Polarization of final electrons/positrons during multiple Compton backscattering process.

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Cross-section of Compton scattering of CP laser photons

$$\frac{d\sigma}{dy}=\frac{\pi r_0^2}{2x}\left\{\frac{1}{1-y}+1-y-s^2\mp\xi_{0z}P_c cy\frac{2-y}{1-y}\mp\xi_z P_c\left[s_z scy+c_z\left(\frac{y}{1-y}+yc^2\right)\right]+\right.$$

$$\left.+\xi_{0z}\xi_z\left[s_z s\left(1+c^2-yc^2\right)+c_z c\left(\frac{1}{1-y}+(1-y)c^2\right)\right]\right\}\qquad\begin{cases}-\text{electrons}\\+\text{positrons}\end{cases}$$

$$x=2pk\approx\frac{4\gamma_0\hbar\omega_0}{mc^2},\qquad y=1-\frac{pk'}{pk}\approx\frac{\hbar\omega}{\gamma_0 mc^2}$$

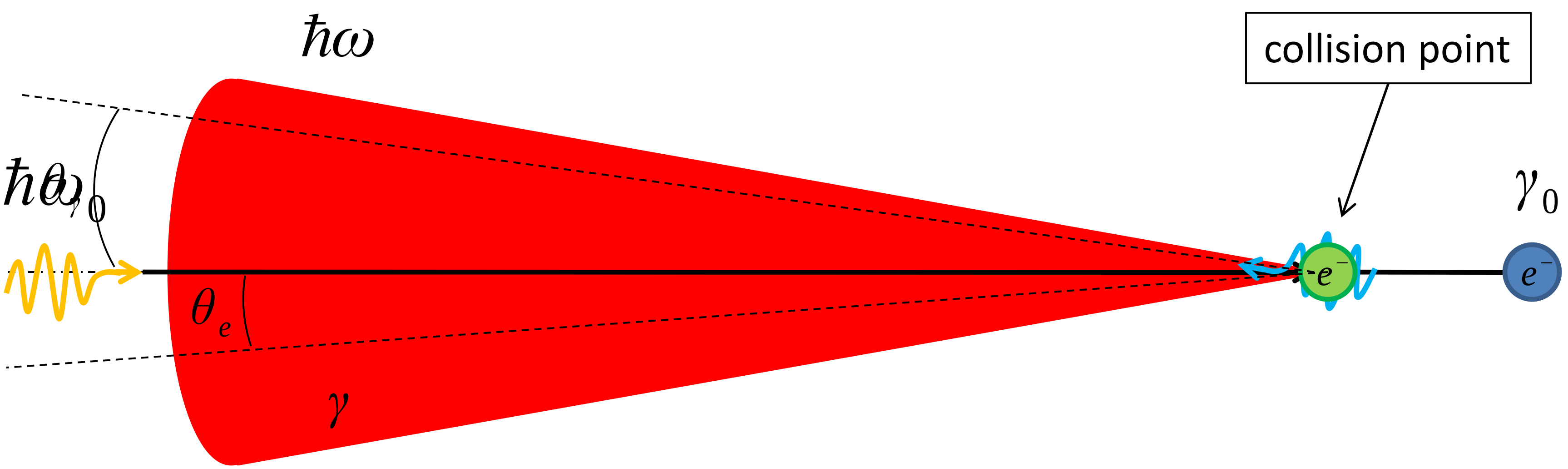
$$s=2\sqrt{r(1-r)},\quad c=1-2r,\quad r=\frac{y}{x(1-y)}$$

$$s_z=s-c\theta_e,\quad c_z=c+s\theta_e$$

Within an accuracy $\sim\gamma_0^{-1}$ $\quad s_z=s;\quad c_z=c$

Geometry of Compton backscattering process

$$\theta_e = \frac{1}{\gamma_0}\frac{\sqrt{y(x-y-xy)}}{1-y} \approx \frac{x}{\gamma_0}$$



Spin-dependent cross section

Unpolarized e^+ beam may be consideredas a sum of twopolarized componentswith a half of the initial intensity and polarized in opposite directions (indice + means parallel orientation of positron spin and momentum,+ - antiparallel one)

In this case for $P_c = +1, \left|\xi_z\right| = \left|\xi_{z0}\right| = 1$

$$\frac{d\sigma_+}{dy} = \frac{d\sigma_{++}}{dy} + \frac{d\sigma_{+-}}{dy} = 2\left(\frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy}\right)$$

$$\frac{d\sigma_-}{dy} = \frac{d\sigma_{--}}{dy} + \frac{d\sigma_{-+}}{dy} = 2\left(\frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy}\right)$$

where

$$\frac{d\sigma_{++}}{dy} = \frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy} + \frac{d\sigma_2}{dy} + \frac{d\sigma_3}{dy}; \quad \text{- non spin - flip term}$$

$$\frac{d\sigma_{+-}}{dy} = \frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy} - \frac{d\sigma_2}{dy} - \frac{d\sigma_3}{dy}; \quad \text{- spin - flip term}$$

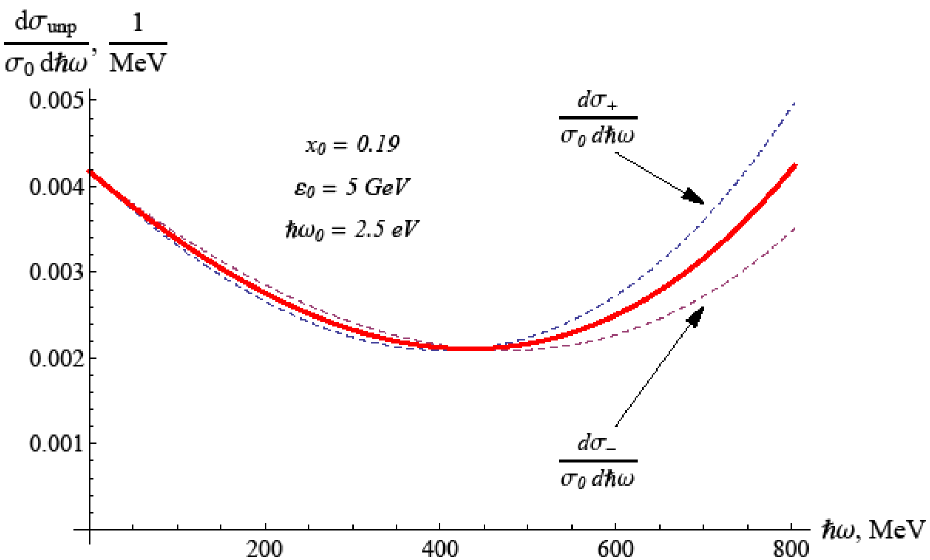
$$\frac{d\sigma_{--}}{dy} = \frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy} - \frac{d\sigma_2}{dy} + \frac{d\sigma_3}{dy}; \quad \text{- non spin - flip term}$$

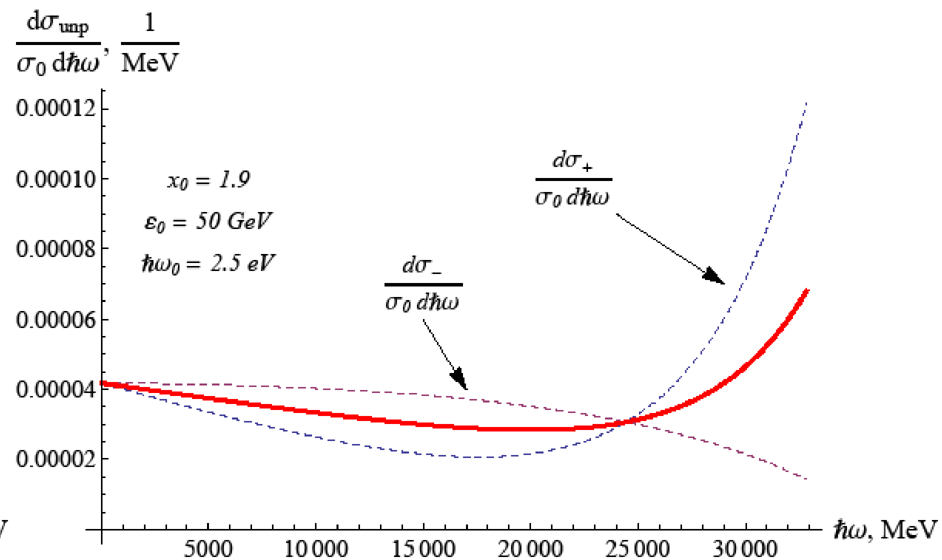
$$\frac{d\sigma_{-+}}{dy} = \frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy} + \frac{d\sigma_2}{dy} - \frac{d\sigma_3}{dy}; \quad \text{- spin - flip term}$$

For unpolarized beam (after averaging over initial polarization states).

$$\frac{d\sigma_{unp}}{dy} = 2\frac{d\sigma_0}{dy} \approx \frac{\pi r_0^2}{x_0}\left\{\frac{1}{1-y} + 1 - y - s^2\right\}$$

$$\hbar\omega_{\max} = \gamma_0 mc^2 \frac{x_0}{1+x_0}$$

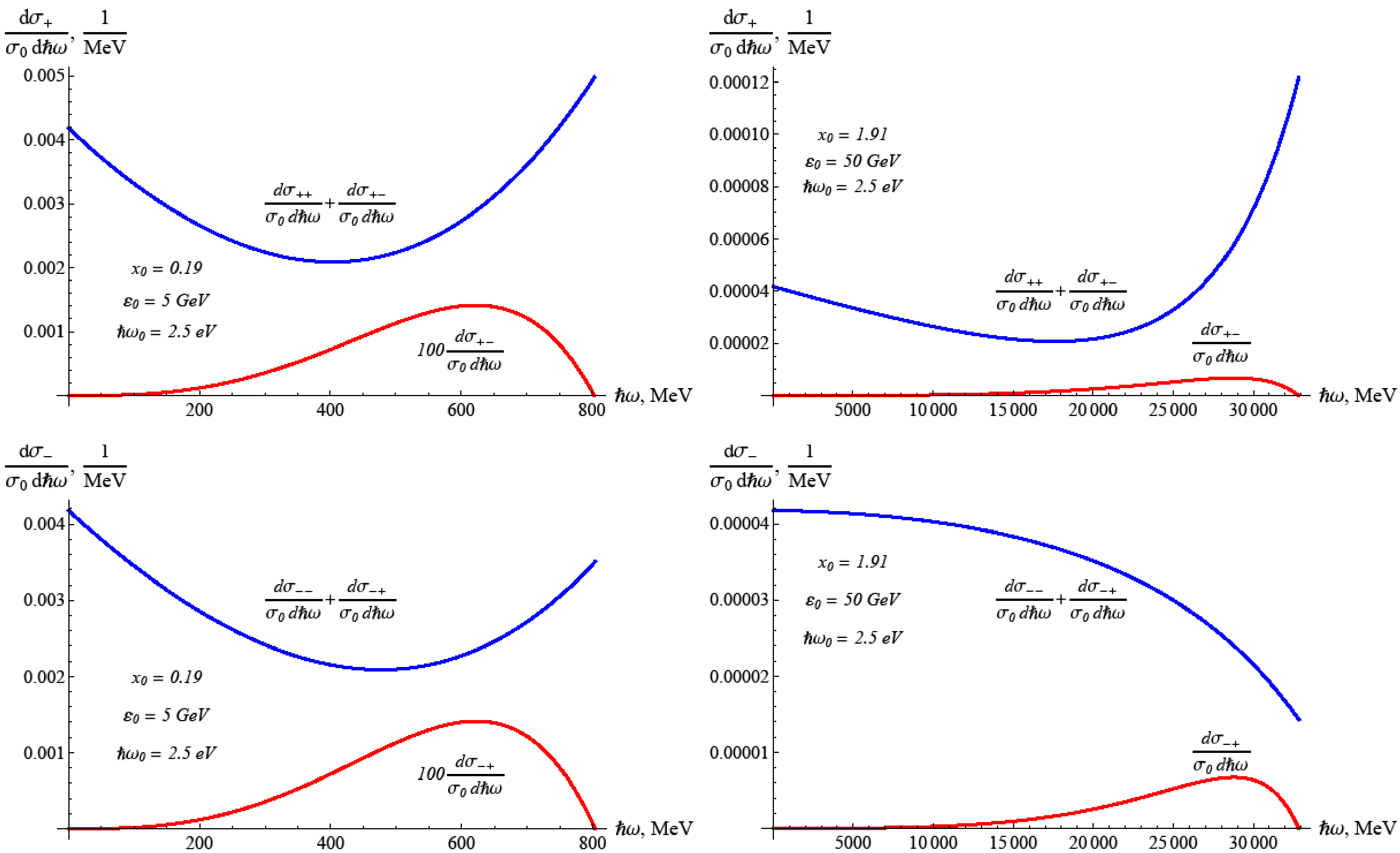




$\hbar\omega_{\max} = 803.546$ MeV

$\hbar\omega_{\max} = 32846.3$ MeV

Comaprison of both parts of cross-section for different electron energies



It is evidently for any electron/positron energy

$$\frac{d\sigma_{+-}}{dy} = \frac{d\sigma_{-+}}{dy}$$

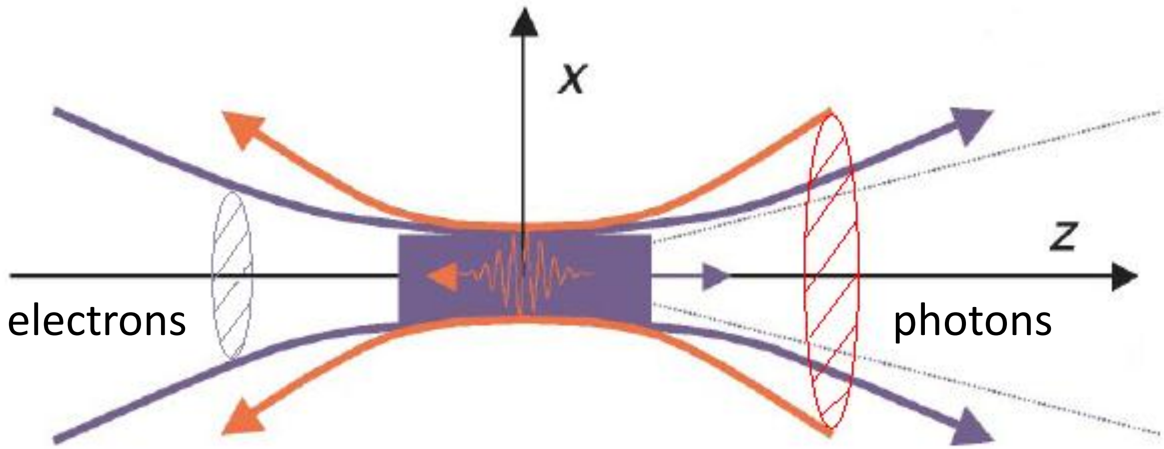
within an accuracy γ_0^{-1}.

It means there is no polarisation of the final beam as whole.

But for the case when each positron in a beam will interact with *CP* laser photons a few times (multiple Compton backscattering proccess) the final positron beam will have the non - zeroth polarization for a part of beam due to difference in cross-sections:

$$\boxed{\frac{d\sigma_{+}}{dy} \neq \frac{d\sigma_{-}}{dy}}$$

Mean collision number per each initial electron/positron. (number of photons per particle)



$$f_L \sim \exp\left(-\frac{\rho^2}{\rho_L^2}\right),$$

$$f_e \sim \exp\left(-\frac{\rho^2}{\rho_e^2}\right)$$

F. Hartemann et al. PRST - AB, (2005)100702)

$$\rho_L^2 = \rho_0^2(1+\frac{z^2}{z_R^2}), \quad \rho_e^2 = \rho_b^2(1+\frac{z^2}{\beta_f^2}).$$

$\rho_0(\rho_b)$ is the minimum radius of photon (electron) beam. z_R is the Rayleigh length, β_f is the beta function.

Longitudinad distributions of beams were approximated by Gaussians with parameters l_L and l_e.

In this model the mean number of scattered photons is determined by the flollowing expression:

$$\bar{k} = \frac{16\sqrt{\pi}}{3}\frac{r_e^2}{\rho_0^2}N_L f(l_e, l_L, r), \text{ where}$$

$$f(l_e,l_L,r) = \exp[(1+r^2)/(\mu^2+r^2\eta^2)]/[(\mu^2+r^2\eta^2)(1+r^2)]^{1/2} \times \left\{1-\Phi\left[(1+r^2)^{1/2}/(\mu^2+r^2\eta^2)^{1/2}\right]\right\},$$

$$\mu = \frac{l_L}{2\sqrt{2}z_R}, \quad \eta = \frac{l_e}{2\sqrt{2}\beta_f}, \quad r = \frac{\rho_b}{\rho_0}, \quad \Phi(x) \text{ is the error function.}$$

Simplest case $\rho_L = const, \rho_e = const$

In the limit $z_R \to \infty, \beta_f \to \infty \ (\mu, \eta \to 0)$

$$1 - \Phi(x) \approx \frac{1}{\sqrt{\pi}} \frac{\exp(-x^2)}{x},$$

$$\bar{k} \approx \frac{16}{3} N_L \frac{r_e^2}{\rho_0^2 + \rho_e^2}$$

or using luminosity L: $\bar{k} = \frac{L\sigma}{N_e}$

Introducing a mean photon concentration n_0 and effective length l_L of a laser flash:

$$n_0 = \frac{N_L}{V_{eff}} = \frac{W_L}{\hbar\omega_0} \frac{1}{\pi \rho_{eff}^2 l_L}:$$

$$\bar{k} \approx l_L 2\sigma_T (1 - x_0) n_0 \approx \frac{l_L}{l_T} \quad for \ x_0 << 1$$

Where $l_T = \frac{1}{2n_0\sigma_T}$ – so-called Thomson free path electron length in a "light target".

Scheme for production of polarized positrons

[T.Omori et al. NIMA 500(2003)232]

The mean number of collisions $\bar{k}$ may be estimated for parameters:

$\rho_0 \approx 20\,\mu m,\ z_R \approx 220\,\mu m,\ l_L \approx 3\,mm$

$\rho_b \approx 20\,\mu m,\ \beta_f \approx 3600\,mm,\ le \approx 1\,mm$

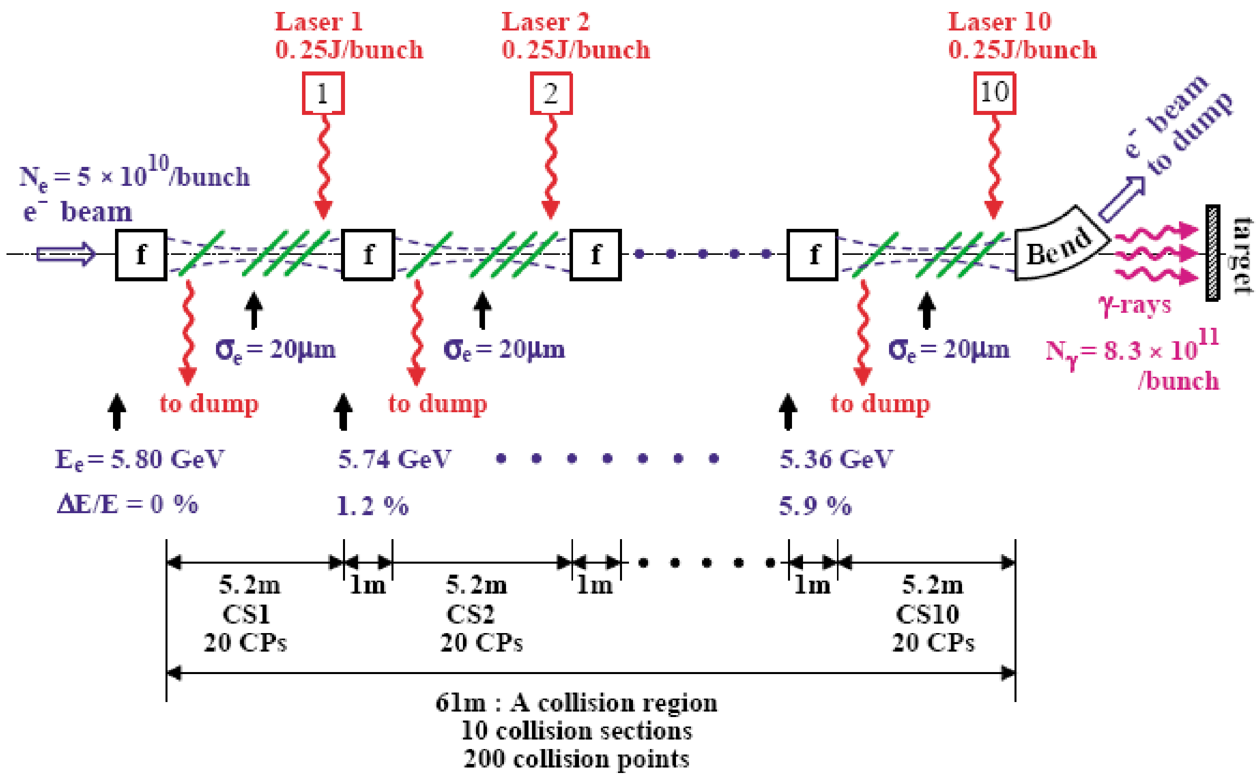
$\overline{k_0} \approx 0.2$

and for 200 collision points $\bar{k} \approx 200\overline{k_0} \approx 40$

Due to non-gaussian laser beam authors have got the value:

$\bar{k} = N_\gamma / N_e = 16.6$

$(N_\gamma = 8.3 \cdot 10^{11}\ photons/bunch,\ N_e = 5 \cdot 10^{10}\ e^- / bunch)$

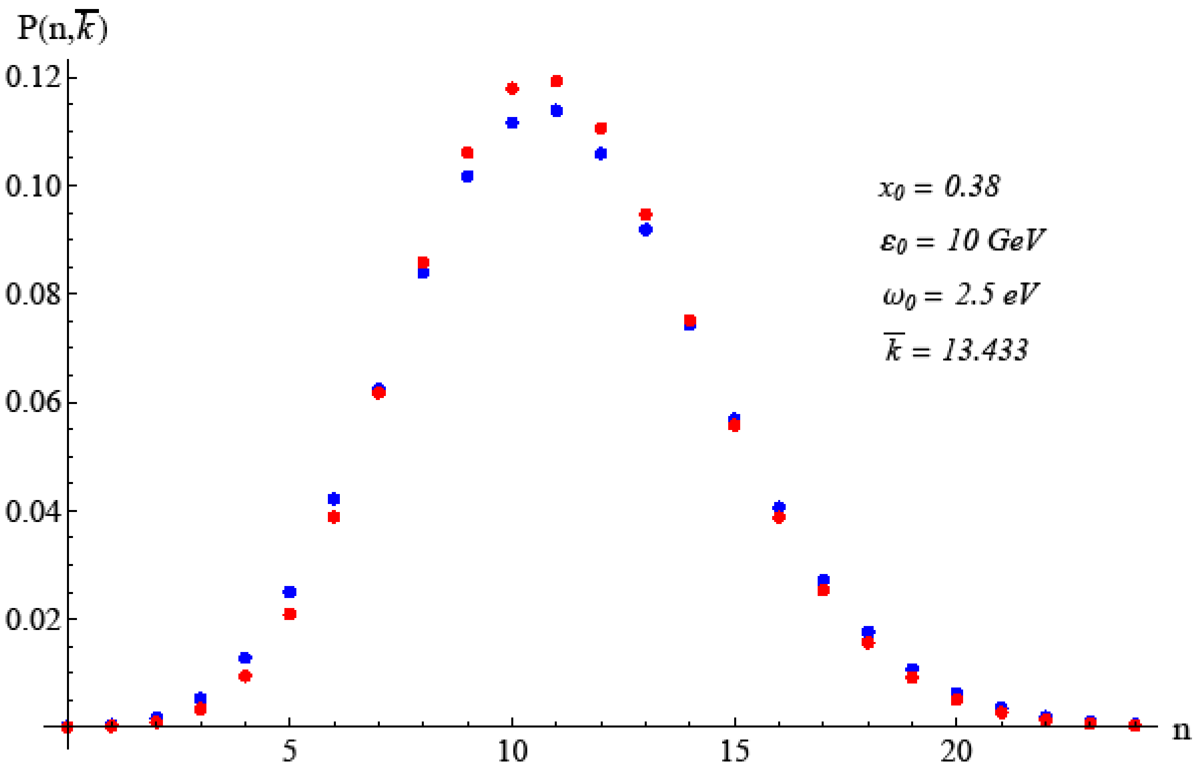


Number of photons per electron:

$\bar{k} = 2.4$ CLICHE [D. Asher et al. Eur. Phys. J. C., 28 (2003) 27]

$\bar{k} = 1.2$ SAPPHiRE [S. A. Bogacz et. al. arXiv: 1208.2827]

Distribution over the collision number *k*



Blue – Poisson law, red – simulation rezults

$\sigma_k^2(l) = \bar{k}(l)$ - main characteristics of the Poisson law

$P(n,\bar{k}) = \bar{k}^2 \exp(-\bar{k})/n!$

For any value of $\bar{k}$ there is a part of electrons passing through a light target without interaction (n = 0):

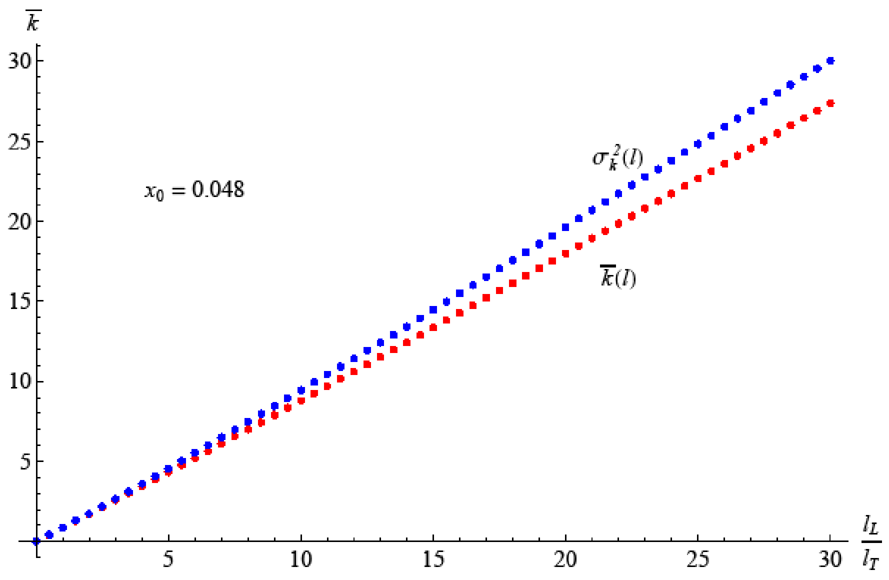
$P(0,\bar{k}) = \exp(-\bar{k})$

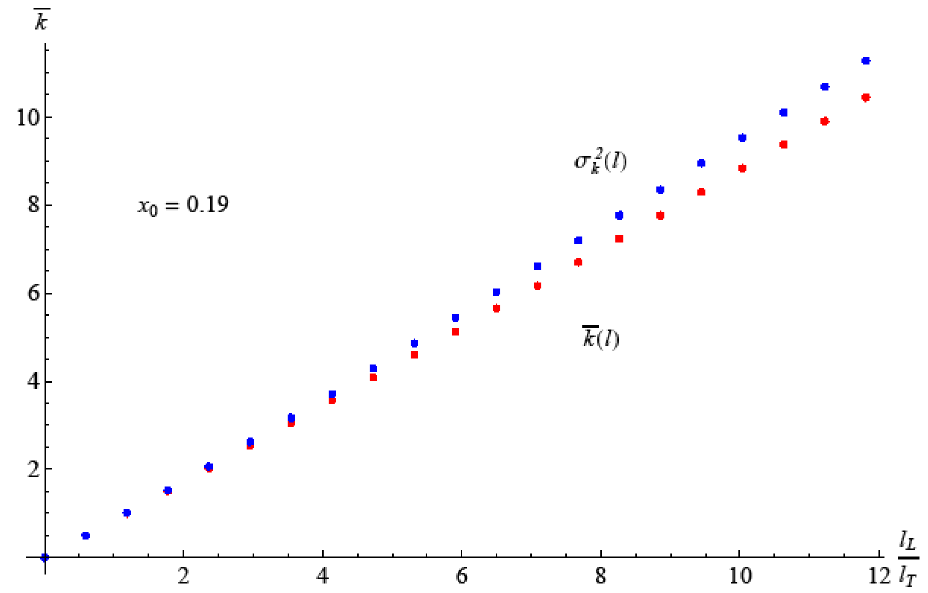
For $\bar{k} = 1$, $\exp(-\bar{k}) = 0.37 = 37\,\%$

Monte – Carlo simulation

A random path length and a random energy loss were successively simulated in each collision

$$\text{Comparison of } \bar{k} \text{ and } \sigma_k^2$$



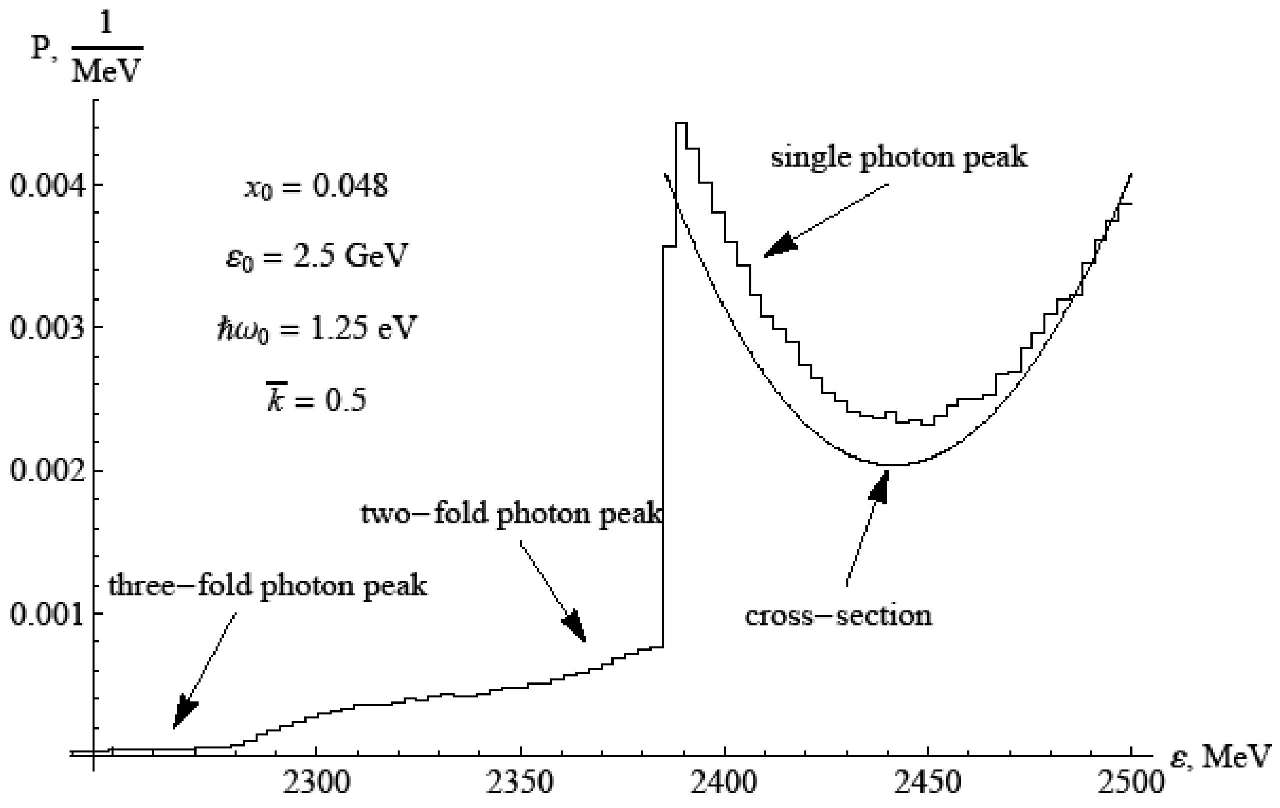


Dependencies $\bar{k}$ and σ_k^2 on a light target thickness.

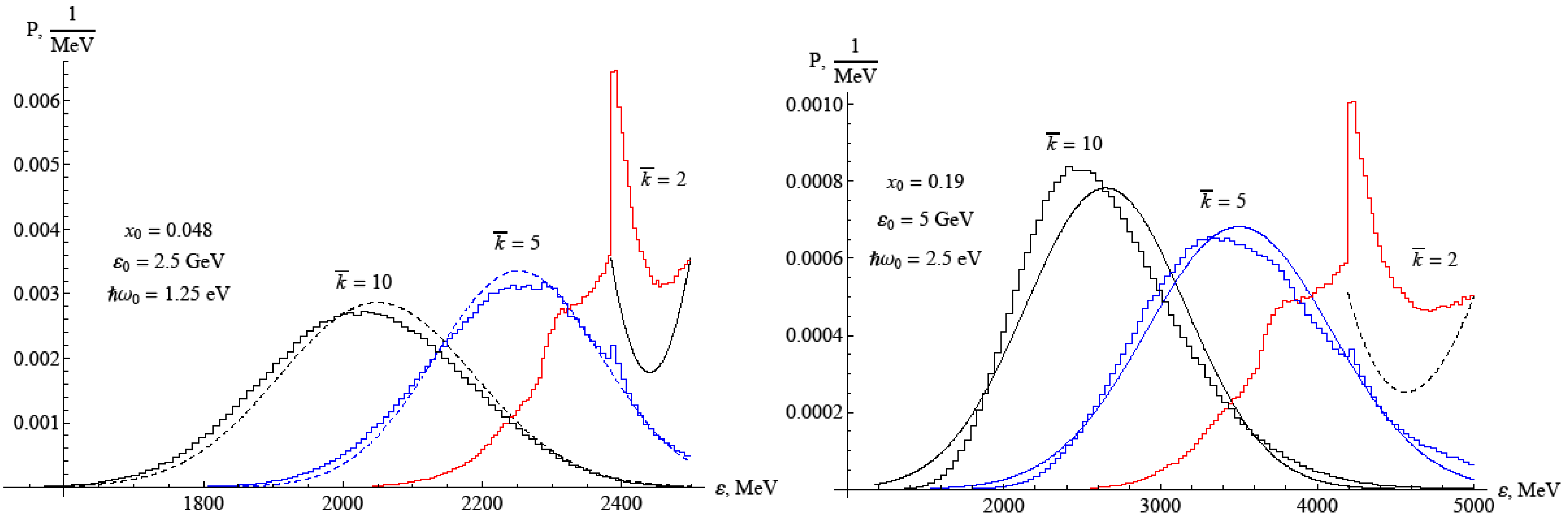
$$\bar{k} \approx \sigma_k^2 \text{ for small } x_0 \text{ and small thickness}$$

Energy distribution of recoil electrons/positrons

for small collision number $\left(\bar{k} = 0.5\right)$



Energy distribution of unpolarized beam passed through a light target

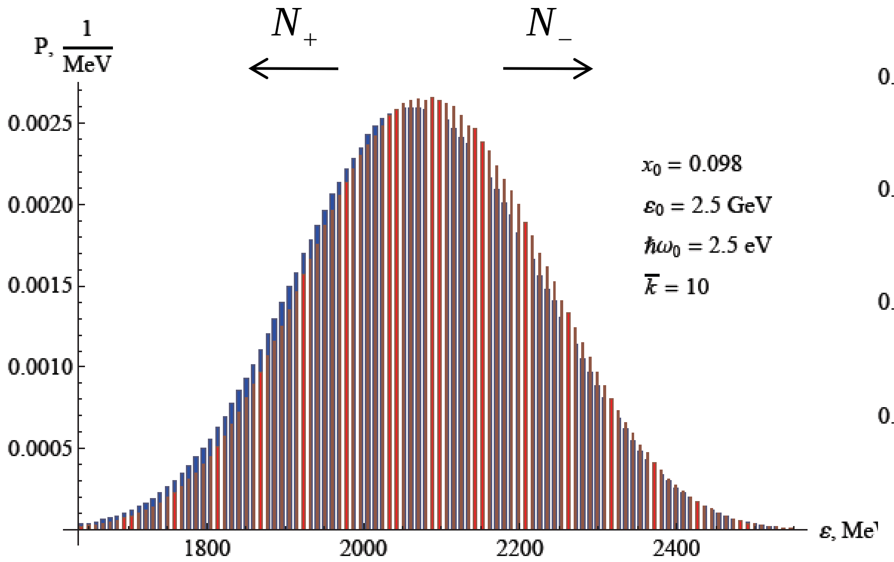


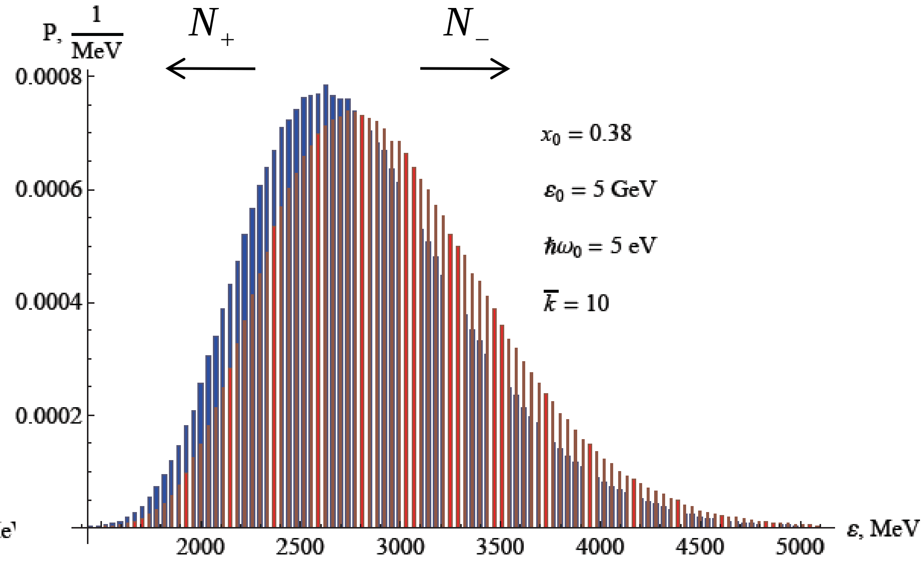
Curves – gaussian distribution with parameters

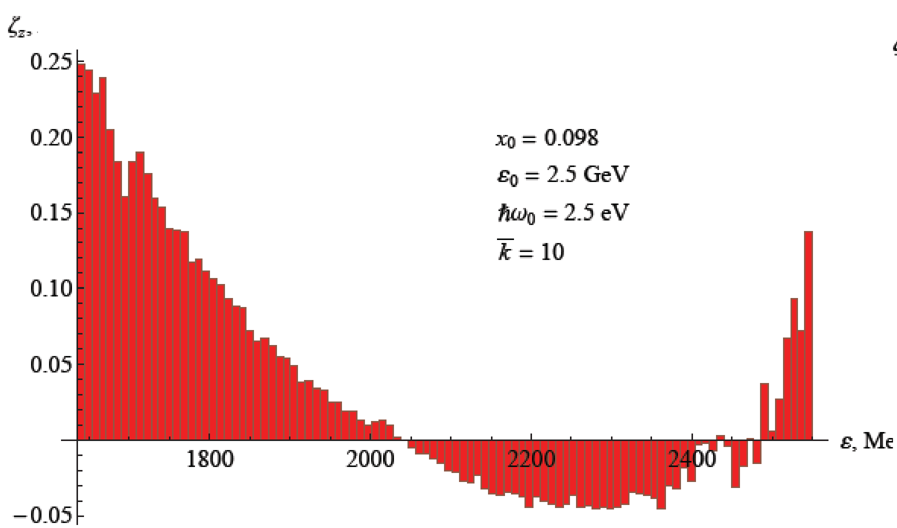
$$\bar{\varepsilon}(\varepsilon_0, l_L) \approx \frac{\varepsilon_0}{1 + \frac{1}{2}\bar{k}x_0\left(1 - \frac{21}{10}x_0\right)}, \quad \Delta(\varepsilon_0, l_L) \approx \frac{7}{20}\frac{\bar{k}\varepsilon_0^2 x_0^2}{\left(1 + \frac{1}{2}\bar{k}x_0\right)^4}$$

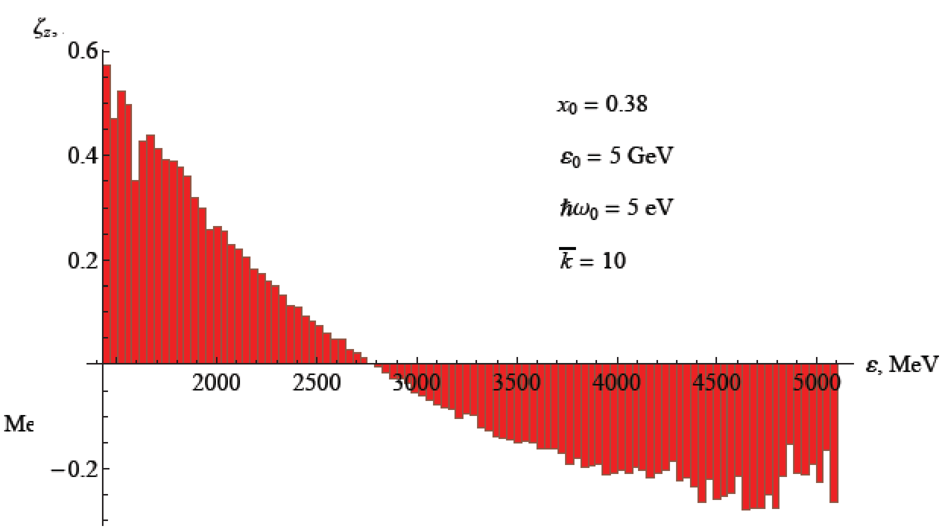
Analitical solution are valid for $x_0 \ll 1$ (see Kolchuzhkin et al. NIMB 201 (2003) 307)

Polarization of a final beam

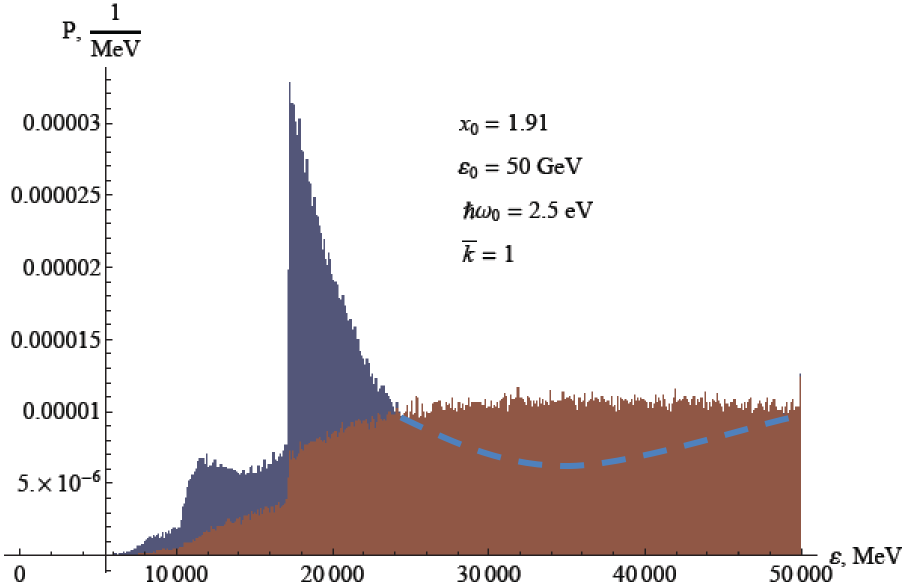


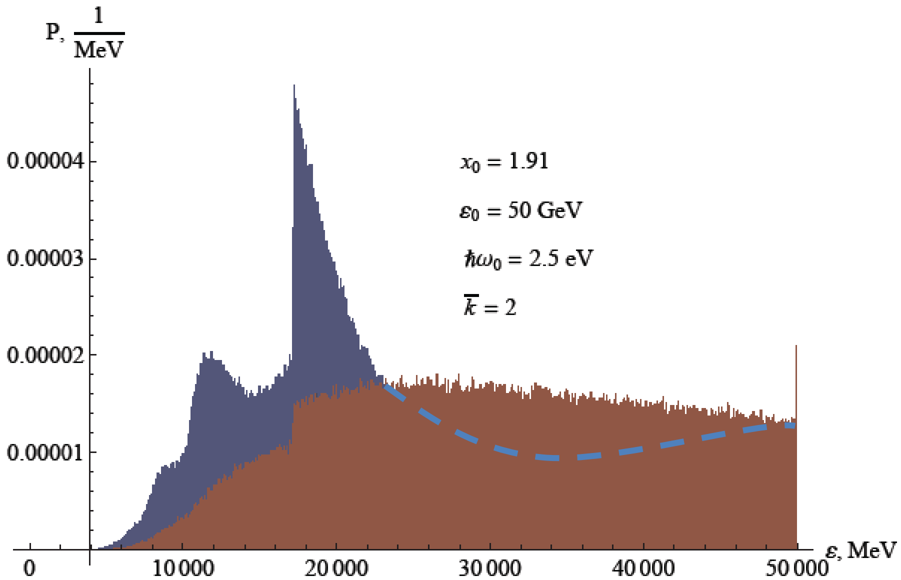


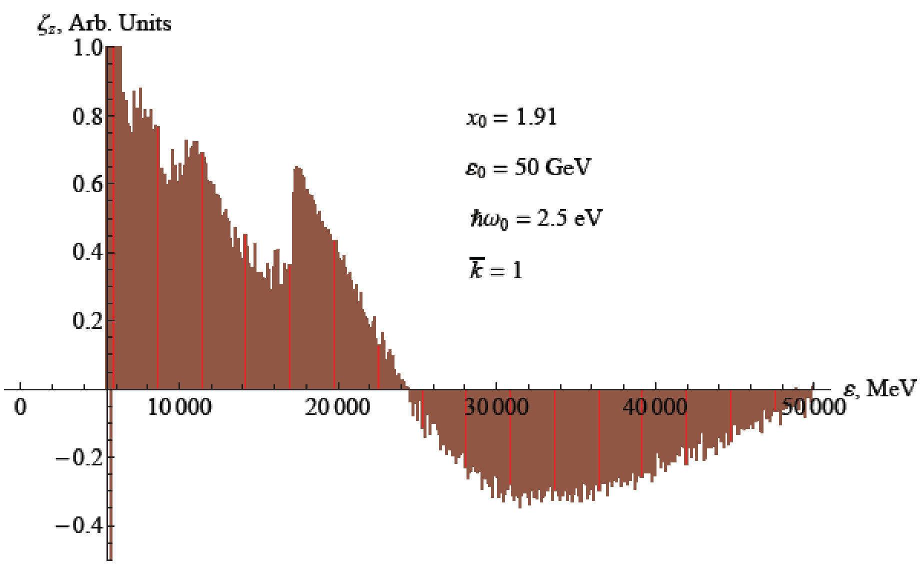


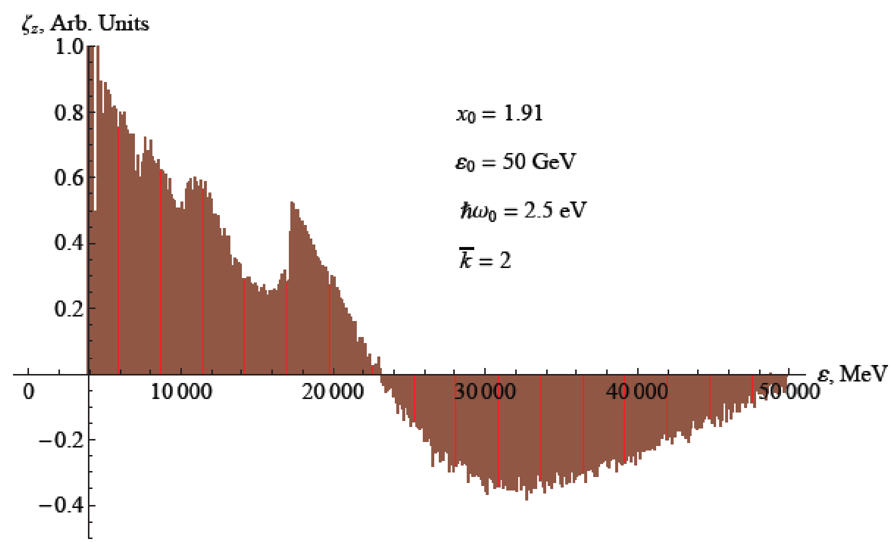


Initial positron energy 50 GeV









All simulation results were obtained for

laser field in the plane - wave approximation (paraxial approximation)

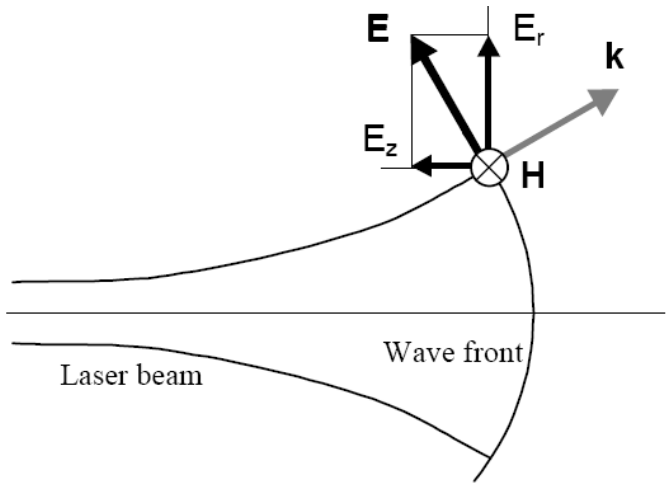
$$(\vec{E} \perp \vec{H}, E_z = H_z = 0)$$

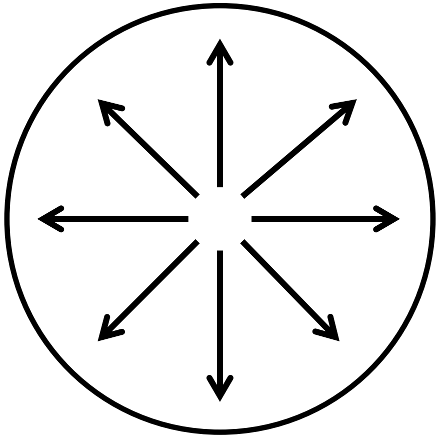
For a tightly focused laser beam ($\rho_0 \leq \lambda$) such an approximation is not valid.

Beyond the paraxial approximation longitudinal fields cannot be ignored

$$|E_z| \sim \frac{\lambda}{2\pi\omega_0} |\vec{E}_\perp|$$

Tightly focused laser beam





Schematic view of relative location of the electromagnetic field and the wave vector k for a real field of beam.

The helical modes with circular polarization. Arrows show instant direction of electric vector and distribution of its phase over the cross-section.

Comparison of longitudinal and radial field components

Magnetic field on the plane z = 0

$$H_z \approx E_0 \exp\left\{-\frac{\rho^2}{\rho_0^2}\right\}\frac{\rho_0}{z_R}$$

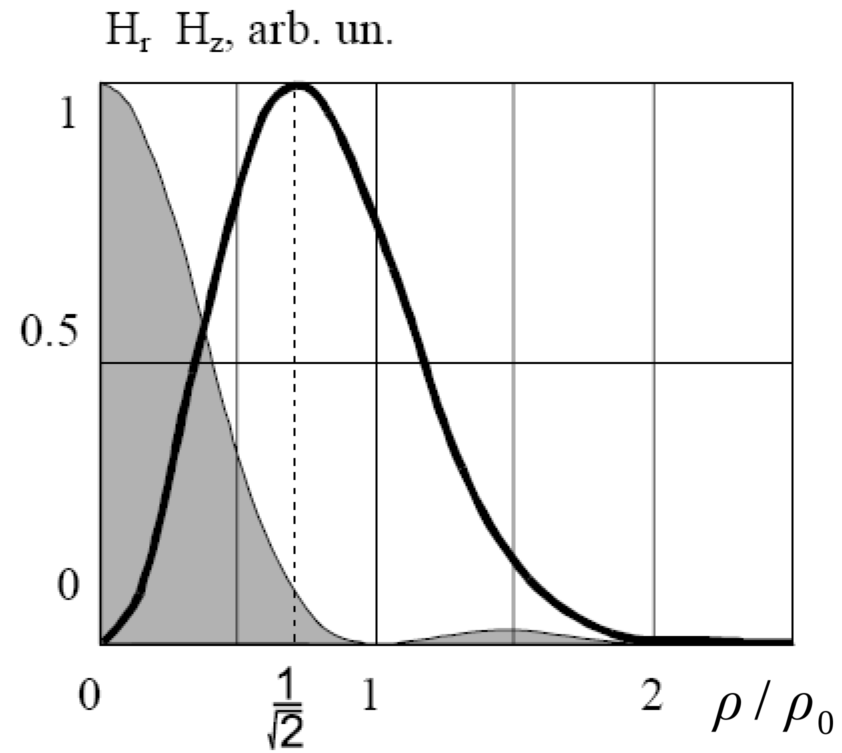
[Y. Salamin et a. PRL 88, (2002) 095005]

For $I_L \sim 10^{19}\ \mathrm{W/cm^2}$, $\lambda \sim 1\ \mu\mathrm{m}$, $E_0 \sim 10^{11}\ \mathrm{V/cm}$

magnetic field

$$H_z \sim 10^{-5}\ \mathrm{Hs} = \chi H_S,$$

$$H_S = \frac{m^2c^3}{e\hbar} - \text{Schwinger field}$$



Radial distribution of longitudinal components H_Z (filled region) and radial H_ρ (curve)

Laser beam fields
(linear polarization on)

Laser beam with weak focusing
Paraxial approximation

$$\mathrm{E}_x \approx E_0 \exp\left\{-\frac{\rho^2}{\rho_L}\right\}$$

$$\mathrm{E}_y = 0$$

$$\mathrm{E}_z = 0$$

$$\mathrm{H}_x = 0$$

$$\mathrm{H}_y = \mathrm{E}_x / c$$

$$\mathrm{H}_z = 0$$

Tightly focused beam

$$\mathrm{E}_x \sim E_0 \exp\left\{-\frac{\rho^2}{\rho_L^2}\right\}\frac{\rho_0}{\rho_L}$$

$$\mathrm{E}_y \sim E_0 \exp\left\{-\frac{\rho^2}{\rho_0}\right\}\frac{x}{\rho_0}\left(\frac{\lambda}{\pi\rho_0}\right)^2\left(\frac{\rho_0}{\rho}\right)^3$$

$$\mathrm{E}_z \sim E_0 \exp\left\{-\frac{\rho^2}{\rho_0}\right\}\frac{y}{\rho_0}\left(\frac{\lambda}{\pi\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)^2$$

$$H_x = 0$$

$$H_y \sim E_x / c$$

$$H_z \sim E_0 \exp\left\{-\frac{\rho^2}{\rho_0^2}\right\}\frac{\lambda}{\pi\rho_0}\left(\frac{\rho_0}{\rho}\right)^2$$

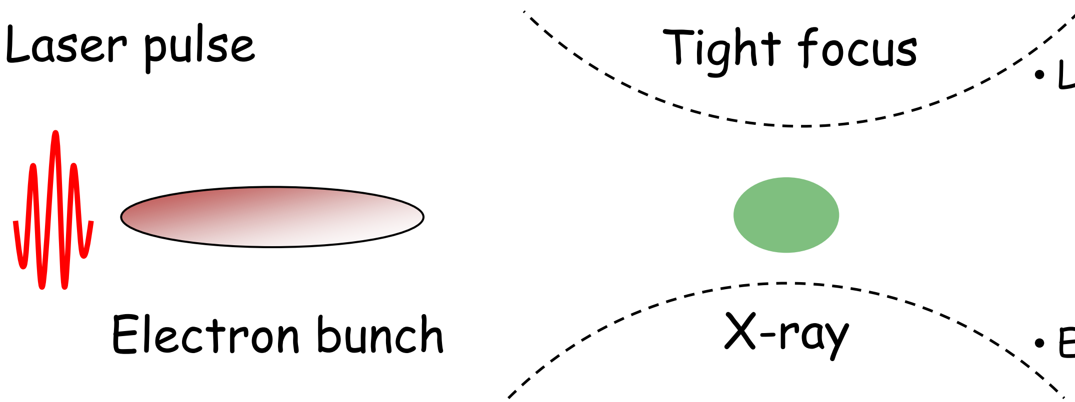
Electron/positron strongly interacts with laser radiation nearly the focus region.

There is no detailed model describing this process.

Simulation results

[H. Lee et al. New Journ. of Phys. 10(2008) 093024]

Electron Bunch : fs / as X-ray Pulse



- Laser
  - Pulse width : 5 fs
  - Wave length : 800 nm
  - Intensity : 2.2X10^{20} W/cm^2
  - Beam waist : 5um
- Electron bunch
  - Energy : 50 MeV
  - Charge : 1 nC
  - Normalized emittance : 2 mm mrad
  - Bunch length : 14 um
  - Bunch radius : 30 um

The effect of non-paraxial high-order fields due to tight focusing turns out to be dramatic. An electron radiates more strongly when the electron is initially located off the laser axis by about the beam waist than when on the laser axis. An enhancement by a factor of 2000 is observed for the focused (w0 = 5μm) laser intensity of 5×10^18 Wcm^−2 compared with a paraxial Gaussian beam case. The longitudinal field (*Ez*) *near the focus* plays an important role, greatly changing the radiation pattern.

Effect of longitudinal magnetic field

One may expect that near tight focus due to longitudinal magnetic field spin-flip probabilities for both components of unpolarized electron/positron beam will differ:

$$\frac{d\sigma_{+-}}{dy} \neq \frac{d\sigma_{-+}}{dy} \quad (*)$$

Increasing of radiation intensity(photon multiplicity) plus spin-flip transitions (*) may lead to radiative polarization of a beam as whole.

Rough analogy with pure synchrotron radiation

The self – polarization time due to spin-flip transition:

$$T_0 \sim \frac{\hbar}{mc^2}\frac{1}{\gamma^2\chi^3} \quad \text{for polarization along magnetic field} \sim 90\%$$

In the R-system where a positron is in a rest

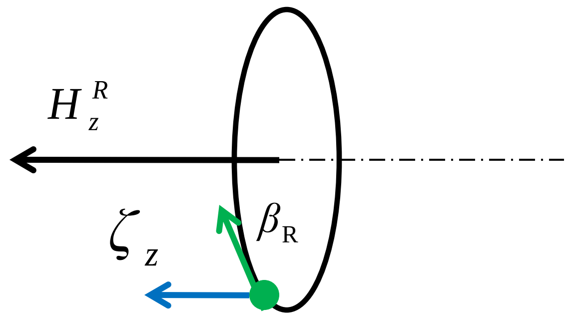
(in average)

$\beta_R \to \beta_\perp,$

$\gamma_R \to \gamma_\perp \sim 1,$

$H_z^R = H_z$

$\chi_R \to \chi \sim 10^{-5}$



Estimation of positron polarization

In R-system $T_0^R \sim \frac{\hbar}{mc^2}\frac{1}{\chi^3}$ and in a lab system $T_0 \sim \frac{\hbar}{mc^2}\frac{1}{\gamma\chi^3}$ or,

introducing "radiative polarization length" $L_P = cT_0 \sim \frac{\lambda\!\!\!^{-}_e}{\gamma\chi^3}$.

Passing the length L_p positron achieves polarization $\zeta_z \sim 90\%$.

For $\gamma = 10^5$, $\chi \sim 10^{-5}$

$$L_P \sim 4\ mm\,!$$

ROUGH ANALOGY AND ROUGH ESTIMATION!

Conclusion

- Even for small mean number of collisions (<k>~1) there is significant contribution of events with k = 2,3… photons from each electron/positron
- The ordinary multiple Compton – backscattering process (plane – wave approximation) may provide polarization of [illegible] unpolarized [illegible]
- Rough estimation of “radiative polarization length” looks as promising
- A spin – dependent model describing interaction of electrons/positrons with tightly focused laser beam should be developed

Thanks for your attention!