Strong field beam-beam simulations

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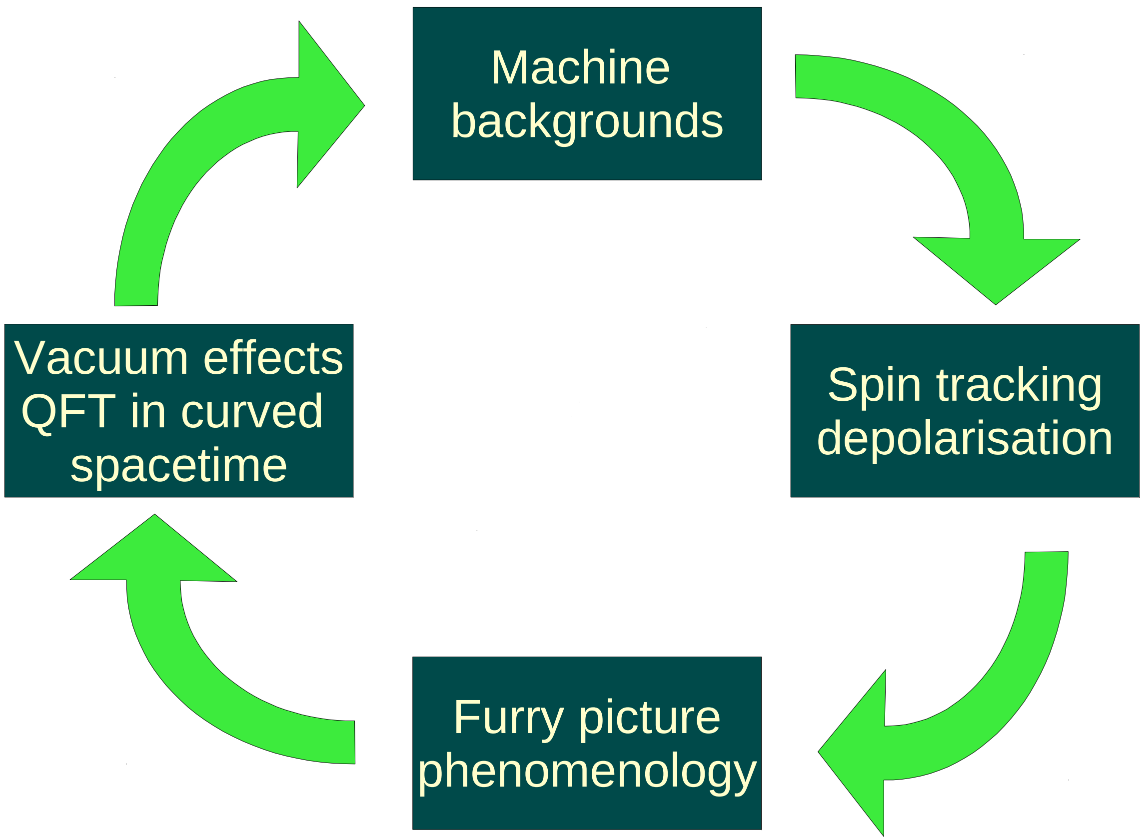
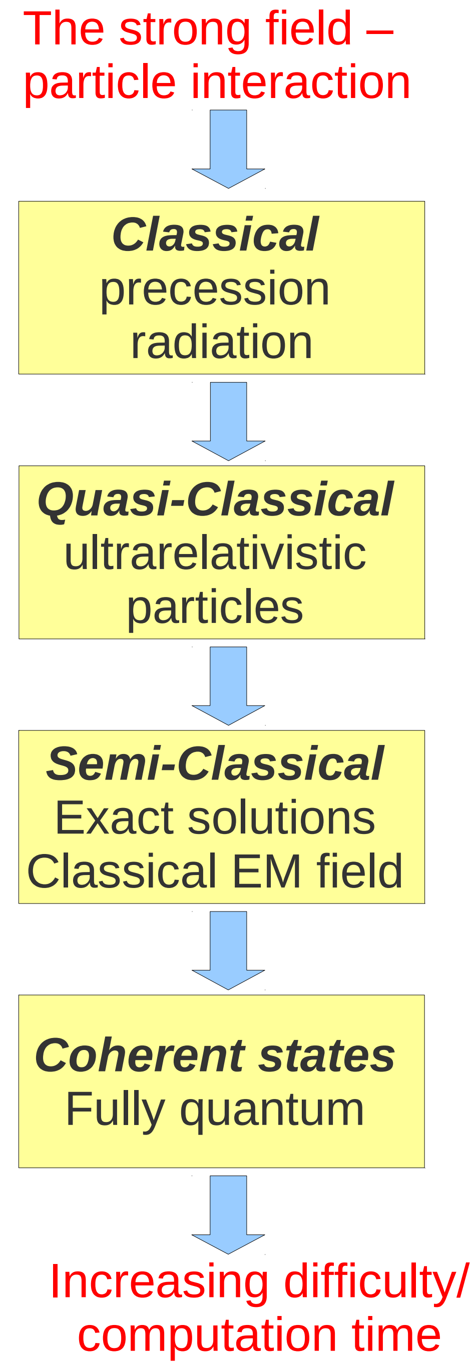
Synopsis

- Motivation/definitions
- IP waist scan, spin tracking
- Beamstrahlung radiation angle
- Strong field formalism
- Higher order effects
- New simulation tool

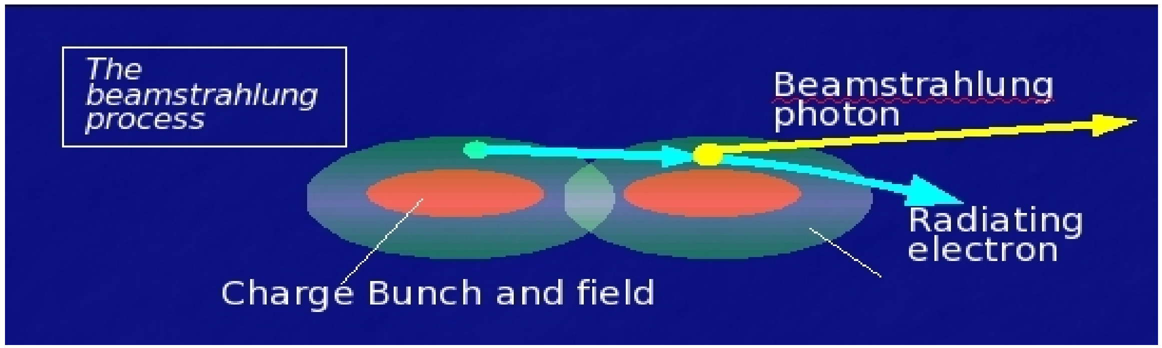
Motivation – precision





Beam-beam processes

(processes which take place in the electromagnetic fields of both charge bunches)



Strong field 'Upsilon' parameter

Vector potential of beam field

particle momentum

$$\Upsilon = \frac{e\hbar|\boldsymbol{a}|}{mc^2 E_{\mathrm{cr}}}(k \cdot p_i)$$

particle mass

Schwinger critical field (10^{18} V/m)

momentum of beam field

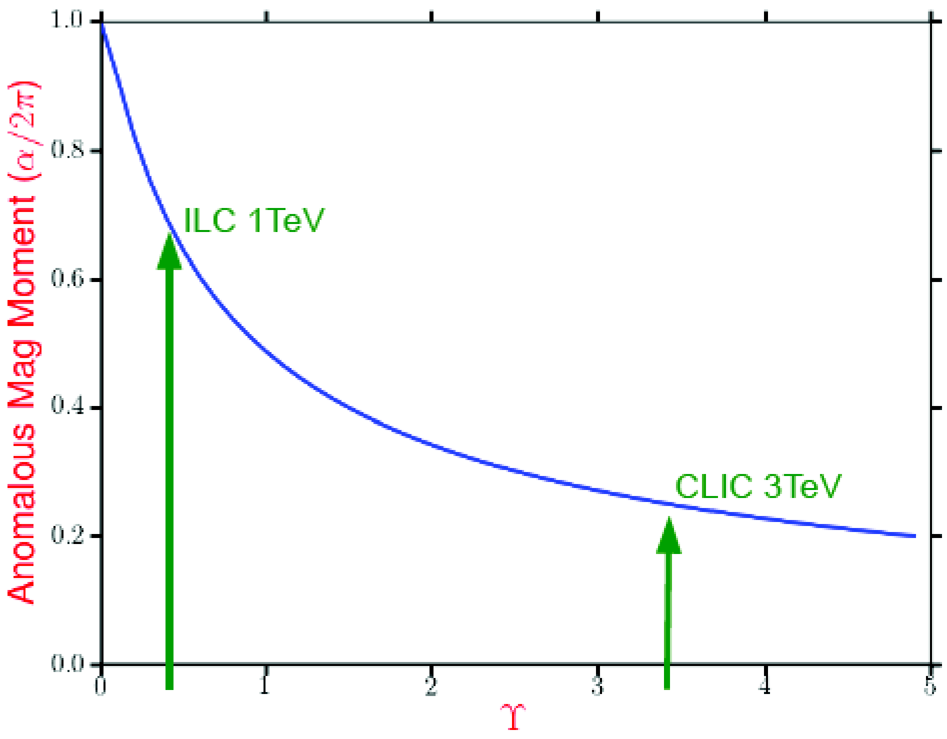
Simulated processes

- Incoherent: Equiv Photon Approx,
  - Breit Wheeler, Bethe Heitler, Landau Lifshitz, Bremstrahlung
- Coherent: Strong field (Furry picture)
  - Beamstrahlung, pair production, AMM, higher orders
- Simulation tools:
  - CAIN2.42 with full spin components, Guinea-Pig, Custom sims

Strong field vacuum polarisation

- anomalous magnetic moment (one-loop) in a const crossed field (V Baier, V.I. Ritus)

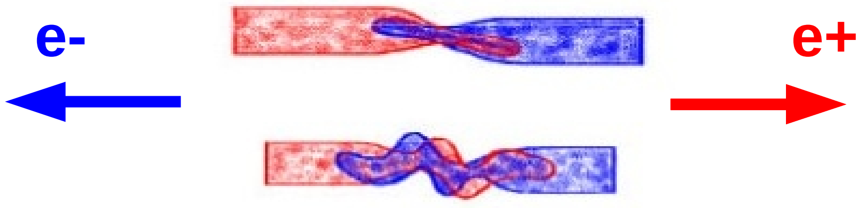
$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^\infty \frac{2\pi\, dx}{(1+x)^3}\left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$



Particular studies

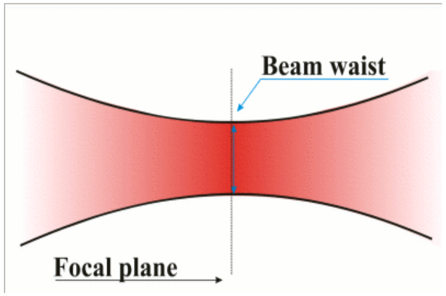
- waist scans
- spin tracking
- radiation angle

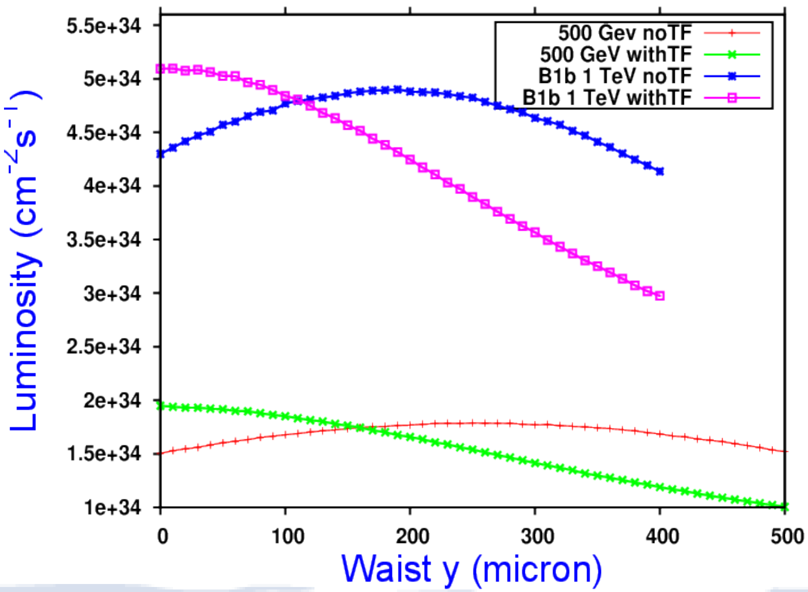
Beam waist luminosity gain

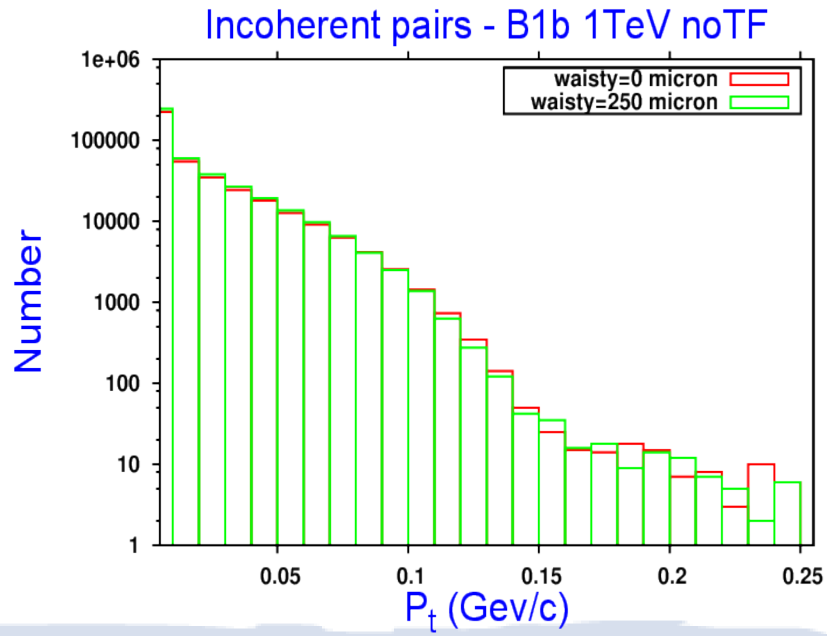


1. Travelling focus
- Focus moves through collision
- Theoretical, hard to implement

2. y_waist z shift
- Set to a fixed z value
- Easy to implement, Well tested







IP depolarisation

Spin precession

Beam field strengths

Spin vector

$$\frac{d\boldsymbol{s}}{dt} = -\left[(a+\gamma^{-1})(\boldsymbol{B}-\boldsymbol{v}\times\boldsymbol{E}) - \boldsymbol{v}\frac{a\gamma}{\gamma+1}\boldsymbol{v}\cdot\boldsymbol{B}\right]\times\boldsymbol{s}$$

Anomalous magnetic moment in external field

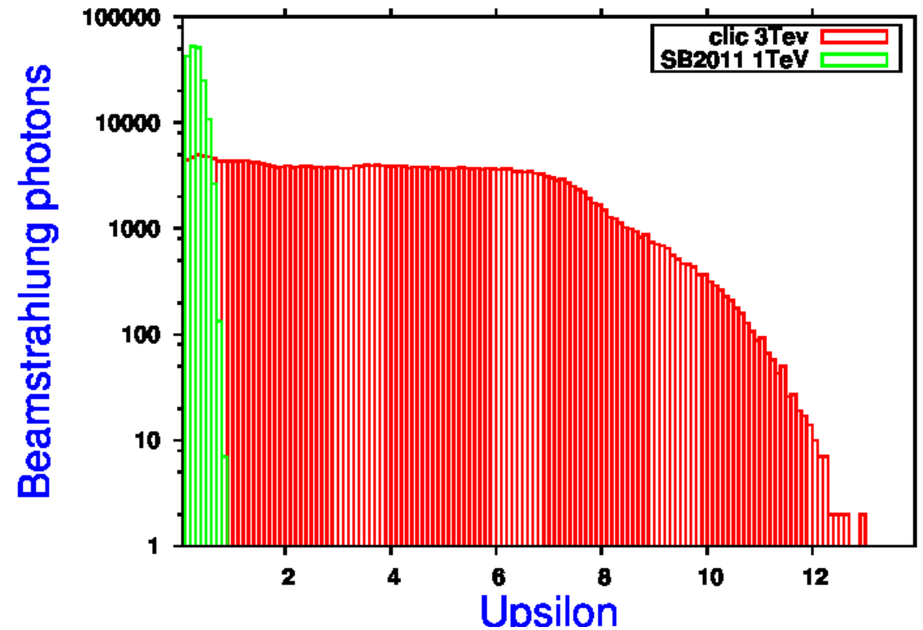
Spin-flip process

Constant crossed field – Airy functions

Spin vector

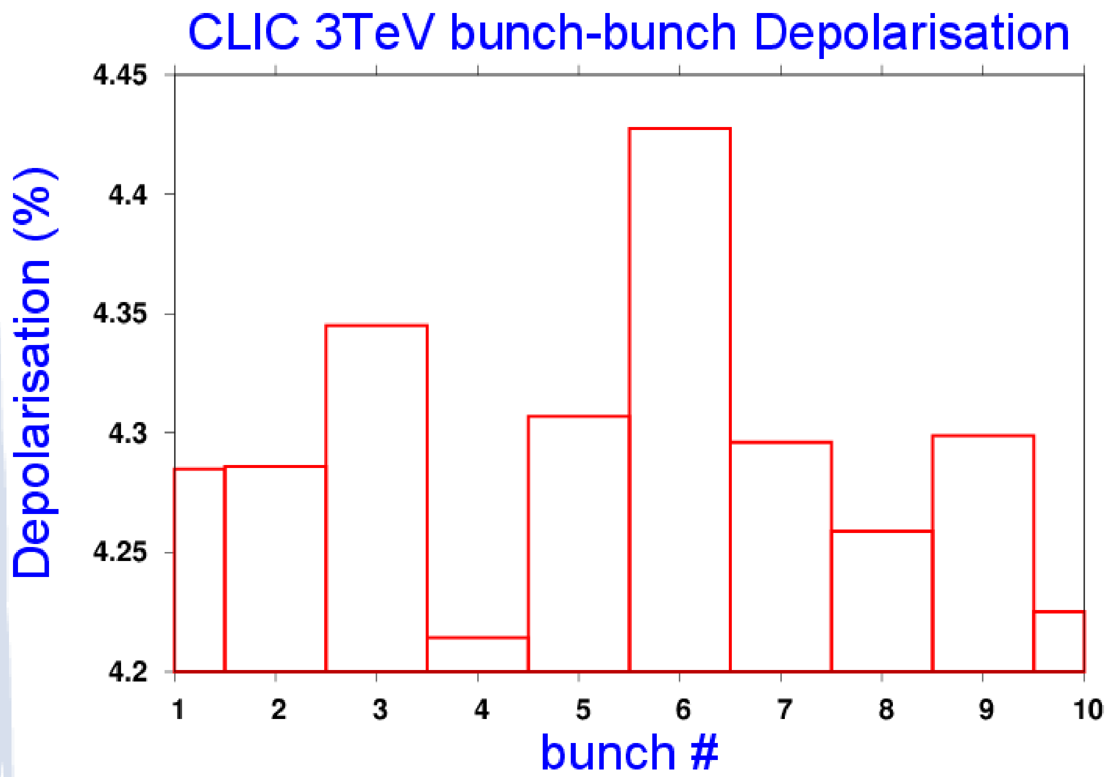
$$W(\Upsilon,\boldsymbol{\xi}) = \frac{\alpha m_e^2}{\pi\epsilon}\int_0^\infty \frac{du}{(1+u)^3}\left[\frac{e}{m^3}F^{*\mu\nu}p_\mu s_\nu \frac{z\mathrm{Ai}(z)}{1+u} - \mathrm{Ai}_1(z) - \frac{2+2u+u^2}{z\,(1+u)}\mathrm{Ai}'(z)\right]$$

beam field tensor



Parameter	ILC 1TeV	CLIC 3 TeV
$\mathcal{L}(\times 10^{34})$	4	3.6
N(incoh)	3.9e5	3.8e5
N(coh)	0	6.8e8
Υ (ave)	0.27	3.34
Υ (max)	0.94	10.9
δE_{bs}	10%	28%
$\langle \mathrm{depol}\rangle_{LW}$	0.62%	3.5%

IP bunch-bunch depolarisation



- Generate 10 CLIC 3 TeV e^+e^- bunches with design energy spread and initial 0.001% depolarisation
- Assume head-on collision
- Process the 10 bunch collisions in CAIN
- Uncertainty ~5% of ΔP

- CLIC 3TeV ΔP=4.3% and $\delta\Delta P$~0.2%
- ILC 1TeV ΔP=2.0% and $\delta\Delta P$~0.1%

Spin tracking eg: incoherent pairs

CAIN default spin tracking - circularly polarised photons only

so... modify CAIN to include all spin components

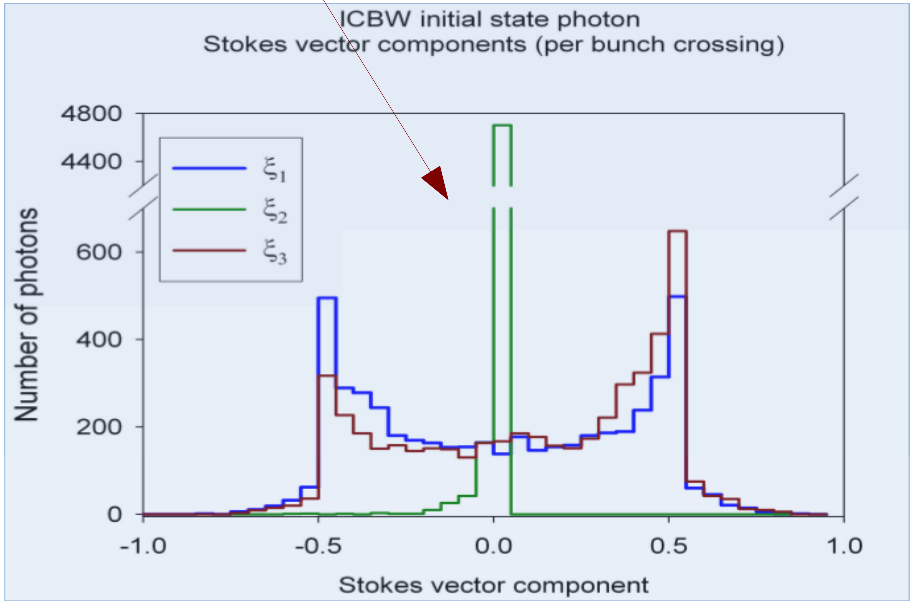
$$d\sigma(\text{Breit-Wheeler}) \propto d\Omega_f \sum_{iji'j'} F^{ii'}_{jj'} \, \xi_j \xi_{j'} \zeta_i \zeta_{i'}$$

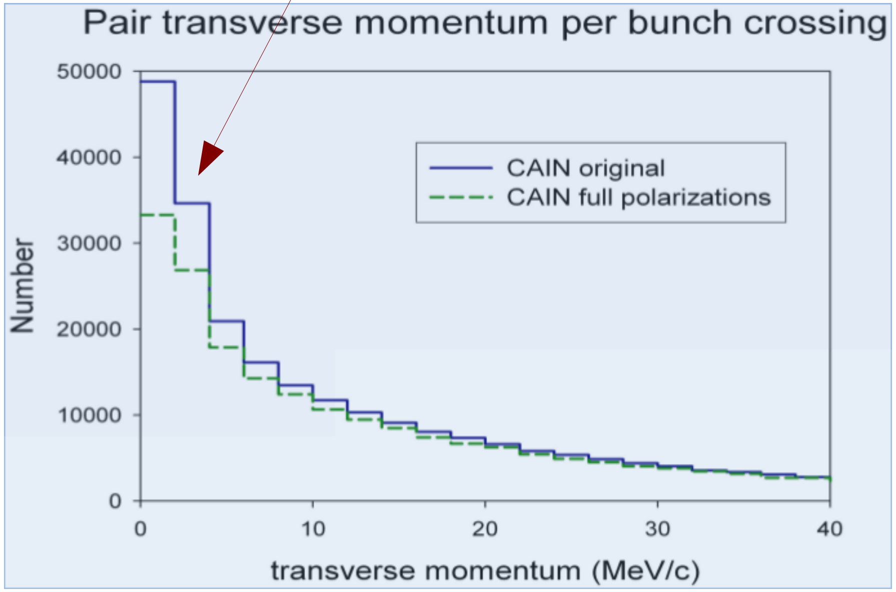
Final fermion spin components

Initial photon stokes components

Initial beamstrahlung photons are linearly polarised

Reduction in low P_t, low E incoherent pairs





Beamstrahlung radiation angle

The beamstrahlung equation is written in terms of Bessel functions

$$\frac{dW}{du} = \frac{\alpha m^2}{\pi\sqrt{3}\epsilon_i}\frac{1}{(1+u)^2}\left[\int_\chi^\infty K_{5/3}(y)dy - \frac{u^2}{1+u}K_{2/3}(\chi)\right]$$

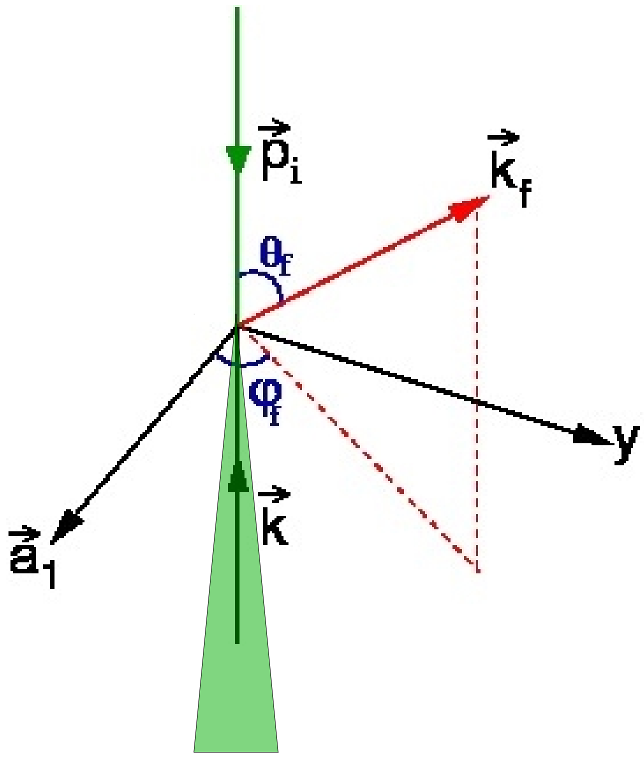
Quasi-classical: $\chi, u \propto \frac{\omega_f}{\epsilon_i - \omega_f}$

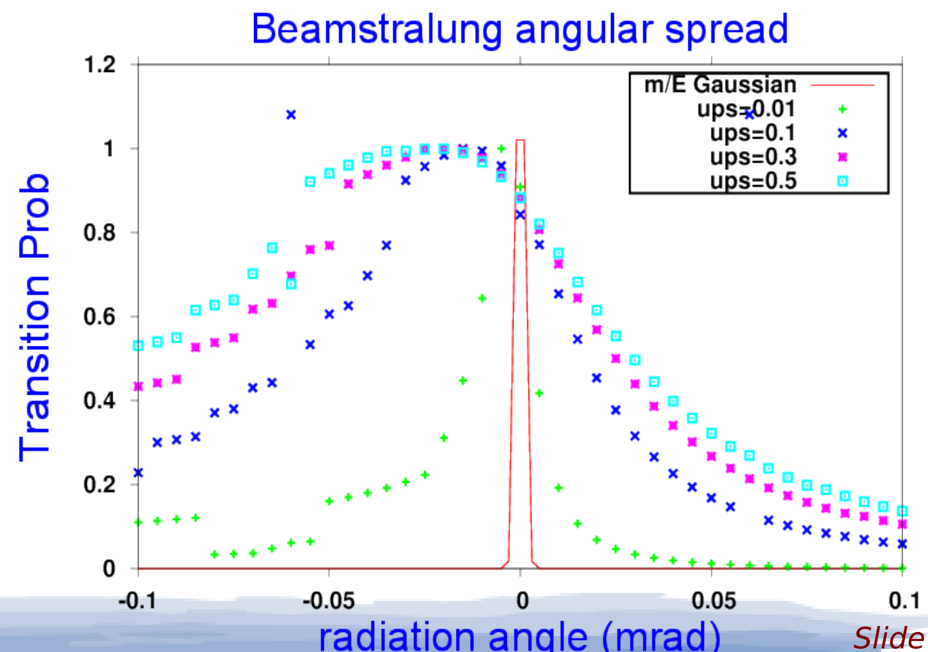
Semi-classical: $\chi, u \propto \frac{\omega_f(1-\cos\theta_f)}{2\epsilon_i - \omega_f(1-\cos\theta_f)}$

Analytically reformulate

Mathematica simulation – cutoff IR divergence

Strong field broadens radiation cone

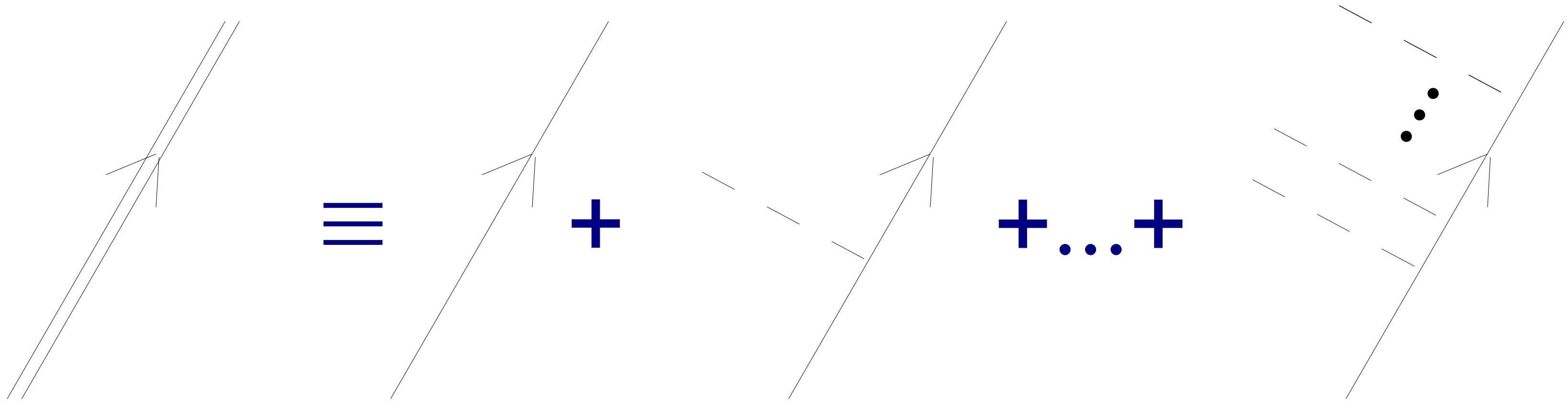




Developments

- Furry picture
- 2nd order processes
- exact solution in 2 fields

Exact solutions in strong fields



All Feynman diagrams with double fermion lines are potential coherent processes

Exact (Volkov) solution for Dirac equation in a plane wave em field-

$$\psi^V = \left[1 + \frac{e}{2(k\cdot p)} \not{k} \not{A}^e\right] \exp\left(-i\int_0^{kx} \left[\frac{e(A^e\cdot p)}{(k\cdot p)} - \frac{e^2 A^{e2}}{2(k\cdot p)}\right] d\phi\right) \exp(-ip\cdot x)\, u_s(p)$$

Υ,χ dependent part including extra dirac matrices

An additional phase factor (Υ,χ)

The usual free fermion part

Furry Interaction picture

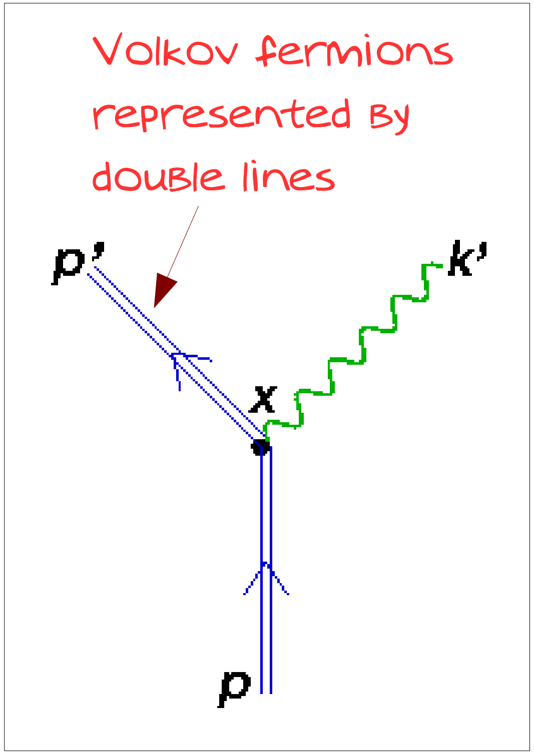
Interaction Lagrangian in Furry picture constitutes a modified vertex

$$\mathcal{L}_{\mathrm{I}} = \bar{\psi}^V \gamma^\mu \psi^V A_\mu \equiv \bar{\psi}\, \gamma^{e\,\mu}(p, p', k\cdot x)\, \psi\, A_\mu$$

$$\psi^V \equiv E_p\, u_{p,s}\, e^{-ip\cdot x}$$

external field 4-momentum

$$\gamma^e_\mu(p, p', k\cdot x) \equiv \bar{E}_p(k\cdot x)\gamma_\mu E_{p'}(k\cdot x)$$



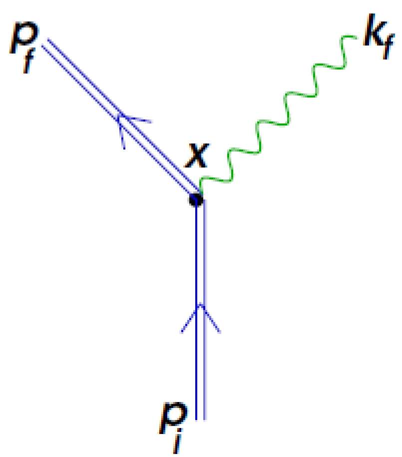
$$\delta(p + rk - p' - k')$$

Transform the modified vertex to momentum space to get a contribution from external field photons

$$\gamma^e_\mu(p, p', k\cdot x) = \int_{-\infty}^{\infty} dr\, d\phi\, e^{-ir(k\cdot x - \phi)} \gamma^e_\mu(p, p', \phi)$$

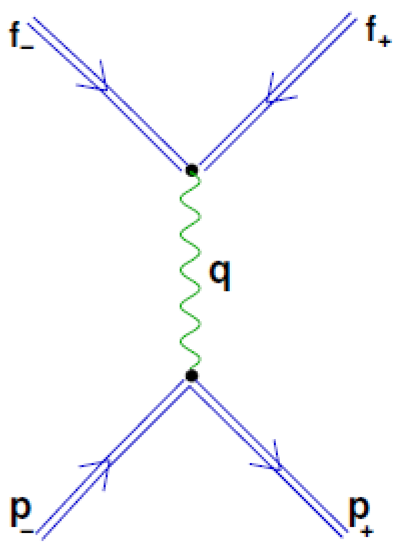
- **<u>New</u> processes possible**
- **<u>Existing</u> processes modified**

Strong field processes at IP



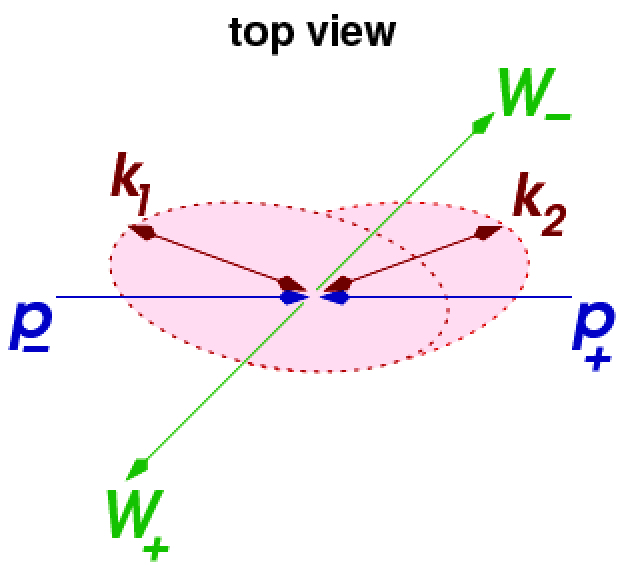
- **1st order:**
  - Beamstrahlung & coherent pair production
  - beam-beam simulations (CAIN, Guinea-PIG)
  - basis of ISR/FSR simulations
  - 1-vertex permitted $p_i + rk - p_f - k_f = 0$

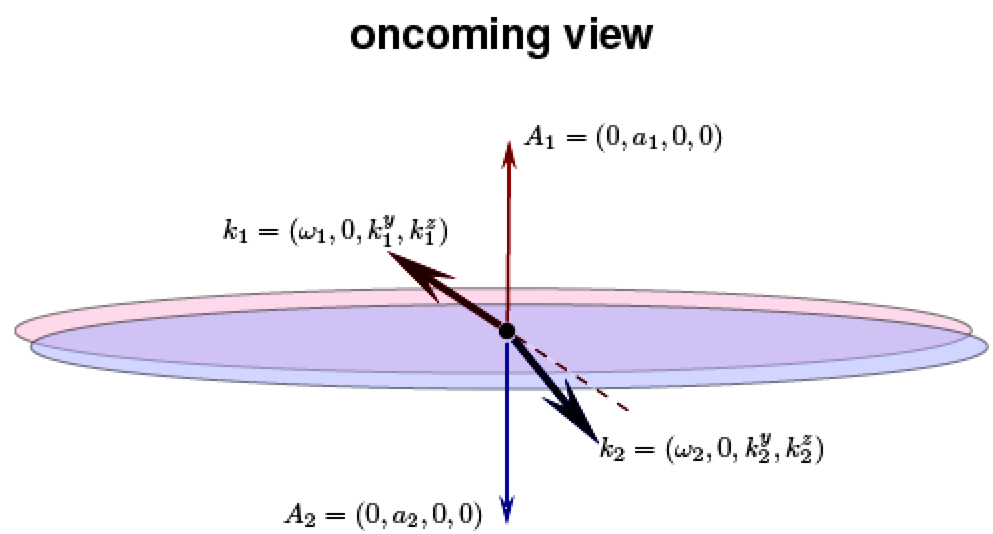
- **ALL** processes at the IP are "strong field" processes



- **2nd order:**
  - "normal processes" in limit $E \to 0$
  - Need Volkov solution in fields of both bunches
  - Need to obtain the cross-section for a generic 2nd order process

Exact solutions in both bunch fields





$(D^2 + m^2)\,\phi_e = 0,$ propose $\phi^e = e^{-ip\cdot x}F(\phi_1, \phi_2), \quad \phi_1 = k_1 \cdot x, \quad \phi_2 = k_2 \cdot x$

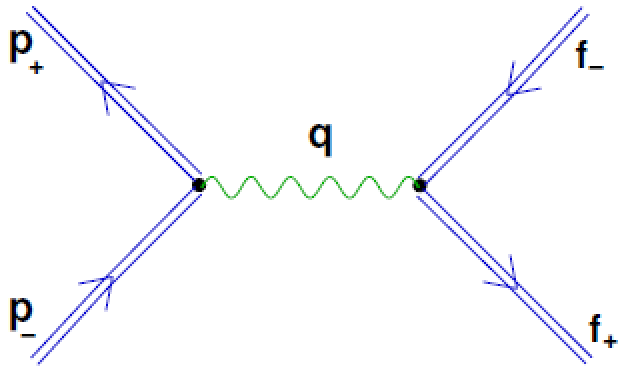
$$-2(k_1 \cdot k_2)\frac{\partial F^2}{\partial\phi_1\partial\phi_2} + 2i\left[k_1 \cdot (p - eA_2)\frac{\partial F}{\partial\phi_1} + k_2 \cdot (p - eA_1)\frac{\partial F}{\partial\phi_2}\right] - (eA_1 + eA_2)^2 F = 0$$

Equation	1 field Volkov	2 fields anti-collinear A	2 fields general case
Klein-Gordon	exponential	parabolic cyl	2 parabolic cyl
2nd order Dirac	✓	✓	
Dirac Equation	✓		
Proca equation	✓		

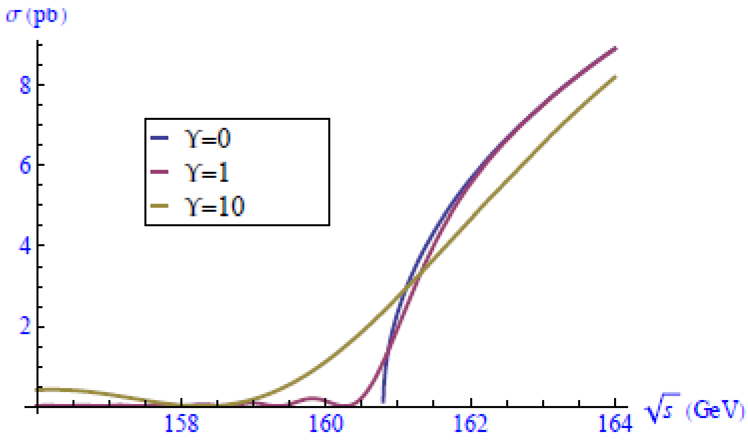
Two vertex Furry picture process

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 \; \bar{v}_{p_+} \gamma^{\text{FP}\mu} u_{p_-} \bar{\epsilon}_{f_+} \gamma^{\text{FP}}_{\mu} \epsilon_{f_-} \frac{\delta(F - I - (r_1 + r_2)(k_1 + k_2))}{(I + (r_1 + r_2)(k_1 + k_2))^2}$$

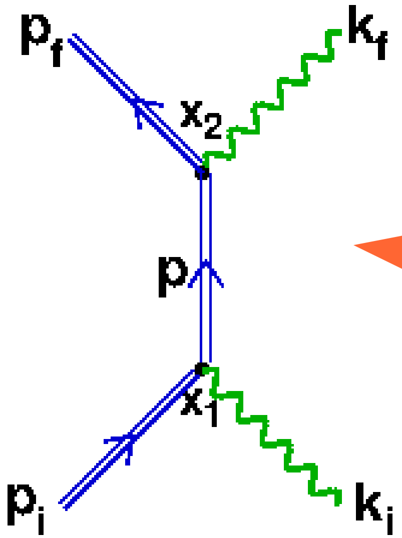
- final states momentum $F \equiv f_- + f_+$ initial state momentum $I \equiv p_- + p_+$
- usual coupling constants and spinors/polarisation
- two modified (Furry) vertices γ^{FP}
- r_1, r_2 momentum contribution from (two) external fields



- eg. W pair production threshold
  - strong field in initial states
  - leading trace term
  - S-channel only
  - 1 external field solution

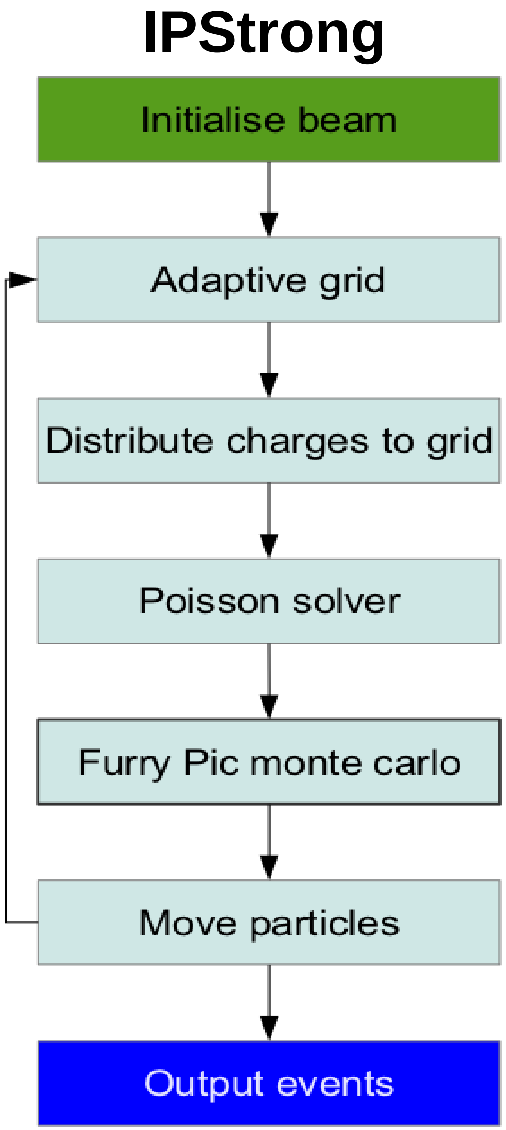


IPStrong – a new beam-beam evt gen



Requires:

Theoretical calculations in the Furry Picture, a new Beam-Beam simulator and strong field Physics Event generator



Summary

- Upsilon sets the scale of the strong field regime ILC 1TeV Y=0.27, CLIC 3TeV Y=3.34
- beam-beam processes can be studied at different theoretical levels – classical, quasi-classical, semi-classical or full quantum
- We want to understand LC beam-beam processes precisely
- Beam-beam Studies
  - y_waist scan -> 10-15% luminosity gain
  - Depolarisation -> ILC 1Tev ΔP=2.03±0.6% CLIC 3Tev ΔP=4.8±1.3%
  - Full spin components -> fewer low p_t low energy pairs
  - Beamstrahlung radiation angle -> **Y** dependent broadening
  - Generic second order processes -> **Y** dependent threshold smearing
- In progress:
  - Exact solutions in both bunch fields
  - *A new event generator* ***IPstrong***