Positive Ion Effects

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Main Results

by D.Arai (WP#145, Feb. 23, 2012)

- Primary Ions: small ~ 8.5um
- Ion back-drift (ion disk)
  ~ 60um w/o gate (Gate needed)
- Ions in Gate-MPGD gap: negligible

Remark:

Based on A.Vogel's simulations of beam induced BGs. Two-photon hadrons are not included.

Principle

Poisson's equation

The E field in a region (D) is the sum of the E field (E0) without space charge in the corresponding region defined by the field shaping strips and the two terminating plates and the field (Eion) calculated with space charge in the virtual grounded conducting boundary of D.

$$\triangle\phi_0(\boldsymbol{x}) = 0$$
$$\triangle\phi_{\mathrm{ion}}(\boldsymbol{x}) = -4\pi\,\rho_{\mathrm{ion}}(\boldsymbol{x})$$

in $\boldsymbol{x} \in D$

$$\phi(\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \phi_{\mathrm{ion}}(\boldsymbol{x})$$

$$\longrightarrow \quad \begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0 + \boldsymbol{E}_{\mathrm{ion}} \\ &= \boldsymbol{E}_0 - \nabla\phi_{\mathrm{ion}}(\boldsymbol{x}) \end{aligned}$$

Boundary Conditions

$$\boxed{\phi_0(\boldsymbol{x}) = V_i \quad \boldsymbol{x} \in C_i}$$

$$\boxed{\phi_{\mathrm{ion}}(\boldsymbol{x}) = 0 \quad \boldsymbol{x} \in \partial D}$$

All we need is Green's function for

$$\boxed{\triangle G(\boldsymbol{x}, \boldsymbol{x}') = -4\pi\delta(\boldsymbol{x} - \boldsymbol{x}')}$$

$$\boxed{G(\boldsymbol{x}, \boldsymbol{x}') = 0 \quad \boldsymbol{x} \in \partial D}$$

E-field distortion is then given by superposition:

$$\boxed{\phi_{\mathrm{ion}}(\boldsymbol{x}) = \int_D d^3\boldsymbol{x}\, G(\boldsymbol{x}, \boldsymbol{x}')\,\rho_{\mathrm{ion}}(\boldsymbol{x}')}$$

Superposition makes life easy!

Green's function

Since the boundaries are most naturally expressed in the cylindrical coordinates (rin=a, rout=b, z=0, Z=L), the corresponding Green function is most conveniently expanded in terms of modified Bessel function as follows:

$$G(r,\varphi,z;r',\varphi',z') = \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} g_{mn}(r,r')\frac{1}{2\pi}e^{im(\varphi-\varphi')}\frac{2}{L}\sin(\beta_n z)\sin(\beta_n z')$$

where

$$g_{mn}(r,r') = \frac{4\pi\left[K_m(\beta_n a)I_m(\beta r_<) - I_m(\beta_n a)K_m(\beta_n r_<)\right]\left[K_m(\beta_n b)I_m(\beta r_>) - I_m(\beta_n b)K_m(\beta_n r_>)\right]}{\beta_n r'\left[I_m(\beta_n a)K_m(\beta_n b) - I_m(\beta_n b)K_m(\beta_n a)\right]\left[K_m(\beta_n r')I'_m(\beta_n r') - K'_m(\beta_n r')I_m(\beta_n r')\right]}$$

$$\beta_n = n\pi/L$$

If the charge distribution is uniform in phi as under our assumption, the phi-integral is trivial and we get

$$\phi_{\rm ion}(r,z) = \sum_{n=1}^{\infty}\frac{8\pi}{\beta_n}\int_a^b dr'\,\frac{\left[K_0(\beta_n a)I_0(\beta r_<) - I_0(\beta_n a)K_0(\beta_n r_<)\right]\left[K_0(\beta_n b)I_0(\beta r_>) - I_0(\beta_n b)K_0(\beta_n r_>)\right]}{\left[I_0(\beta_n a)K_0(\beta_n b) - I_0(\beta_n b)K_0(\beta_n a)\right]\left[K_0(\beta_n r')I'_0(\beta_n r') - K'_0(\beta_n r')I_0(\beta_n r')\right]}$$
$$\sin(\beta_n z)\int_0^L \frac{dz'}{L}\,\sin(\beta_n z')\rho_{\rm ion}(r',z')$$

Derivatives of the modified Bessel functions can be rewritten in terms of those of different orders:

$$I'_0(x) = I_1(x) \quad \text{and} \quad K'_0(x) = -K_1(x)$$

If we can assume that the charge distribution can be factorized into the phi part and the z part as $\rho_{\rm ion}(r', z') = \bar{\rho}_r(r')\,\hat{\rho}_z(z')$ with $\int dz'\,\hat{\rho}_z(z') = L$

we can further simplify the calculation and get the following for Er:

$$\boxed{\begin{aligned} E_r(r,z) = -8\pi \sum_{n=1}^{\infty} & \frac{\sin(\beta_n z)}{I_0(\beta_n a)K_0(\beta_n b) - I_0(\beta_n b)K_0(\beta_n a)} \int_0^L \frac{dz'}{L}\,\sin(\beta_n z')\hat{\rho}_z(z') \\ & \Bigg[[K_0(\beta_n b)I_1(\beta r) + I_0(\beta_n b)K_1(\beta_n r)] \int_a^r dr'\,\frac{K_0(\beta_n a)I_0(\beta r') - I_0(\beta_n a)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')}\,\bar{\rho}_r(r') \\ & + [K_0(\beta_n a)I_1(\beta r) + I_0(\beta_n a)K_1(\beta_n r)] \int_r^b dr'\,\frac{K_0(\beta_n b)I_0(\beta r') - I_0(\beta_n b)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')}\,\bar{\rho}_r(r') \Bigg] \end{aligned}}$$

where

$$\beta_n = n\pi/L$$

In the practical calculations, we have to sum up the series up to high enough “n”, which is determined by the ratio of the shortest and the largest scales that specify the charge distribution and the geometry of the boundary of the region in question.

For a thin disk or in the MPGD-gate gap, summation up to 500 or more is necessary, which in turn requires quadruple precision calculations for the modified Bessel functions.

Principle (continued)

Assuming that E0 is parallel with the B field, it will not contribute to the ExB effect.
(c.f.) the Langevin Equation

$$\boldsymbol{\omega} := \frac{(-e)\boldsymbol{B}}{mc}$$

$$\omega\tau \simeq 10 \text{ for T2K gas at B=3.5T}$$

$$\langle \boldsymbol{v} \rangle = \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[1 + (\omega\tau)\hat{\boldsymbol{B}} \times + (\omega\tau)^2 \hat{\boldsymbol{B}}\,\hat{\boldsymbol{B}} \cdot \right] \frac{e}{m}\boldsymbol{E}$$

If we write down the distortion of the velocity due to the distortion of the E-field in the longitudinal and transverse directions, we get

$$\Delta \langle \boldsymbol{v} \rangle = \frac{e}{m} \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[(1+(\omega\tau)^2)\Delta\boldsymbol{E}_{\parallel} + \boldsymbol{E}_{\perp} - (\omega\tau)\boldsymbol{E}_{\perp} \times \hat{\boldsymbol{B}} \right]$$

Numerically integrating this over the drift time by noting $\delta l_i = \langle v_{\parallel} \rangle \delta t_i$, we get the following formula for the distortion:

$$\boxed{\begin{aligned} \langle \Delta \boldsymbol{x} \rangle &= \sum_{i=1}^{n} \frac{\Delta \langle \boldsymbol{v} \rangle_i}{\langle v_{\parallel} \rangle_i} \delta l_i \\ &\simeq \sum_{i=1}^{n} \delta l_i \left[-\frac{\Delta \boldsymbol{E}_{\parallel i}}{E_{\text{nom}}} - \left(\frac{1}{1+(\omega\tau)^2} \right) \frac{\boldsymbol{E}_{\perp i}}{E_{\text{nom}}} + \left(\frac{\omega\tau}{1+(\omega\tau)^2} \right) \frac{\boldsymbol{E}_{\perp i} \times \hat{\boldsymbol{B}}}{E_{\text{nom}}} \right] \end{aligned}}$$

Key point: distortion is linear w.r.t. E-field distortion, and hence also w.r.t. space charge for a drift from the same z to the anode: Superposition makes life easy!

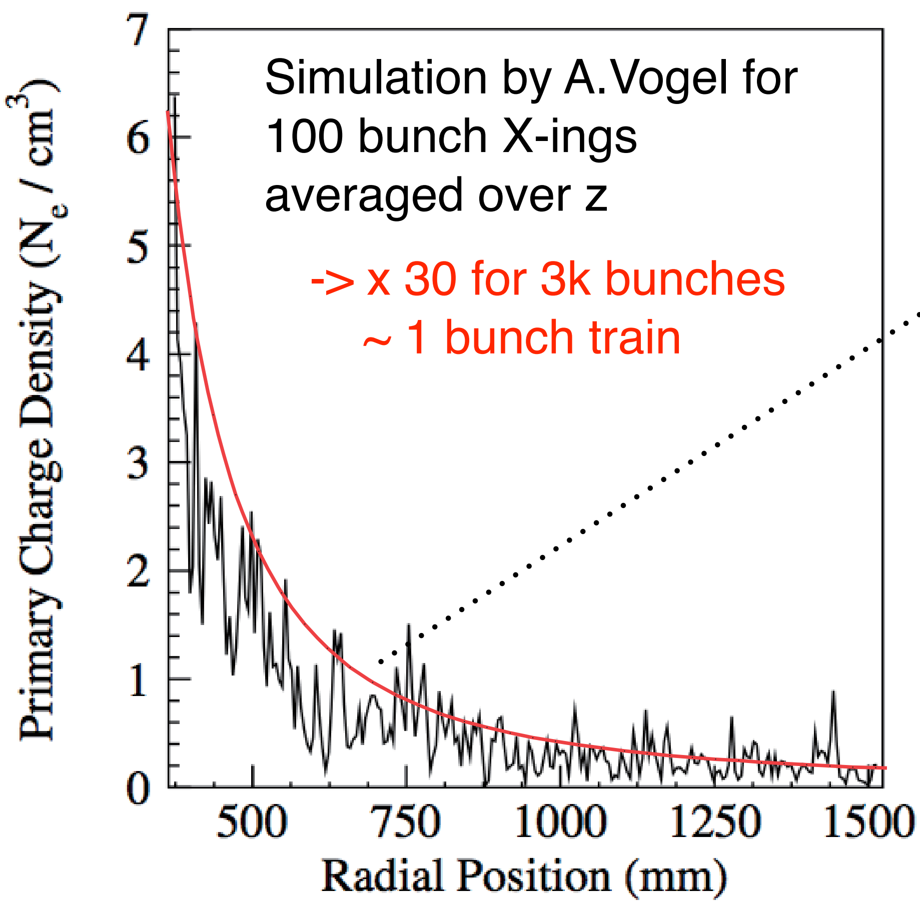
Effect of Primary Ions in the Drift Region

Input from BG simulation

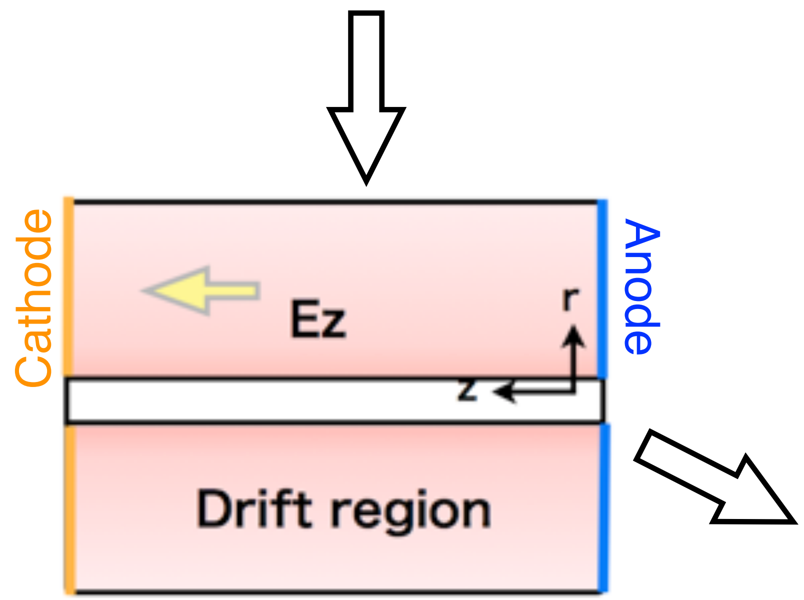
Charge density in the drift volume

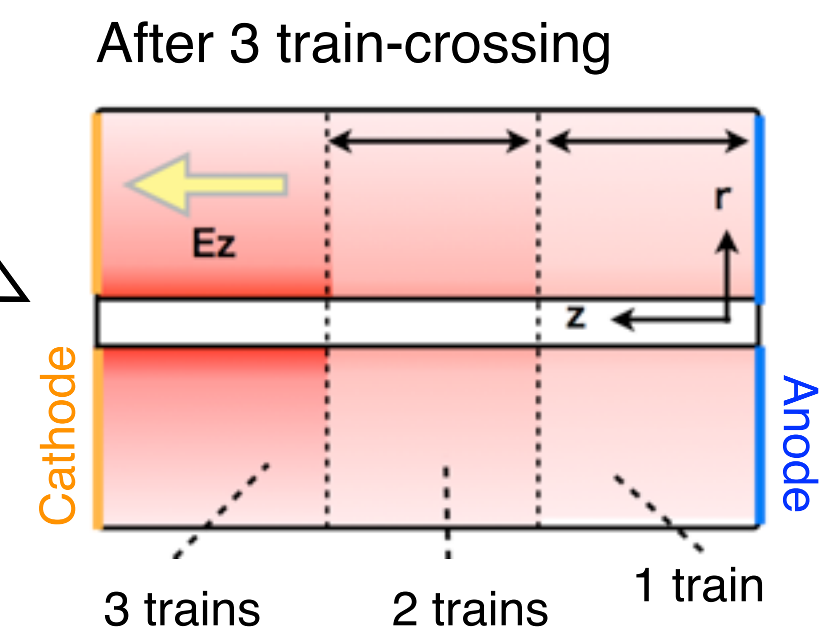
Fit by D.Arai

$$\bar{\rho}_r(r) = 1.6 \times \frac{1}{4(r-0.2)^2} \boxed{\times 30} \times 10^{-13} \quad [\mathrm{C/m^3}]$$



In what follows we assume that the charge distribution is uniform in both phi and z directions for a single train.





No two-photon hadron BG included.
-> underestimate

rin=0.4m
rout=1.8m
L=2.2m

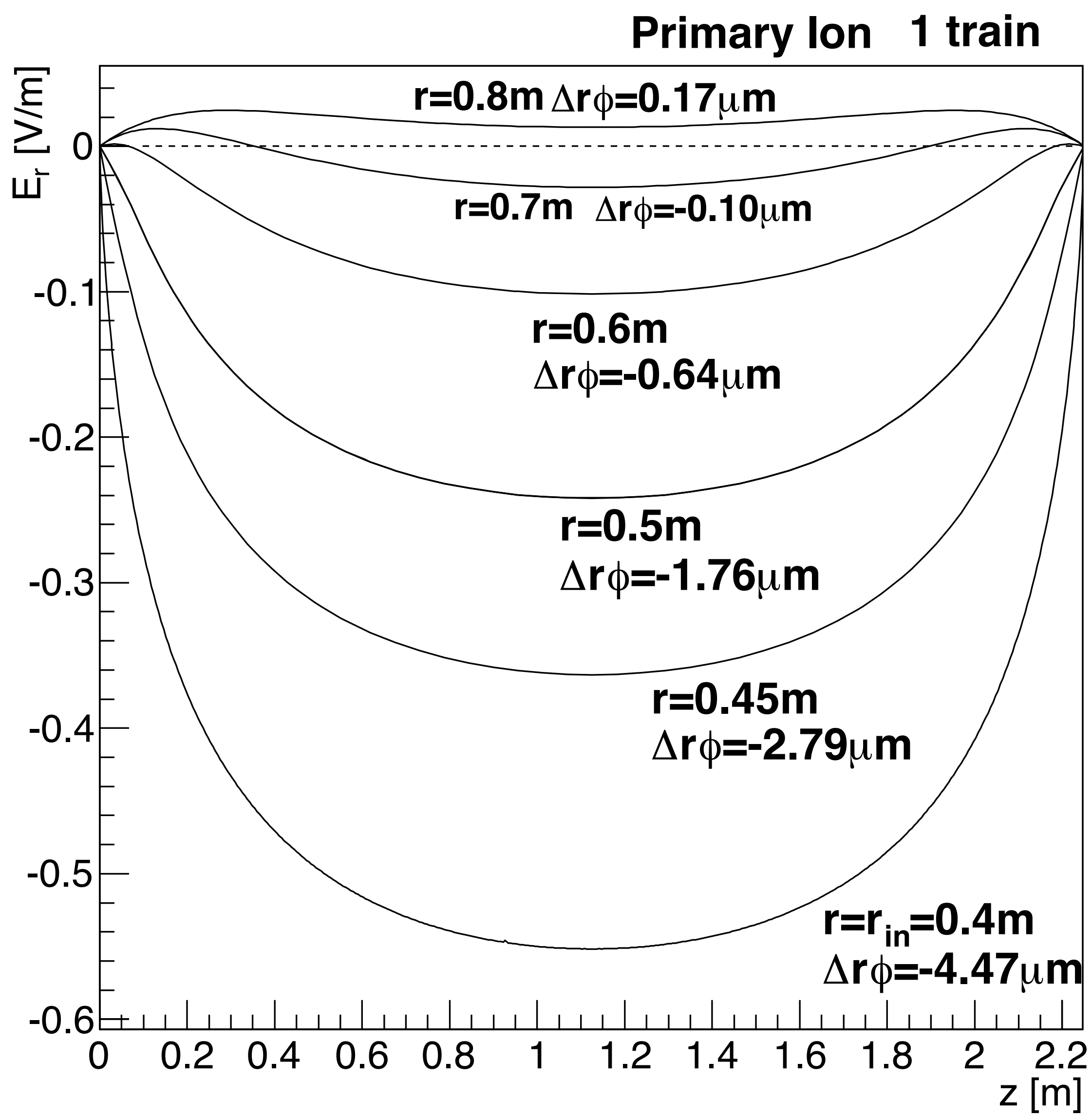
Spikes are from Jacobean peaks due to loopers.

Singularities due to micro-curlers?

Since we readout all the hits in the TPC we can calculate the positive ion distribution in principle.

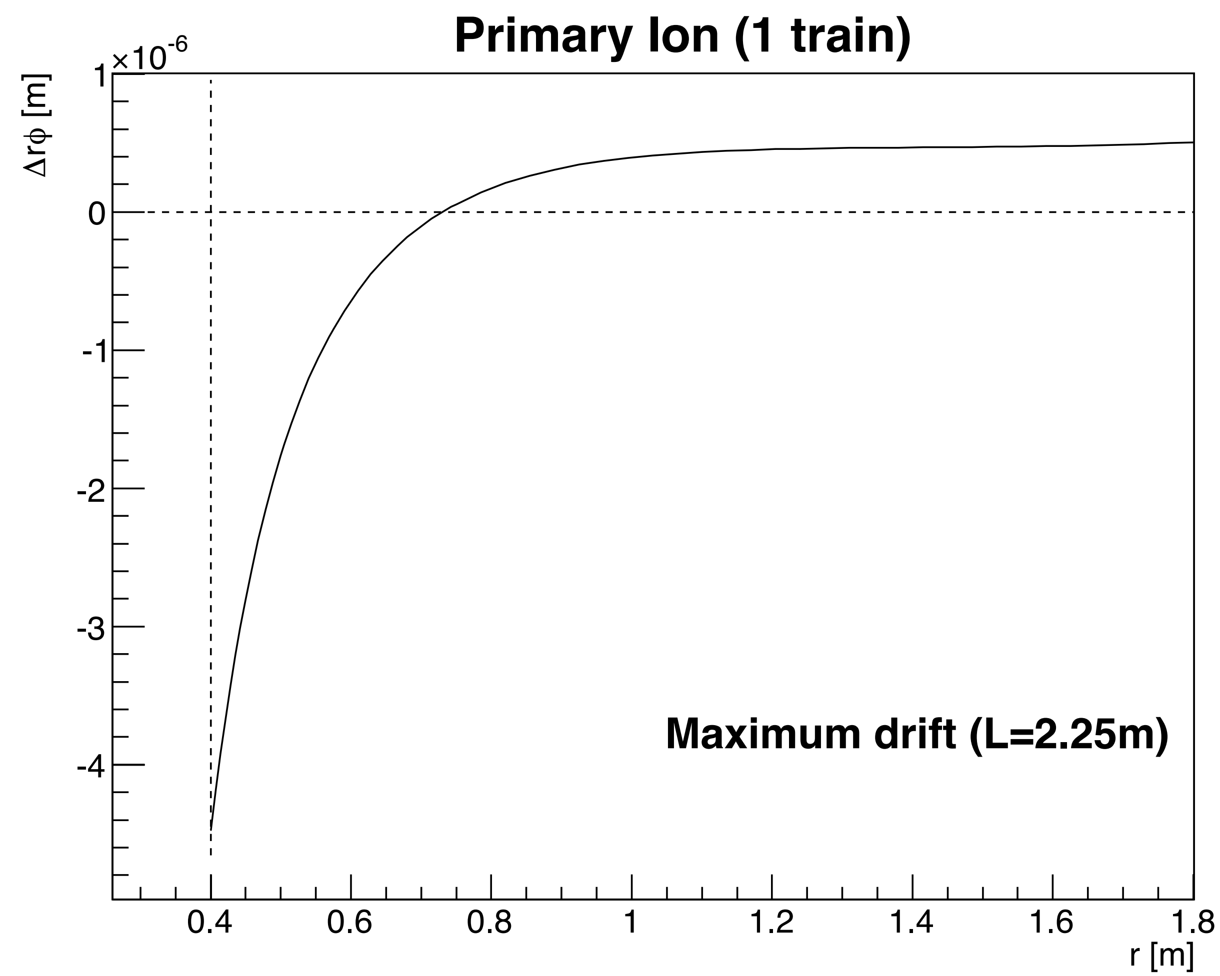
Distortion due to a single train

E-field distortion (r-component)



E-field distortion (emergence of radial component) is a moderate function of "z" for the primary ions for a single train, since we assume charge homogeneity in "z". Notice that Er=0 at the anode and at the cathode.

The E-field distortion and hence the space point distortion (ExB effect) is the largest near the inner cylinder because of the -ve charge induced on the inner cylinder by the +ve ions.

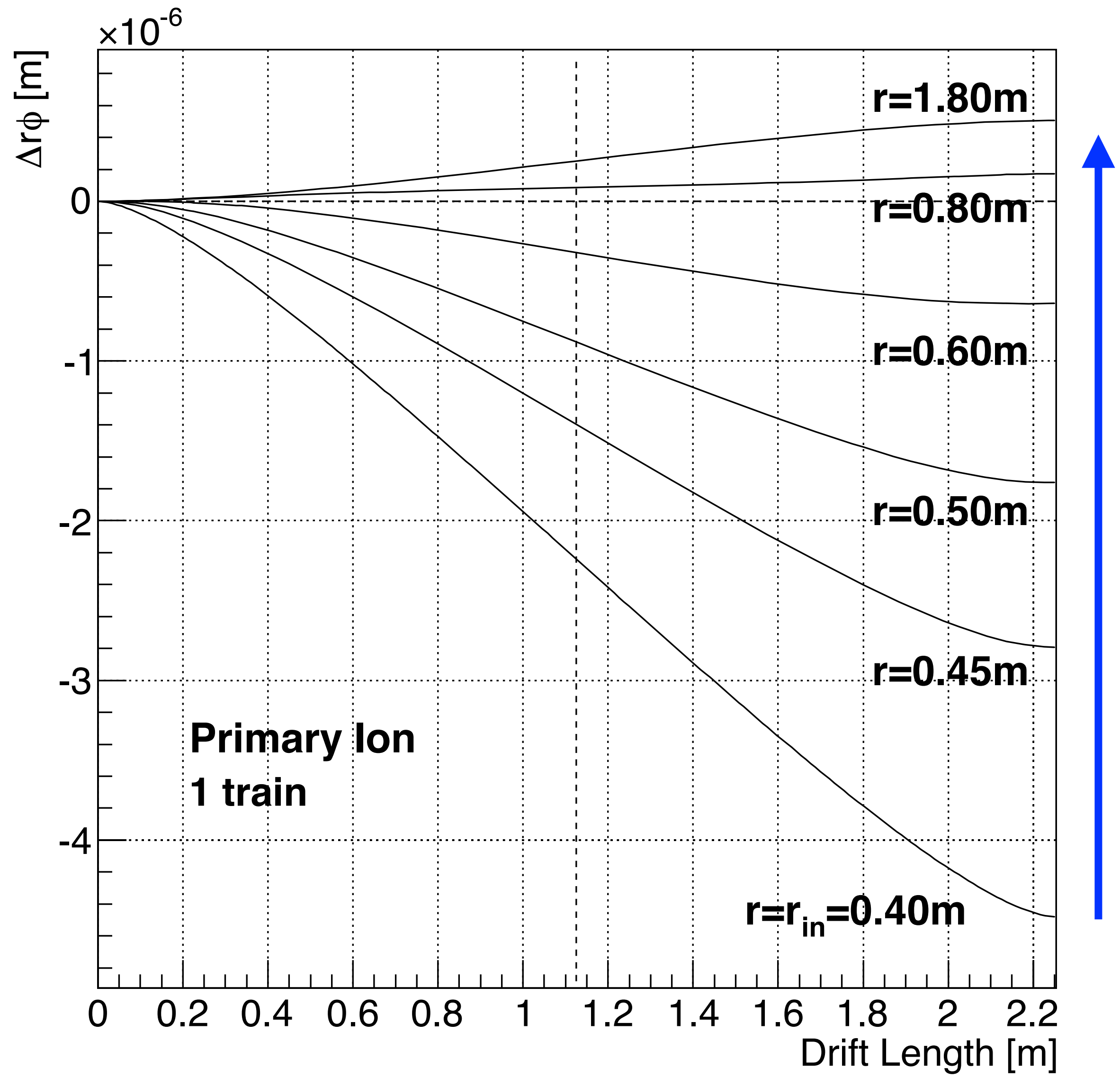


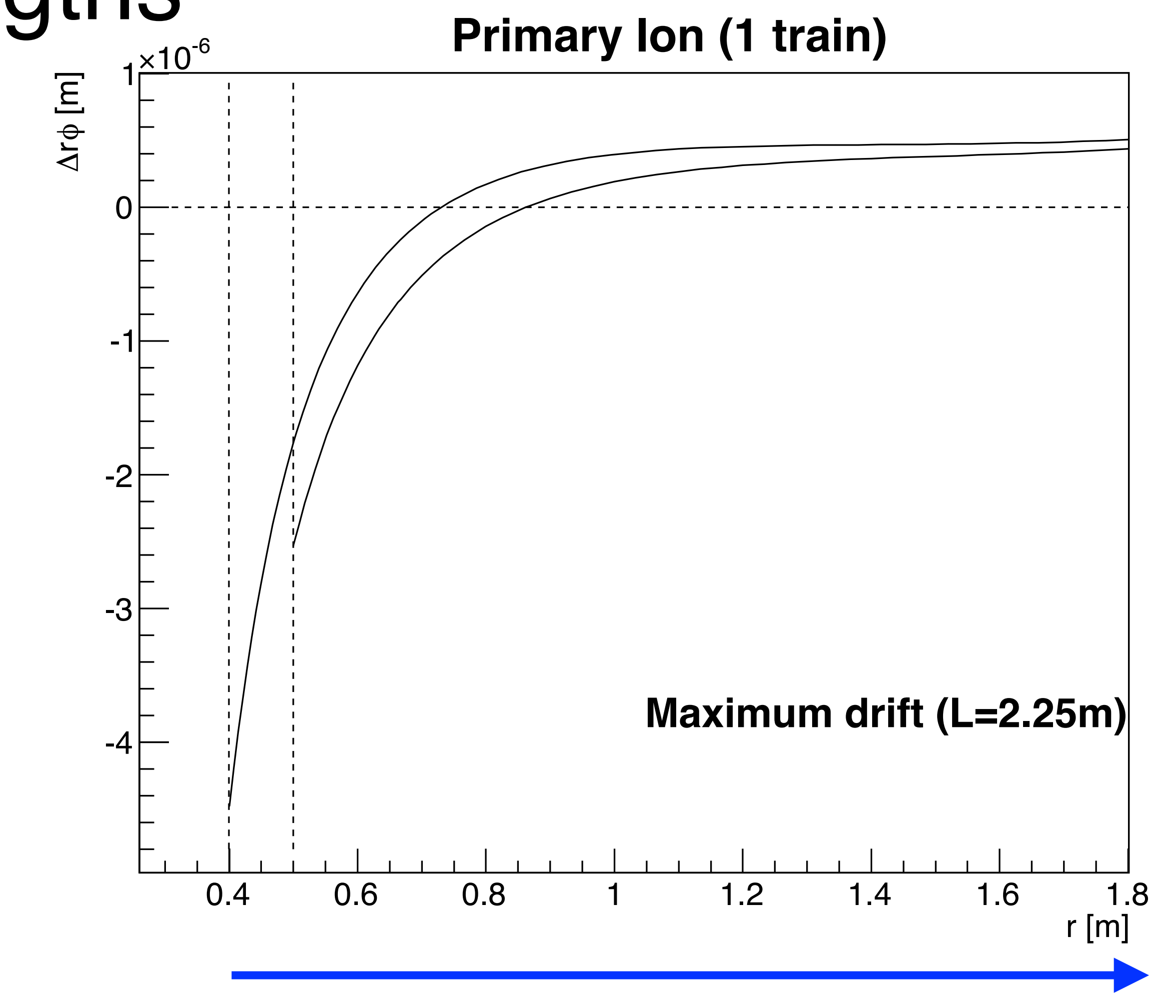
The maximum distortion takes place near the inner cylinder and is ~4.5um for a full drift, for +ions from a single bunch train.

Distortion due to a single train

Drift distance dependence of the ExB effect

Distortion for different drift lengths



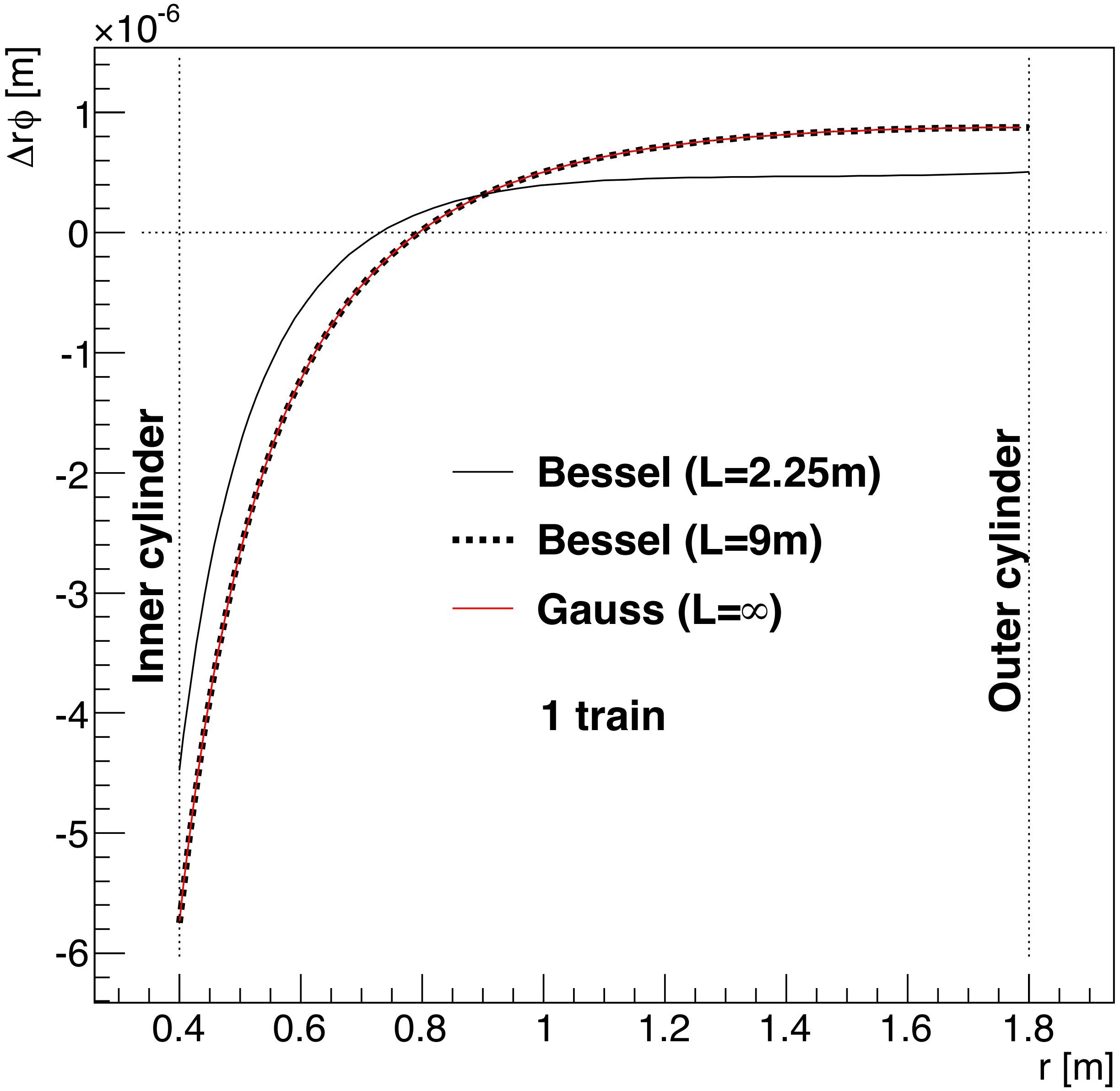


Since the E-field distortion is a moderate function of “z” for the primary ions for a single train, the drift-length dependence of the space point distortion is almost linear w.r.t “z”.

Notice that the distortion flips its sign at around r=0.75m. This is because the -ve charge induced on the inner cylinder by the +ve ions are electro-statically shielded by the +ve ions at around there and beyond there the +ve ions dominate.
The worst distortion at r=rin is caused by the induced charge on the inner cylinder, so increasing rin pushes the problem to the larger r.

Comparison with Gaussian Result

Cross check with the result from a simple Gaussian approximation



Approximating the TPC drift volume as a infinitely long cylinders (rin=a, rout=b) filled with a +ve ions distributed uniformly in "z" and applying the Gauss law, we can analytically estimate the E-field distortion (=Er) and hence the space point distortion (=drphi):

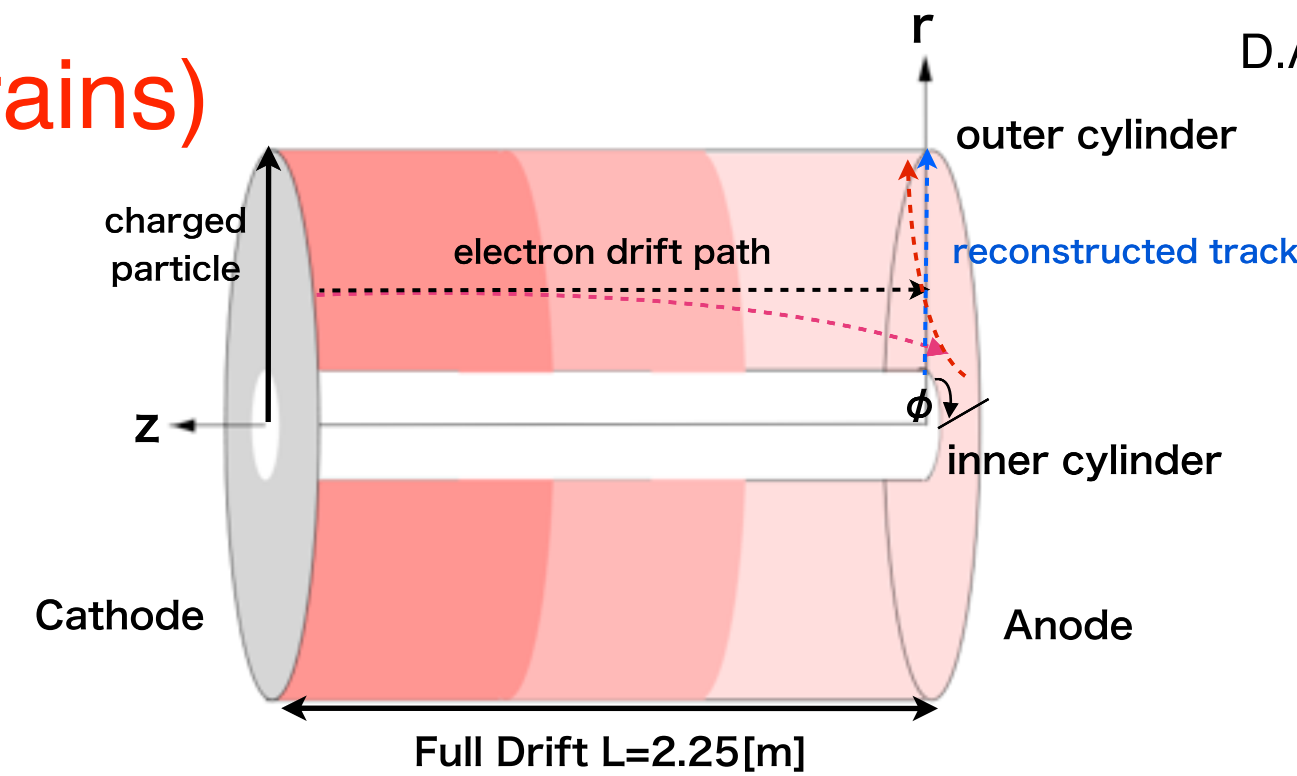
$$E_r = \frac{4\pi}{r}\int_a^r r' dr' \, \bar{\rho}_r(r') - \frac{4\pi}{\ln(b/a)\, r}\int_a^b \frac{dr'}{r'}\int_a^{r'} r'' dr'' \, \bar{\rho}_r(r'')$$

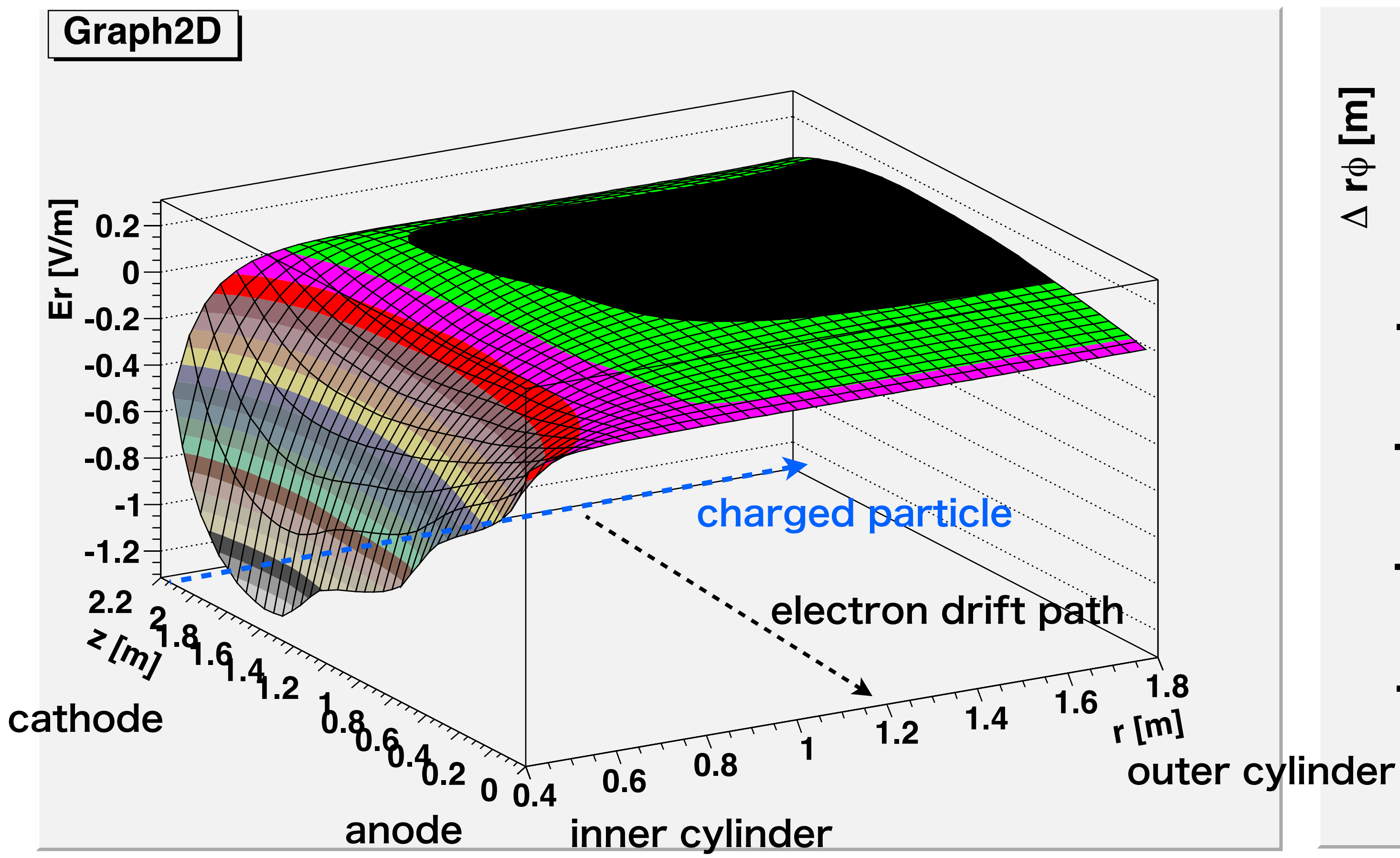
The result from this Gaussian approximation shows perfect matching with the result using Er obtained with the modified Bessel function assuming (L=9m), validating the formulation based on the latter.

Result (3 trains)

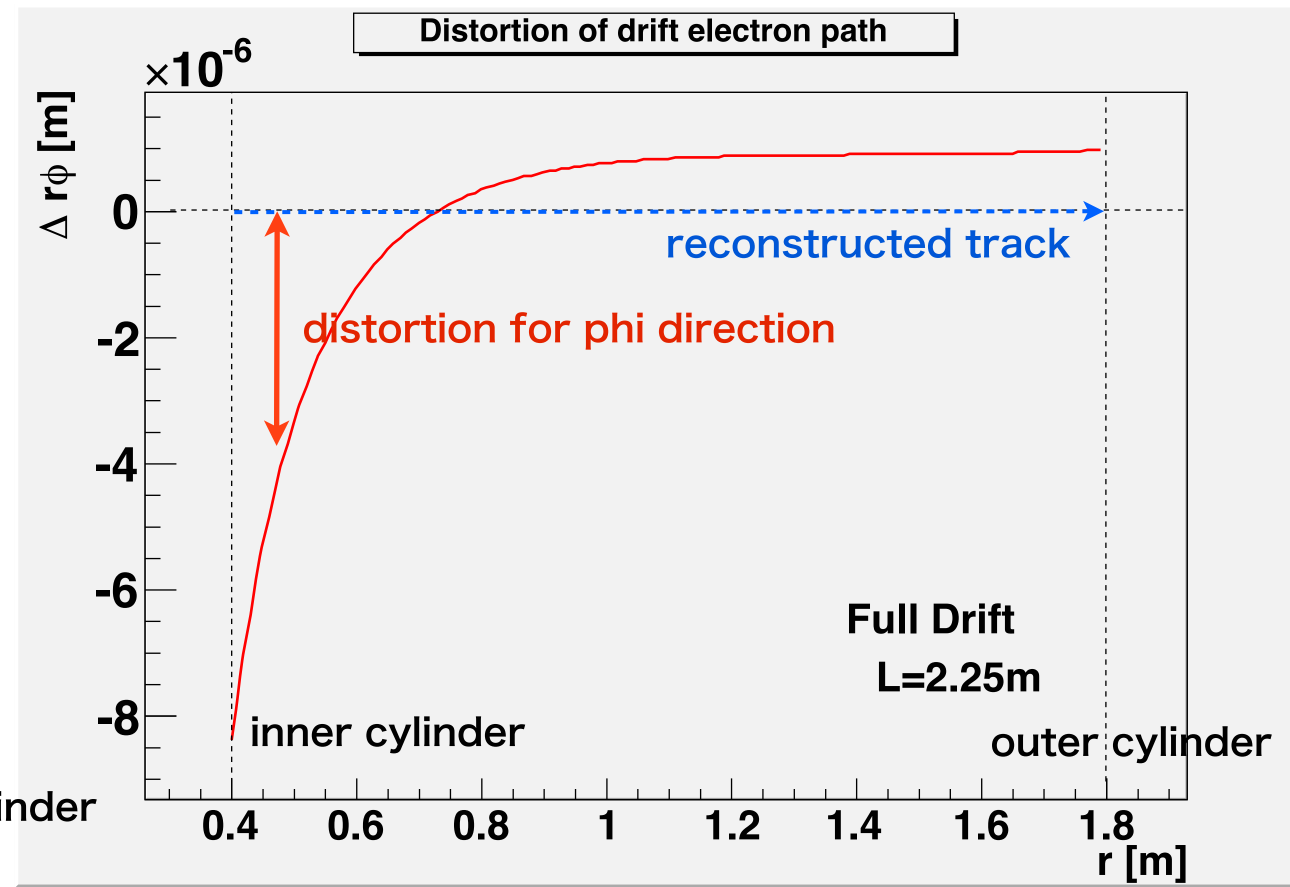
D.Arai Feb. 23, 2012 (wp#145)

Primary Ion





Efield map



Distortion of drift electron path (Full drift)

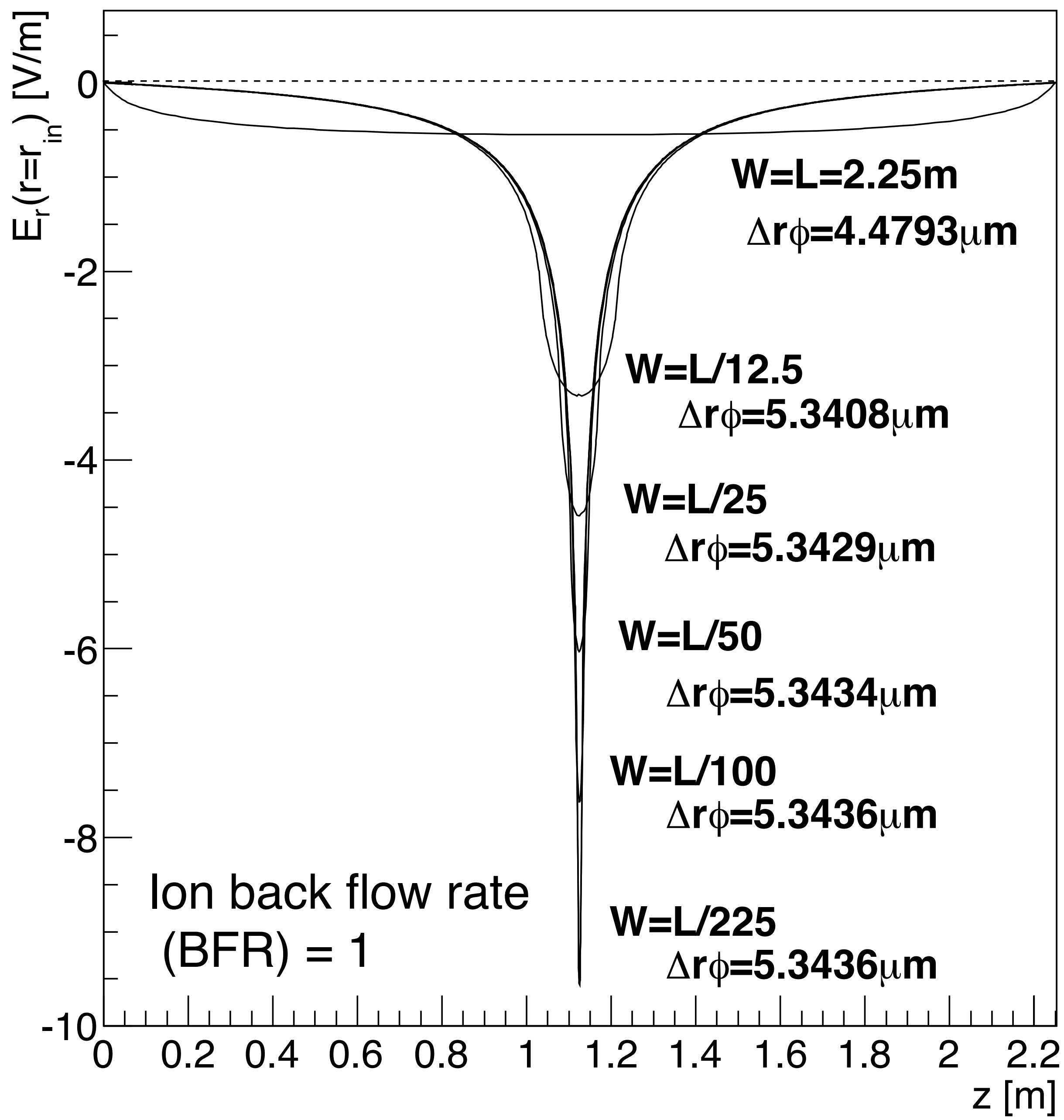
point resolution 100 μm > Maximum distortion is 8.5 μm. Small !!

Effect of Ion Disks in the Drift Region

Ion Disk

E-field distortion due to an ion-disk at the middle of the drift region

E-field distortion (r-component)



If there is no gating device to shut out the back flow of the 2ndary +ve ions created in the amplification region to the drift volume, thin (~1cm) disks of +ve ions will migrate towards the cathode and distort the drift path of track electrons.

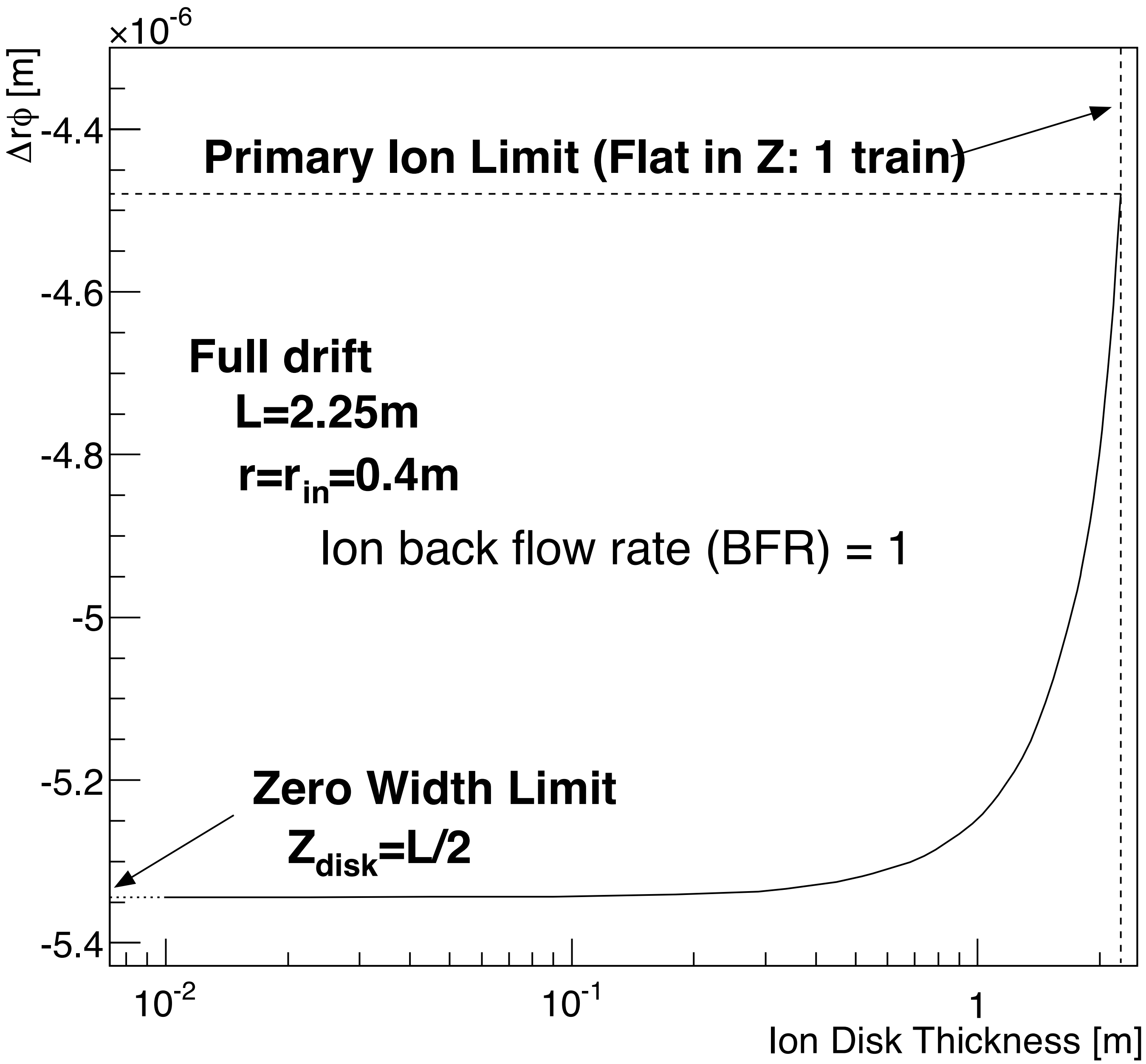
Since the charge density in the disk is inversely proportional to the width (w) of the disk, the narrower the width, the more the E-field distortion (Er) peaks at the "z" location of the disk.

The ExB effect is, however, proportional to the integration of Er over the drift path and hence the net effect becomes independent of the disk width in the zero width limit (see next slide).

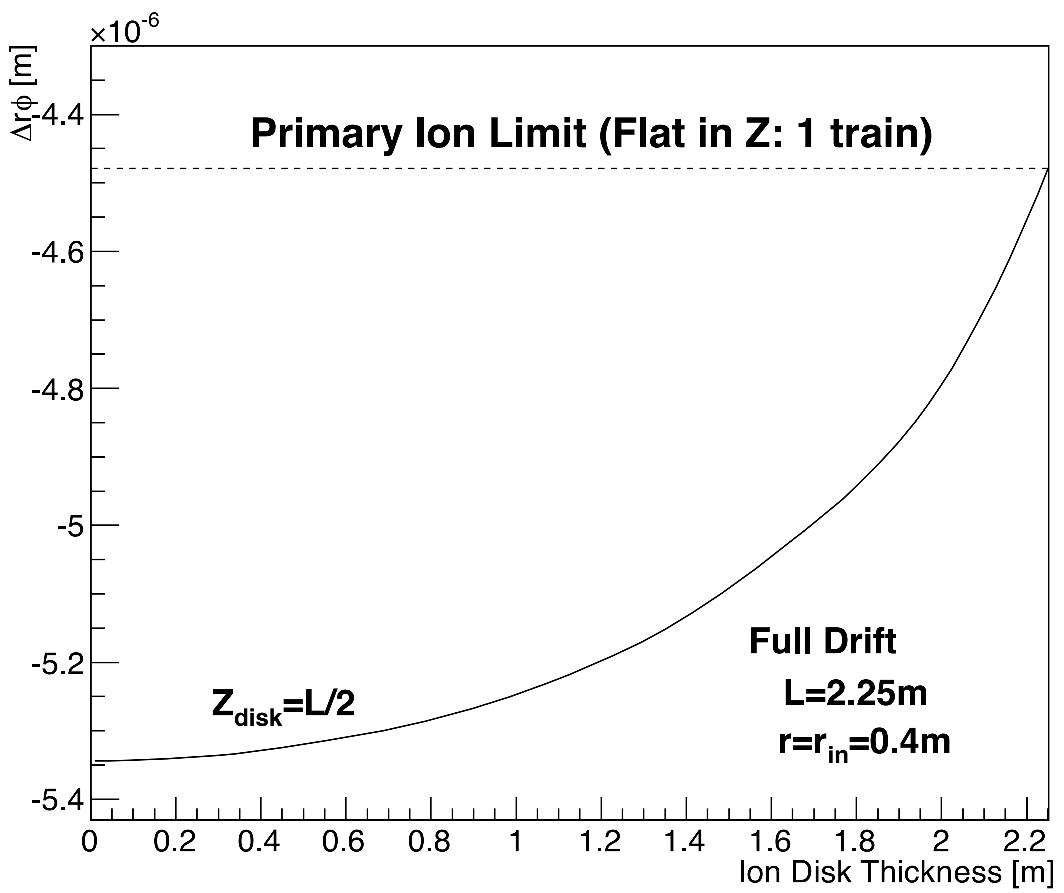
For a single train (FBR=1), the difference between the primary ion case is only 20% level!

Ion Disk

Disk thickness dependence of the ExB effect



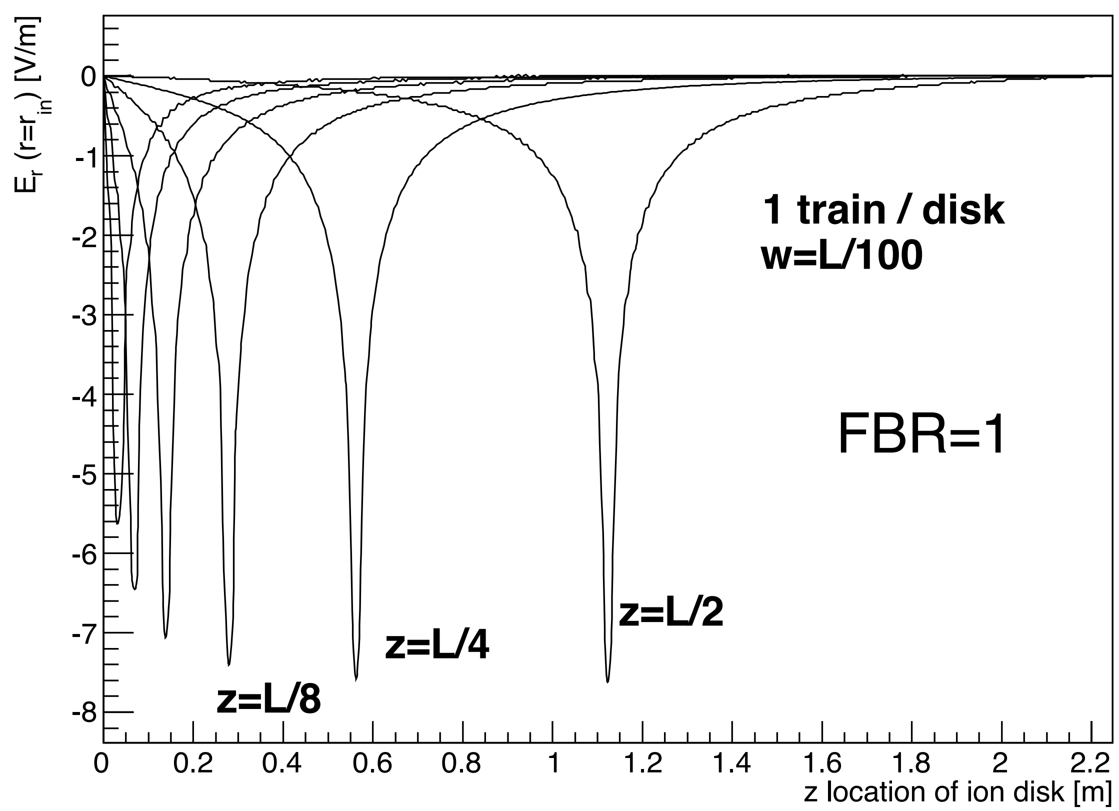
Same plot with linear scale for the horizontal axis



Ion Disk

Dependence on the “z” location of the ion disk

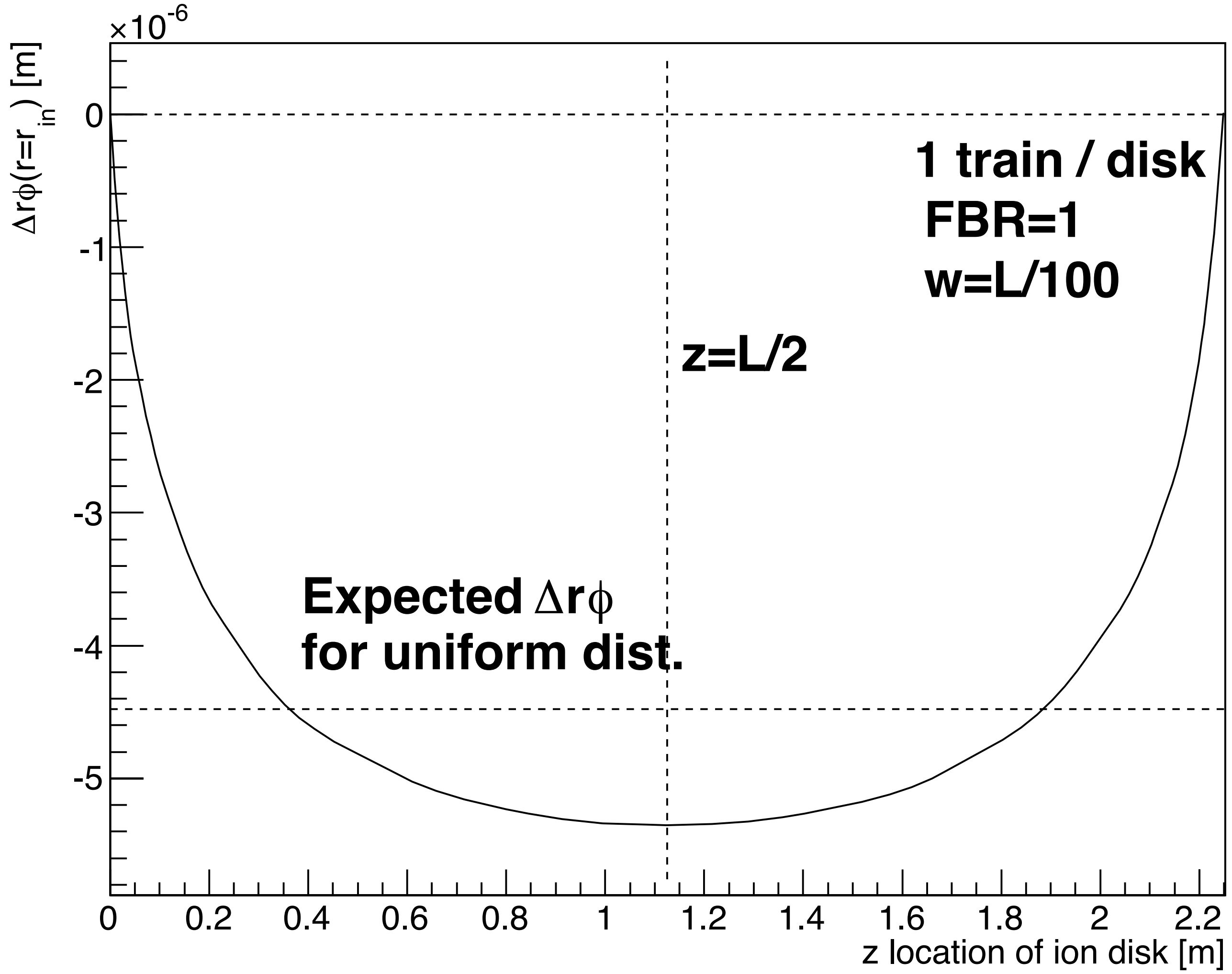
Er(r=rin, z) for different disk locations in “z”



Near the electrode (z=0), Er drops to zero, because the electrode sucks the E-field.

Remember the superposition principle, the primary ion case can be reproduced by summing up the contributions from many disks at different “z” locations.

Distortion (rphi) after maximum drift for different disk locations in “z”



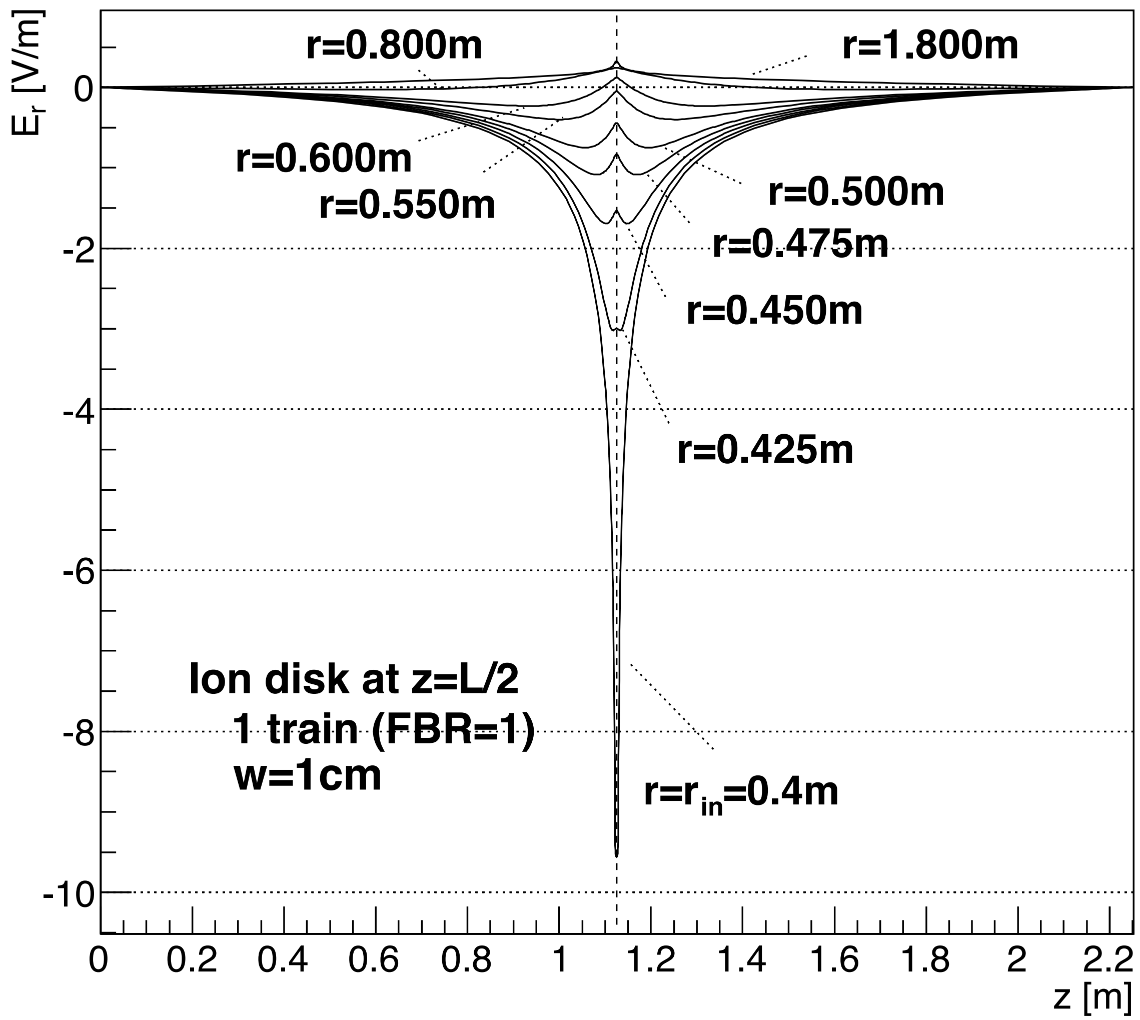
Remember the superposition principle, the effect of multiple disks is simply obtained by adding up the contributions from the individual disks.

Superposition makes life easy!

Ion Disk

E-field distortion and dependence on and drift distance

E-field distortion (r-component)

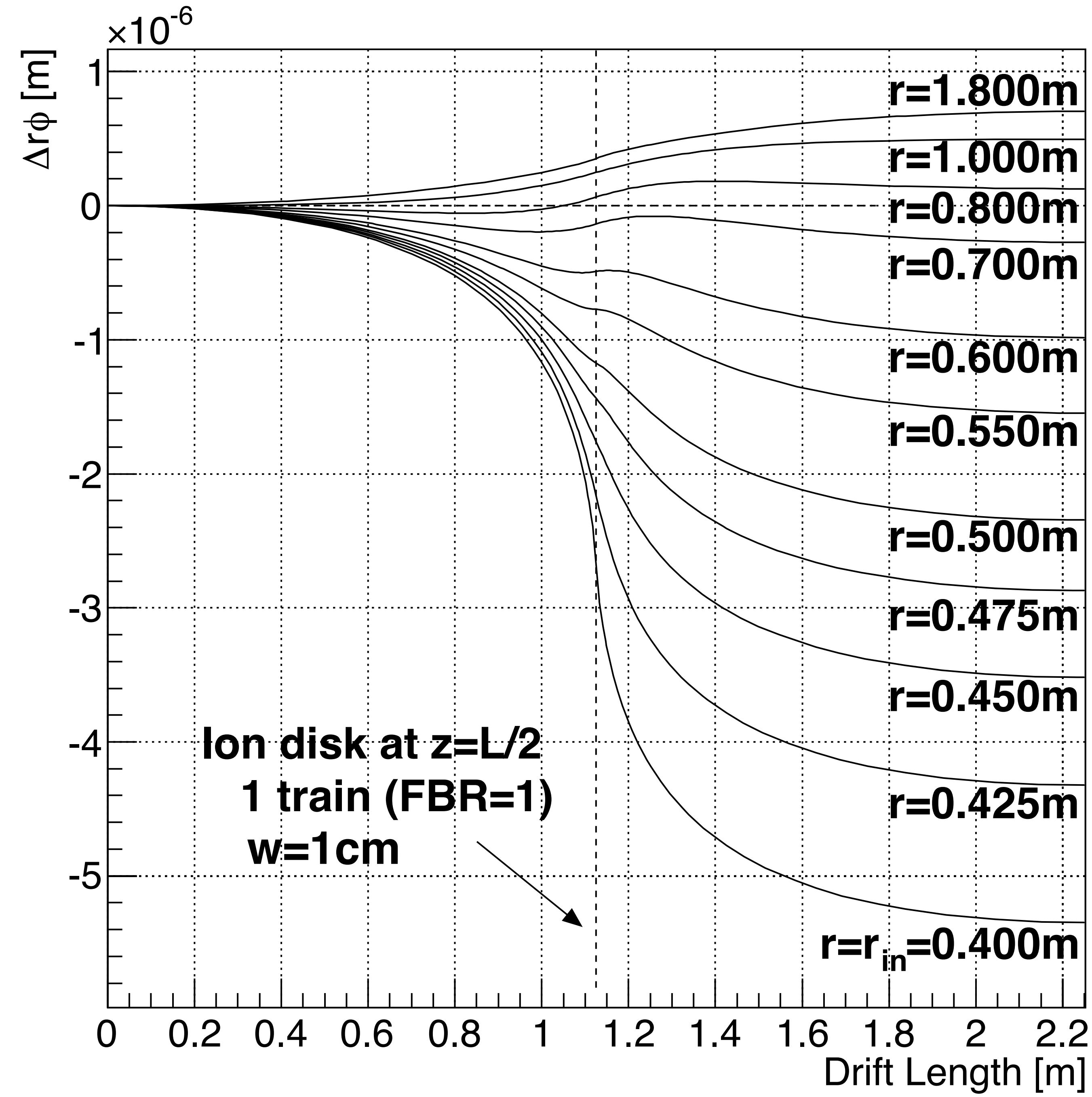


E-field distortion (emergence of radial component) is a sharply peaked function of "z" for an ion disk and changes its sign as r increases at some point.

The peak is mostly due to the induced charge on the inner cylinder!

Notice that Er=0 at the anode and at the cathode.

Distortion (rphi) vs drift length



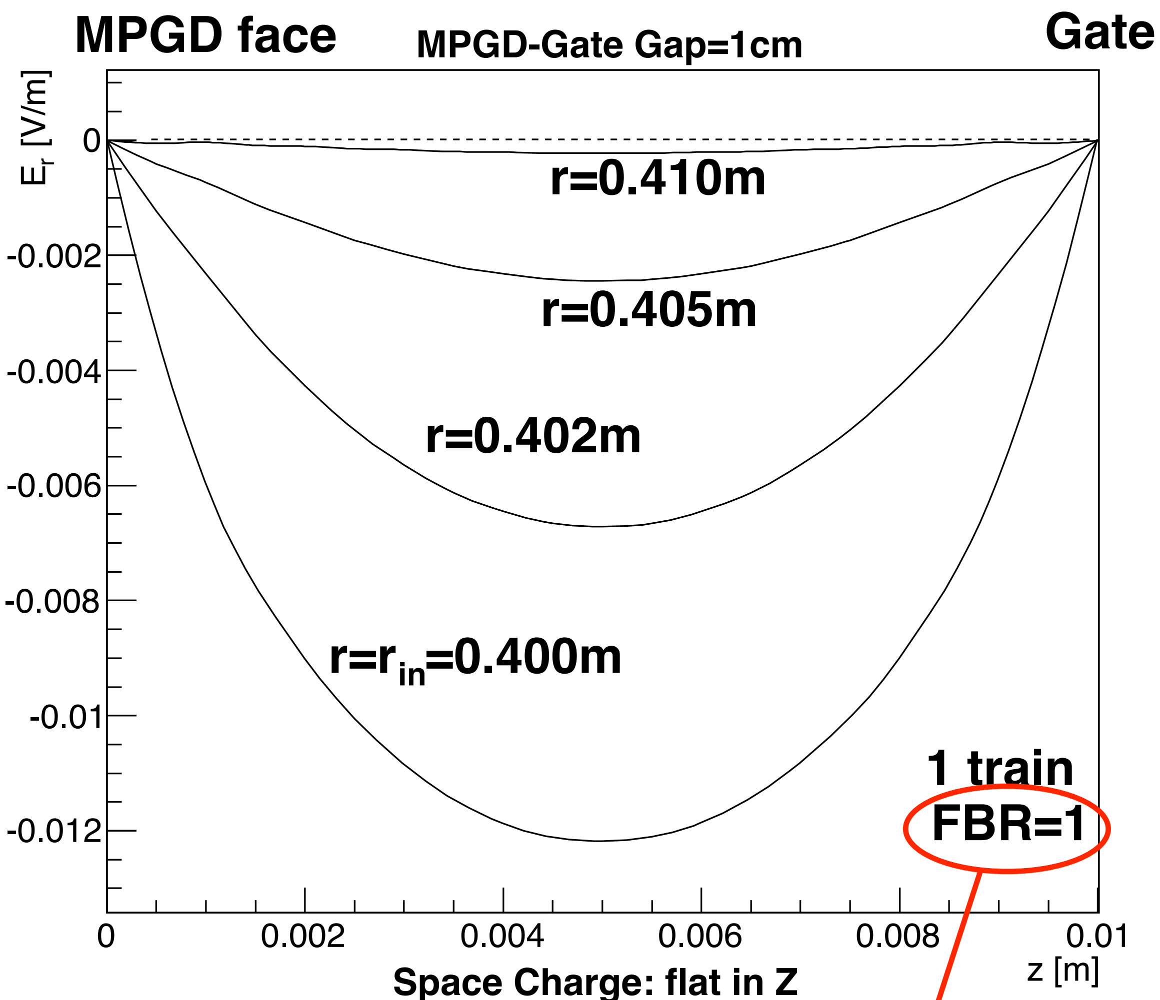
As expected, the distortion gets significantly larger when a track electron has to traverse the ion disk.

Effect of Ions behind the Gate

Ions in the gate-MPGD gap

Drift distance dependence of the ExB effect

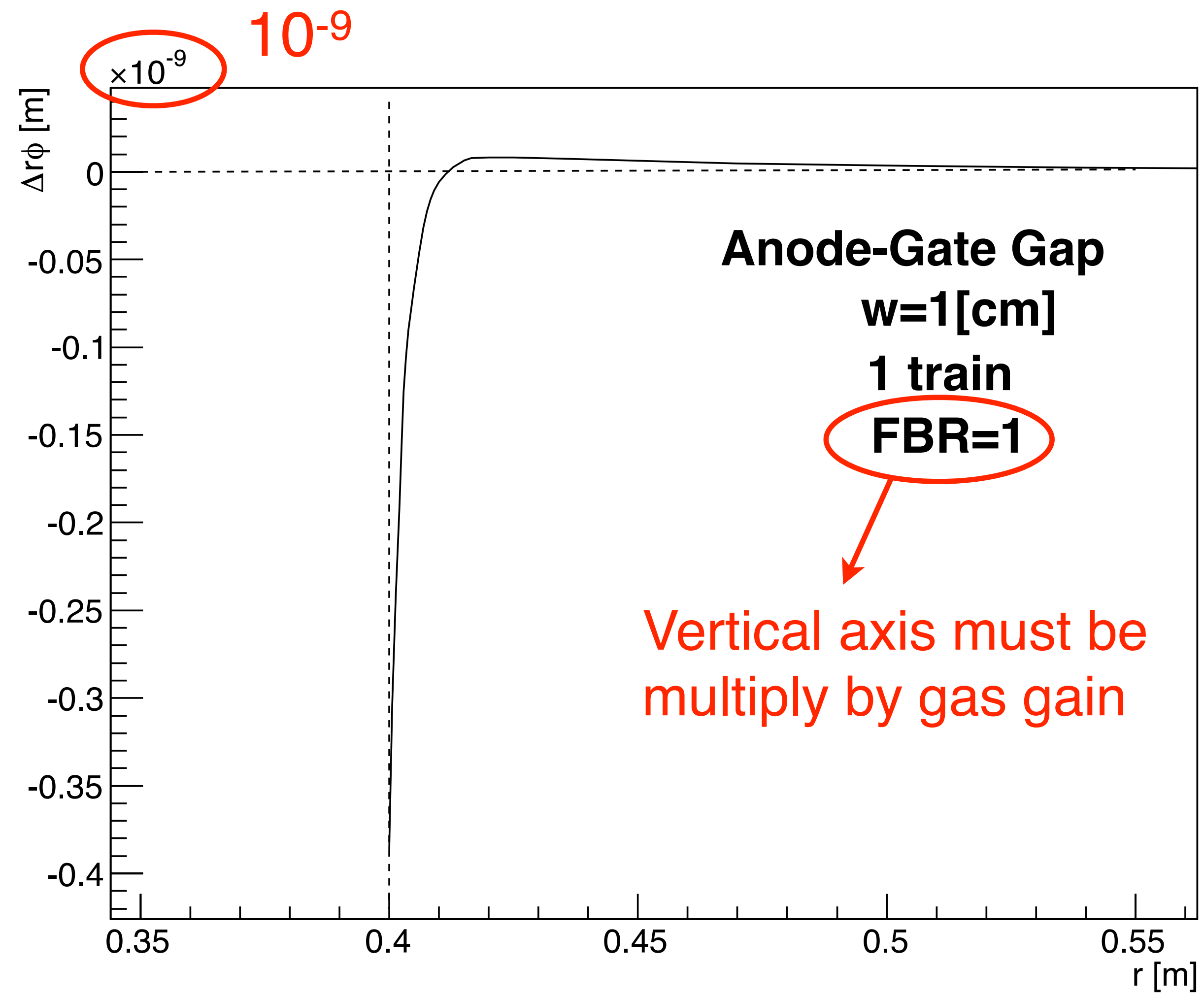
E-field distortion (r-component)



Vertical axis must be multiply by gas gain

Notice that Er is much smaller than those in the case of primary ions or ion disks in the drift region. This is because the MPGD front face and the gating device facing each other at a small distance strongly suck the E-field.

Distortion (rphi)



For gas gain of 1000, the maximum distortion is less than 1um, completely negligible if the distance from the inner cylinder becomes > 2cm!

Summary

Main Results

by D.Arai (WP#145, Feb. 23, 2012)

- Primary Ions: small ~ 8.5um
- Ion back-drift (ion disk)
  ~ 60um w/o gate (Gate needed)
- Ions in Gate-MPGD gap: negligible

KF's addition (this talk)

- Superposition makes life easy
- Increasing rin does not help
- The current BG estimate is an underestimate: two-photon hadron is not included, etc.
  --> After all, the most important is the BG simulation.