2010 LINEAR COLLIDER SCHOOL
SUPERCONDUCTIN RF
HOMEWORK

Problem 1

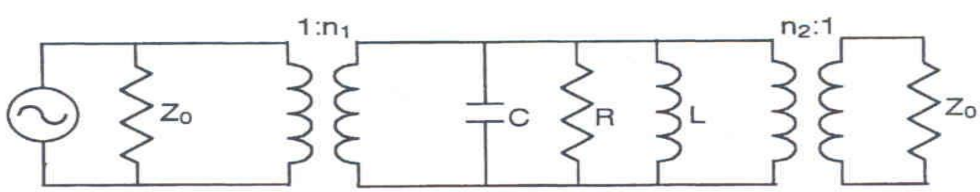
A film of thickness $2d$ made of a London superconductor (of penetration depth λ) is placed in a parallel magnetic field H_0.

Calculate the magnetic field and supercurrent in the film.

Use the coordinate system where the film extends from $x=-d$ to $x=+d$, and the magnetic field is in the $+z$ direction.

Plot both for $\frac{d}{\lambda} = 0.5, 1,$ and $5.$

Problem 2



Assume a 2-port cavity with coupling coefficients β_1 (input) and β_2 (output)
Calculate the dissipated, transmitted, and reflected power for a given incident power.
Note: With a little bit of thinking you should be able to write down the answers directly with almost no calculations.
What happens if we interchange input and output?
What happens when $\beta_1=\beta_2$?

Problem 3

The power required to operate a cavity in the presence of beam loading is given by

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1+\beta+b)^2 + \left[(1+\beta)\tan\psi - b\tan\phi \right]^2 \right\}$$

where $b = \dfrac{R_{sh} i_0 \cos\phi}{V_c}$

Derive that expression and show that the required rf power is minimal when

$(1+\beta_{opt})\tan\psi_{opt} = b\ \tan\phi,$ and $\beta_{opt} = |1+b|$

and is given by $P_g^{opt} = \dfrac{V_c^2}{R_{sh}} \dfrac{|1+b| + (1+b)}{2}$

If additionally we need to control microphonics, then show that the optimal coupling and rf power are given by

$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \dfrac{\delta\omega_m}{\omega_0} \right)^2}$ and $P_g^{opt} = \dfrac{V_c^2}{2R_{sh}} \left[(b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \dfrac{\delta\omega_m}{\omega_0} \right)^2} \right]$

Problem 4

Assume that we have a cavity providing a voltage V_0 to a very high current I_0. If the detuning and coupling are optimized, show that the required rf power is $V_0 I_0 \cos\phi$ where ϕ is the phase between the beam and the cavity voltage.
We now want the cavity to provide a voltage V using the same amount of rf power, and we want to determine what is the maximum current I we will be able to accelerate.
Define $v = \dfrac{V}{V_0}$ and $i = \dfrac{I}{I_0}$
Show that, if we can reoptimize the coupling, then the relationship between v and i is $vi = 1$.
For example, this means that if we want to increase the gradient to 1.5 times its original value then we must decrease the current to 2/3 and still provide the same beam power.
On the other hand, if the power coupler is fixed and we cannot reoptimize the coupling, then show that the relationship between v and i is $v + i = 2$.
In this case we would have to decrease the current to 1/2 and only 2/3 of the power will be transferred to the beam and 1/3 will be reflected.

Problem 5

A cavity has the following ratio of peak to accelerating fields:

E_p/E_{acc}=2.3, B_p/E_{acc}=3.6 mT/(MV/m)

When it is running at 50 MV/m, what is the radiation pressure where the magnetic field is maximum (assuming there is no electric field) and where the electric field is maximum (assuming there is no magnetic field)? Compare to atmospheric pressure.

Problem 6

The following values of $Q(E_p)$ data were measured for a Nb SRF cavity.

Field emission was detected starting at E_p=29.5 MV/m. Calculate the field enhancement factor β from the data. Take a value of 4 eV for the work function of Nb.

E_p (MV/m)	Q_0
1.55	1.73E+10
2.36	1.76E+10
3.77	1.79E+10
4.63	1.81E+10
6.26	1.78E+10
7.20	1.79E+10
9.71	1.75E+10
10.17	1.71E+10
11.99	1.72E+10
14.09	1.69E+10
17.17	1.66E+10
15.40	1.68E+10
18.84	1.67E+10
20.46	1.62E+10
23.56	1.65E+10
22.02	1.64E+10
27.81	1.59E+10
25.73	1.61E+10
29.47	1.51E+10
31.30	1.39E+10
32.69	1.29E+10
35.24	9.81E+09
34.26	1.10E+10
36.90	7.89E+09
38.00	6.84E+09