Introduction
Top Physics, QCD, Loopverein

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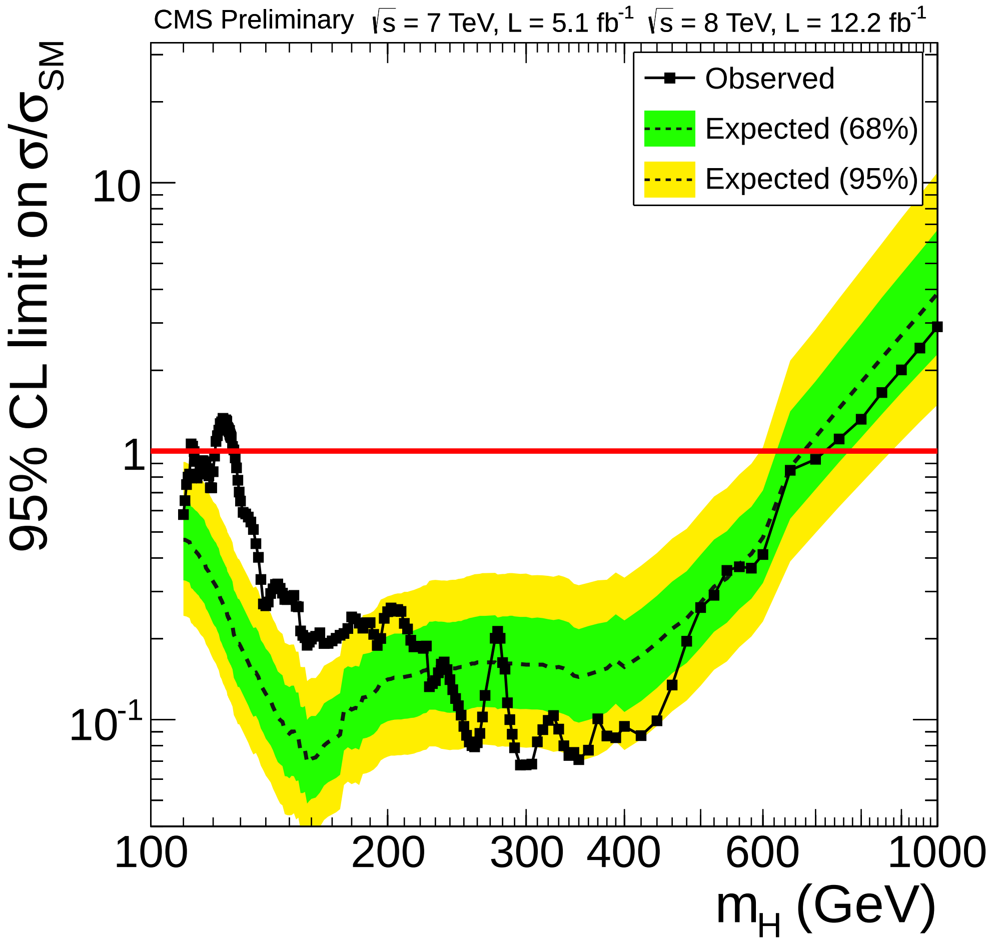
Example from LHC Higgs measurements

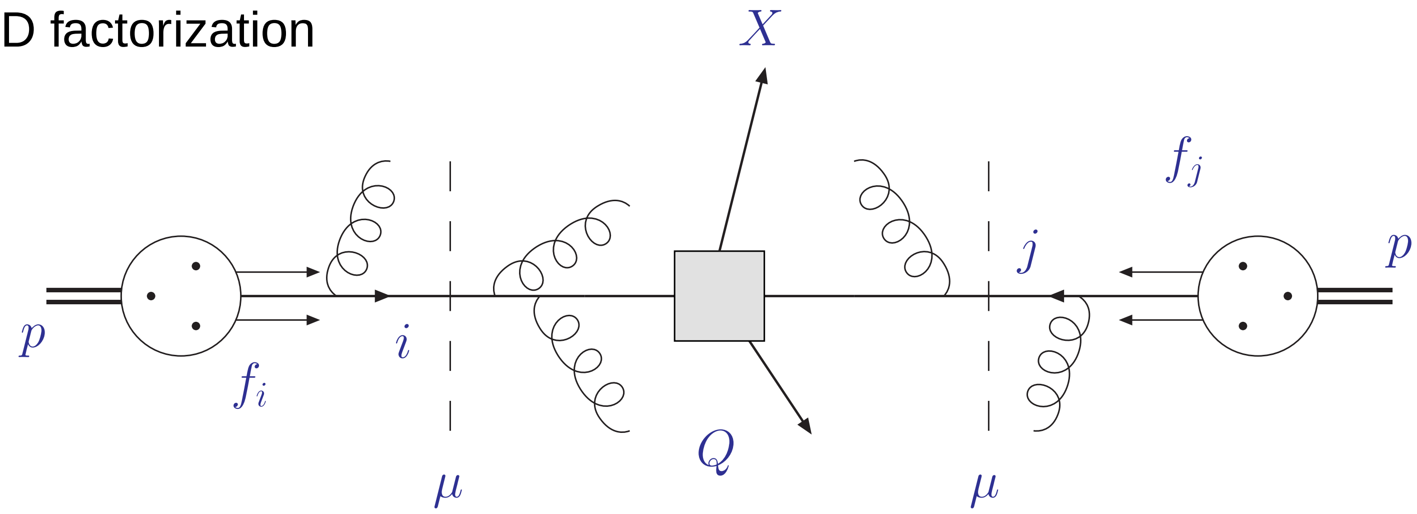
CMS coll. Dec 2012



- Signal strength of all analyzed decay modes
  - normalization to Standard Model expectation
  - accuracy of $\sigma_{\rm SM}$ crucial

QCD factorization

- QCD factorization

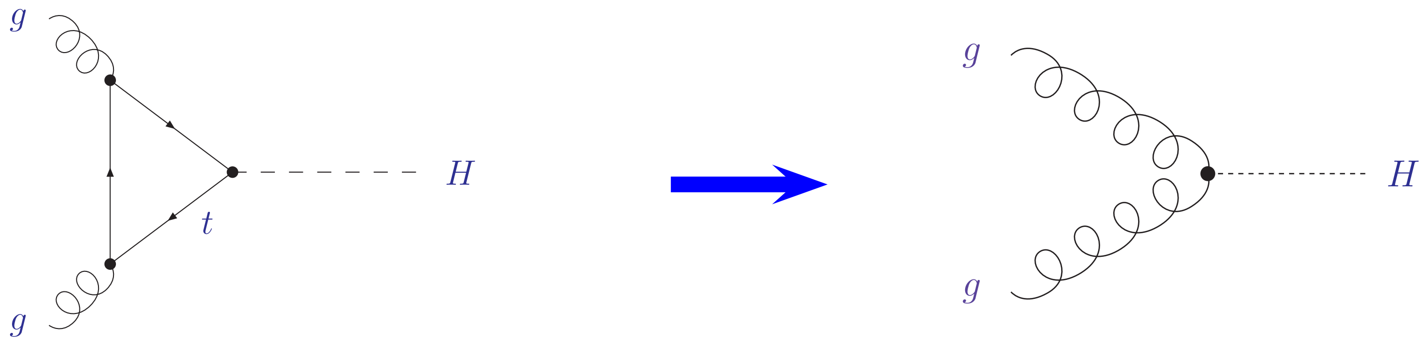


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Higgs production in gg-fusion

Effective theory

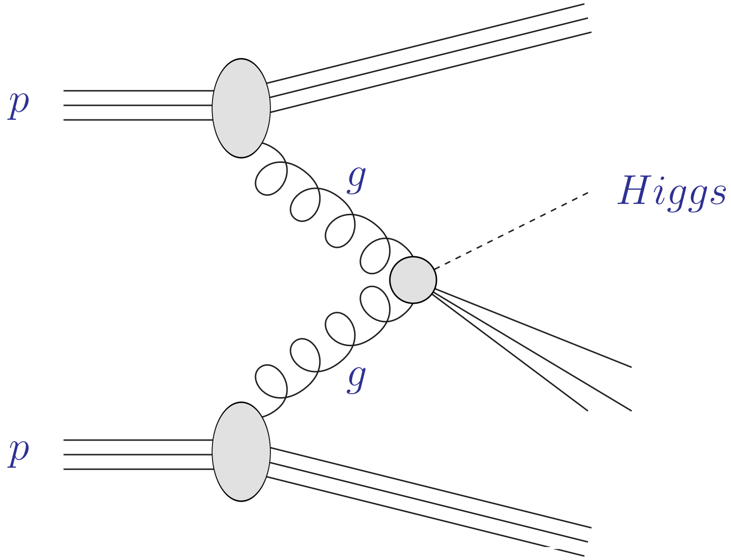


- Integration of top-quark loop (finite result)
  - decay width $H \to gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H \to gg} \; = \; \frac{G_\mu \, m_H^3}{64 \, \sqrt{2} \, \pi^3} \, \alpha_s^2 \;\, f\left(\frac{m_H^2}{4 m_t^2} \right)$$

- Effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4} \, \frac{H}{v} \, C_H \, G^{\mu\nu \, a} \, G^a_{\mu\nu}$
  - operator $H G^{\mu\nu \, a} \, G^a_{\mu\nu}$ relates to stress-energy tensor
  - additional renormalization proportional to QCD β-function required
    Kluberg-Stern, Zuber ‘75; Collins, Duncan, Joglekar ‘77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

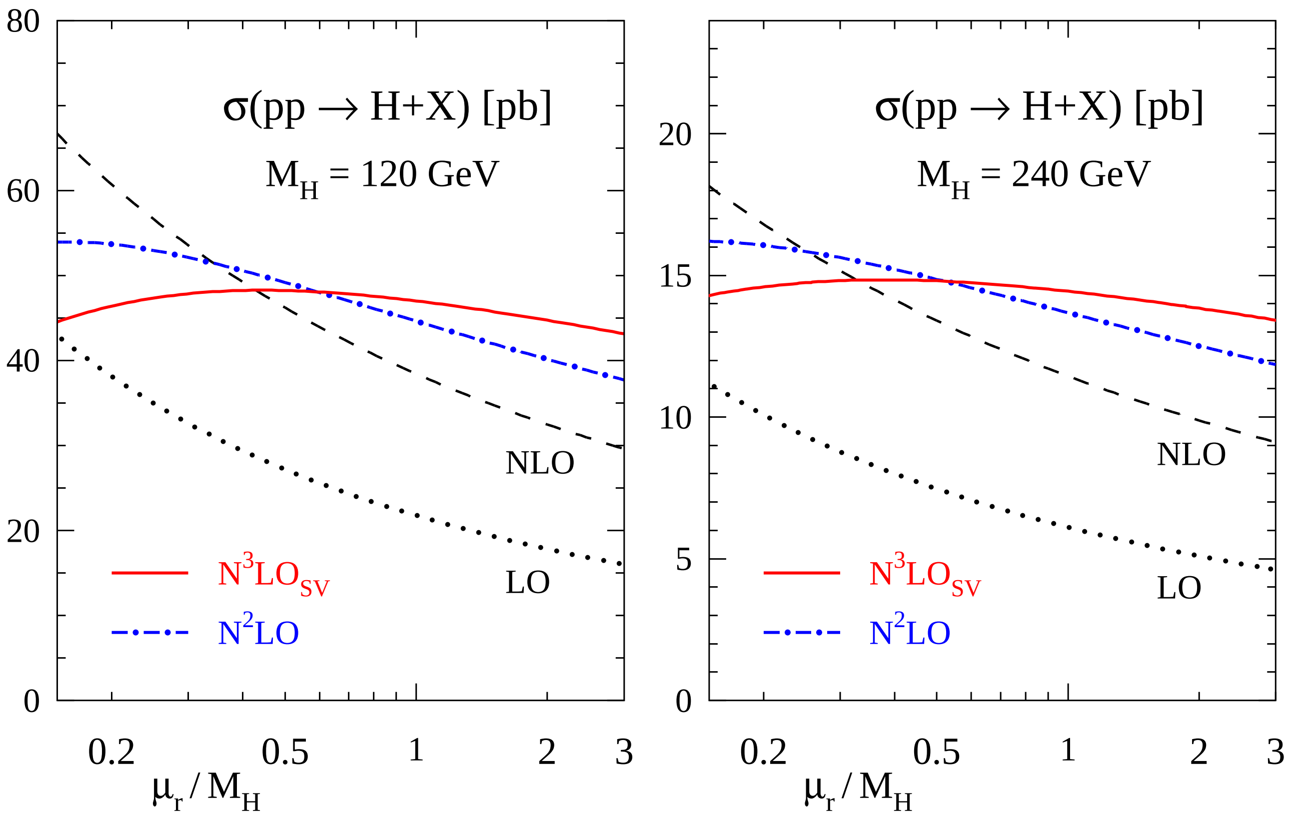
$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

- Partonic cross section $\hat{\sigma}_{ij\to H}$

$$\hat{\sigma}_{ij\to H} = \underbrace{\alpha_s^2 \left[\hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to H}\right.}_{\text{NLO: standard approximation (large uncertainties)}} \left. + \; \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to H} + \ldots \right]$$

NLO: standard approximation (large uncertainties)

Perturbation theory at work



- Apparent convergence of perturbative expansion
  - NNLO corrections still large

    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement through complete soft N^3LO corrections S.M., Vogt '05 or NNLL resummtion Catani, de Florian, Grazzini, Nason '03, Ahrens et al. '10
- Perturbative stability under renormalization scale variation

Non-perturbative parameters

Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
  - masses of heavy quarks m_c, m_b, m_t
  - parton distribution functions $f_i(x, \mu^2)$
  - strong coupling constant $\alpha_s(M_Z)$

Interplay with perturbation theory

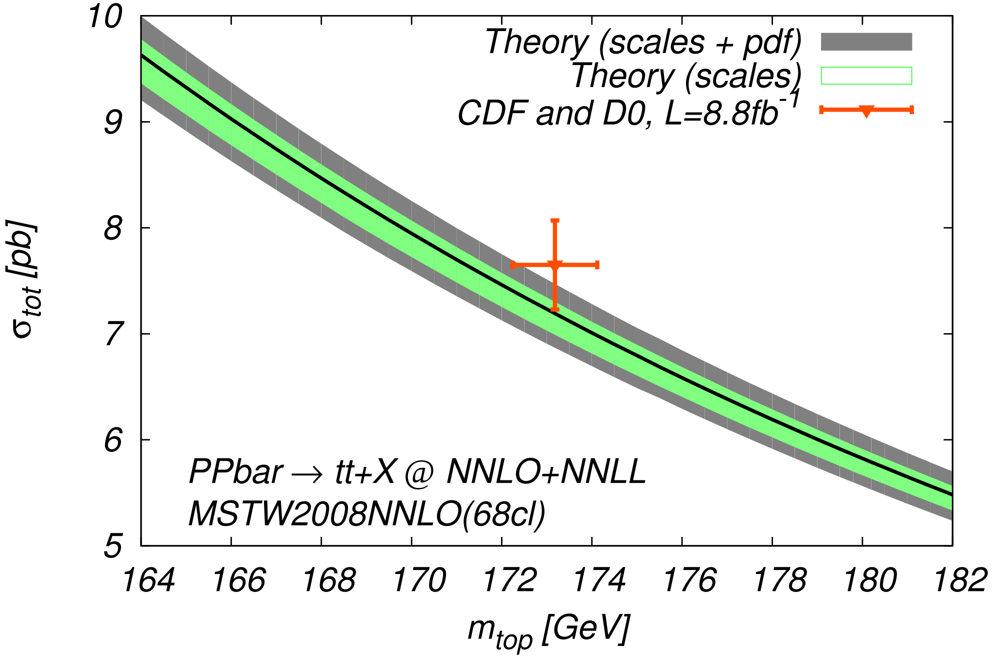
- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales μ_R, μ_F
  - chosen scheme (e.g., $\overline{MS}$ scheme)
  - ...

Top-quark pair-production

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13

- Illustration of mass dependence for Tevatron



- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Heavy-quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{b} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^{a}_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^{a}_{\mu}$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

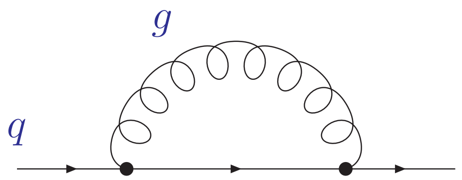
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Heavy-quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2 = m_q^2}$$



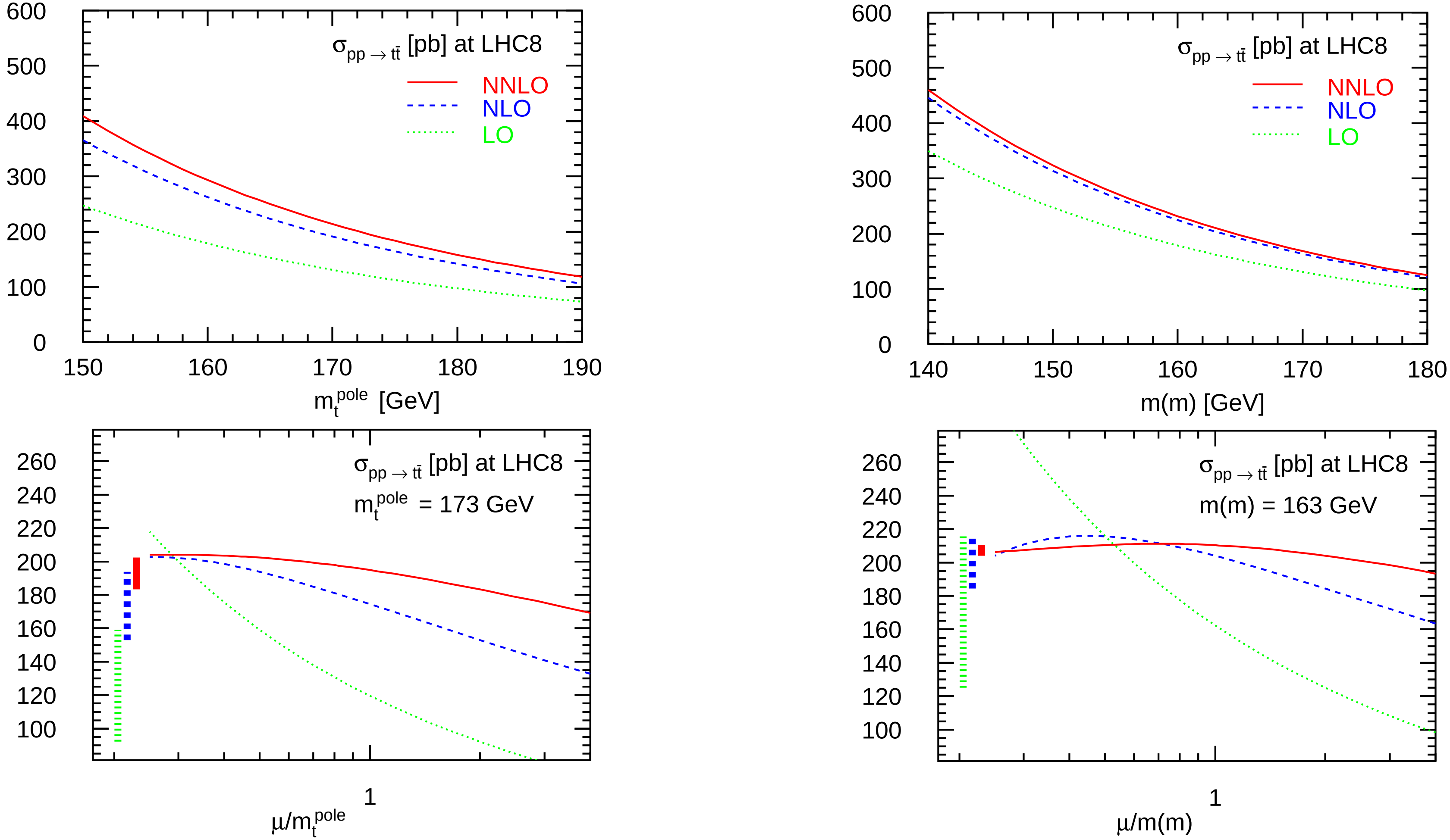
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV
    Bauer, Bali, Pineda '11

Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ perturbation theory

Total cross section with running mass

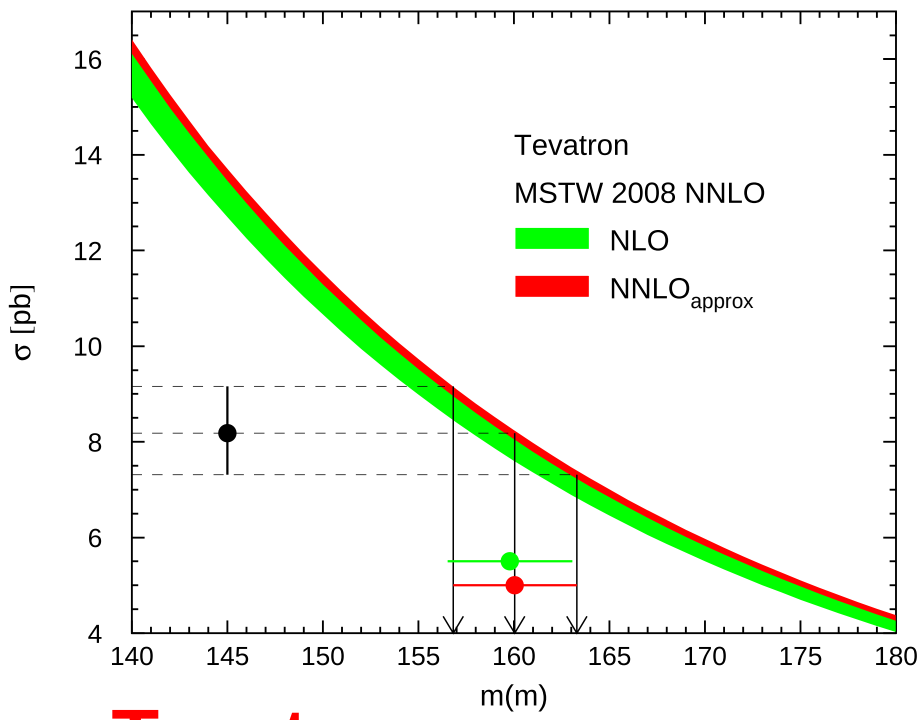
Comparison pole mass vs. $\overline{MS}$ mass



- good apparent convergence of perturbative expansion
- small theoretical uncertainity form scale variation

Top mass from total cross section

- Total top quark cross section as function of $\overline{MS}$ mass
  Langenfeld, S.M., Uwer ‘09



Tevatron

Tevatron

- Determine top quark mass from Tevatron cross section data
  - $\sigma_{t\bar{t}} = 7.56^{+\,0.63}_{-\,0.56}$ pb D0 coll. arXiv:1105.5384
  - $\sigma_{t\bar{t}} = 7.50\,^{+0.48}_{-0.48}$ pb CDF coll. CDF-note-9913
- Fit of m_t for individual PDFs
  - parton luminosity at Tevatron driven by $q\bar{q}$
  - $\overline{MS}$-scheme for $m_t^{\overline{\rm MS}}(m_t)$, then scheme transformation to pole mass $m_t^{\rm pole}$ at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\rm MS}}(m_t)$	162.0 $^{+2.3}_{-2.3}\,^{+0.7}_{-0.6}$	163.5 $^{+2.2}_{-2.2}\,^{+0.6}_{-0.2}$	163.2 $^{+2.2}_{-2.2}\,^{+0.7}_{-0.8}$	164.4 $^{+2.2}_{-2.2}\,^{+0.8}_{-0.2}$
$m_t^{\rm pole}$	171.7 $^{+2.4}_{-2.4}\,^{+0.7}_{-0.6}$	173.3 $^{+2.3}_{-2.3}\,^{+0.7}_{-0.2}$	173.4 $^{+2.3}_{-2.3}\,^{+0.8}_{-0.8}$	174.9 $^{+2.3}_{-2.3}\,^{+0.8}_{-0.3}$
$(m_t^{\rm pole})$	(169.9 $^{+2.4}_{-2.4}\,^{+1.2}_{-1.6}$)	(171.4 $^{+2.3}_{-2.3}\,^{+1.2}_{-1.1}$)	(171.3 $^{+2.3}_{-2.3}\,^{+1.4}_{-1.8}$)	(172.7 $^{+2.3}_{-2.3}\,^{+1.4}_{-1.2}$)

- Good consistency within errors for $m_t^{\rm pole} = 171.7 \ldots 174.9$ at NNLO

The fine print

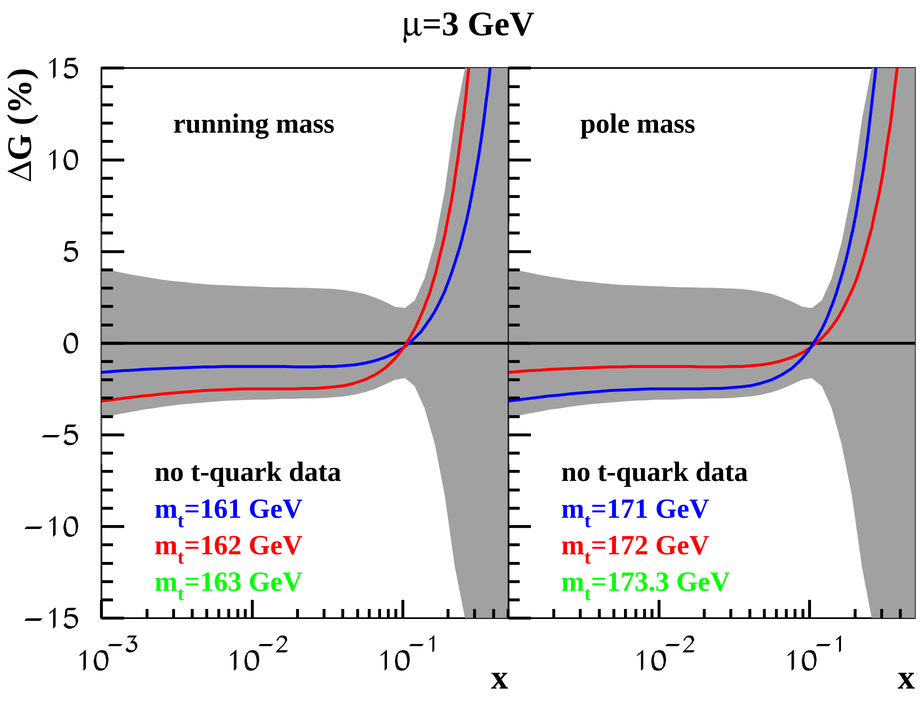
- Intrinsic limitation of sensitivity in total cross section

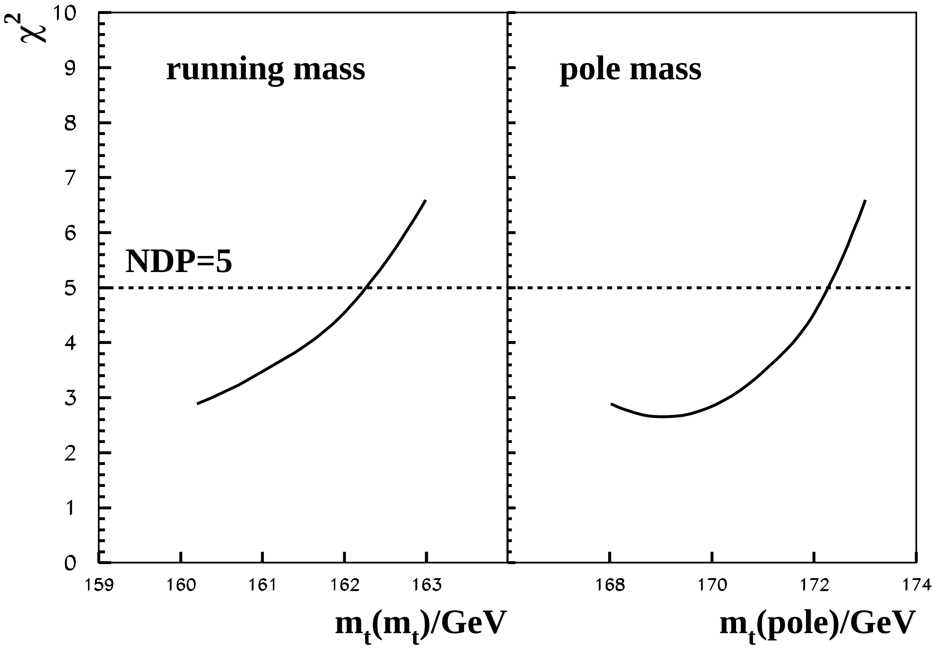
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$, gluon PDF
  $\sigma_{t\bar{t}} \sim \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo ‘13

The fine print

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162$ GeV Alekhin, Blümlein, S.M. [in progress]





Top mass from leptonic decay

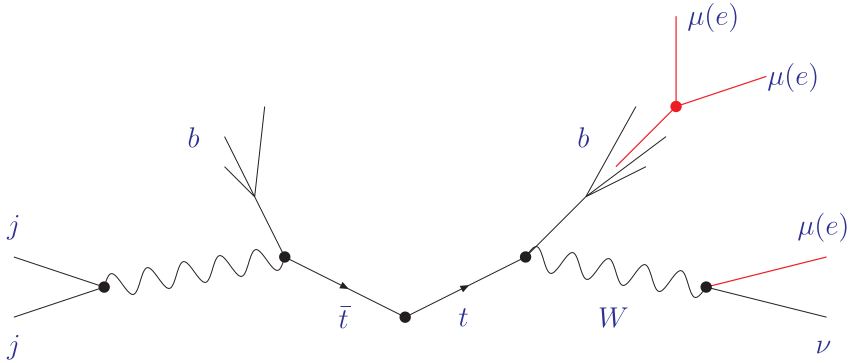
- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

  - identification of μ-pair in J/ψ decay; leptonic or hadronic decay of W

    Kharchilava ‘00

    Chierici, Dierlamm ‘06

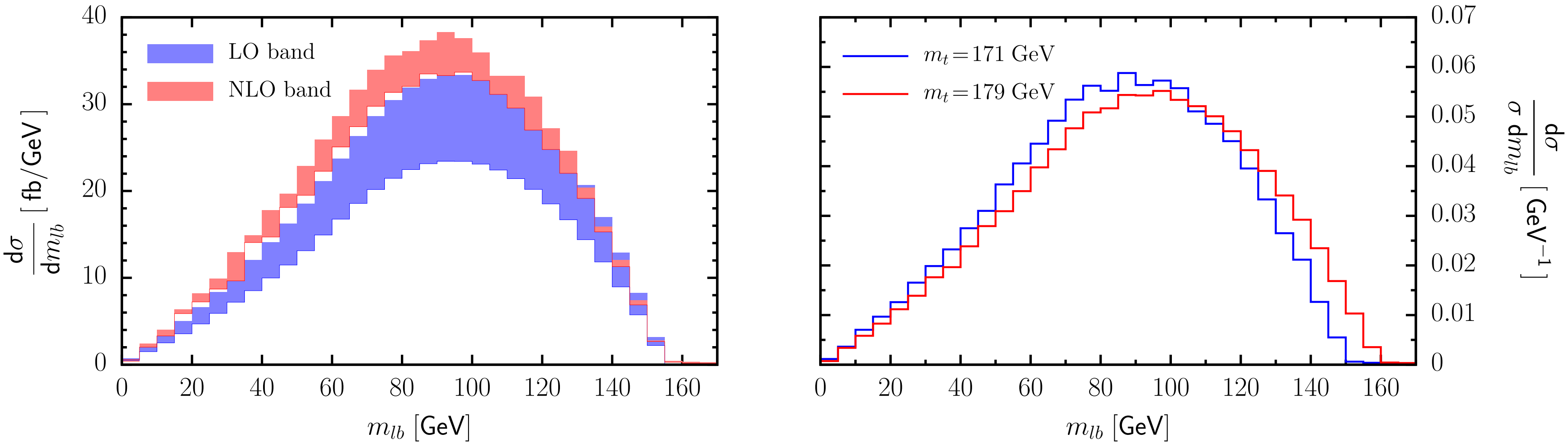


Top mass from leptonic decay

- Top mass from exclusive hadronic states

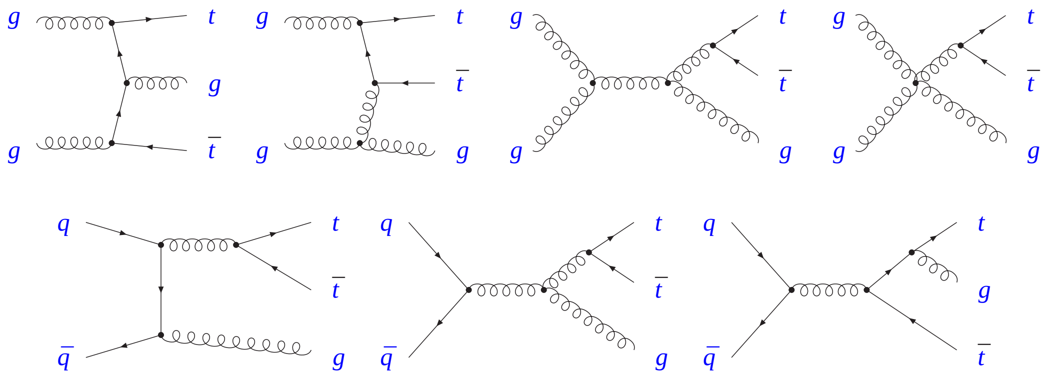
$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

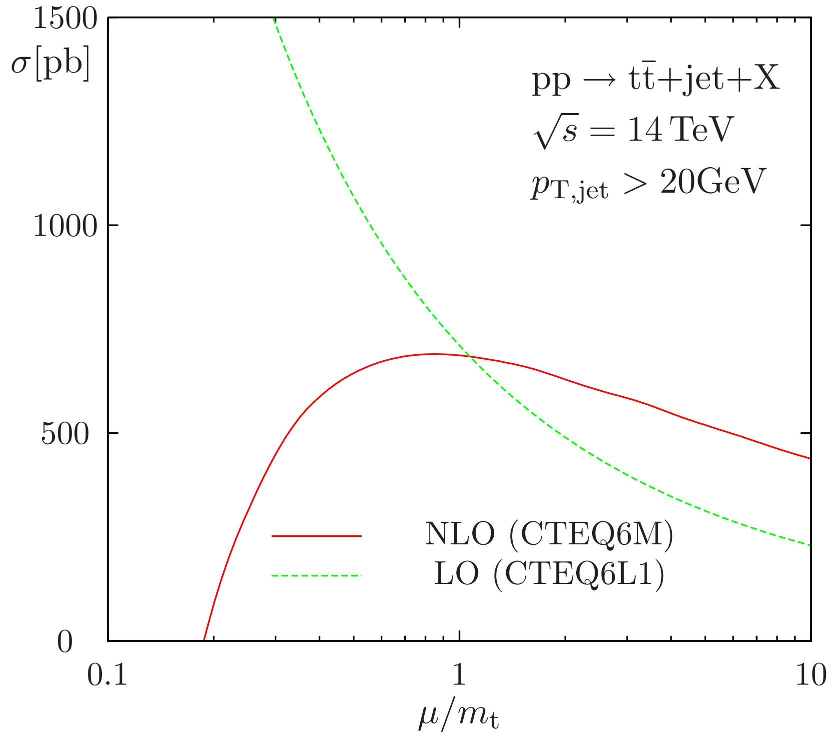
- Study of m_{lb} distribution at NLO in QCD Biswas, Melnikov, Schulze '10
  - NLO QCD corrections to production **and** decay very important for value of m_t (effects of order $\Delta m_t = \mathcal{O}$(few) GeV
- Invariant mass distribution of lepton and b-jet (LHC14)
  - scale dependence at LO and NLO (left)
  - normalized m_{lb} distributions, $m_t = 171$ GeV and 179 GeV (right)



Top mass from jet rates

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

Mass measurement with $t\bar{t}+\mathrm{jet}$-samples

- Mass measurement with new observable

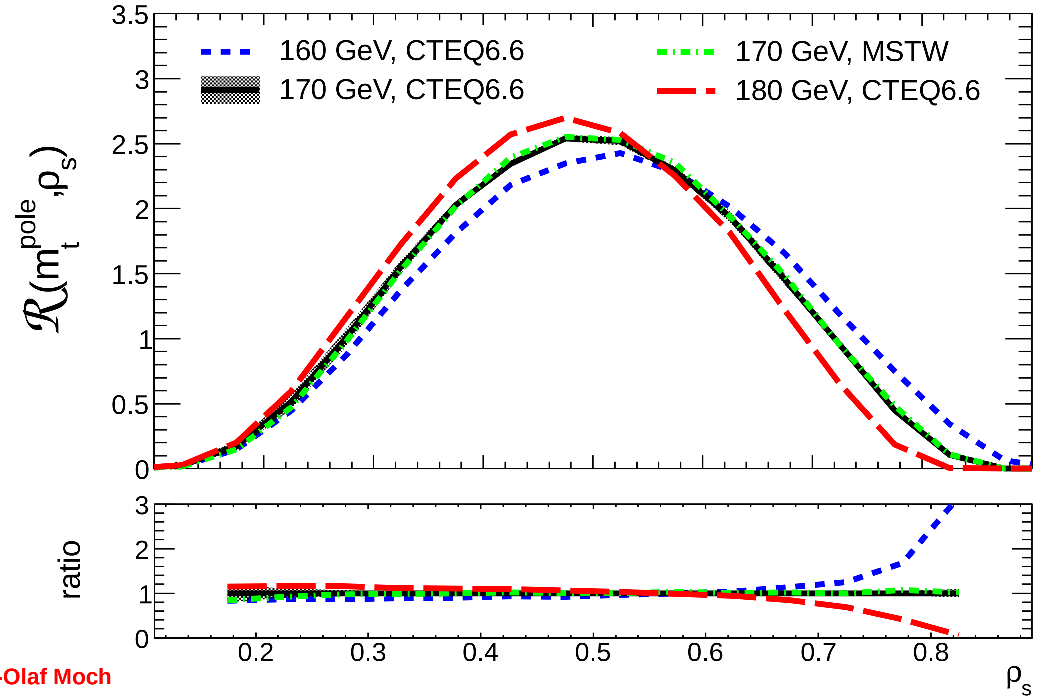
Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos ‘13

  - variable $\rho_s = \frac{2\cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t}+1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$

- Normalized-differential $t\bar{t}+\mathrm{jet}$ cross section

$$\mathcal{R}(m_t,\rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t,\rho_s)$$

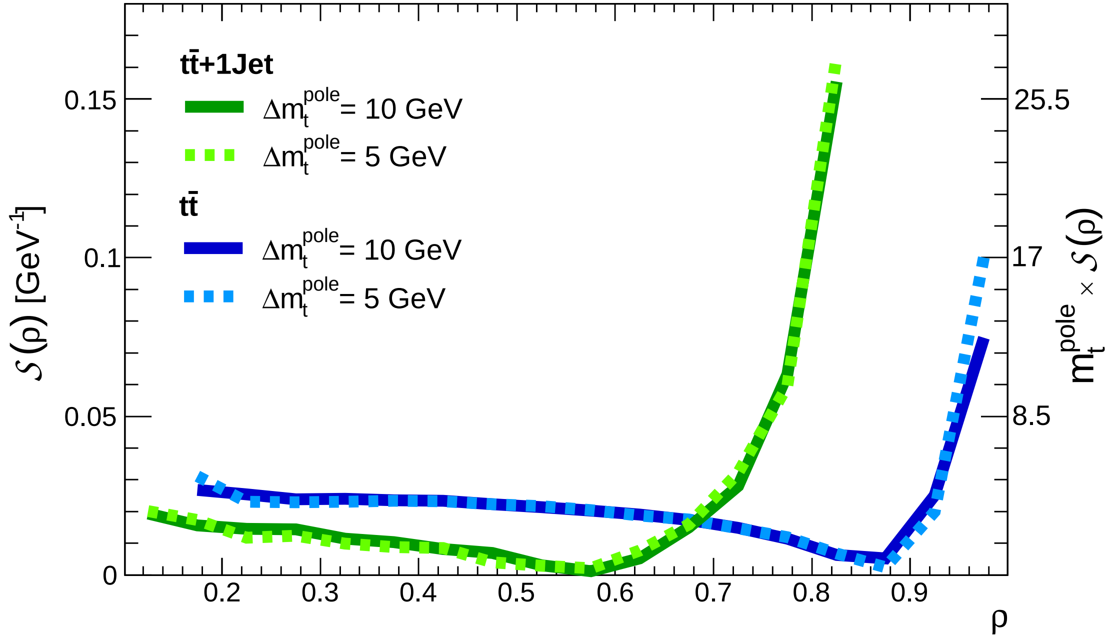
  - significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Sensitivity to top-quark mass very good

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t \mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$

  - increased sensitivity for system $t\bar{t} + \text{jet}$ compared to $t\bar{t}$



Upshot

- Precision determination of well-defined top-quark mass m_t possible
  - alternative to inclusive cross sections

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left| \Phi^\dagger \Phi - \frac{v}{2} \right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \le v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

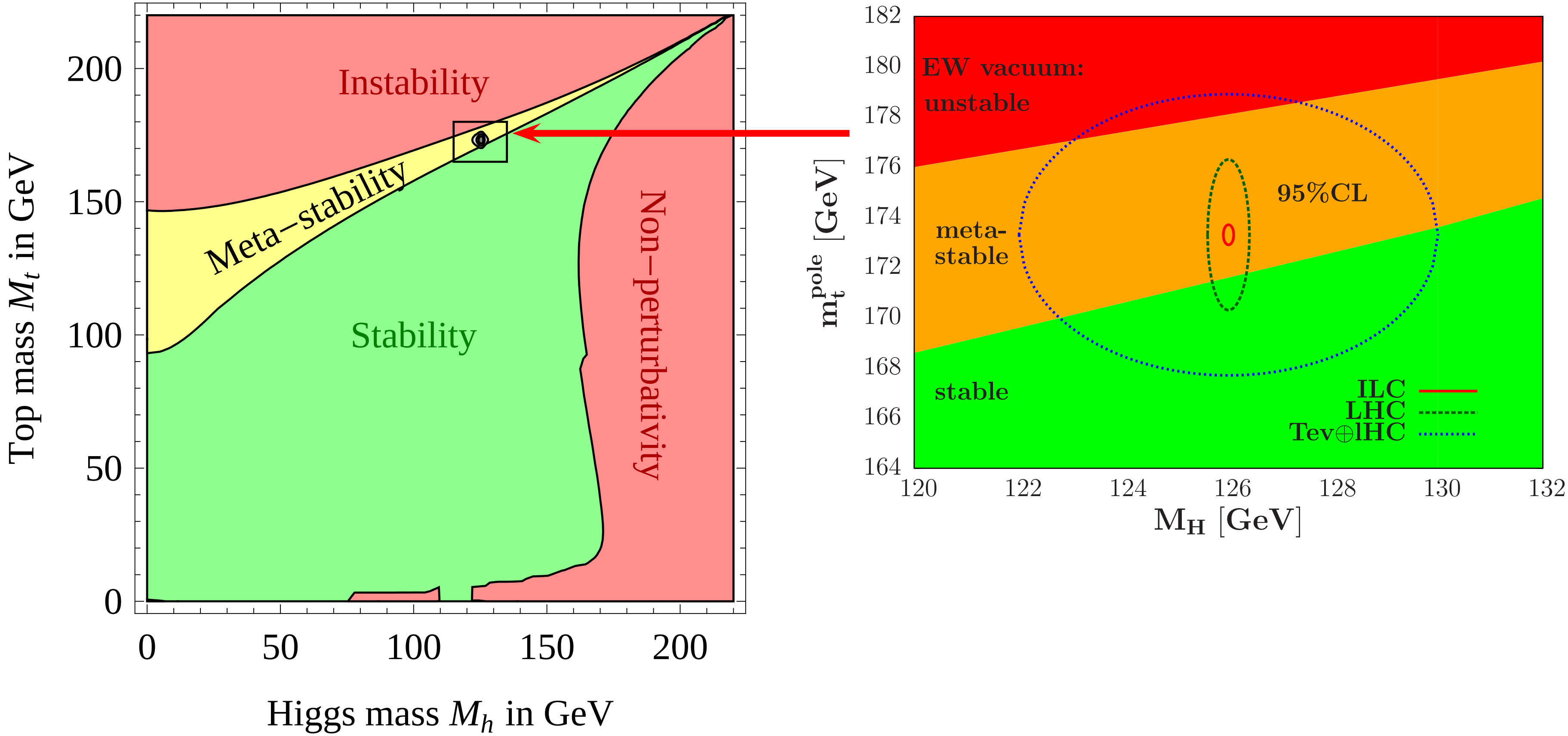
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\rm pole} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\rm MS}}(m_t) = 163.3 \pm 2.7$ GeV implies in pole mass scheme $m_t^{\rm pole} = 173.3 \pm 2.8$ GeV
  - good consistency of mass value between different PDF sets

Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 129.4 \pm 5.6$ GeV
  - "fate of universe" still undecided

Summary

Physics at the Terascale

- Discovery of (SM like) Higgs boson opens new avenue for studies of Standard Model physics and beyond
- Precision determinations of non-perturbative parameters is essential
  - masses m_t, M_W, m_H, ...
  - coupling constants $\alpha_s(M_Z)$
  - parton content of proton (PDFs)
- Precision measurements require careful definition of observable
  - top-quark mass m_t in well defined scheme
- Radiative corrections at higher orders in QCD and EW are mandatory
  - continuous benchmarking mandatory
  - theory improvements driven by experimental precision
- Lots of challenging tasks for young researchers