Study on Angle Pad Effect

Status report

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Analytic Expression of the Spatial Resolution

Helpful to understand how the point resolution is determined.

This work is based on the past work, in which only a perpendicular track to pad-rows was discussed. **(Nucl.Instrum.Meth.A641:37-47,2011)**

New points :

- de-clustering effect ・・・ diffusion and pad response function in a direction of pad-rows act on Neff
- angular pad effect ・・・ track angle is a factor affecting the spatial resolution

We will also check the validity of approximation used in our calculation by a Monte-Carlo simulation.

Resolution

definition:

$$\left\langle (\boldsymbol{x}-\tilde{\boldsymbol{x}})(\boldsymbol{x}-\tilde{\boldsymbol{x}})^T \right\rangle = \int_v \frac{d\tilde{\boldsymbol{x}}}{v} \int d\boldsymbol{x}\ P(\boldsymbol{x};\tilde{\boldsymbol{x}})(\boldsymbol{x}-\tilde{\boldsymbol{x}})(\boldsymbol{x}-\tilde{\boldsymbol{x}})^T$$

$\boldsymbol{x}$: measured values

$\tilde{\boldsymbol{x}}$: true values to be measured

$P(\boldsymbol{x};\tilde{\boldsymbol{x}})$: probability to be measured $\boldsymbol{x}$

v : readout unit (pad , pixel , voxel , ...)

Application to TPC case

TPC:

Two dimensional readout
x coordinate is determined by a charge centroid method.
y coordinate is determined by pad-row numbers.

(In the case of standard readout pad)

measured values:

$$\boldsymbol{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_n \end{pmatrix}$$

(subscript gives pad-row number)

<u>Strategy:</u>
Concentrate on one pad-row and discuss x resolution in the pad-row.
We would also like to consider correlations between pad-rows due to de-clustering effect.

If there is correlations between pad-rows,
$\langle(\boldsymbol{x}-\tilde{\boldsymbol{x}})(\boldsymbol{x}-\tilde{\boldsymbol{x}})^T\rangle$ has off-diagonal elements.

In that case, σ_x^2 is not enough to discuss spatial resolution.

Spatial Resolution (Standard readout pad case):

$$\sigma_{\bar{x}}^2 = \int_{-1/2}^{+1/2} d\left(\frac{\bar{x}}{w}\right) \int d\bar{x} P(\bar{x};\tilde{x})(\bar{x}-\tilde{x})^2$$

w : pad width

$P(\boldsymbol{x}; \tilde{\boldsymbol{x}})$ Components

- Primary ionization

$P_{PI}(N; n\Delta Y)$

N : # of primary electrons
n : gas density
ΔY : projected track length to y axis, to be considered
In general $\Delta Y \to \infty$

- y position of i-th primary ionization along a track

$$P(y_i) = \frac{1}{\Delta Y}$$

y_i : projected position to y of i-th cluster

- Secondary ionization

$P_{SI}(M_i)$

M_i : # of secondary electrons from i-th primary electron

- Diffusion

$P_D(\Delta x_{ij}), P_D(\Delta y_{ij})$

Δx_{ij}, Δy_{ij} : displacement of the j-th electron in i-th cluster, by the diffusion in drift region

- Gas amplification

$P_G(G_{ij})$

G_{ij} : gain of the j-th electron in i-th cluster

- Electric noise

$P_E(\Delta Q_a; \sigma_E)$

ΔQ_a : noise charge
a : pad number

- Pad response function

x direction : $F_a(x_{ij})$

x_{ij} : position where j-th electron in i-th cluster arrives at

$$x_{ij} = \tilde{x} + y_i \tan\phi + \Delta x_{ij}$$

ϕ : track angle
y_i : projected position to y of i-th cluster

y direction : $R_r(y_{ij})$

r : row ID (omit if not necessary)
y_{ij} : position where j-th electron in i-th cluster arrives at

In this report, I would like to skip the details of calculation.
(Instead I give some notes in the backup slides.)

Following is a expression after some calculation.

$$
\begin{aligned}
\sigma_{\bar{x}} &= \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
&+ \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \prod_{i=1}^{N} \left[\int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} \bar{P}_{SI}(k_i, y_i) \right] \times \\
&\left[\left(\sum_a \sum_b (abw^2) \sum_{i=1}^{N} k_i \left(< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}^2}{(\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij})^2} \right\rangle_{G_{ij}} \right.\right.\right. \\
&\qquad \left.\left. - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}}^2 \right) \right) \\
&\left. + \left(\sum_a (aw) \sum_{i=1}^{N} k_i < F_a(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \tilde{x} \right)^2 \right]
\end{aligned}
$$

Summary and Plans

My understanding at this moment

Electric noise part

Ionization probability and de-clustering effect part

Width of pad response function part (Diffusion、track angle)

Displacement of the center of pad response (S-shape systematics, Hodoscope effect)

$$\frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} + \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \prod_{i=1}^{N} \left[\int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} \bar{P}_{SI}(k_i, y_i) \right] \times$$

$$\left[\left(\sum_a \sum_b (abw^2) \sum_{i=1}^{N} k_i \left(< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}^2}{(\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij})^2} \right\rangle_{G_{ij}} - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}}^2 \right) \right) + \left(\sum_a (aw) \sum_{i=1}^{N} k_i < F_a(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \tilde{x} \right)^2 \right]$$

- To obtain more detailed relation, we need further calculation.
  --> Need to continue this work.

- Approximations used in the calculation should be validated by a Monte-Carlo simulation.

- Try to find the possibility to calculate resolution in the case of arbitrary angle of a track faster than a Monte-Carlo simulation.

- We would also like to study on the correlation effect between pad-rows.

Backup

ΔQ_a Integration

$$
\begin{aligned}
P(\bar{x};\tilde{x}) &= \sum_{N=1}^{\infty} P_{PI}(N;n\Delta Y) \sum_{M_1,M_2,\cdots,M_N} \prod_{i=1}^{N} \left[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \right. \\
& \prod_{j=1}^{M_i} \left[\int_{-\infty}^{\infty} d\Delta x_{ij} P_D(\Delta x_{ij};\sigma_d) \int_{-\infty}^{\infty} d\Delta y_{ij} P_D(\Delta y_{ij};\sigma_d) \int_{0}^{\infty} dG_{ij} P_G(G_{ij}) \right] \\
& \left. \prod_{a} \left[\int d\Delta Q_a P_E(\Delta Q_a;\sigma_E) \right] \right] \left(\frac{\sum_a (Q_a + \Delta Q_a)(aw)}{\sum_a (Q_a + \Delta Q_a)} - \tilde{x} \right)^2
\end{aligned}
$$

$$
\begin{aligned}
\approx & \quad \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ & \quad \sum_{N=1}^{\infty} P_{PI}(N;n\Delta Y) \sum_{M_1,M_2,\cdots,M_N} \prod_{i=1}^{N} \left[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \right. \\
& \left. \prod_{j=1}^{M_i} \left[\int_{-\infty}^{\infty} d\Delta x_{ij} P_D(\Delta x_{ij};\sigma_d) \int_{-\infty}^{\infty} d\Delta y_{ij} P_D(\Delta y_{ij};\sigma_d) \int_{0}^{\infty} dG_{ij} P_G(G_{ij}) \right] \right] \left(\frac{\sum_a Q_a (aw)}{\sum_a Q_a} - \tilde{x} \right)^2
\end{aligned}
$$

The first-order term of ΔQ_a vanishes after integration.

$\sum_a \Delta Q_a \approx 0$ have been used.

$Q_a = \sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} F_a(x_{ij}) R(y_{ij})$ which corresponds to the charges arriving at a-th pad.

$\sum_a Q_a = \sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij})$ because $\sum_a F_a(x_{ij}) = 1$

Δx_{ij} Integration

$$
\begin{aligned}
= & \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ & \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \sum_{M_1, M_2, \cdots, M_N} \prod_{i=1}^{N} \left[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \right. \\
& \left. \prod_{j=1}^{M_i} \left[\int_{-\infty}^{\infty} d\Delta y_{ij} P_D(\Delta y_{ij}; \sigma_d) \int_0^{\infty} dG_{ij} P_G(G_{ij}) \right] \right] \left\langle \left(\frac{\sum_a (aw) \sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} F_a(x_{ij}) R(y_{ij})}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij})} - \tilde{x} \right)^2 \right\rangle
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ & \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \sum_{M_1, M_2, \cdots, M_N} \prod_{i=1}^{N} \left[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \right. \\
& \left. \prod_{j=1}^{M_i} \left[\int_{-\infty}^{\infty} d\Delta y_{ij} P_D(\Delta y_{ij}; \sigma_d) \int_0^{\infty} dG_{ij} P_G(G_{ij}) \right] \right] \times \\
& \left[\left(\frac{\sum_a \sum_b (abw^2) \sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij}^2 R(y_{ij})^2 (< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x})}{(\sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij}))^2} \right) \right. \\
& \left. + \left(\frac{\sum_a (aw) \sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij}) < F_a(x_{ij}) >_{\Delta x}}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij})} - \tilde{x} \right)^2 \right]
\end{aligned}
$$

The difficulty to go further is that denominator includes the integration variables.
--> Need approximations or variable transformation

There is still room for improvement the following formulation.

In this report, I would like to show a formulation that is easy to understand the meaning but is not always demand mathematical strictness.

y_{ij} is more convenient as a integration variable than Δy_{ij} due to the de-clustering effect because the charges arriving at the pad-row depend on only y_{ij}

$$y_{ij} = y_i + \Delta y_{ij}$$

$$\begin{aligned} dy_i \, d\Delta y_{ij} &= \left| \begin{array}{cc} \frac{\partial y_i}{\partial y_i} & \frac{\partial \Delta y_i}{\partial y_i} \\ \frac{\partial y_i}{\partial \Delta y_i} & \frac{\partial \Delta y_i}{\partial \Delta y_i} \end{array} \right| dy_i \, dy_{ij} \\ &= dy_i \, dy_{ij} \end{aligned}$$

y_{ij} **Integration** (originally Δy_{ij} integration)

$$
\begin{aligned}
= \quad & \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ \quad & \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \sum_{M_1, M_2, \cdots, M_N} \prod_{i=1}^{N} \Bigg[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \\
& \prod_{j=1}^{M_i} \left[\int_{-\infty}^{\infty} dy_{ij} (R(y_{ij}) + (1 - R(y_{ij}))) P_D(y_{ij} - y_i; \sigma_d) \int_0^{\infty} dG_{ij} P_G(G_{ij}) \right] \Bigg] \times \\
& \left[\left(\frac{\sum_a \sum_b (abw^2) \sum_{i=1}^{N} (< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x}) \sum_{j=1}^{M_i} G_{ij}^2 R(y_{ij})^2}{(\sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij}))^2} \right) \right. \\
& \left. + \left(\frac{\sum_a (aw) \sum_{i=1}^{N} < F_a(x_{ij}) >_{\Delta x} \sum_{j=1}^{M_i} G_{ij} R(y_{ij})}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} G_{ij} R(y_{ij})} - \tilde{x} \right)^2 \right]
\end{aligned}
$$

Supposing $R(y_{ij}) = \theta(L/2 + y_{ij}) \theta(L/2 - y_{ij})$

$\sum_i^{M_i}$ is replaced by $\sum_i^{k_i}$ and $R(y_{ij}) = 1$ in the integrand.

Using $\eta(y_i) = \int_{-\infty}^{+\infty} dy_{ij} P_D(y_{ij} - y_i) R(y_{ij})$

$$
\begin{aligned}
= \quad & \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ \quad & \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \sum_{M_1, M_2, \cdots, M_N} \prod_{i=1}^{N} \Bigg[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} {}_{M_i}C_{k_i} \eta(y_i)^{k_i} (1 - \eta(y_i))^{M_i - k_i} \\
& \prod_{j=1}^{k_i} \left[\int_0^{\infty} dG_{ij} P_G(G_{ij}) \right] \Bigg] \times \\
& \left[\left(\frac{\sum_a \sum_b (abw^2) \sum_{i=1}^{N} (< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x}) \sum_{j=1}^{k_i} G_{ij}^2}{(\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij})^2} \right) \right. \\
& \left. + \left(\frac{\sum_a (aw) \sum_{i=1}^{N} < F_a(x_{ij}) >_{\Delta x} \sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} - \tilde{x} \right)^2 \right]
\end{aligned}
$$

G_{ij} Integration

$$
\begin{aligned}
&= \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
&+ \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \sum_{M_1, M_2, \cdots, M_N} \prod_{i=1}^{N} \left[P_{SI}(M_i) \int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} {}_{M_i}C_{k_i} \eta(y_i)^{k_i} (1-\eta(y_i))^{M_i - k_i} \right] \times \\
&\left[\left(\sum_a \sum_b (abw^2) \sum_{i=1}^{N} (< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x}) \times \right.\right. \\
&\qquad \left. \sum_{j=1}^{k_i} \left\langle \frac{G_{ij}^2}{(\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij})^2} \right\rangle_{G_{ij}} \right) \\
&+ \left(\sum_a (aw) \sum_{i=1}^{N} < F_a(x_{ij}) >_{\Delta x} \sum_{j=1}^{k_i} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \tilde{x} \right)^2 \\
&+ \sum_a \sum_b (abw^2) \sum_{i=1}^{N} < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x} \times \\
&\qquad \left. \sum_{j=1}^{k_i} \left(\left\langle \frac{G_{ij}^2}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}}^2 \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
&+ \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \prod_{i=1}^{N} \left[\int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} \bar{P}_{SI}(k_i, y_i) \right] \times \\
&\left[\left(\sum_a \sum_b (abw^2) \sum_{i=1}^{N} k_i \left(< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}^2}{(\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij})^2} \right\rangle_{G_{ij}} \right.\right.\right. \\
&\qquad \left.\left. - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}}^2 \right) \right. \\
&+ \left.\left(\sum_a (aw) \sum_{i=1}^{N} k_i < F_a(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \tilde{x} \right)^2 \right]
\end{aligned}
$$

Using $\bar{P}_{SI}(k_i, y_i) = \sum_{M_i=1}^{\infty} P_{SI}(M_i)_{M_i}C_{k_i}\eta(y_i)^{k_i}(1-\eta(y_i))^{M_i-k_i}$

and

$$\sum_{M_i=1}^{\infty}\sum_{k_i=0}^{M_i} = \sum_{k_i=0}^{\infty}\sum_{M_i=1}^{\infty}$$

$$\begin{aligned}
= \quad & \frac{\sigma_E^2 \sum_a (aw)^2}{\sum_a Q_a} \\
+ \quad & \sum_{N=1}^{\infty} P_{PI}(N; n\Delta Y) \prod_{i=1}^{N} \left[\int_{-\Delta Y/2}^{\Delta Y/2} \frac{dy_i}{\Delta Y} \sum_{k_i=0}^{M_i} \bar{P}_{SI}(k_i, y_i) \right] \times \\
& \left[\left(\sum_a \sum_b (abw^2) \sum_{i=1}^{N} k_i \left(< F_a(x_{ij}) F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}^2}{(\sum_{i=1}^{N}\sum_{j=1}^{k_i} G_{ij})^2} \right\rangle_{G_{ij}} \right. \right. \right. \\
& \left. \left. - < F_a(x_{ij}) >_{\Delta x} < F_b(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}}^2 \right) \right. \\
& \left. + \left(\sum_a (aw) \sum_{i=1}^{N} k_i < F_a(x_{ij}) >_{\Delta x} \left\langle \frac{G_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{k_i} G_{ij}} \right\rangle_{G_{ij}} - \tilde{x} \right)^2 \right]
\end{aligned}$$