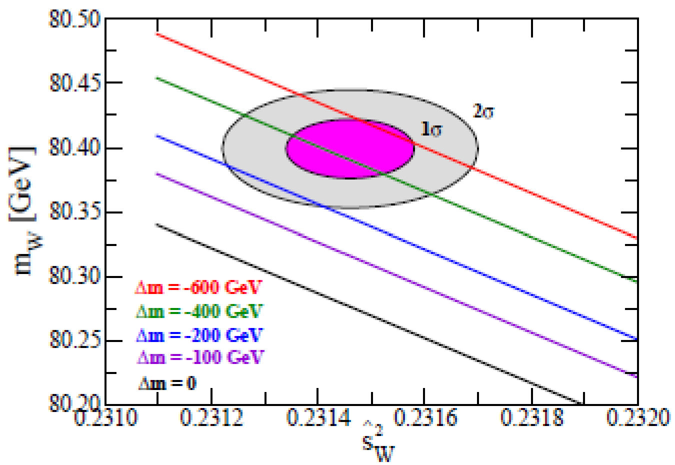
m_h = 125 GeV and mH^{++} = 300 GeV



m_h = 700 GeV and mH^{++} = 150 GeV



m_h = 700 GeV and mH^{++} = 300 GeV



Renormalized ρ and m_W

$$\boxed{\begin{aligned}\Delta r_{\rho\neq 1} = {} & \frac{\Pi_T^{WW}(0) - \Pi_T^{WW}(m_W^2)}{m_W^2} + \frac{d}{dp^2}\Pi_T^{\gamma\gamma}(p^2)\Big|_{p^2=0} + \frac{2\hat{s}_W}{\hat{c}_W}\frac{\Pi_T^{\gamma Z}(0)}{m_Z^2} + \delta_{VB} \\ & + \frac{\hat{c}_W}{\hat{s}_W}\frac{\Pi_T^{\gamma Z}(m_Z^2)}{m_Z^2} + \delta'_V\end{aligned}}$$

One-loop corrected ρ and m_W are given by:

$$\rho = \frac{\pi\alpha_{\rm em}}{\sqrt{2}G_F m_Z^2 \hat{s}_W^2 \hat{c}_W^2}(1+\Delta r) \quad , \quad m_W^2 = \frac{\pi\alpha_{\rm em}}{\sqrt{2}G_F \hat{s}_W^2}(1+\Delta r)$$

Approximately formulae of Δr

Case I:
mA > mH$^+$ > mH^{++}

$$\Delta r \simeq \frac{g^2}{16\pi^2}\left(\underline{\ln m_{H^+} + \ln m_A} - 2\ln m_{H^{++}}\right) + \cdots$$

Case II:
mH^{++} > mH$^+$ > mA

$$\Delta r \simeq \frac{g^2}{16\pi^2}\left(\underline{\ln m_{H^{++}} + \ln m_{H^+}} - 2\ln m_{H^{++}}\right) + \cdots$$

By using $m_{H^{++}}^2 - m_{H^+}^2 = m_{H^+}^2 - m_A^2$

Case I:

$$\Delta r \simeq \frac{g^2}{16\pi^2}\ln\frac{\sqrt{2}(|\Delta m|^2 + 2|\Delta m| m_{\text{lightest}} + m_{\text{lightest}}^2)}{m_{\text{lightest}}^2} + \cdots$$

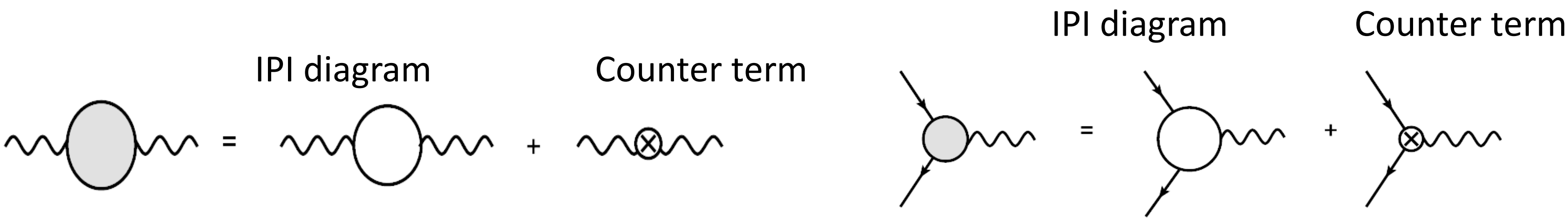
with m_{lightest} = mH^{++}

Case II:

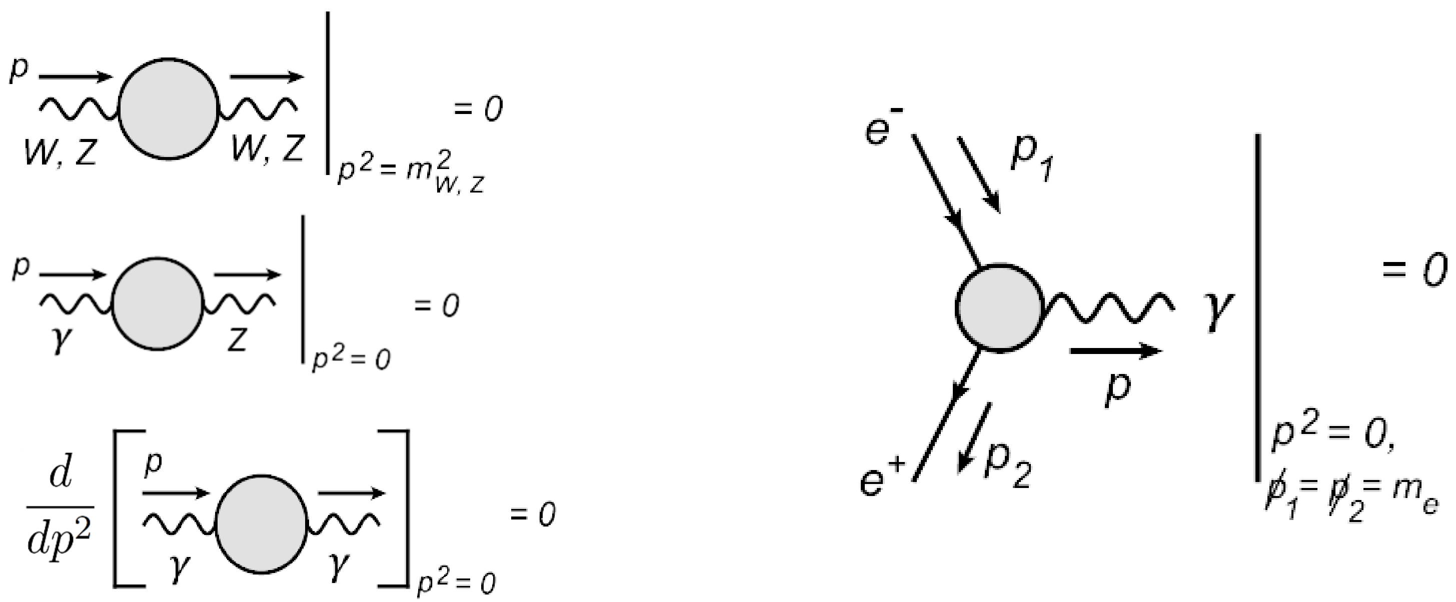
$$\Delta r \simeq \frac{g^2}{16\pi^2}\ln\frac{1+\sqrt{2}+m_{\text{lightest}}^2/(4\Delta m^2)}{2+\sqrt{2}+m_{\text{lightest}}^2/(4\Delta m^2)} + \cdots$$

with m_{lightest} = mA

On-shell renormalization scheme



On-shell renormalization conditions



From these 5 conditions, 5 counter terms (δg, $\delta g'$, δv, δZ_B, δZ_W) are determined.

Radiative corrections to the EW parameters

The deviation form $m_W^2 s_W^2 = \frac{\pi\alpha_{\rm em}}{\sqrt{2}G_F}$ can be parametrized as:

$$\boxed{m_W^2 s_W^2 = \frac{\pi\alpha_{\rm em}}{\sqrt{2}G_F}(1+\Delta r)}$$

$$\Delta r = -\frac{\delta G_F}{G_F} + \frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} - \frac{\delta s_W^2}{s_W^2} - \frac{\delta m_W^2}{m_W^2}$$

From the renormalization conditions;

$$\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} = \frac{d}{dp^2}\Pi_T^{\gamma\gamma}(p^2)\Big|_{p^2=0} + \frac{2s_W}{c_W}\frac{\Pi_T^{\gamma Z}(0)}{m_Z^2}$$

$$\frac{\delta G_F}{G_F} = -\frac{\Pi_T^{WW}(0)}{m_W^2} - \delta_{VB}$$

$$\frac{\delta m_W^2}{m_W^2} = \frac{\Pi_T^{WW}(m_W^2)}{m_W^2}$$

In models with ρ = 1 at the tree level, $\mathbf{s_W^2}$ is the dependent parameter.
Therefore, the counter term for δs_W^2 is given by the other conditions.

$$\boxed{s_W^2 = 1 - \frac{m_W^2}{m_Z^2}} \Rightarrow \frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left[\frac{\Pi_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_T^{WW}(m_W^2)}{m_W^2}\right]$$

This part represents the violation of the custodial symmetry by the sector which is running in the loop.

Y=1 Higgs Triplet Model:Kinetic term

$$\mathcal{L}_{\text{kin}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \text{Tr}[(D_\mu \Delta)^\dagger (D^\mu \Delta)]$$

Covariant Derivative

$$D_\mu \Phi = \left(\partial_\mu + i\frac{g}{2}\tau^a W_\mu^a + i\frac{g'}{2}B_\mu \right) \Phi$$

$$D_\mu \Delta = \partial_\mu \Delta + i\frac{g}{2}[\tau^a W_\mu^a, \Delta] + ig' B_\mu \Delta$$

VEVs

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}$$

$$\langle \Delta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

Masses for Gauge bosons

$$m_W^2 = \frac{g^2}{4}(v_\Phi^2 + 2v_\Delta^2)$$

$$m_Z^2 = \frac{g^2}{4\cos^2\theta_W}(v_\Phi^2 + 4v_\Delta^2)$$

ρ parameter

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{1 + \frac{2v_\Delta^2}{v_\Phi^2}}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}$$

The Higgs Triplet Model (HTM)

The Higgs potential

$$\boxed{\begin{aligned} V = & m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + \left[\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.}\right] \\ & + \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}[(\Delta^\dagger\Delta)^2] \\ & + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi. \end{aligned}}$$

$$\mathbf{\Phi} = \left[\begin{array}{c} \varphi^+ \\ \frac{1}{\sqrt{2}}(\varphi + v + i\chi) \end{array}\right]$$

$$\mathbf{\Delta} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{\Delta^0} = \frac{1}{\sqrt{2}}(\delta + v_\Delta + i\eta)$$

There are 10 degrees of freedom of scalar states.

$$\Delta^{\pm\pm} = H^{\pm\pm}$$

$$\begin{pmatrix} \varphi^\pm \\ \Delta^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta_\pm & -\sin\beta_\pm \\ \sin\beta_\pm & \cos\beta_\pm \end{pmatrix} \begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix} \qquad \tan\beta_\pm = \frac{\sqrt{2}v_\Delta}{v}$$

$$\begin{pmatrix} \chi^0 \\ \eta^0 \end{pmatrix} = \begin{pmatrix} \cos\beta_0 & -\sin\beta_0 \\ \sin\beta_0 & \cos\beta_0 \end{pmatrix} \begin{pmatrix} z^0 \\ A^0 \end{pmatrix} \qquad \tan\beta_0 = \frac{2v_\Delta}{v}$$

$$\begin{pmatrix} \varphi^0 \\ \delta^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} \qquad \tan 2\alpha = \frac{v_\Delta}{v}\frac{2v^2(\lambda_4+\lambda_5) - 4M_\Delta^2}{2v^2\lambda_1 - M_\Delta^2 - v_\Delta^2(\lambda_2+\lambda_3)}$$

10 scalar states can be translated to **3 NG bosons, 1 SM-like scalar boson** and **6 Δ-like scalar bosons.**

6 Δ-like scalar bosons → $H^{\pm\pm}$, $H^\pm$, A and H

Doubly-charged, Singly-charged, CP-odd, CP-even

Custrodial Symmetry

The Higgs doublet can be written by the 2×2 matrix form

$$\Sigma \equiv (\widetilde{\Phi}, \Phi) = \begin{pmatrix} -\phi_0^\star & \phi^+ \\ \phi^- & \phi_0 \end{pmatrix}$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\mathrm{Tr}\left[(\tilde{D}_\mu\Sigma)^\dagger(\tilde{D}^\mu\Sigma)\right] \qquad \tilde{D}_\mu\Sigma = \partial_\mu\Sigma + i\frac{g}{2}\tau\cdot W_\mu\Sigma - i\frac{g'}{2}B_\mu\Sigma\tau_3$$

In the limit of g' → 0,
The kinetic term is invariant under $SU(2)_L \times SU(2)_R$

$$\Sigma \to \Sigma' = U_L \Sigma U_R^\dagger$$

After the Higgs field gets the VEV,
Only the symmetry of $SU(2)_L = SU(2)_R = SU(2)_V$ remain.
This $SU(2)_V$ is called the custodial symmetry.

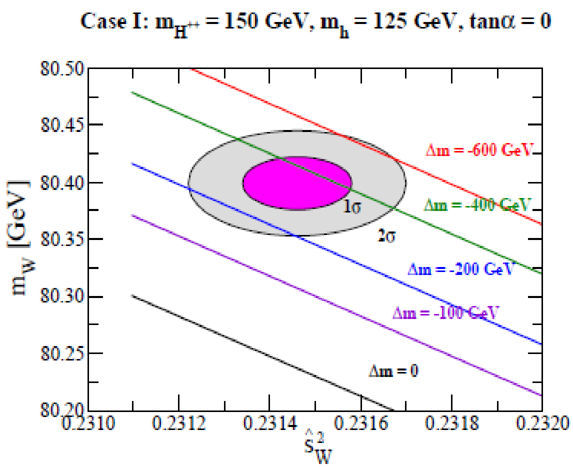
$$\Sigma \to \langle\Sigma\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Kanemura, Yagyu, arXiv: 1201.6287 [hep-ph]

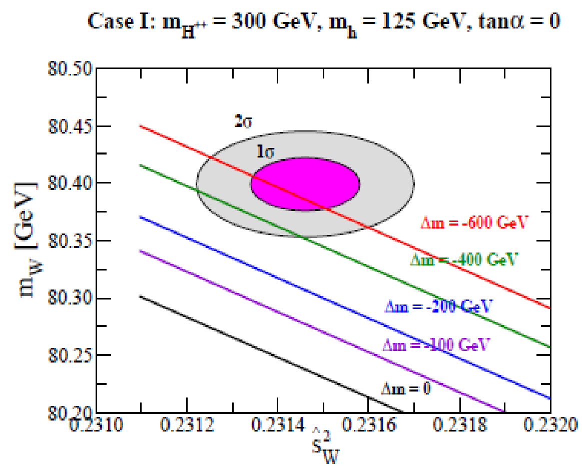
Prediction to the W boson mass at the 1-loop level

$\Delta m = m_{H^{++}} - m_{H^+}$

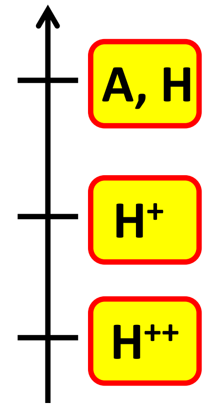
mH^{++} = 150 GeV

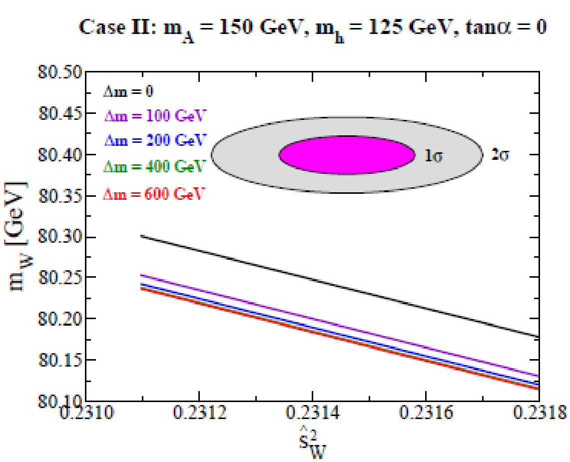


mH^{++} = 300 GeV

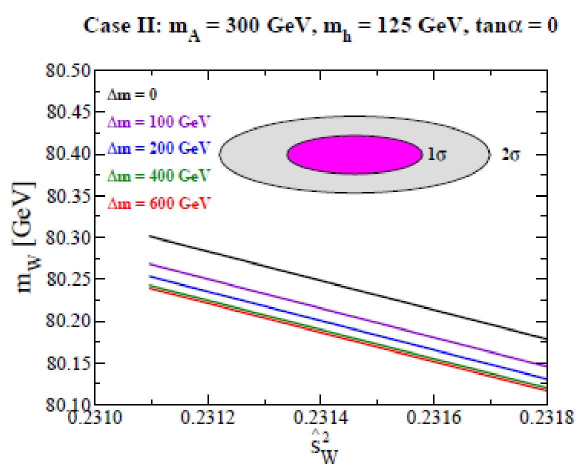


Case I



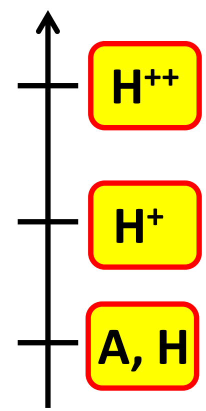


mA = 150 GeV



mA = 300 GeV

Case II



In Case I, by the effect of the mass splitting, there are allowed regions .
Case II is highly constrained by the data.

Kanemura, Yagyu, arXiv: 1201.6287 [hep-ph]

Heavy mass limit

$\xi = m_{H++}{}^2 - m_{H+}{}^2$

Case I

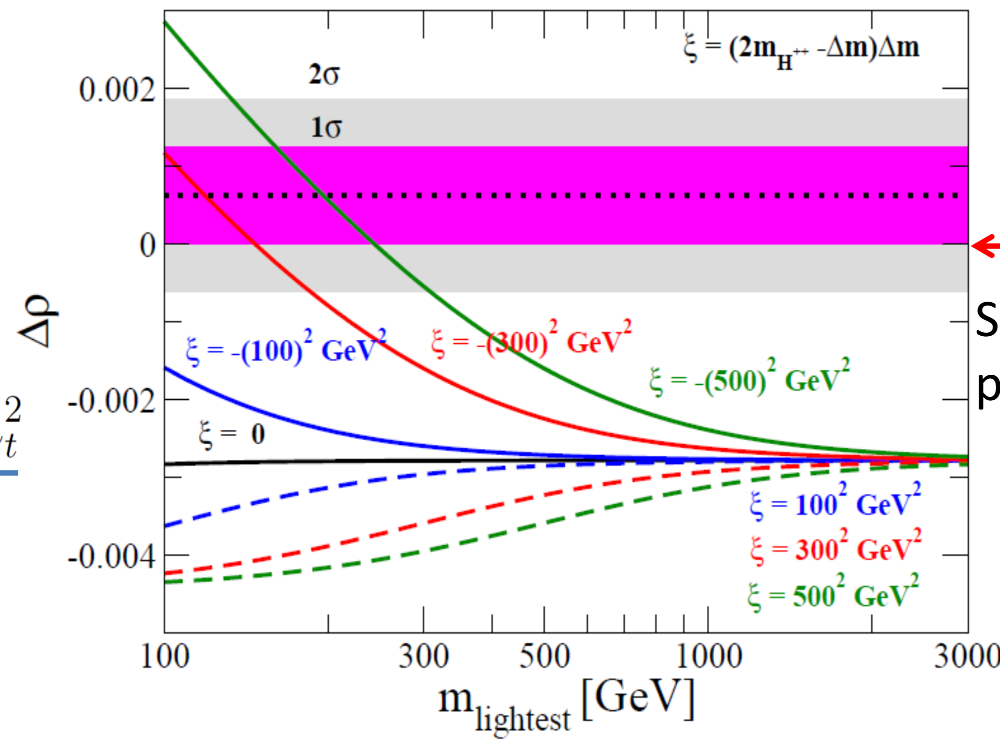
Case II

$$\rho_{\mathrm{SM}} \simeq 1 + \frac{\hat{c}_W^2}{\hat{c}_W^2 - \hat{s}_W^2}\frac{N_c\sqrt{2}}{16\pi^2}G_F m_t^2$$

$$\rho_{\mathrm{HTM}} \simeq 1 + \frac{g^2 N_c}{16\pi^2}\ln m_t$$

$$\Delta\rho \equiv \rho - \rho_{\mathrm{SM}}(m_h^{\mathrm{ref}} = 125\ \mathrm{GeV})$$

$$\Delta\rho^{\exp} = 0.000632 \pm 0.000621$$



SM prediction

When we take heavy mass limit, loop effects of the triplet-like scalar bosons disappear. Even in such a case, the prediction does not coincide with the SM prediction.

Decoupling property of the HTM

$$\mu\Phi\cdot\Delta^\dagger\Phi$$

HTM with ~~L~~# ($\mu \neq 0$)

- $\rho \neq 1$ at the tree level ($v_\Delta \neq 0$)
- 4 input parameters (α_{em}, G_F, m_Z and $s_W{}^2$)

HTM with L# ($\mu = 0$)

- $\rho = 1$ at the tree level ($v_\Delta = 0$)
- 3 input parameters (α_{em}, G_F and m_Z) with $s_W{}^2 = 1-m_W{}^2/m_Z{}^2$

Heavy Δ-like fields limit

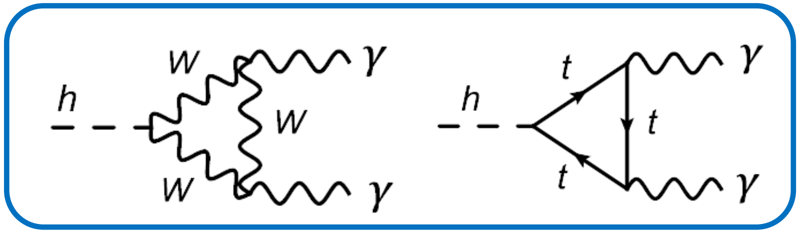
Heavy Δ-like fields limit

SM

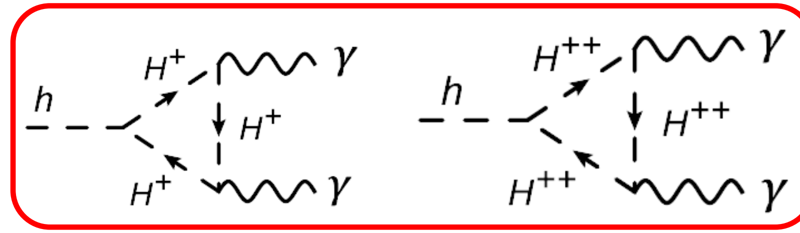
- L# is conserved.
- $\rho = 1$ at the tree level
- 3 input parameters (α_{em}, G_F and m_Z) with $s_W{}^2 = 1-m_W{}^2/m_Z{}^2$

Kanemura, Yagyu, arXiv: 1201.6287 [hep-ph]

Higgs → two photon decay

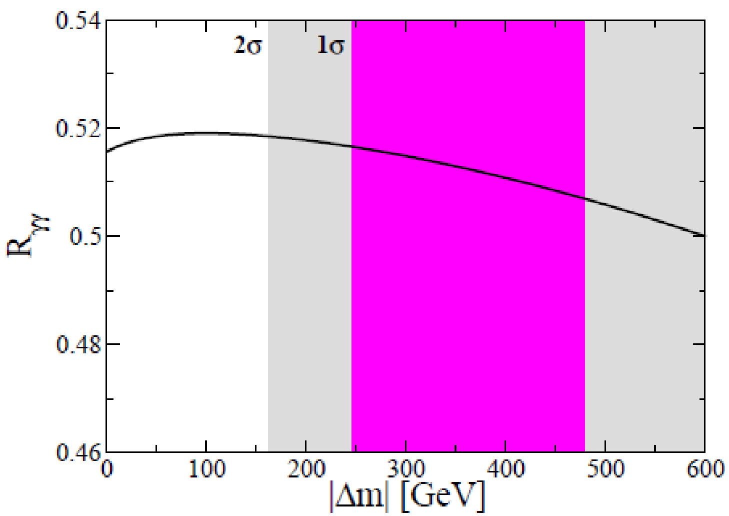


+

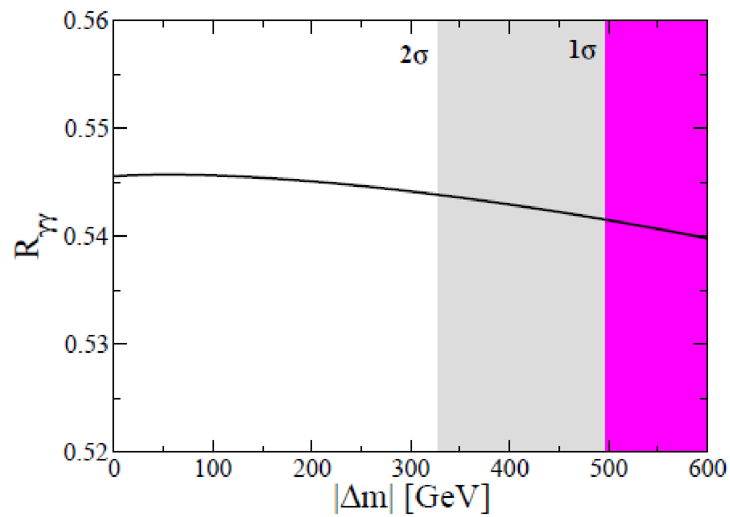


SM contribution + **Triplet-like scalar loop contribution**

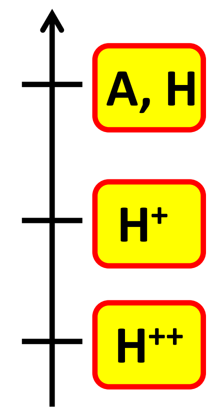
mH^{++} = 150 GeV, mh = 125 GeV



mH^{++} = 300 GeV, mh = 125 GeV



Case I



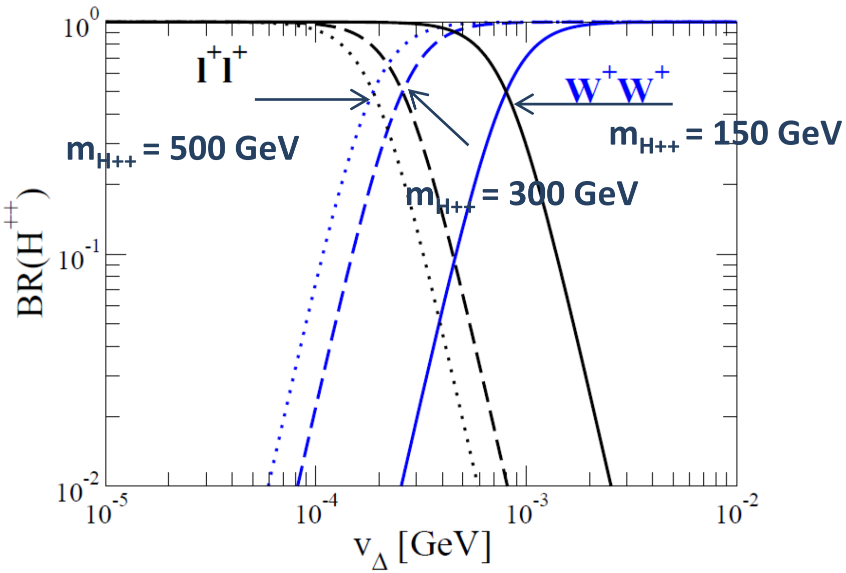
$$\boxed{R_{\gamma\gamma} \equiv \frac{\Gamma(h \to \gamma\gamma)_{\rm HTM}}{\Gamma(\phi_{\rm SM} \to \gamma\gamma)_{\rm SM}}}$$

$$\lambda_{hH^+H^-} \simeq \frac{2m_{H^+}^2}{v} \qquad \lambda_{hH^{++}H^{--}} \simeq \frac{2m_{H^{++}}^2}{v}$$

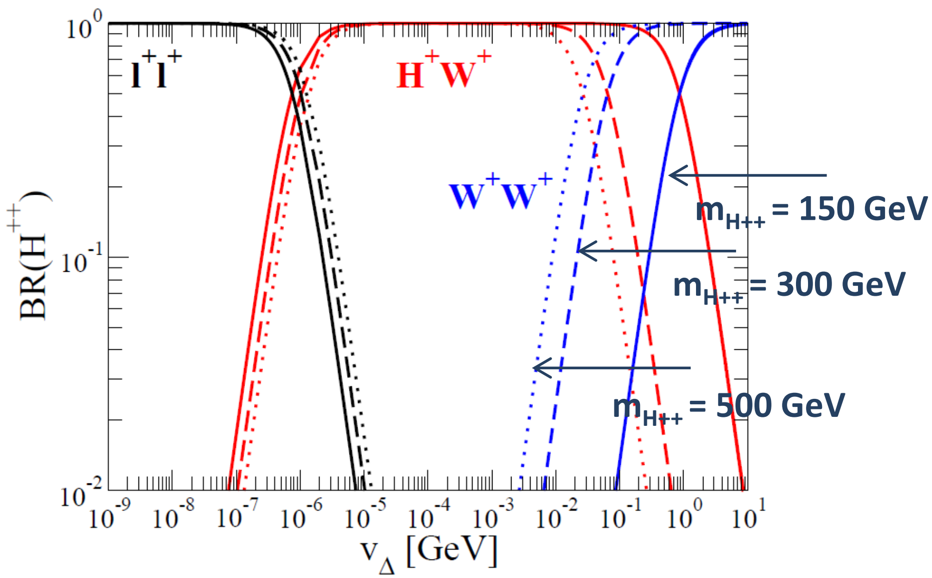
The decay rate of h → γγ is around half in the HTM compared with that in the SM.

Branching ratio of H^{++}

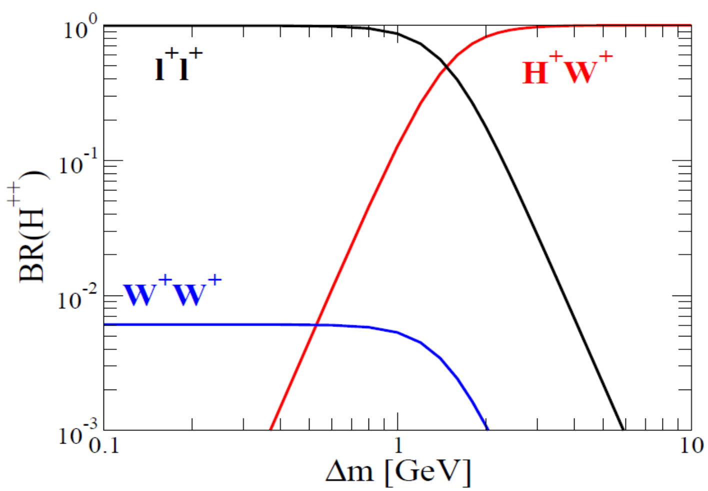
$\Delta m = 0$



Δm = 10 GeV



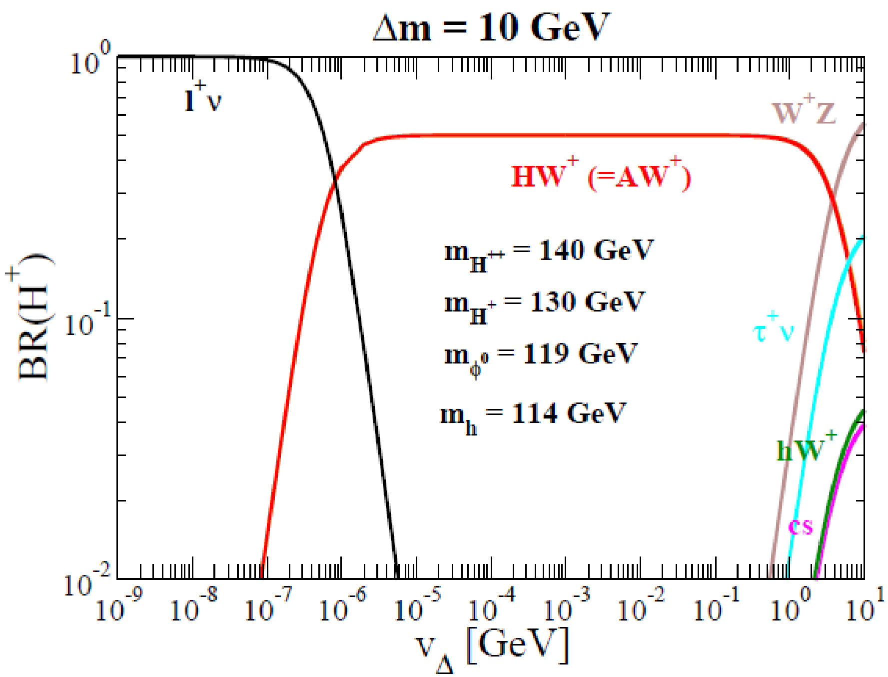
v_Δ = 0.1 MeV, m_{H++} = 200 GeV

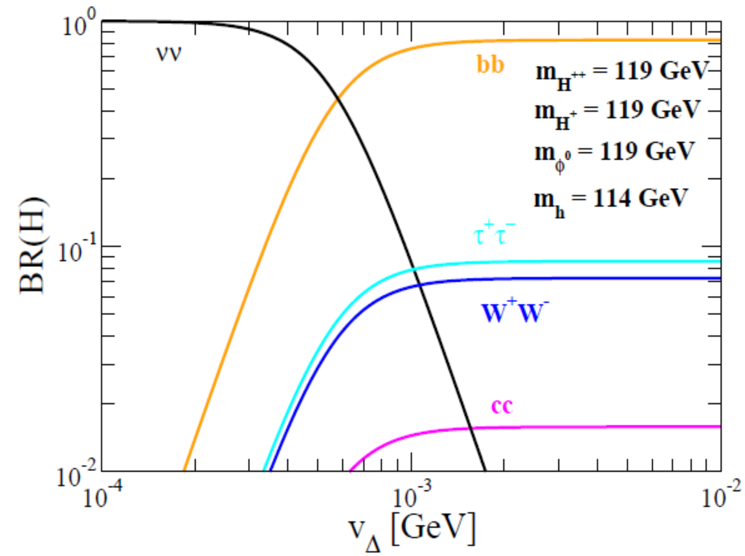


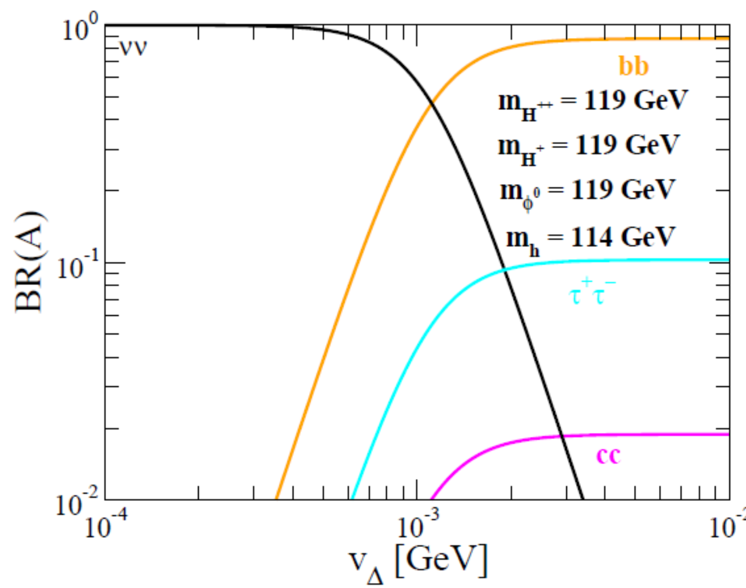
Chakrabarti, Choudhury, Godbole, Mukhopadhyaya, (1998);
Chun, Lee, Park, (2003);
Perez, Han, Huang, Li, Wang, (2008);
Melfo, Nemevsek, Nesti, Senjanovic, (2011)

Phenomenology of $\Delta m \neq 0$ is drastically different from that of $\Delta m = 0$.

Branching ratios of H^+, H and A







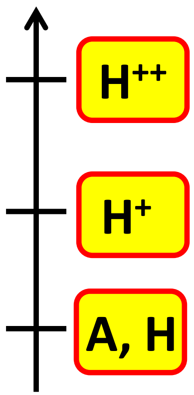
★ The $H^+ \to \phi^0 W^+$ mode can be dominant in the case of $\Delta m \neq 0$.

★ The $\phi^0 \to bb$ mode can be dominant when $v_\Delta >$ MeV.

Phenomenology of HTM with the mass splitting at the LHC

Aoki, Kanemura, Yagyu , Phys. Rev. D, in press (2011)

Case II

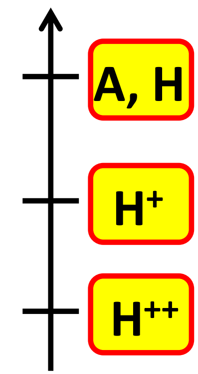


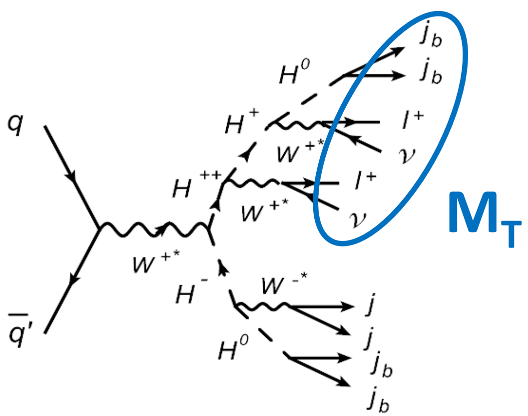
Cascade decays of the Δ-like scalar bosons become important.

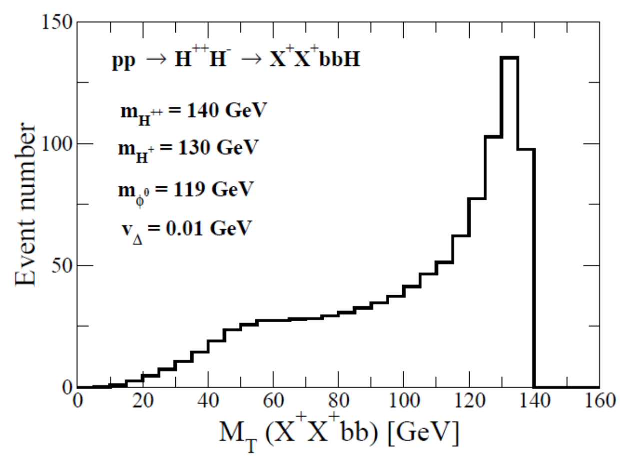
$H^{++} \rightarrow H^{+}W^{+} \rightarrow A\,(H)W^{+}W^{+}$
$H^{+} \rightarrow A\,(H)W^{-}$
$A\,(H) \rightarrow \nu\nu$ or bb
(mA~100 GeV case)

$A\,(H) \rightarrow H^{+}W^{-} \rightarrow H^{++}W^{-}W^{-}$
$H^{+} \rightarrow H^{++}W^{-}$
$H^{++} \rightarrow l^{+}l^{+}$ or $W^{+}W^{+}$

Case I







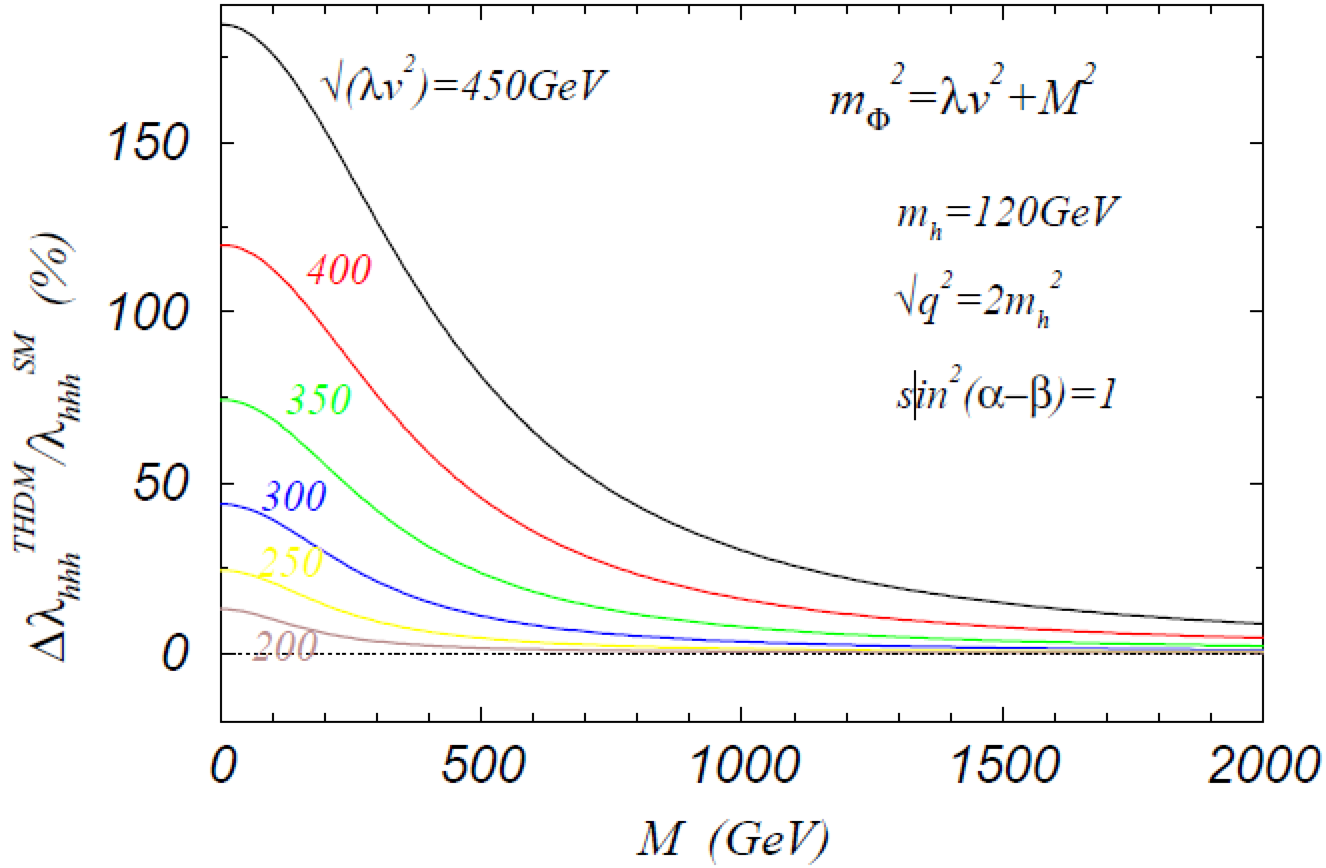
Transvers mass

$$\boxed{\begin{aligned} M_T^2 &= (\not{E}_T + p_T)^2 \\ &\simeq 2|\not{E}_T||p_T|(1-\cos\varphi) \end{aligned}}$$

By using the M_T distribution, we may reconstruct the mass spectrum of Δ-like scalar bosons.
→ We would test the Higgs potential in the HTM.

2HDM の *hhh*結合

Kanemura, Okada, Senaha, Yuan (2004)



$$m_\Phi^2 = \lambda v^2 + M^2$$