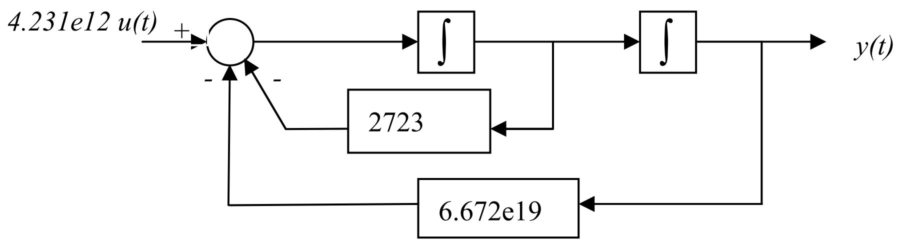
Homework Solutions 2012

1. Solution:

a) The cavity state space equation is

$$\dot{x} = Ax + Bu$$
$$y = Cx$$
$$A = \begin{bmatrix} 0 & 1 \\ -6.672\times10^{19} & -2723 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 4.231\times10^{12} \end{bmatrix}$$

b) The block diagram is



c) We need to calculate the poles of the equation

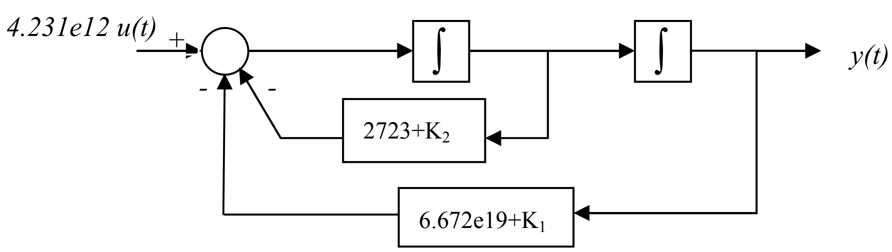
$$|\lambda I - A| = 0 \quad \Rightarrow$$
$$\begin{vmatrix} \lambda & -1 \\ 6.672\times10^{19} & \lambda + 2723 \end{vmatrix} = 0 \quad \Rightarrow$$
$$\lambda^2 + 2723\lambda + 6.672\times10^{19} = 0 \quad \Rightarrow$$
$$\lambda \approx -1361 \pm j8.163\times10^{9}$$

There are two conjugate poles which located in the left part of the s plane, so the open loop system is stable.

d) The closed loop state space equation is

$$\dot{x} = Ax + B(u + u_{fb}) = (A - BK)x + Bu = A_{cl}x + Bu$$
$$y = Cx$$
$$A_{cl} = \begin{bmatrix} 0 & 1 \\ -6.672\times10^{19} - K_1 & -2723 - K_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 4.231\times10^{12} \end{bmatrix}$$

e) Block diagram for the closed loop system



f) The characteristic equation of the closed loop system is

$$\lambda^2 + (2723 + K_2)\lambda + 6.672\times10^{19} + K_1 = 0$$

We can ignore K_1 because the resonance frequency is large and we do not want to change the resonance frequency. The system is stable when

$$2723 + K_2 > 0$$

g) To extend the closed loop system bandwidth by 4 times, we get

$$\frac{2723 + K_2}{2723} = 4$$

So K_2 = 8169, and we can set K_1 = 0.

2. Solution:

a) The closed loop function is

$$T_{cl} = \frac{KG(s)}{1 + KG(s)} = \frac{1360K}{s + 1360(K+1)}$$

The closed loop system is stable if the feedback gain $K > 0$.

b) Consider the stability criteria in frequency domain.
The 1 μs loop delay will generate -180 degree phase delay at the frequency of 500 KHz and -90 degree at the frequency of 250 KHz, and at those frequencies, the system open loop transfer function will make a phase delay close to -90 degree. So at the frequency of 250 KHz, the loop phase is -180 degree, and the open loop gain is 1360/(2*pi*250e3) = 8.658e-4. The system will be stable when the overall loop gain is smaller than 1. So

K * 8.658e-4 < 1 => K < 1155.

c) To reduce the disturbance by 100 times, we need to set the loop gain to 100. In this case, the unity gain frequency of the open loop transfer function is calculated by the equation below

$$\left| \frac{1360 * 100}{j\omega + 1360} \right| = 1.$$

Approximately, ω = 100 * 1360, so the frequency is 21.65 KHz. If the loop delay generates a phase delay greater than (in absolute value) -90 degree at this frequency, the system will be unstable, so the delay should be smaller than ¼ * 1/21.65e3 = 11.55 μs.

3. Solution:

a) We will get maximum steady state voltage when the cavity detuning is zero.
The cavity half bandwidth and the loaded resistance are

$$\omega_{1/2} = \frac{\omega_0}{2Q_L} = \frac{\pi f_0}{Q_L} = 1.3614 \times 10^3 rad$$

$$R_L = \frac{1}{2}\left(\frac{r}{Q}\right) Q_L = 1.554 \times 10^9 \Omega .$$

The cavity driving current can be calculated from the forward power

$$I_g = 2\sqrt{\frac{2P_{for}}{R_L}} = 0.0321A$$

So the maximum cavity voltage is

$$V_{\max} = R_L I_g = 49.86MV$$

b) If the cavity is detuned by 200 Hz, the detuning angle will be (the phase difference between the cavity input and output)

$$\psi = tg^{-1}\left(\frac{\Delta\omega}{\omega_{1/2}}\right) = 0.7454rad$$

The cavity steady state voltage will be

$$V = R_L I_g \cos\psi = 36.64MV$$

c) The beam injection time is calculated as

$$t_{inj} = \frac{1}{\omega_{1/2}} \ln\left(\left|\frac{I_g}{2I_{b0}}\right|\right) = 511\mu s$$

4. Solution:

When there is no detuning, the filling power of the cavity is decided by the desired cavity voltage, filling time and quality factor of the cavity

$$P_{fill} = \frac{V_0^2}{4\left(\frac{r}{Q}\right)Q_L \cdot \left[1 + e^{-\frac{\omega_0 T_{fill}}{Q_L}} - 2e^{-\frac{\omega_0 T_{fill}}{2Q_L}}\right]} = 206.24KW\ .$$

To accelerate a beam of 5mA at 25 MV, the RF power required at the flattop is

$$P_{flattop} = \frac{V_0^2}{4\left(\frac{r}{Q}\right)Q_L}\left(1 + \frac{\left(\frac{r}{Q}\right)Q_L I_{b0}}{V_0}\right)^2 = 132.2KW$$

The RF power delivered to the beam is P_{beam} = 25 MV * 5 mA = 125 KW. Assume we can ignore the power dissipation on the superconducting cavity wall, so the reflected power at the RF flattop is 7.2 KW.

5. Solution:

The optimized detuning and loaded quality factor are

$$\Delta f_{opt} = -\frac{\left(\frac{r}{Q}\right) I_{b0} f_0}{2V_0} \sin\phi_b = -107.7Hz$$

$$Q_{L,opt} = \frac{V_0}{\left(\frac{r}{Q}\right) I_{b0} \cos\phi_b} = 3.483 \times 10^6$$

The required klystron power is

$$P_{flattop} = \frac{V_0^2}{\left(\frac{r}{Q}\right) Q_{L,opt}} = 173.21KW .$$

In this case, all the RF power is delivered to the beam, so there is no reflection power during flattop.

6. Solution:

The SEL system works at the frequency with the loop phase integer times of 2π. So the working frequency nearest to 1.3 GHz satisfies the condition of

$$\frac{f}{1.3GHz} = \frac{1000 * 360}{1000 * 360 + 10} .$$
$$f = 1.3GHz - 36.11KHz$$

7. Solution:

The signal to noise ratio caused by the clock jitter is

$$SNR_{jitter} = -20 \log_{10}\left(2\pi f_{RF} t_{jitter\_rms}\right) = 47.48dB .$$

Compared to the ADC quantization noise, the noise caused by the clock jitter is dominated, which will cause phase measurement error of 10^(-47.48/20) * 180 / pi = 0.24 degree, and amplitude measurement error of 0.42%.

8. Solution:

From the equation provided, the loaded quality factor can be calculated as

$$\omega_{1/2} = 2720 \qquad \Rightarrow$$
$$Q_L = \frac{\omega_0}{2\omega_{1/2}} = 1.5 \times 10^6$$

The detuning can be calculated from the slope of the phase equation

$$\Delta f = \frac{1}{2\pi} \frac{d\varphi}{dt} = -33Hz$$

9. Solution:

a) The measured vector sum is

$$V_{sum,mea} = V_1 + V_2 \cdot 1.01 e^{j\pi/180} = (44.22,102.1)$$

The vector sum seen by the beam is

$$V_{sum,true} = V_1 + V_2 = (50,100)$$

b) If there is microphonics, the cavity field will be changed to

$$V_1' = V_1 \cdot 1.05 e^{j5\pi/180} = (122.9, -200.05)$$
$$V_2' = V_2 \cdot 1.02 e^{-j3\pi/180} = (-34.92, 308.25)$$

So the new measured vector sum and vector seen by the beam are

$$V'_{sum,mea} = V_1' + V_2' \cdot 1.01 e^{j\pi/180} = (82.21, 110.62)$$
$$V'_{sum,true} = V_1' + V_2' = (87.99, 108.2)$$

The feedback will compensate the changes of the measured vector sum perfectly, which means, the measured vector sum will be scaled and rotated back to the original value, the complex gain generated by the controller is

$$G = \frac{V_{sum,mea}}{V'_{sum,mea}},$$

The gain will be applied on the vector sum seen by the beam, so after the feedback, the vector sum seen by the beam will be

$$V''_{sum,true} = G V'_{sum,true} = (49.21, 101.24)$$

So the amplitude error is 0.68% and the phase error is 0.64 degree.

10. Solution:

a) The beam induced voltage is

$$\left|V_{beam,MV}\right| = I \cdot \Delta t \cdot \left(\frac{r}{Q}\right) \cdot \pi \cdot f_0 = 0.381MV$$

b) The cavity voltage can be calculated as

$$\left|V_{cav,MV}\right| = \frac{\left|V_{cav}\right| \cdot \left|V_{beam,MV}\right|}{\left|V_{beam}\right|} = 9.41MV$$

And be the beam phase is

$$\varphi_b = 180^o - (\angle V_{beam} - \angle V_{cav}) = -61.98^o$$