

RESONANT DEPOLARIZATION AT THE ILC DR WITH RF DIPOLES

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MOTIVATION

- The undulator based positron source will provide $|P_{e^+}| \geq 30\%$.
- It is important to *identify and reduce any systematic errors* arising from the accelerator itself in the experiments with polarised beams.
- The direction of polarisation should be reversed or even some *bunches can be intentionally depolarised completely* in order to compare data for unpolarised and polarised beams under same machine conditions. The background processes can be identified thus reducing the systematic errors in the experiments.
- So it would be a good solution to have *possibility of destroying the positron polarization to reduce systematic errors and level of uncertainties* for experiment.

RESONANT DEPOLARIZATION

- **Resonant depolarization** is produced by exciting the beam with an oscillating magnetic field (**kicker**).
- The exciting field of the **kicker** is perpendicular to the beam axis and situated on the plane of the ring.
- A **resonance occurs** when the rf magnetic field's frequency is synchronized with the spin tune and the circulation frequency:

$$f_r = f_c (n \pm \nu_s)$$

- When the kicker frequency is close to the resonant frequency, the **kicks add up coherently**, and the cumulative effect of the kicks is to tilt the spins strongly away from the vertical.
- **Positron spins** are coherently **swept away from the vertical direction** and **polarization disappears**.

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The Single Resonance Model assumes that *one of frequencies of Fourier decomposition is nearly in resonance with the spin tune*.

Spin will see the same component of perturbing field turn after turn leading to build-up tilt of the spin. The *amplitude* of this single component is often called the *resonance strength*.

FROISSART-STORA FORMULA

Froissart and Stora (1960) found the *exact analytical solution* for the depolarization of a spinpolarized beam caused by the crossing of a *single resonance driving term*, i.e. only one Fourier harmonic in the perturbing Hamiltonian.

The Froissart–Stora formula assumes that the *depolarizing resonances are narrow and well-separated*. Hence the beam *crosses only one resonance* at a time.

$$\frac{P_f}{P_i} = 2 \exp\left\{\frac{-\pi |\varepsilon|^2}{2\alpha}\right\} - 1 \quad \varepsilon = \frac{(1 + G\gamma) \int B dl}{4\pi B \rho} \text{ is resonance strength.}$$

α is the rate of resonance crossing (crossing speed).

If the rf dipole tune is swept across an interval ΔQ in N turns, then $\alpha = \frac{\Delta Q}{2\pi N}$

Three distinct conditions for the variation rate crossing ΔQ are:

Rate	Polarization	Effect
Fast crossing	$P_f \approx P_i$	Little or no depolarization
Medium crossing	$P_i > P_f > -P_i$	Depolarization
Slow crossing	$P_f \approx -P_i$	Spin-flip

SIMPLIFIED MODEL

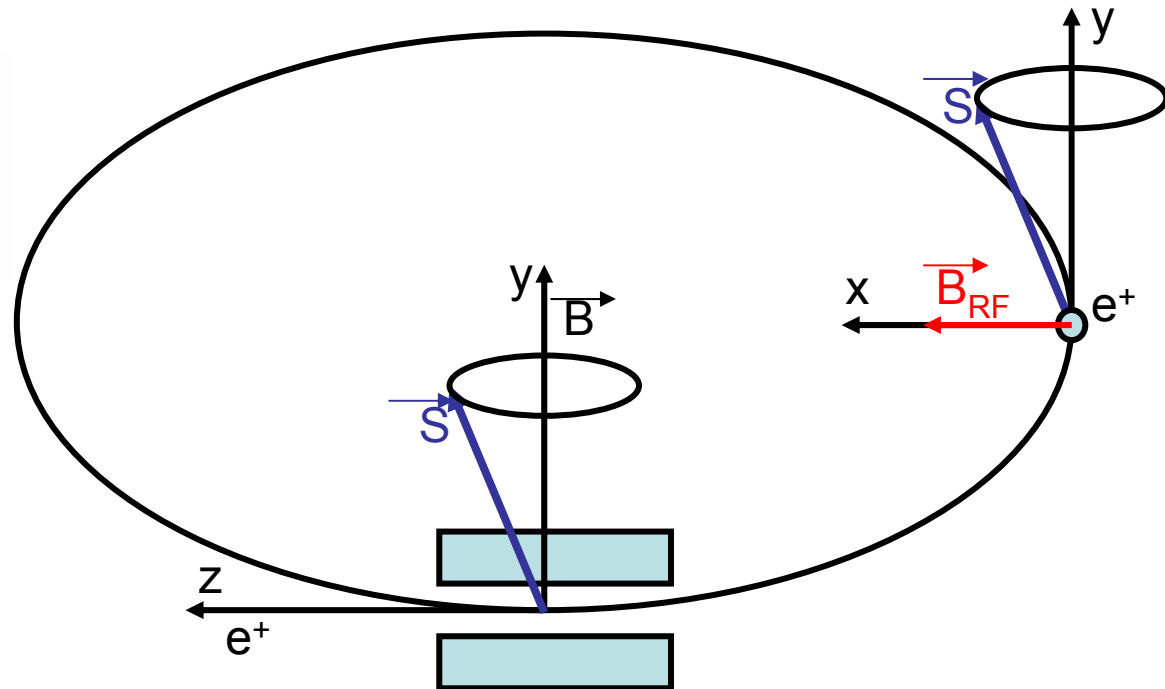
- The model consists of an RF dipole followed by continuous bending magnet.
- Spin behaviour of bunch of particles is described by applying rotation matrices at each revolution turn.

RF Dipole

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

Dipole Magnet

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$



NUMERICAL SIMULATION

- Initial polarization (vertical): 30%
- Spin tune $G\gamma=11.35$
- Revolution frequency=92.5 kHz
- Resonance frequency=60.17 kHz
- Number of turns (N)=9256

To depolarize beam, we scan the dipole frequency across the spin resonance.

Spin vector gets precessed around the horizontal X-axis at every m th turn

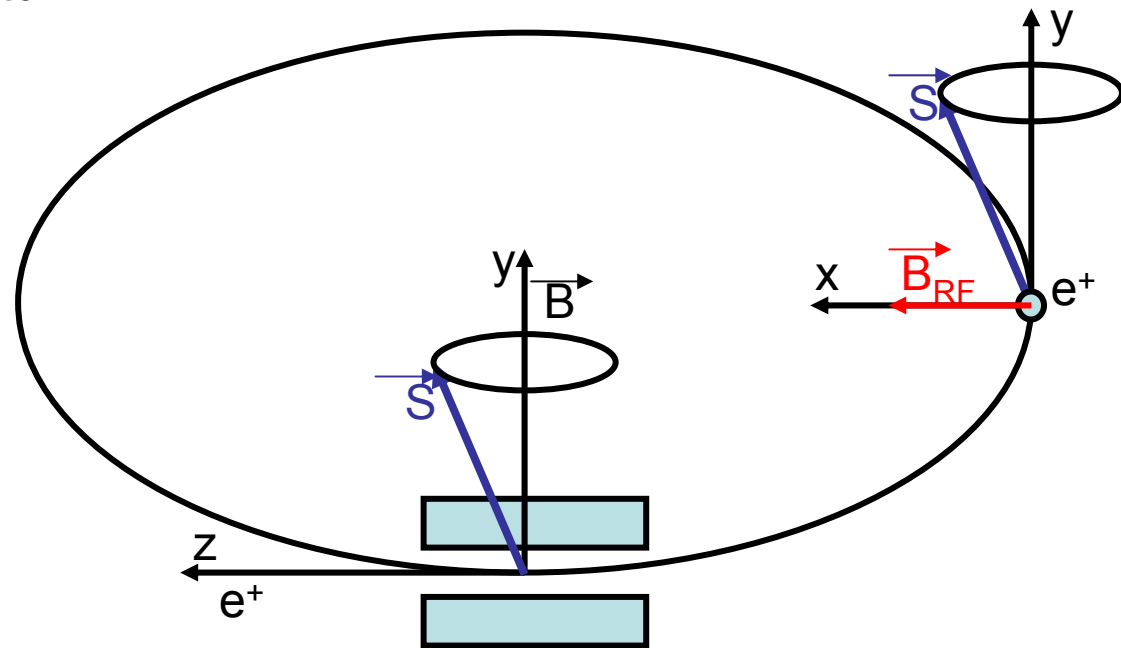
$$\theta_{xm} = \frac{(1 + G\gamma) \cdot B_{RFm} L_{RF}}{B\rho}$$

$$B_{RFm} L_{RF} = BL \cos(2\pi mQ)$$

A spin resonance is observed

$$G\gamma - \text{Integer}(G\gamma) = Q$$

Speed of crossing: $\alpha = \frac{\Delta Q}{2\pi N}$

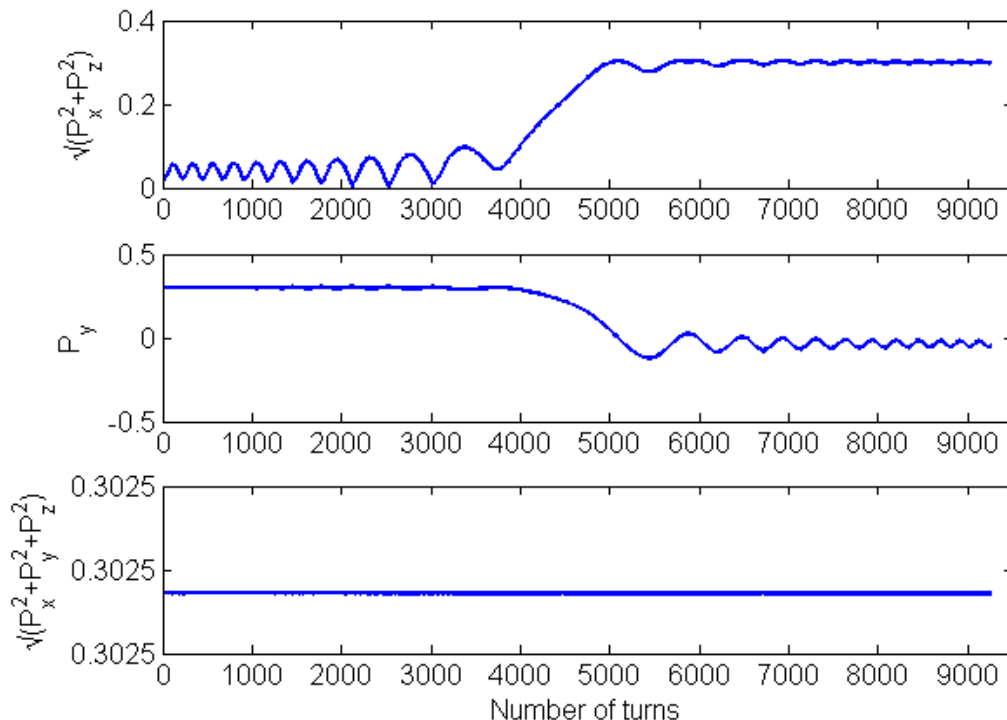


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Energy spread = 0%

A spin resonance is observed

$$G\gamma - \text{Integer}(G\gamma) = Q$$

Mean polarization is

$$\sqrt{P_x^2 + P_y^2 + P_z^2}$$

DEPOLARIZATION CONDITIONS

- Initial polarization (vertical): 30%
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- Resonance frequency=60.17 kHz
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Integrated field of rf dipole was taken from the COSY experiment $\int Bdl$

$$5.8 \times 10^{-4} T m$$

Resonance strength is changed as follows:

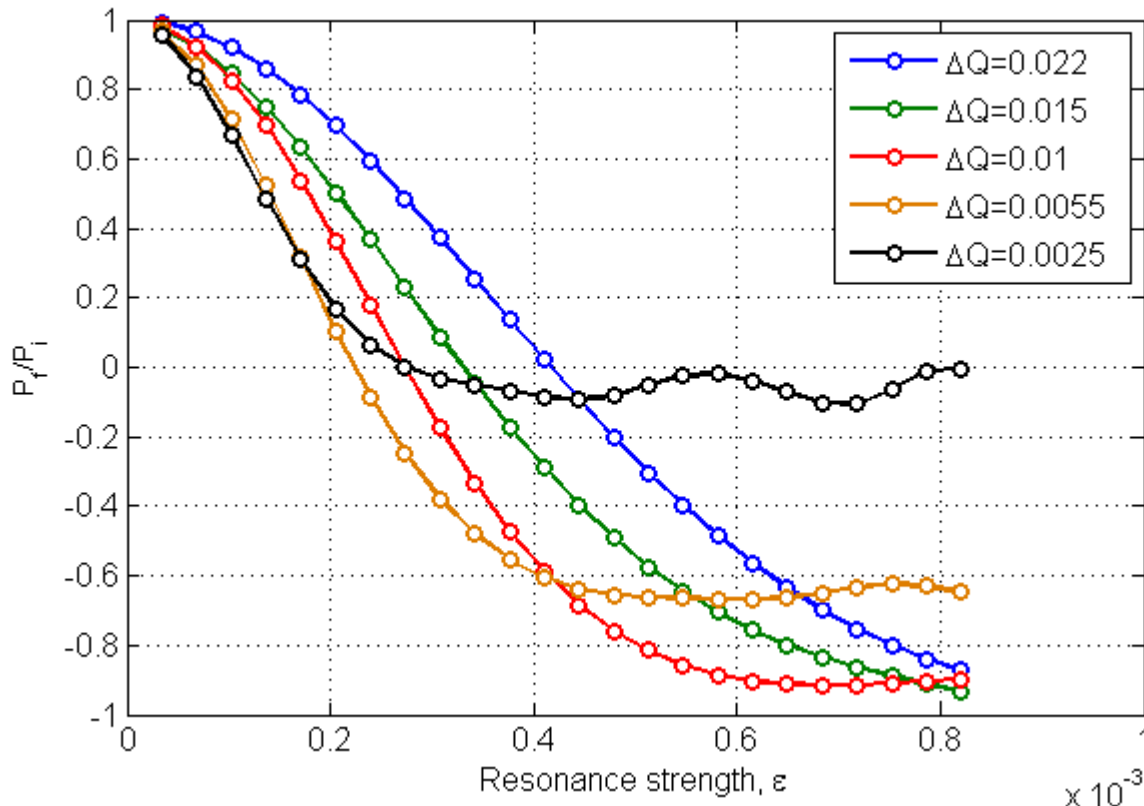
$$\varepsilon = \frac{(1 + G\gamma)n \int Bdl}{4\pi B\rho}$$

$$n = 1, 2, 3, \dots$$

Energy spread = 0.5%

Speed of crossing:

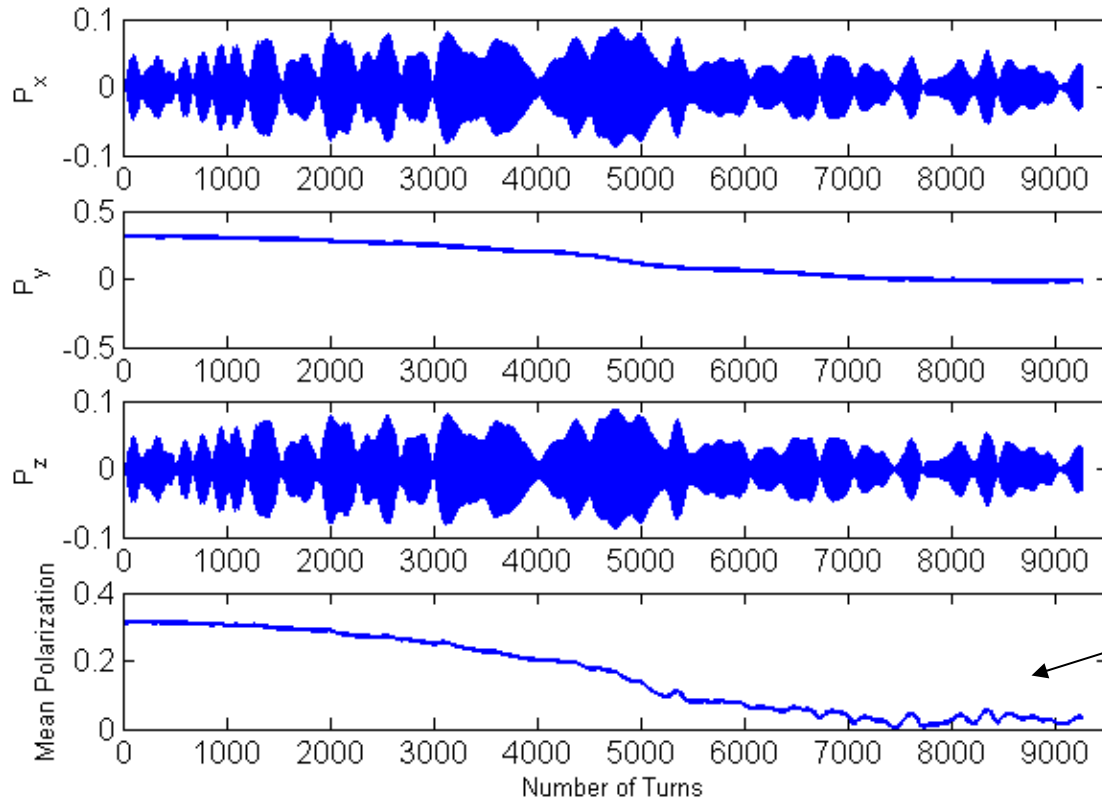
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Integrated field of rf dipole:
 $n \cdot \int B dl = 8 * 5.8 \times 10^{-4} = 0.0046 \text{ T m}$



Resonance strength:

$$\varepsilon = \frac{(1 + G\gamma)n \int B dl}{4\pi B \rho} = 2.73 \cdot 10^{-4}$$

Mean polarization goes to 0

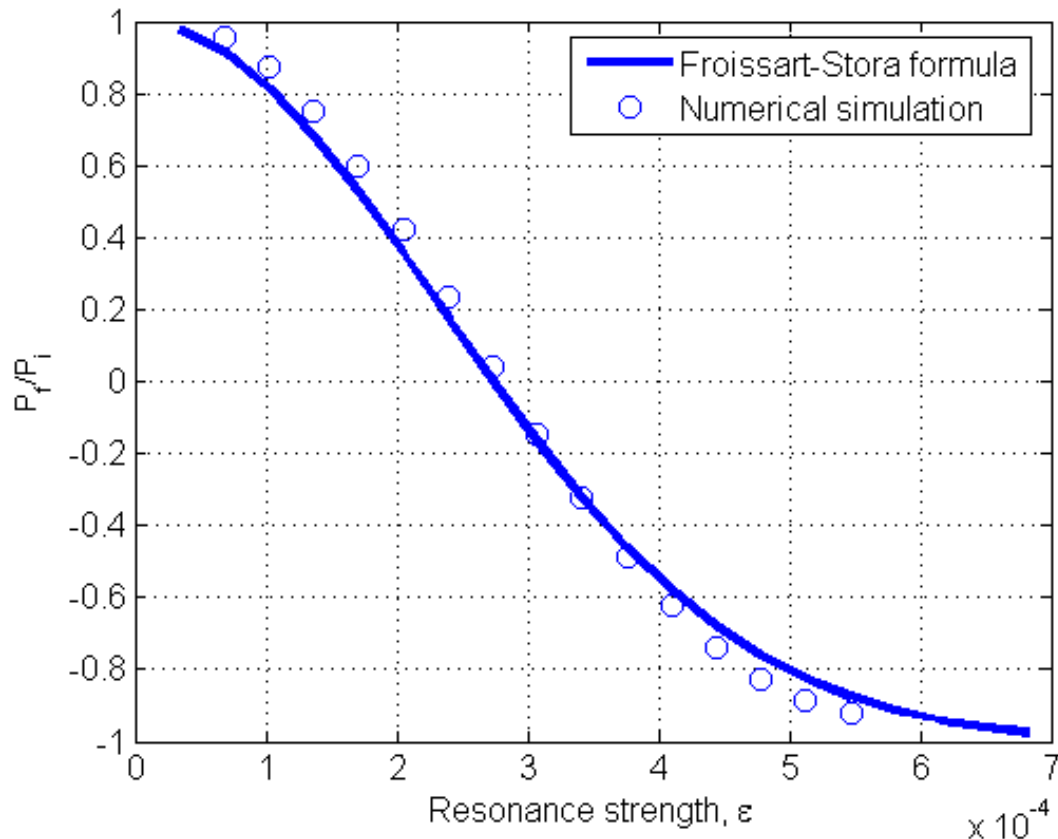
$$\sqrt{P_x^2 + P_y^2 + P_z^2}$$

NUMERICAL SIMULATION AND ANALYTICAL FORMULA

- Spin tune $G\gamma=11.35$
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Froissart-Stora formula:

$$\frac{P_f}{P_i} = 2 \exp\left\{ \frac{-\pi |\varepsilon|^2}{2\alpha} \right\} - 1$$



CONCLUSIONS

- An rf dipole with oscillating magnetic field can be used to induce a spin resonance.
- As a first approach the simplest model was investigated and the appropriate value for resonance strength was found in order to depolarize the beam. The results are in a good agreement with analytical Froissart-Stora formula.

FUTURE PLANS

- The rf dipole also excites a vertical coherent betatron oscillation. This coherent oscillation introduces additional perturbations on the spin motion due to the quadrupole fields the particle experiences along its trajectory.
- This effect can either cancel or enhance the rf dipole driven resonances depending on the lattice parameters.
- Therefore spin tracking through the real lattice including the rf dipole has to be simulated.