



Probing Strongly Interacting W 's at the ILC with Polarized Beams

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- "What is the mechanism of **ElectroWeak Symmetry Breaking** (EWSB)"?
One of the most important questions in particle physics yet to be answered at **Large Hadron Collider** (LHC)
- On a broad scale theorists have two different ideas for this question
- "**Weak scenario**", with the assumption that the Higgs sector consists of a light Higgs in the range $130 \leq m_H \leq 180$ GeV
- "**Strong scenario**", characterized in general by the absence of Higgs field or the mass is of the order of the new interaction, i.e. around 1 TeV or when the Higgs is light ~ 125 GeV, consistent with the regions so far not excluded by LHC searches
- One all-around consequence of the "**Strong scenario**" is the longitudinal W 's and Z 's start interacting strongly with themselves at high energies
- This talk mainly concentrates on the theory of WW scattering at high energies, and the feasibility of carrying out such a study in **International Linear Collider** (ILC)



- The longitudinal $W(W_L)$ hold the key to EWSB, as it corresponds to the Goldstone degree of freedom
- The polarization vector of W_L, Z_L grows as the energy of the particle, so the amplitude diverges in the high energy limit
- In gauge theory the cancellation between the graphs, cures the bad high energy behaviour of the amplitude
- At high energies $s \gg m_W^2$, vector boson scattering amplitudes are calculated using **Equivalence Theorem**

Equivalence Theorem

- At high energies there is a correspondence between the longitudinal component of gauge bosons and the corresponding Goldstone bosons
- $\mathcal{M}(W_L W_L \rightarrow W_L W_L) = \mathcal{M}(ww \rightarrow ww)$
 $\mathcal{M}(Z_L Z_L \rightarrow Z_L Z_L) = \mathcal{M}(zz \rightarrow zz)$
 $\mathcal{M}(W_L Z_L \rightarrow W_L Z_L) = \mathcal{M}(wz \rightarrow wz)$
 $\mathcal{M}(W_L W_L \rightarrow Z_L Z_L) = \mathcal{M}(ww \rightarrow zz)$
 where w and z are the Goldstone bosons



Custodial Symmetry

- Experimentally $\rho = 1$ holds upto 1 percent, indicating the actual larger symmetry of Higgs sector

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$$

- Explained naturally in the SM, where the Higgs field is a doublet
- In the strongly interacting scenario, $\rho = 1$ is not natural but is protected from large quantum corrections by custodial symmetry, with the breaking pattern: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
- Using **Equivalence Theorem**, the longitudinal gauge bosons have a close resemblance to pions, both being Goldstone Bosons of same symmetry breaking pattern $SU(2)_L \times SU(2)_R \rightarrow SU(2)$
- Goldstone boson interactions are defined by **Low Energy Theorem** (LET), analogous to the theorem for $\pi\pi$ scattering in the **Chiral Lagrangian**, for $s \ll \Lambda_{SB}^2$



Electroweak Chiral Lagrangian

- At lowest order the Lagrangian is given by:

$$\mathcal{L} = \frac{1}{2} \partial_\mu W^a \partial^\mu W^a + \frac{1}{6v^2} [(W^a \partial_\mu W^a)^2 - W^a W^a (\partial_\mu W^a)^2] + \dots$$

- Scattering amplitudes for W^\pm , Z boson are calculated at $s \gg m_W, m_Z$, using **Equivalence Theorem**, from the scattering of w^\pm, w^3
- The scattering amplitudes are given as (*D. Dominici, Riv. Nuovo Cim. 20N11 (1997) 1*) :

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L) = A(s, t, u)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = A(s, t, u) + A(t, s, u)$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L Z_L) = A(s, t, u) + A(t, s, u) + A(u, t, s)$$

$$\text{with } A(s, t, u) = s/v^2$$

- The Lagrangian is valid at low energies, with the scattering amplitudes violating unitarity at higher energies



Partial Wave Unitarity

- Expanding the scattering amplitude in a series of partial wave amplitude

$$\mathcal{M}^I = 32\pi \sum_l (2l+1) a_{lI} P_l(\cos\theta)$$

with \mathcal{M}^I as the isospin amplitude

- Projecting out the partial waves, the energy where partial wave unitarity is violated is found by requiring $|a_{lI}| \leq 1$ or $|\text{Re } a_{lI}| \leq \frac{1}{2}$,
- Bound on m_H is obtained, by considering the $l=0$ partial wave unitarity for large s for WW scattering in the absence of Higgs

$$\text{Re } a_{00} \sim \text{Re } \frac{G_F s}{16\pi\sqrt{2}} < 1/2$$

$$s < \frac{4\pi\sqrt{2}}{G_F} \approx (1.2 \text{ TeV})^2 \text{ i.e. } m_H < 1.2 \text{ TeV}$$



Saving Unitarity

- Lowest order amplitudes violate unitarity, with the need for including higher order corrections
- The amplitudes calculable to all order is usually hard to compute, with the need to obtain a unitary amplitude from the approximate amplitude which violates unitarity
- From low energy hadron physics, several extrapolation schemes are there incorporating the tree level and one loop results from χ PT along with satisfying unitarity
- They are mainly the **K-matrix**, **Padé** and **N/D methods**, extensively discussed in the literature (*K. -i. Hikasa, In "Saariselkae 1991, Proceedings, Physics and experiments with linear colliders"*)
- The method of unitarization along with the parameters of the Lagrangian decides whether a resonance exists in a particular channel or not



Resonances

- It is impossible to predict the resonances, as the underlying dynamics of strong interactions is not known
- The zeroth order **K- matrix** amplitude, presents a model with non resonant $l = \mathcal{I} = 1$ partial wave, whereas **Padé** gives a resonance
- In terms of partial wave amplitudes :

$$a^K(s) = \frac{a^{LET}(s)}{1 - ia^{LET}(s)}$$

$$a^{Pade}(s) = \frac{a^{LET}(s)}{1 - a^{(1)}(s)/a^{LET}(s)}$$

- The other way of including the resonances i.e vector or scalar is assuming the existence of resonance in a particular channel, and writing down a Lagrangian including resonance as the elementary field
- The BESS (**Breaking Electroweak Symmetry Strongly**) models consider the ρ like resonance as a gauge boson of a hidden $SU(2)$, introducing strong interactions in the weak gauge boson sector

LHC and ILC

- Gauge boson scatterings in processes like $qq \rightarrow qqVV$, where $V = W, Z$ were computed within, the equivalent gauge boson approximation (EGBA) (*Accomando et al. Phys. Rev. D 74 (2006) 073010*)
- For the leptonic colliders, $e^+e^- \rightarrow ll' W^+W^-$, where $l, l' = e, \mu, \nu_e, \nu_\mu$ have been investigated for strong VV scattering (*T. Abe et al., arXiv:1006.3396 [hep-ex]*)
- The process is sensitive to scalar and tensor resonances, as well as the vector resonances arising in gauge boson scattering
- The process $e^+e^- \rightarrow W^+W^-$, which we consider has the advantage that only vector resonances are involved, and also its cross section (at ILC energies) is about three orders of magnitude larger than $e^+e^- \rightarrow ll' W^+W^-$ (*T. L. Barklow, [arXiv:hep-ph/0112286]*)
- Within the framework of the BESS model, the contribution of the additional ρ in the s -channel is done for $e^+e^- \rightarrow W^+W^-$, including W 's decay spectrum (*Casalbuoni et al. [hep-ph/9912377]*)
- These effects are almost negligible unless one is very close to the new vector resonance, owing to their highly constrained fermionic couplings



Form Factors

- The effect of single vector resonance, is parametrized by introducing suitable form factors in the $l = 1$ partial wave amplitude of $e^+e^- \rightarrow W^+W^-$
- The complex form factor scaled in analogy to the pion form factor derived from **Gounaris Sakurai** (GS) or **Breit Wigner** (BW) representation, approximates the effect of the resonance (*Iddir et al. Phys. Rev. D 41 (1990) 22, Werthenbach and Sehgal, Phys. Lett. B 402 (1997) 189*)
- An improved treatment of vector boson resonance is to introduce a suitable Omnès function, to implement the phase shift depending on dynamics to the P partial wave in $e^+e^- \rightarrow W^+W^-$ (*Bernreuther and Schroder, Z. Phys. C 62 (1994) 615*)
- The Omnes formalism is the mathematical exercise of finding functions which are analytic except for $4m_W^2 \leq s \leq \infty$ and considering that the reaction is elastic at the energies considered



Omnès Function

- The phase of the form factor δ obtained either from a **K-matrix**, or from GS or BW parametrization for a given M_ρ and Γ_ρ , gives the Omnès Function

$$\Omega(s) = \exp \left[\frac{s}{\pi} \int_{4m_W^2}^{\infty} \frac{\delta(s') ds'}{s'(s' - s - i\epsilon)} \right]$$

- The form factor for a single channel elastic scattering is related to $\Omega(s)$ by :

$$F(s) = \left(1 + \frac{s}{a} + \frac{s^2}{b^2} + \dots \right) \Omega(s)$$

provided $\lim_{s \rightarrow \infty} \delta(s) = \text{finite}$, $\lim_{s \rightarrow \infty} \frac{|f(s)|}{s} \rightarrow 0$

- The constants a , b , \dots etc. are fixed from additional inputs from the underlying theory, or experiments, which we assume to be large
- GS and BW parametrizations are treated as low energy representation of the form factor to generate δ

Feynmann Diagrams

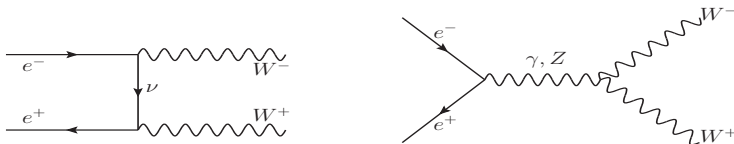


Figure: Feynman diagrams contributing to the process $e^+e^- \rightarrow W^+W^-$ in the SM

- The strong interaction through ρ like resonance, affects only the $l = 1$ partial wave
- The amplitudes involving W_L are modified, through an Omnès function, representing the strong final state interaction



- The γ and Z exchange s -channel contributions are pure P waves, so the amplitude corresponding to s -channel of the $W_L W_L$ production is modified as:

$$M_{LL}^{\gamma+Z}(s, \theta) \rightarrow \Omega(s) M_{LL}^{\gamma+Z}(s, \theta).$$

- The ν exchange t -channel has $l \geq 1$ components, so $l=1$ amplitude is modified, leaving the others unaffected

$$M_{LL}^{\nu,l}(s, \theta) \rightarrow \Omega(s) M_{LL}^{\nu,l=1}(s, \theta) + M_{LL}^{\nu,l>1}(s, \theta)$$

- The helicity amplitude for $W_L W_L$ is defined in terms of partial waves as :

$$M_{LL}^{\nu,l=1}(s, \theta) = \frac{3}{4\pi} d_{m,0}^{1*}(\theta) a_L^1(s), \quad m = \pm 1$$

where

$$d_{m,0}^1(\theta) = -\frac{1}{\sqrt{2}} \sin \theta$$

is the relevant rotation function.



- The $l = 1$ partial wave is isolated by the projection,

$$a_L^1(s) = 2\pi \int_{-1}^{+1} d(\cos \theta) d_{m,0}^1(\theta) M_{LL}^{\nu,1}(s, \theta).$$

- The rest of the amplitude with $l > 1$ is obtained by,

$$M_{LL}^{\nu,l>1}(s, \theta) = M_{LL}^{\nu,l}(s, \theta) - M_{LL}^{\nu,l=1}(s, \theta)$$

- The effect of the strong interaction in the t -channel is rewritten as:

$$M_{LL}^{\nu,l}(s, \theta) \rightarrow M_{LL}^{\nu,l}(s, \theta) + (\Omega(s) - 1) M_{LL}^{\nu,l=1}(s, \theta)$$

- Beam polarization at ILC will significantly benefit the physics program, in searches for new physics with small deviations from SM cross sections
- A beam polarization of $\geq 80\%$ for electrons and $\geq 30\%$ for positrons at the interaction point is proposed, with a possible upgrading to about 60% for the positron beam

Longitudinal Polarization

- The dependence of the cross section to the polarization is parametrized through $P_l = (N_R - N_L)/(N_R + N_L)$, where $N_{L,R}$ denote the number of left-polarized and right-polarized electrons (or positrons), respectively

Transverse Polarization

- At the ILC, with the help of the proposed spin rotator scheme the LP can be reoriented to achieve TP of the same degree

$$\frac{d\sigma}{d\Omega} = \frac{\beta}{64\pi^2 s} \left\{ \frac{1}{4} \left((1 + P_l)(1 - P_t) |M_{+-}|^2 + (1 - P_l)(1 + P_t) |M_{-+}|^2 \right) - \frac{1}{2} P_t P_l (\cos 2\phi \operatorname{Re} M_{+-}^* M_{-+} - \sin 2\phi \operatorname{Im} M_{+-}^* M_{-+}) \right\}$$



W helicities production cross section

| λ_{W^-} | λ_{W^+} | σ (pb) | | |
|-----------------|-----------------|--------------------------------|--------------------------------------|-------------------------------------|
| | | $P_{e^-} = 0$ $P_{e^+} = 0$ | $P_{e^-} = -0.8$ $P_{e^+} = +0.6$ | $P_{e^-} = 0.8$ $P_{e^+} = -0.6$ |
| -1 | -1 | 0.0003 | 0.0010 | 0.0 |
| -1 | 0 | 0.0191 | 0.0541 | 0.0024 |
| -1 | 1 | 3.4943 | 10.063 | 0.2795 |
| 0 | -1 | 0.0032 | 0.0084 | 0.0011 |
| 0 | 0 | 0.0468 | 0.1124 | 0.0263 |
| 0 | 1 | 0.0191 | 0.0541 | 0.0024 |
| 1 | -1 | 0.0921 | 0.2653 | 0.0074 |
| 1 | 0 | 0.0032 | 0.0085 | 0.0011 |
| 1 | 1 | 0.0003 | 0.0010 | 0.0 |

Table: SM cross sections in pb for $\sqrt{s} = 800$ GeV with different beam polarizations and for different W^+W^- helicities



Choice of Parameters

- For our analyses we chose the mass of the resonance to vary from the unitarity limit 1.2 TeV to 2 TeV
- For the choice of width, we consider the relation:

$$\Gamma_\rho = \frac{m_\rho^3}{96\pi v^2}$$

from the constraints employed by low energy chiral QCD on Γ_ρ

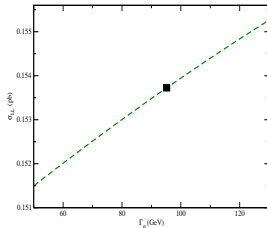


Figure: Total unpolarized cross section for W_L as a function of width for $M_\rho = 1200$ GeV, at $\sqrt{s} = 800$ GeV. The square denotes the value of cross section for Γ_ρ obtained from above eq.



Total Cross Section

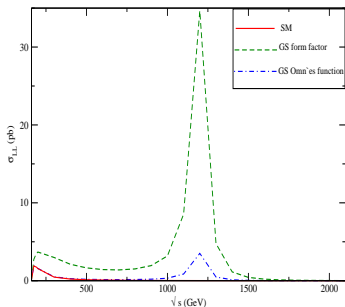


Figure: Total unpolarized c.s for W_L as a function of \sqrt{s} for SM, along with GS form factor and the respective Omnès function for $M_\rho = 1200$ GeV, $\Gamma_\rho = 94$ GeV

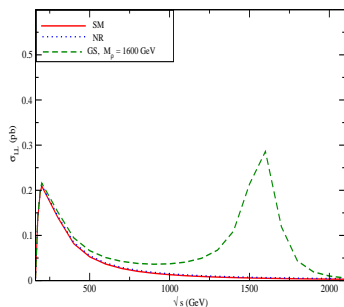


Figure: Cross Section for $W_L W_L$, with $P_{e^-} = 0.8$, $P_{e^+} = -0.6$, as a function of \sqrt{s} in SM, NR along with GS-Omnès parametrization. An angular cut of $|\cos \theta| < 0.5$ is applied



Angular distribution of $W_L W_L$ and $W_L W_T$

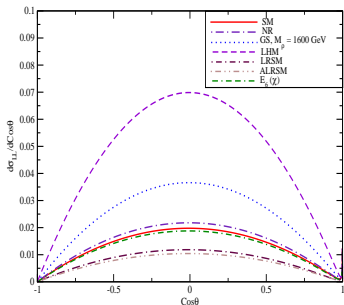


Figure: Polar angle distribution of W_L in $W_L W_L$ production in SM, NR, GS-Omnès parametrization and the different Z' models, with $P_{e^-} = 0.8$ and $P_{e^+} = -0.6$ at $\sqrt{s} = 800$ GeV

- For the Z' models, we consider the mixing angle $\theta_M = 0.003$ and $\Delta M = 0.12$ GeV, whereas for LHM, $f = 1$ TeV and $\cos \theta_H = 0.45$ is considered satisfying the electroweak constraints.
- With both W 's in the final state being transversely polarized, no effect of new physics from Z' or strongly interacting sector is observed
- $W_T W_T$ are mostly produced through the ν -exchanged t -channel, whereas the Z' affects the s -channel only
- With one of the W transversely polarized and the other longitudinally polarized, Z' models sensitivity can be observed with no influence from SFI



Forward Backward Asymmetry

- The fraction of W' 's emitted in the backward hemisphere is considered:

$$f_{back} = \frac{\int_{-1}^0 (d\sigma/d \cos \theta) d \cos \theta}{\int_{-1}^1 (d\sigma/d \cos \theta) d \cos \theta}$$

The above observable is related to FB asymmetry by $A_{FB} = 2f_{back} - 1$

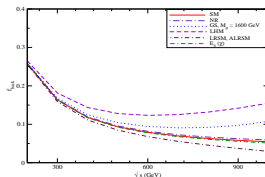


Figure: Fraction of unpolarized W' 's emitted in the backward hemisphere as a function of \sqrt{s} , for the different models considered, with $P_{e^-} = 0.8$ and $P_{e^+} = -0.6$

- At $\sqrt{s} = 800$ GeV, in SM 6% of the events are in the backward region, in the presence of strong interaction this is substantially increased to a 9%
- With the cross section at 0.3 pb, for a moderate integrated luminosity of 100 fb^{-1} , and considering a BR ($\sim 4/27$ for semi-leptonic channel) and reconstruction efficiency ($\sim 65\%$) 6% of the events amount to few hundred events
- Certainly, a measurement of increase by 9% or 13% is conceivable in this case

Azimuthal Distribution of W 's

- The deviation from SM in azimuthal distribution for different scenarios with unpolarized W 's in the final state

$$\Delta \frac{d\sigma}{d\phi} = \frac{d\sigma}{d\phi}(\text{new}) / \frac{d\sigma}{d\phi}(\text{SM}) - 1$$

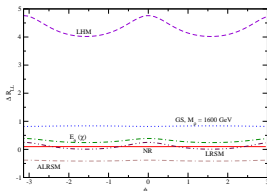


Figure: ϕ distribution of polarized $W_L W_L$ showing the deviation from SM as a function of ϕ at $\sqrt{s} = 800$ GeV, in the different scenarios considered. Purely transversely polarized beams with $P_T = 0.8$ and $P_{\bar{T}} = 0.6$ are considered

- The TP case has an interesting feature of receiving contribution from the imaginary part of the amplitude

- The size of the contribution from $\text{Im}(M_{+-}^* M_{-+})$ can be estimated by considering the following asymmetry

$$A^{\text{img}}(\theta) = \int_{-\pi}^{\pi} \frac{d\sigma}{d\Omega} \sin 2\phi d\phi$$

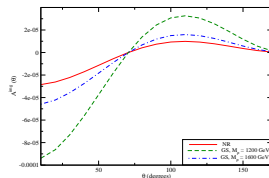


Figure: Asymmetry showing the contribution from imaginary part of NR, GS for different M_ρ



Azimuthal distribution of decay leptons

- The observables considered so far are only sensitive to the modulus of the Omnès function
- Sensitivity to the effect of phase shift, was addressed by the inclusion of TP, but too small to be seen at design ILC energies
- Since W_L are affected by the strong interactions, with no strong interaction for W_T , the interference pattern between the two amplitudes will be sensitive to the phase
- The spins of the weak bosons are analysed by their weak decays, where the longitudinal-transverse spin-spin correlations is obtained by correlating the azimuthal angles of W decay product

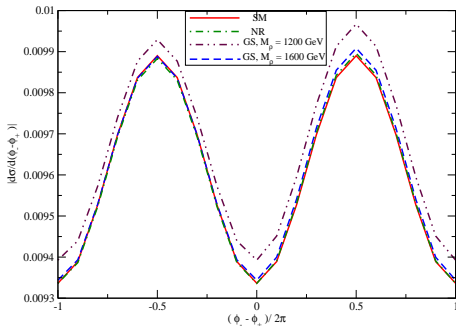


Figure: Expected angular distribution, with and without SFI, with initial beam polarization of $P_{e^-} = -0.8$ and $P_{e^+} = 0.6$

Energy distribution of decay leptons

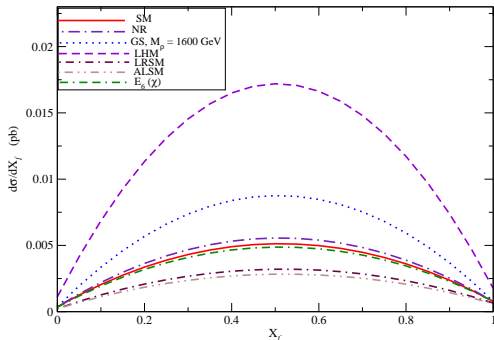


Figure: Laboratory energy distribution of the secondary lepton from W_L^- , while W_{unpol}^+ is allowed to decay into anything. Longitudinally polarized beams with $P_{e^-} = 0.8$ and $P_{e^+} = -0.6$ at $\sqrt{s} = 800$ GeV is considered. $X_{l\pm} = \frac{2}{\beta\sqrt{s}} \left(E_{l\pm} - \frac{\sqrt{s}}{4}(1 - \beta) \right)$

Discussions and Conclusions

- We have focussed on the fingerprinting of a strongly interacting sector at the ILC, which may lead to a formation of vector resonance in $e^+e^- \rightarrow W^+W^-$ with polarized beams
- The method we have used is the inclusion of phase due to the strongly interacting sector which modifies the $l=1$ partial wave, with the phase modelled in terms of GS or BW parametrizations due to the possibility of a resonance, or in terms of a non-resonant background
- We have analyzed the process in great detail for the case of both initial as well as final state polarizations and have studied various observables like the total cross section, angular distribution of W_L , W_T , the FB as well as the LR asymmetry
- A detailed comparison is carried out with the popular models where new physics can arise due to the presence of an additional heavy gauge boson Z' . The behavioral pattern is almost the same with $W_L W_T$ channel acting as a model discriminator for the Z' models from the strongly interacting sector

Discussions and Conclusions

- The case of decays is also considered where the correlation proportional to $(\phi_- - \phi_+)$ is obtained using analytic methods
- With a discovery of resonance at LHC, ILC will be able to disentangle some of the contesting models against strongly interacting sector, whereas in the absence of any information from LHC, it will be a difficult job requiring very high luminosity and large c.m. energy.
- Our work also shows that a strong polarization program at the ILC is very useful in shedding light on the dynamics of EWSB



Thank You