Muon Collider Homework November 29, 2012

Problem 1 – Muon Collider Luminosity

Recall the expression we used to make an estimate of the luminosity for a circular muon collider.

$$\mathcal{L} \approx \frac{\left\langle N^2 \right\rangle_{n_{turns}} n_{turns} f_{bunch}}{4\pi\sigma_{\perp}^2}$$

This expression assumes that a single pair of counter-rotating bunches are injected into the collider with a frequency f_{bunch} and that they collide for n turns.

(a) Since the bunches are decaying away, suggest a better way to write the luminosity in terms of *N(t).*

(b) For a 1.5 TeV collider, what is the exponential decay constant for *N(t)*, the number of particles in the bunch.

(c) Assuming that the decay time is far shorter than the spacing between the injection of the bunches, what is the integrated luminosity we expect from each injection cycle based on the 1.5 TeV collider parameters?
Recall: $N_0=2\times10^{12}$; $\beta^* = 10$ mm; $\varepsilon_{x,y}$ *(norm)* = 25 μm-rad

(d) What is the effective luminosity of the machine assuming a 15Hz injection rate?

Problem 2 – Particle Motion in Solenoid Fields

Consider a long solenoid centered on the z-axis with field B_z=3.33 T. A particle located at the origin of our coordinate system, with initial momenta p_z=200 MeV/c and p_x=20 MeV/c, is moving through the solenoid.

(a) Describe the particle's trajectory in the lab frame. What is the radius of curvature of the particle and what is the maximum radial distance it travels from the z-axis?

(b) At what distance along the z-axis does the particle next cross the z-axis?

(c) What is the wavelength in the Larmor frame, λ_{Larmor}?

(d) What is the lattice parameter $\beta_{\perp}$ for the particle?

Problem 3 – Ionization Cooling

Consider a linear cooling channel with $\beta_{\perp}$ = 40cm and liquid H_2 as the material providing the ionization energy loss. For a 200 MeV/c muon, what is the expected equilibrium emittance?

Problem 4 – Ionization Cooling and Solenoid

Consider, once again, the particle in the solenoid of problem 2. If this were part of an ideal cooling system with continuous energy loss and re-acceleration, the transverse momentum of the particle would eventually be reduced to nearly zero.

(a) Describe the trajectory of the particle as its transverse momentum drops
(b) What is the final distance from solenoid axis?
(c) What does the particle's trajectory in the Larmor frame look like?
(d) Assume that the particle reaches the end of the solenoid (assumed to be a sharp edge where B_z rapidly drops to zero). What happens to the motion of the muon?