

# Pad Response Function PRF



A. Bellerive - M. Dixit - P. Hayman - N. Shiell

December 11, 2012

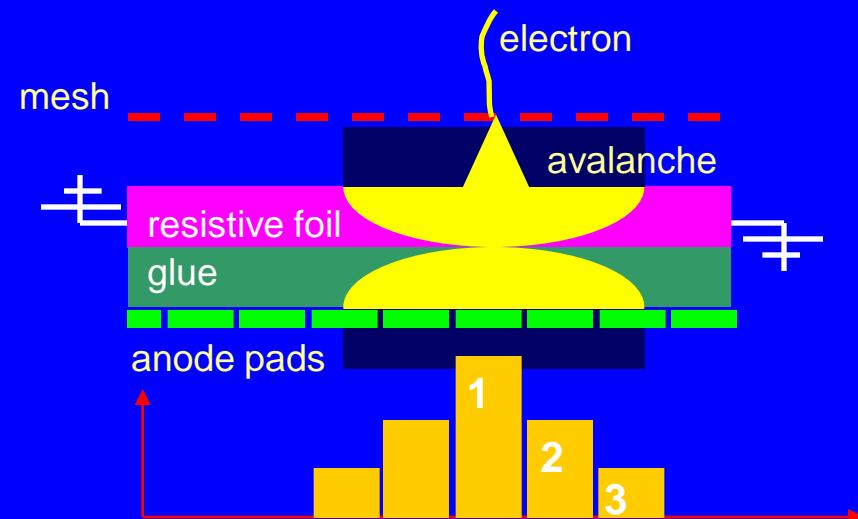
# Outline

- Intro: Signal Pulse
  - Definition of amplitude and time ( $A_i, t_i$ ) for a pad
  - Conceptual Pad Response Function (PRF)
  - Scaling of PRF
- Determination of PRF parameters (calibration)
  - Parameterization of PRF
  - Seed track
  - $\chi^2$  minimization and number of d.o.f.
- Development and Future
  - Handling Error from  $(A, t) \rightarrow (PRF, t) \rightarrow (x, y, z)$  for a hit to unbiased track estimators ( $d_0, \varphi, CU, z, \lambda$ )
  - Calibration PRF Module and Simulation
- Summary

# Charge dispersion

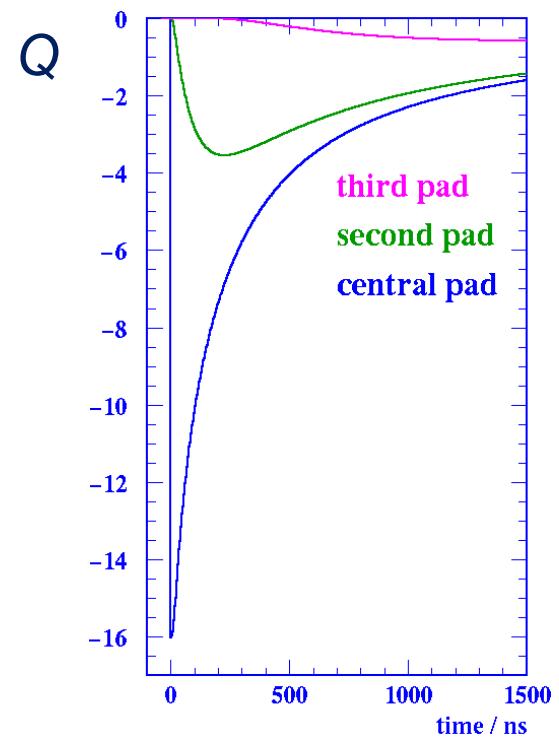
- A high resistivity film bonded to a readout plane with an insulating spacer
- 2D continuous RC network defined by material properties and geometry.
- point charge at  $r = 0$  &  $t = 0$  disperses with time.

## Micromegas + resistive anode



$$\frac{\partial \rho}{\partial t} = \frac{1}{RC} \left[ \frac{\partial^2 \rho}{\partial r^2} + \frac{1}{r} \frac{\partial \rho}{\partial r} \right]$$

$$\Rightarrow \rho(r, t) = \frac{RC}{2t} e^{\frac{-r^2 RC}{4t}}$$



# Pulse shape origin

Transverse diffusion

$$T(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma_x^2}\right)$$

Longitudinal diffusion

$$L(t) = \frac{1}{\sigma_t \sqrt{2\pi}} \exp\left(\frac{-t^2}{2\sigma_t^2}\right)$$

Induction gap

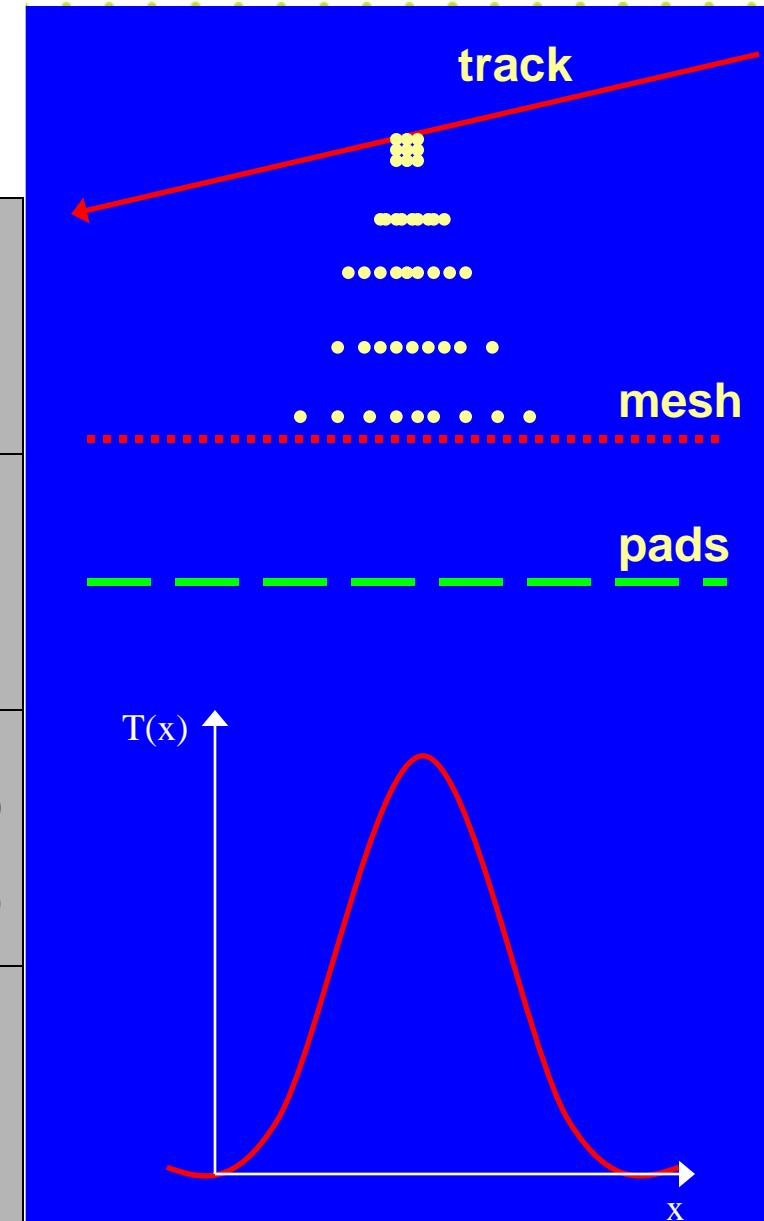
$$\begin{aligned} R(t) &= \frac{t}{T_{rise}} & 0 < t < T_{rise} \\ &= 1 & t > T_{rise} \\ &= 0 & t < 0 \end{aligned}$$

Preamplifier Response

$$\begin{aligned} A(t) &= \exp\left(-\frac{t}{t_f}\right) \left(1 - \exp\left(\frac{t}{t_r}\right)\right) & t > 0 \\ &= 0 & t < 0 \end{aligned}$$

Resistive foil + glue

$$\rho(x, y, t) = \left(\frac{1}{\sigma_t \sqrt{\pi t h}}\right)^2 \exp\left(\frac{-(x^2 + y^2)}{4t h}\right)$$



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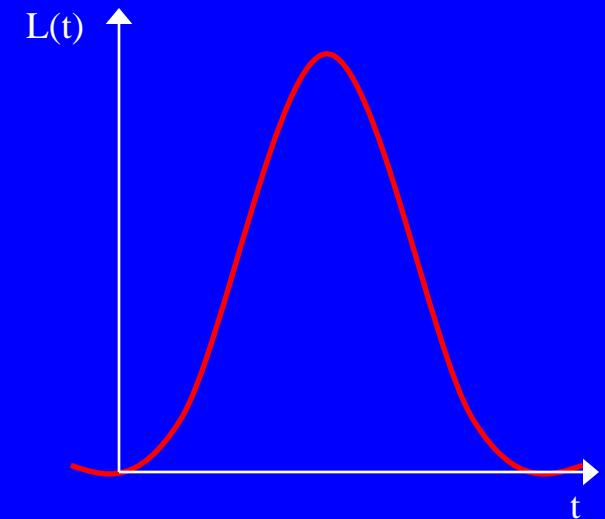
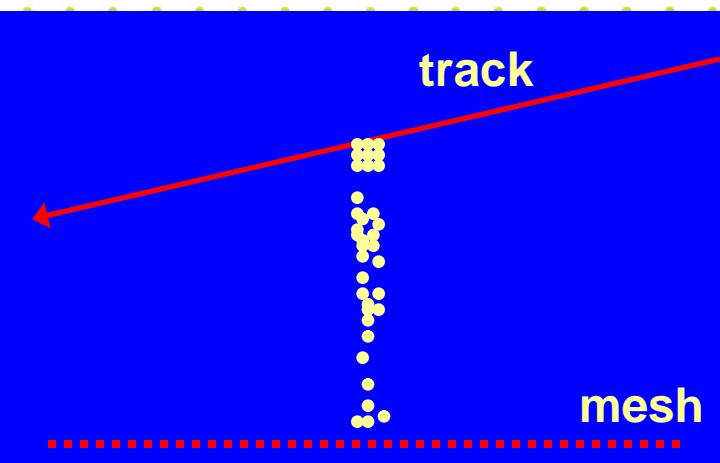
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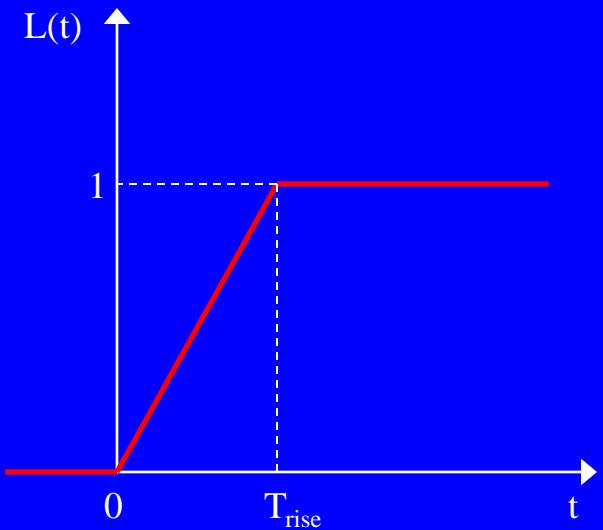
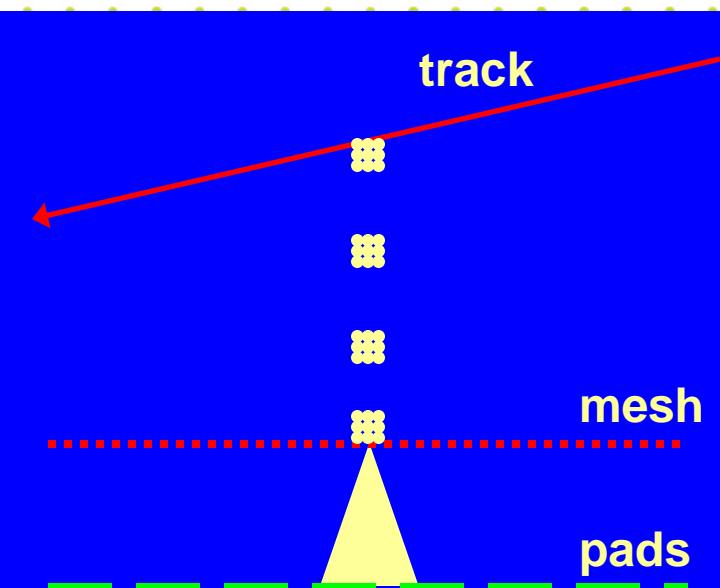
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Induction gap

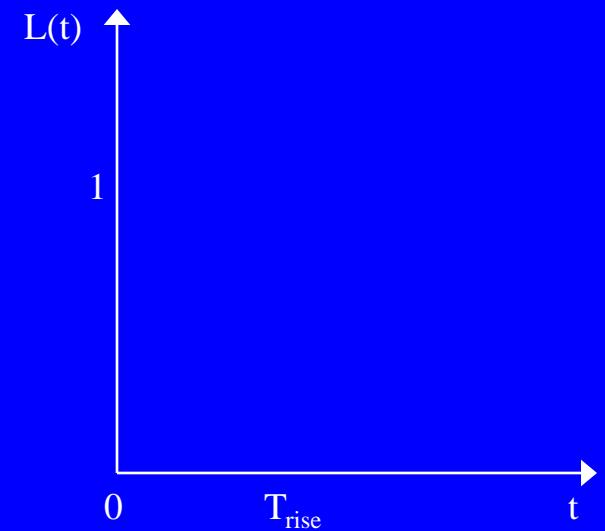
$$\begin{aligned} R(t) &= \frac{t}{T_{rise}} & 0 < t < T_{rise} \\ &= 1 & t > T_{rise} \\ &= 0 & t < 0 \end{aligned}$$

Preamplifier Response

$$\begin{aligned} H(t) &= \exp\left(-\frac{t}{t_f}\right) \left(1 - \exp\left(\frac{t}{t_r}\right)\right) & t > 0 \\ &= 0 & t < 0 \end{aligned}$$

Resistive foil + glue

$$\rho(x, y, t) = \left(\frac{1}{\sigma_t \sqrt{\pi h}}\right)^2 \exp\left(\frac{-(x^2 + y^2)}{4th}\right)$$



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Preamplifier Response

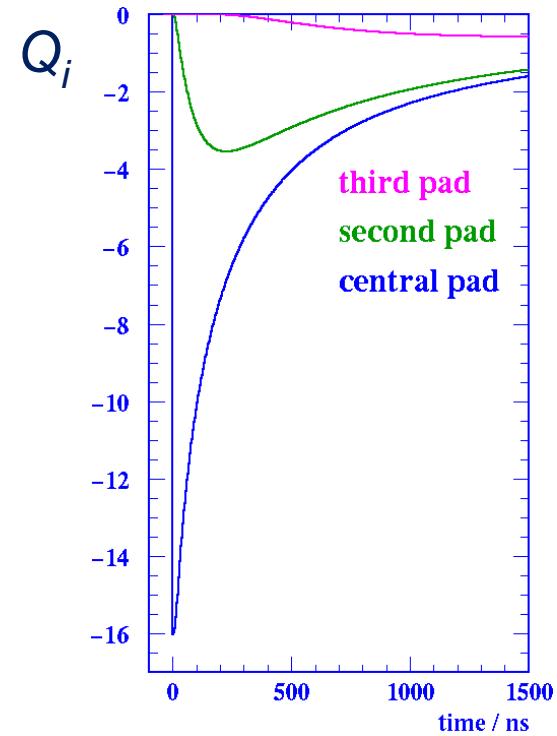
$$\begin{aligned} A(t) &= \exp\left(-\frac{t}{t_f}\right) \left(1 - \exp\left(\frac{t}{t_r}\right)\right) & t > 0 \\ &= 0 & t < 0 \end{aligned}$$

Resistive foil + glue

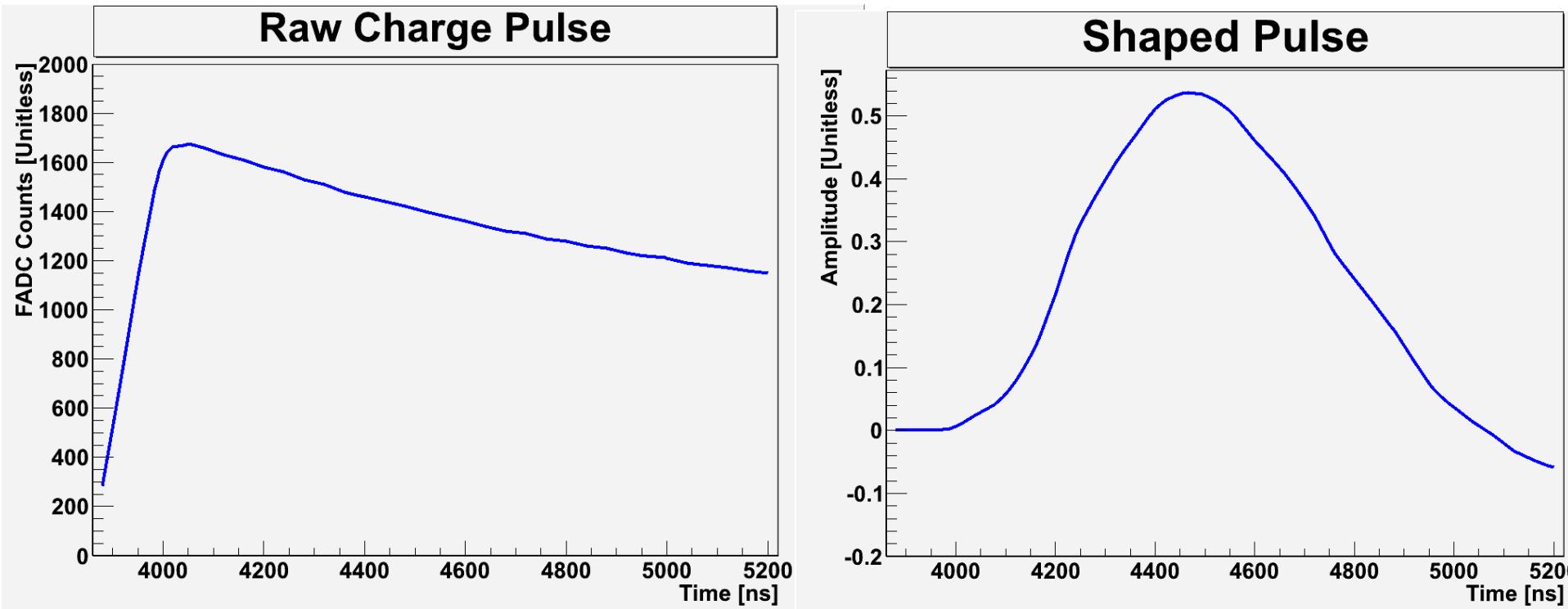
$$\rho(x, y, t) = \left(\frac{1}{\sigma_t \sqrt{\pi h}}\right)^2 \exp\left(\frac{-(x^2 + y^2)}{4th}\right)$$

$$Q_i = \int \rho_i(r) dr$$

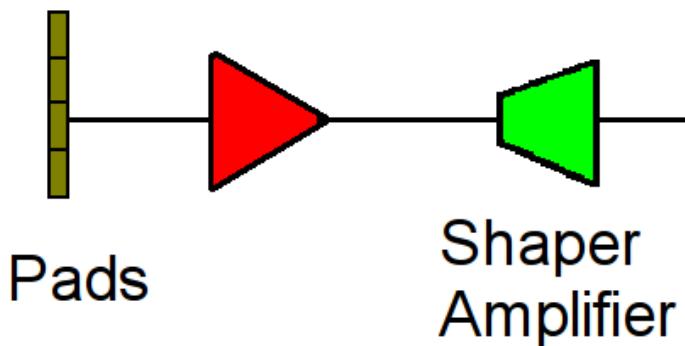
Stand alone simulation



# Raw Charge Shape versus Shaped Pulse



Preamp



FADC

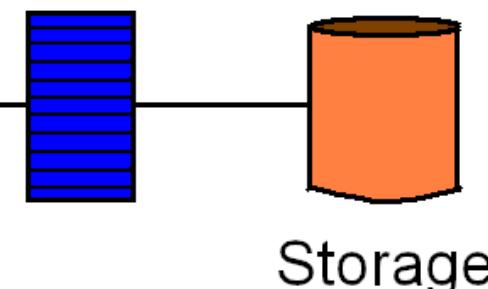
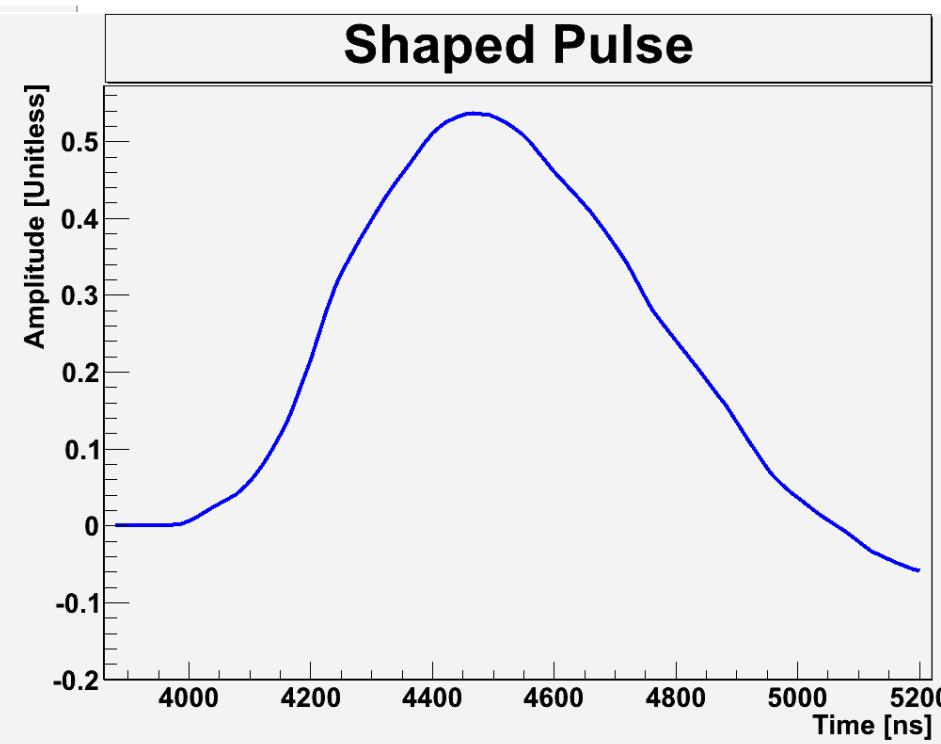
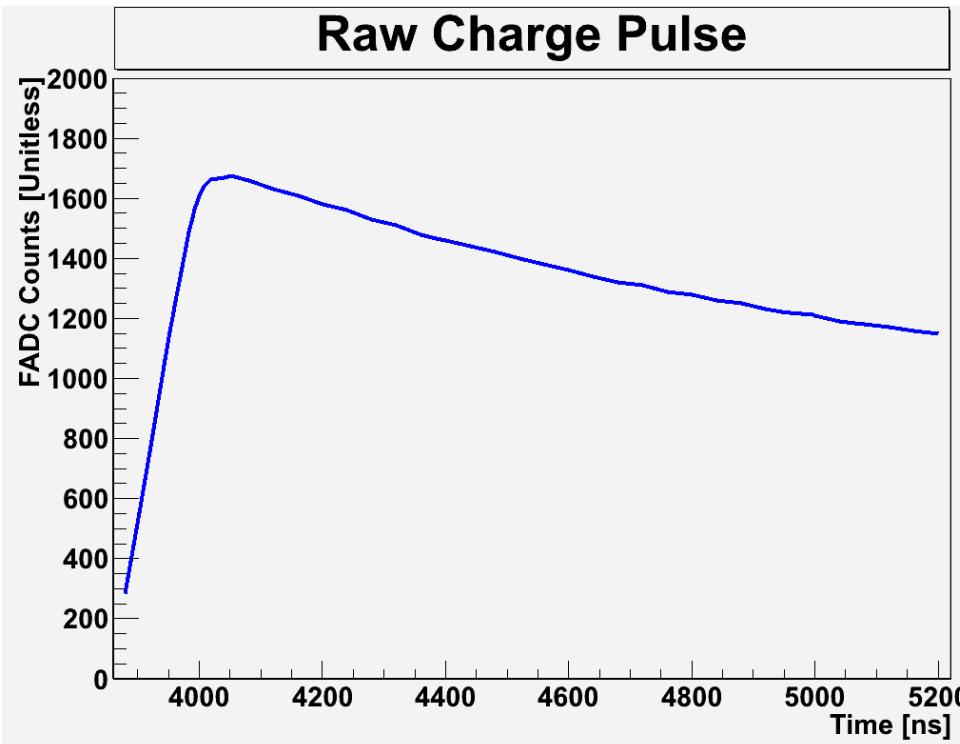


Figure: N. Shiell

# Raw Charge Shape versus Shaped Pulse

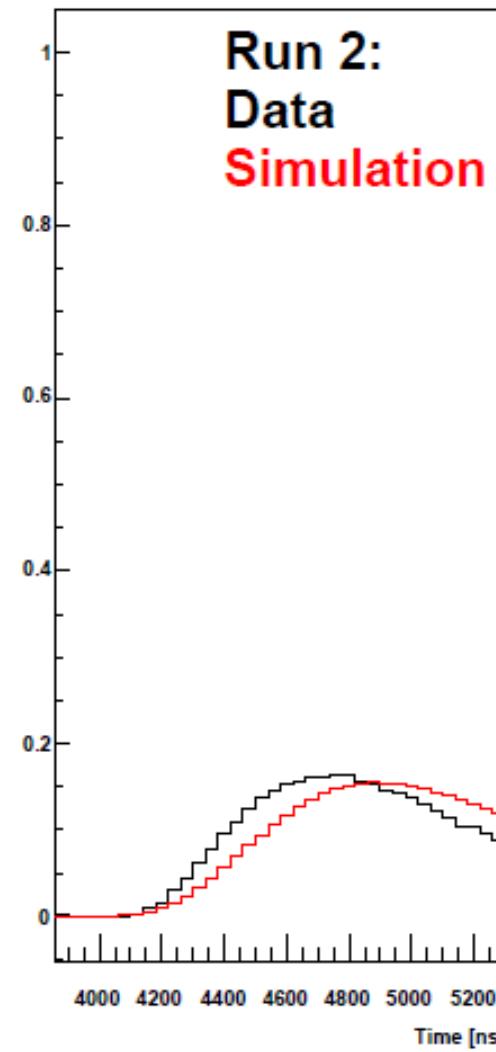
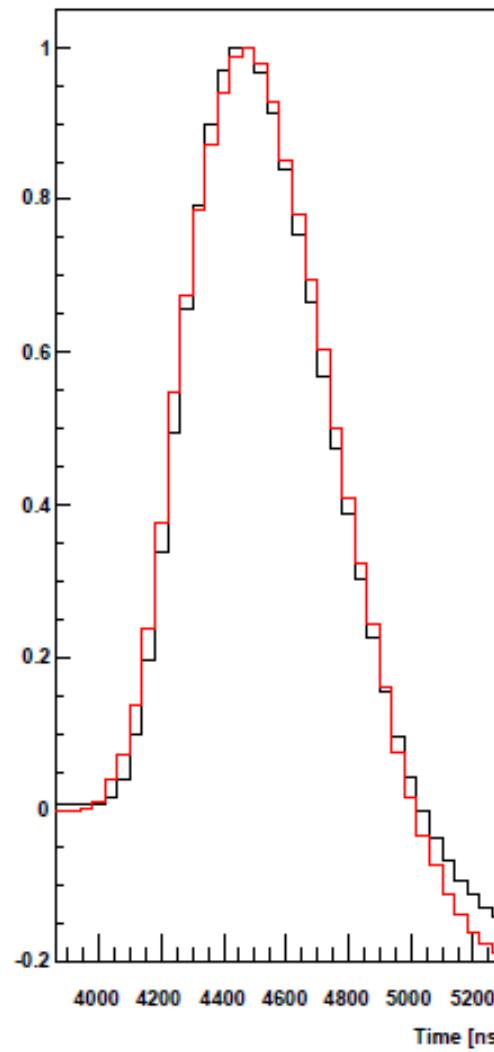
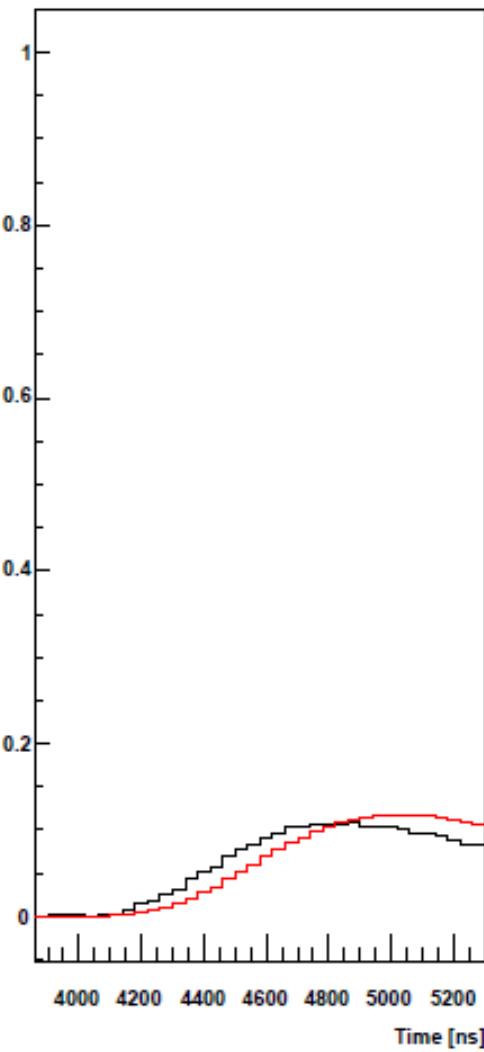


$$H(t) = A_0 \left( \frac{t}{\tau} \right)^3 \sin\left(\frac{t}{b\tau}\right) \exp\left(-\frac{t}{\tau}\right)$$

from Eric Delagnes et al at Saclay

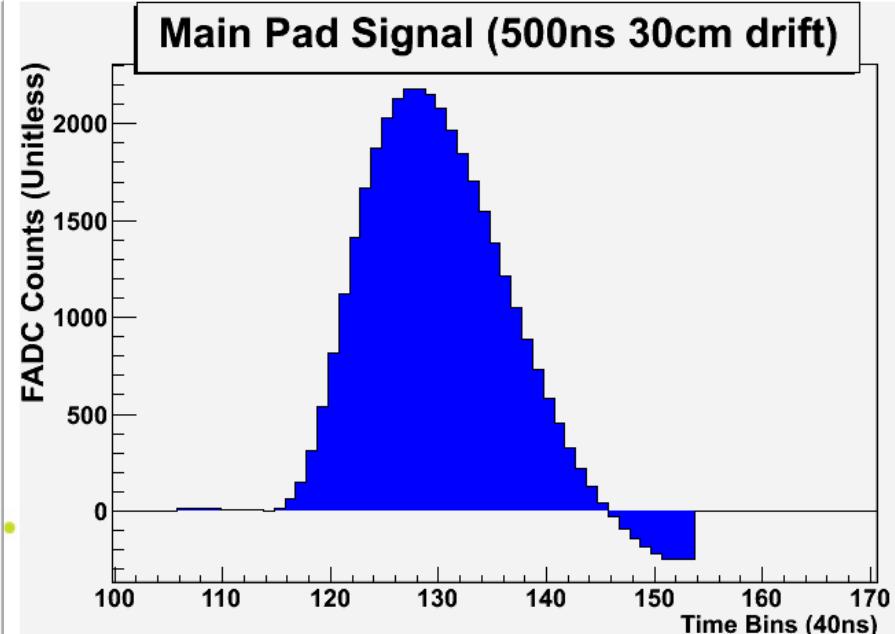
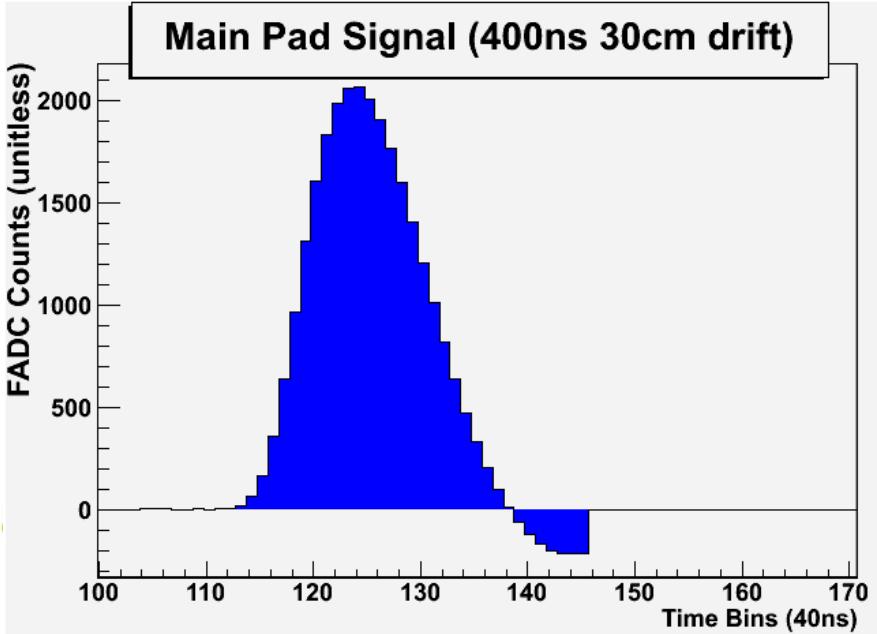
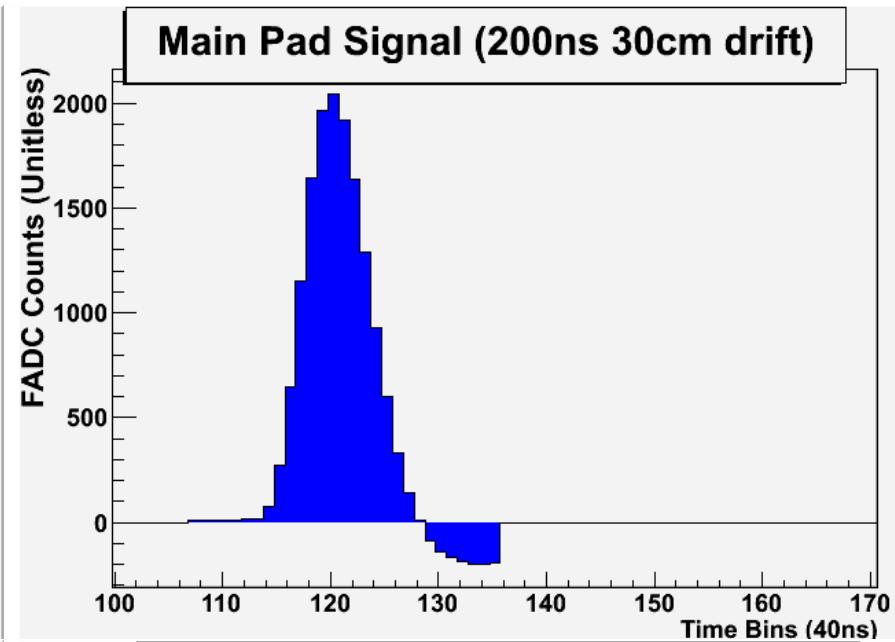
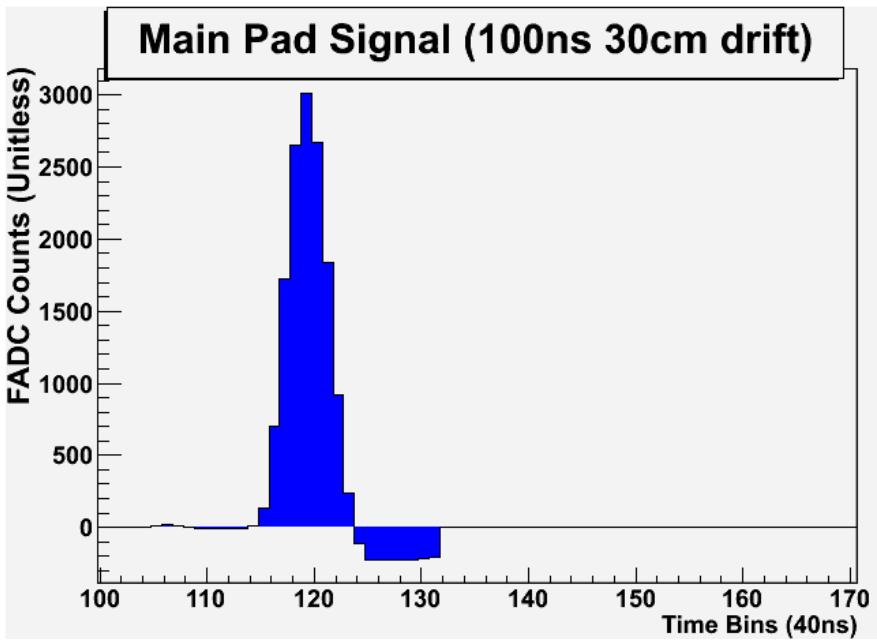
Parameter	Initial value	Final value	???
Drift speed	76.98 um/ns	fixed	Measured value in the lab
Transverse diffusion	95.4 um/root(cm)	fixed	
Longitudinal diffusion	231.289 um/root(cm)	fixed	
Resistivity	2.9 MOhm/sq	fixed	
Glue thickness	75 um	fixed	
Dielectric constant	4.5	2.66 ← → 2.09	
Induction time	120 ns	166 ns ← → 199 ns	
b (shaper)	3.7	3.42 ← → 3.33	
$\tau$ (shaper)	151 ns	151 ns	
Pad angular width	0.001984 rad	fixed	
Pad height	6.84 cm	fixed	
Lower radius of bottom row	1.522457785 m	fixed	
$X_0$ track	event dependent		
$\phi$ track	event dependent		
Drift distance	30 cm	30 cm	

# Stand-Alone Calculation (2011)

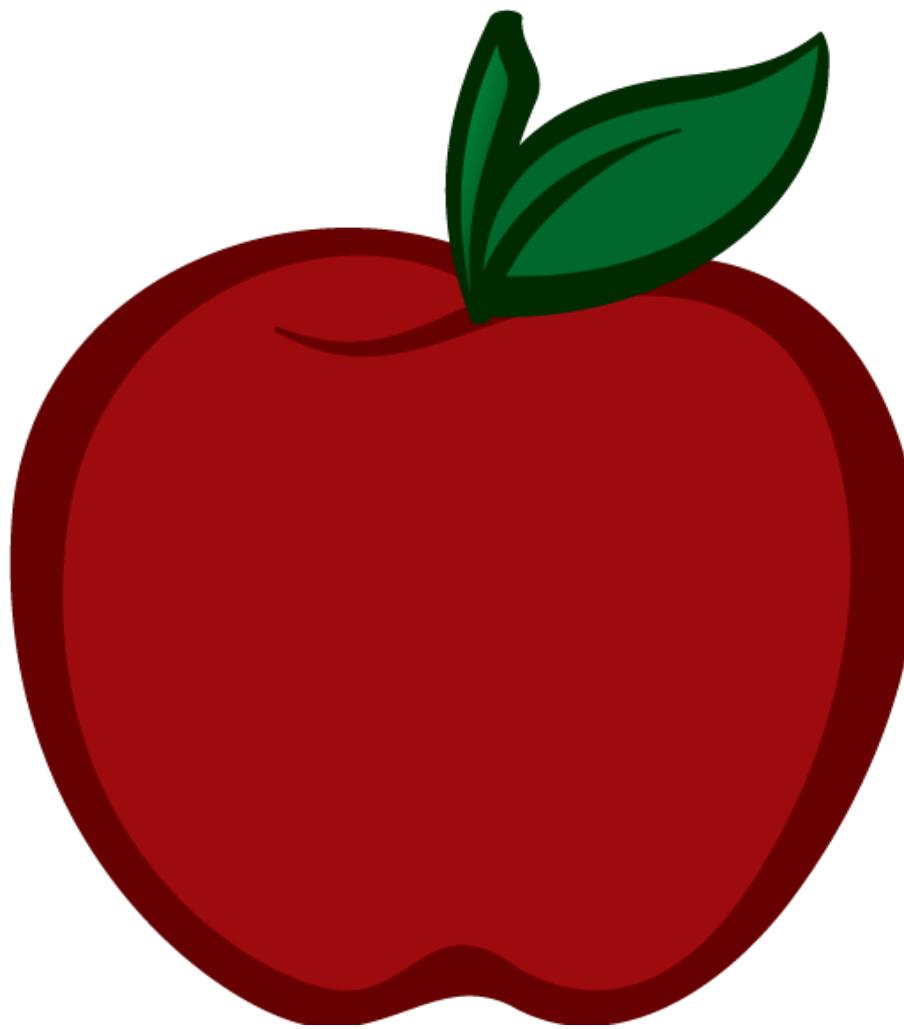


CRUCIAL TO CHARACTERIZE DETECTOR PARAMETERS

# Shaped Pulse (for different shaping time)

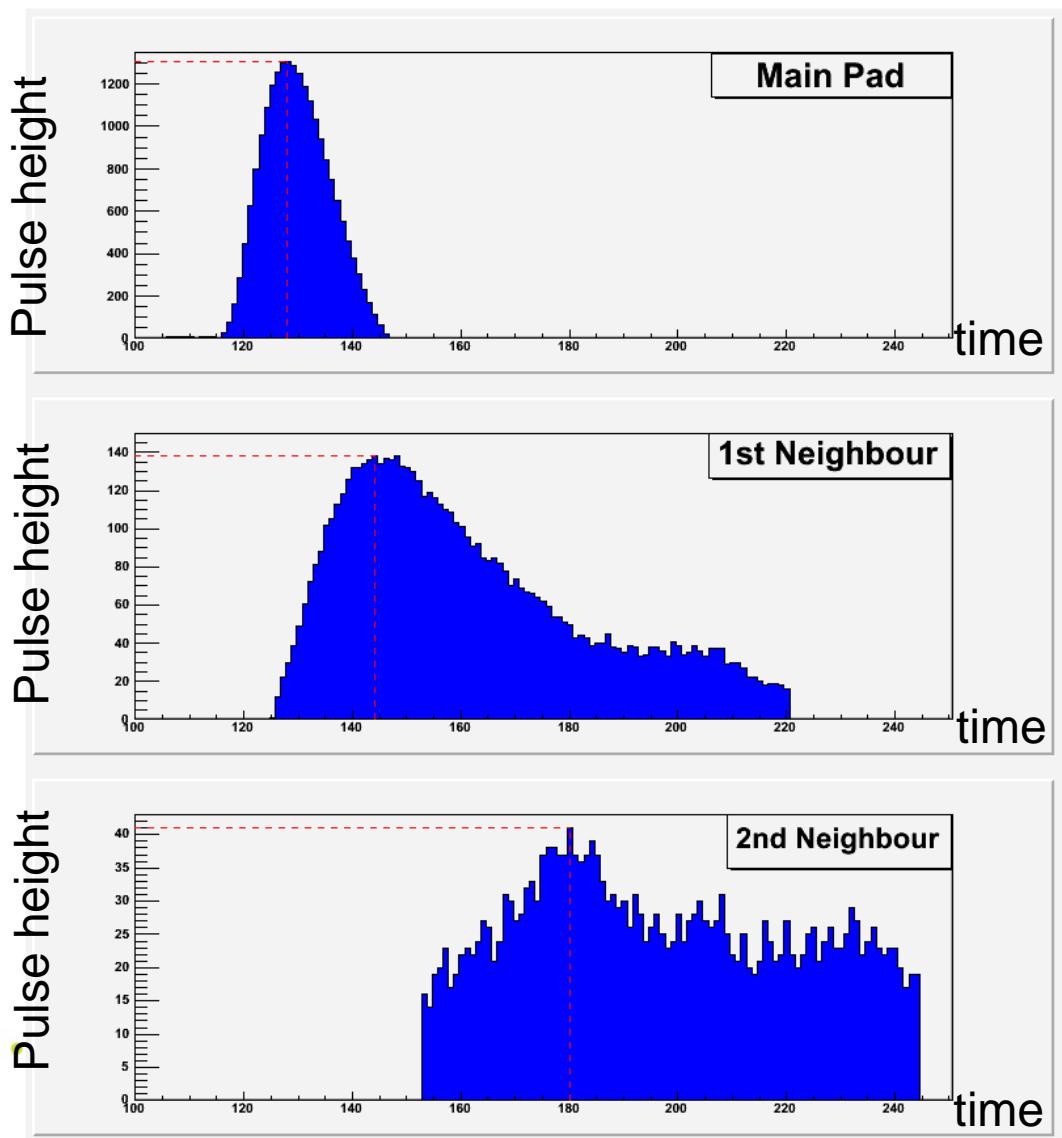


# Pad Amplitude



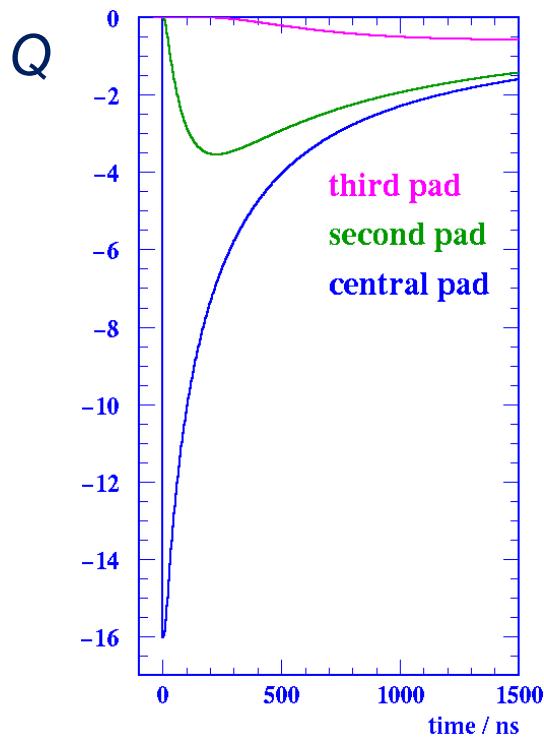
1) Use the maximum as the amplitude  
Single Point Maximum(SPM)

$$A_i = \text{max pulse height } P(i)$$



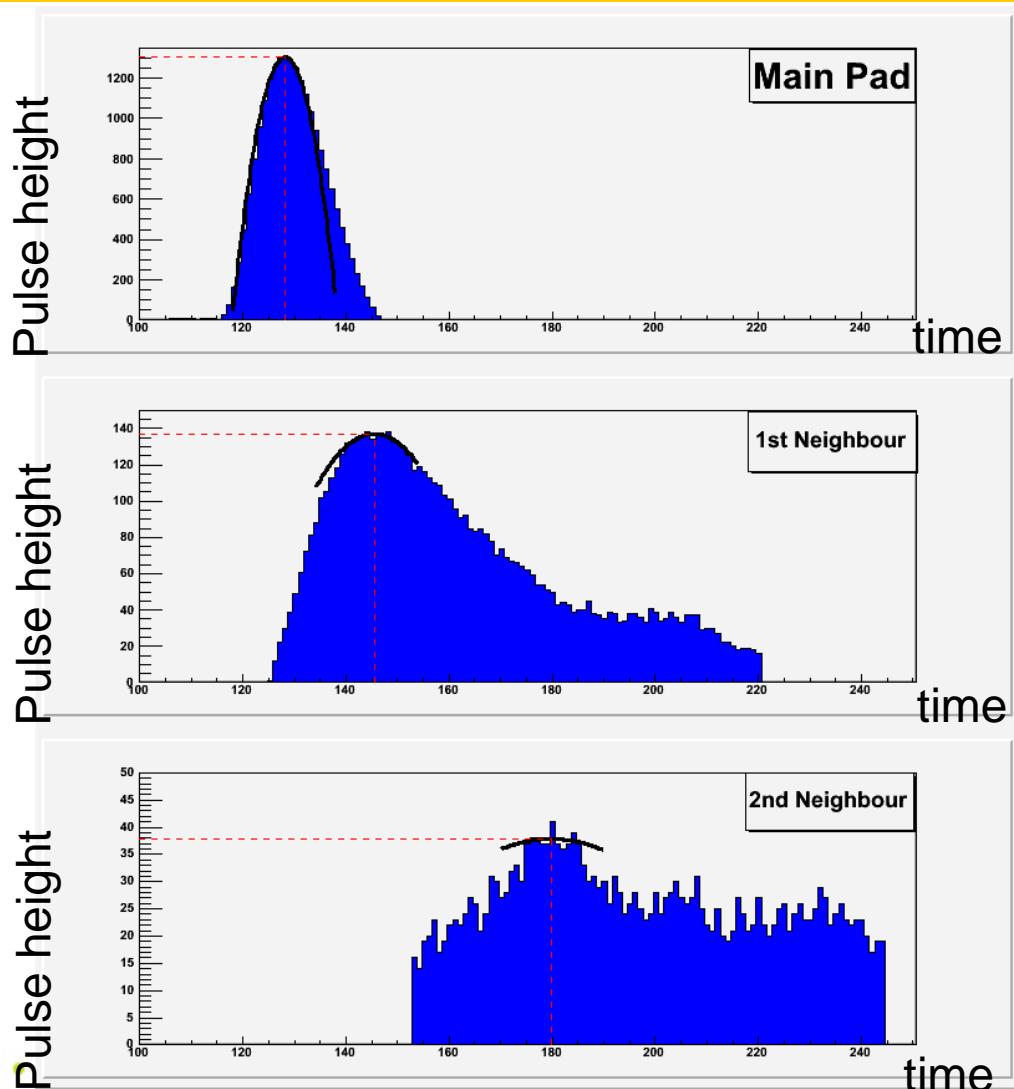
# Pad Amplitude

method used here



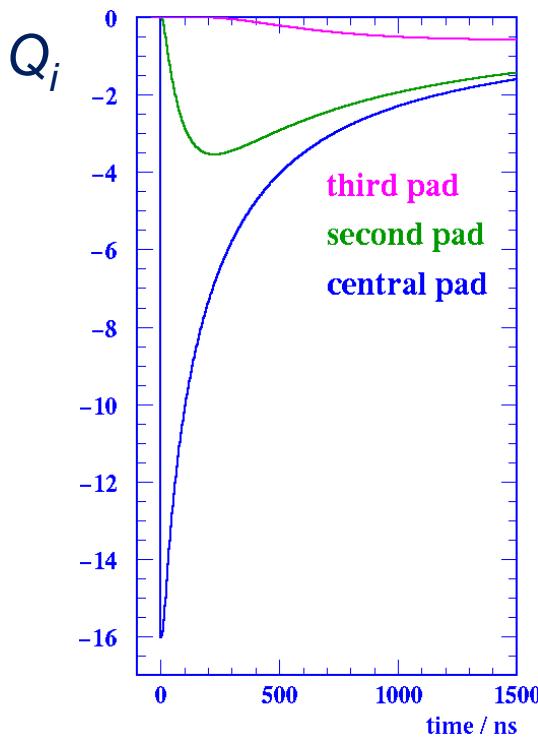
## 2) Maximum of Parabola Quadratic Fit Method (QFM)

$$A_i = \text{max of parabola } P(i)$$



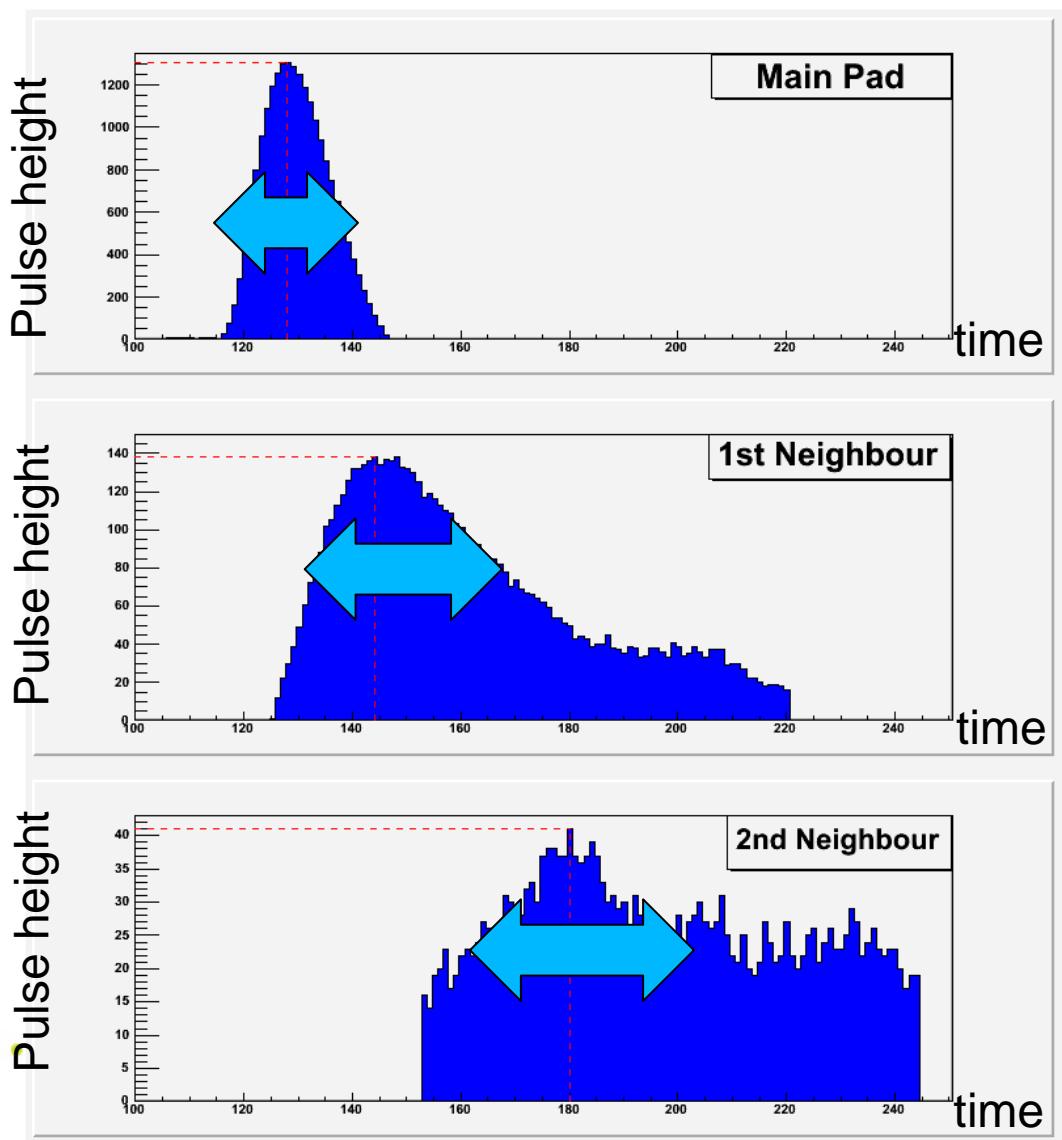
# Pad Amplitude

Method use pre-2011



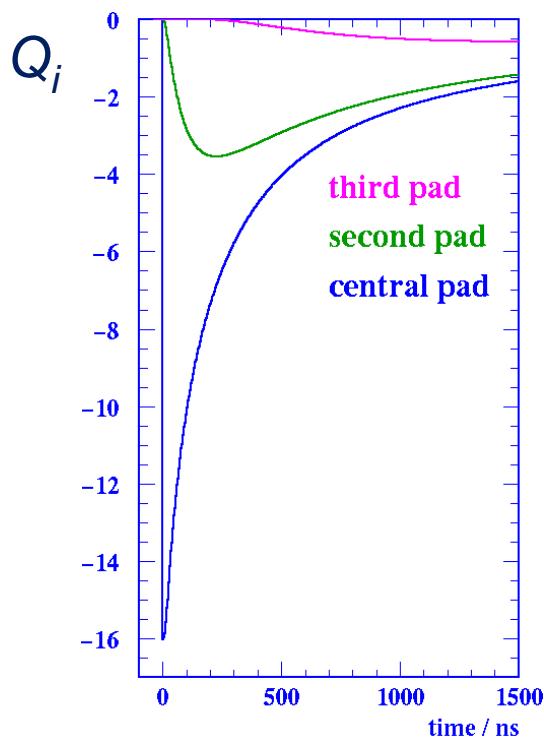
3) Integrate above threshold  
Re-integration method (RM)

$$A_i = \text{Sum } P(i)$$

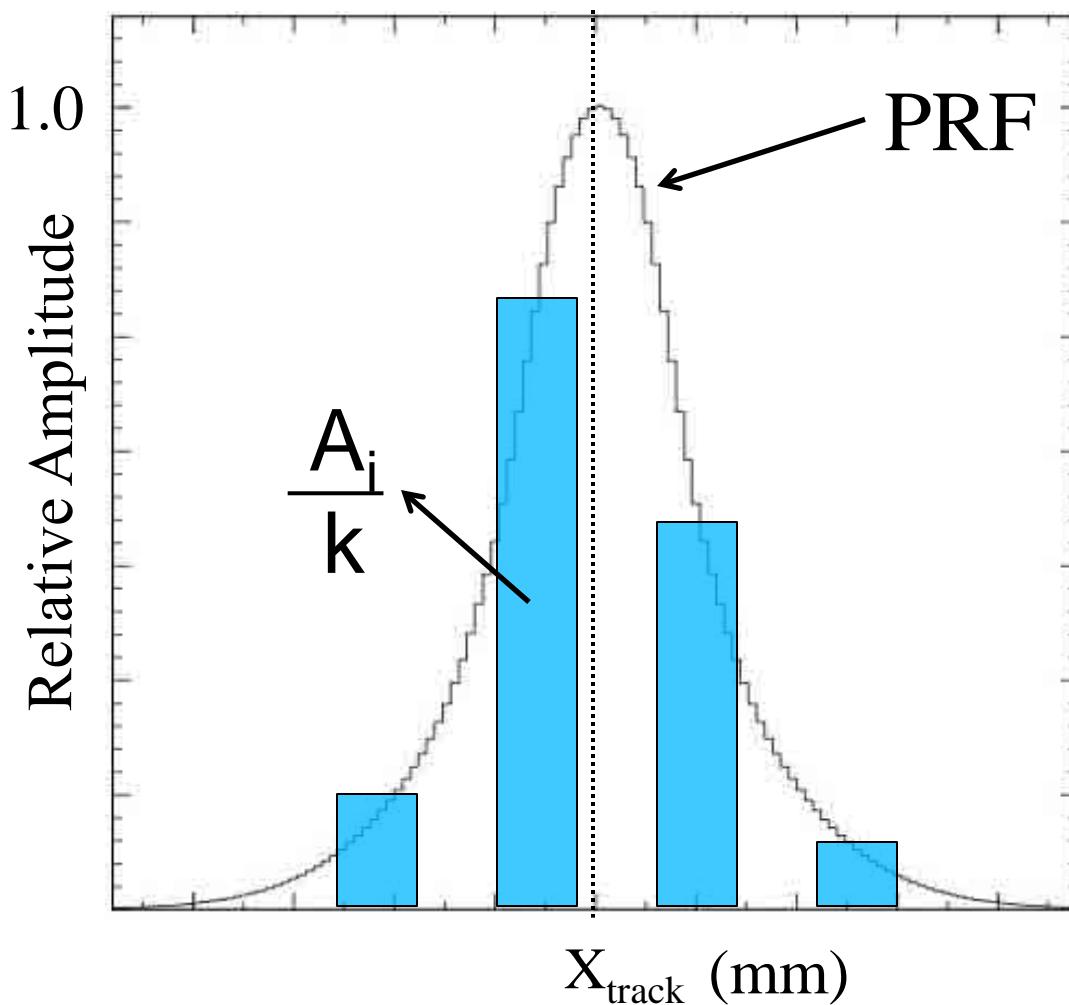


# Pad Amplitude

Method use in 2011

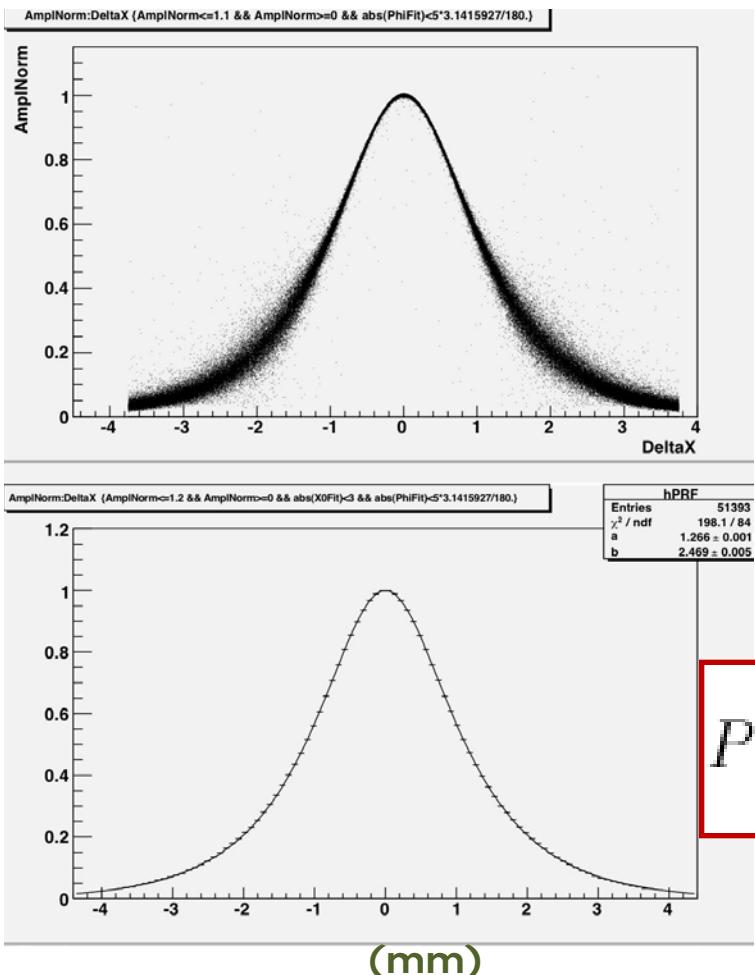


# Pad Response Function (PRF)

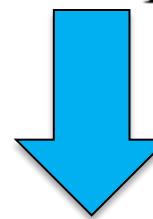


For a given  $X_{\text{track}}$  (known position) the PRF is defined to be unity

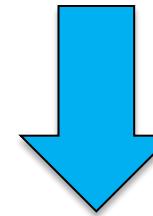
# Pad Response Function (model)



$$\text{PRF}(x, \Gamma, \Delta, a, b) = \frac{1 + a_2 x^2 + a_4 x^4}{1 + b_2 x^2 + b_4 x^4}$$



$$PRF(x, a, b) = b^2 \frac{e^{\frac{-x^2}{2a^2}}}{x^2 + b^2}$$

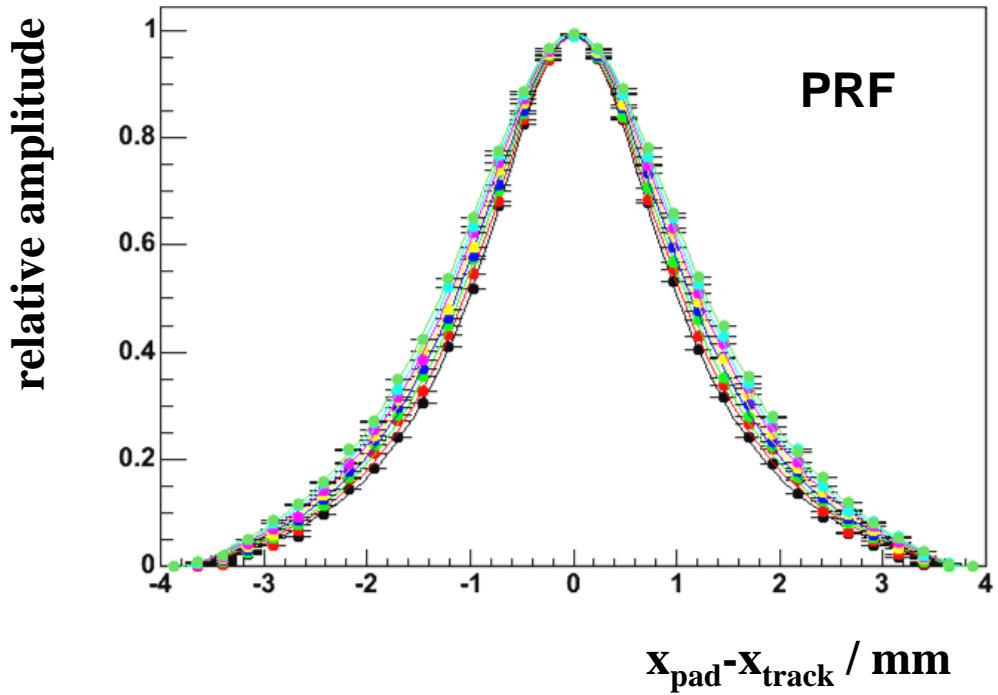
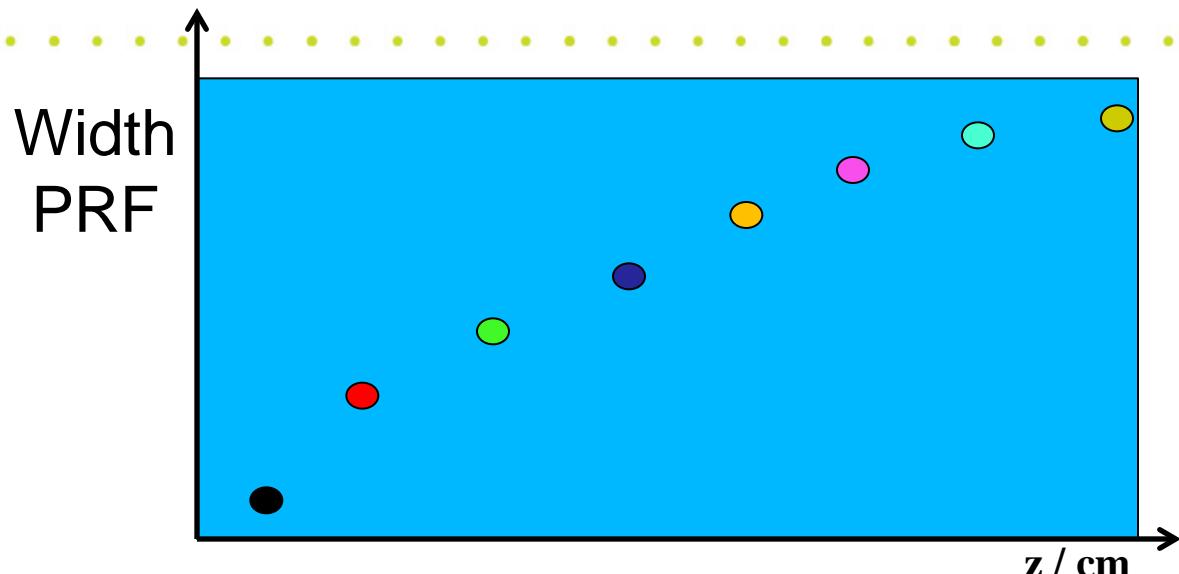
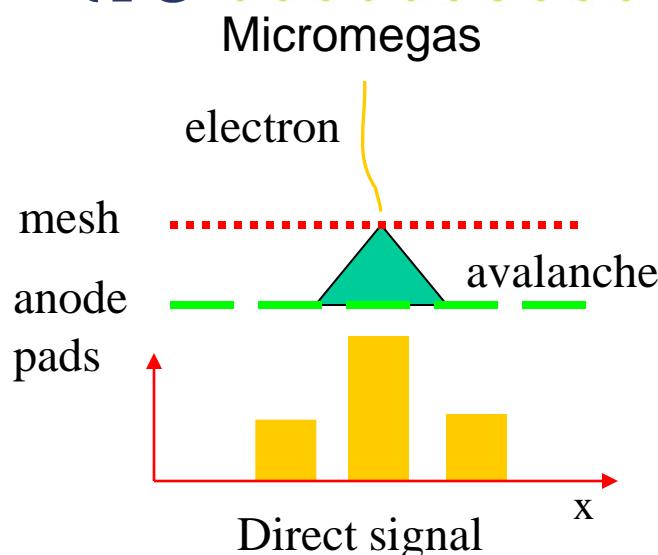


new (used here)

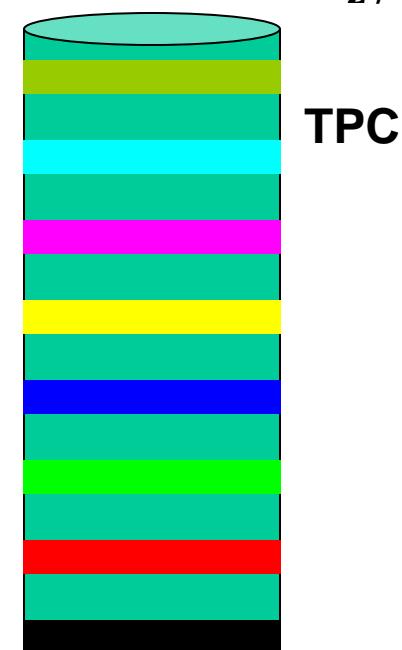
$$PRF(x; r, w) = \frac{\exp(-4\ln(2)(1-r)x^2/w^2)}{1 + 4rx^2/w}$$

- Only two parameters (simpler model)
- Easier to work with
- Better fits to data

# PRF versus Z

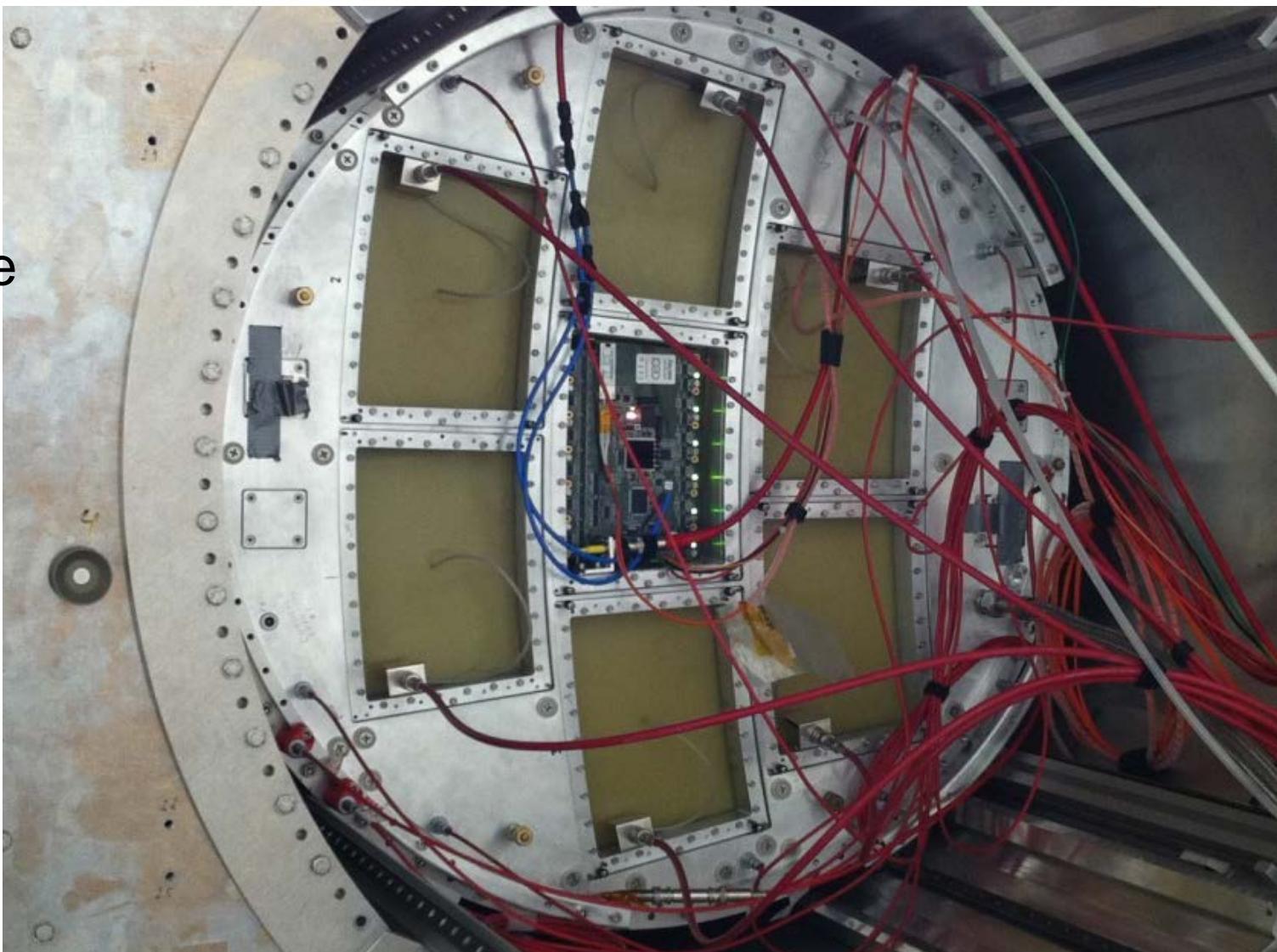


$14 < z < 15\text{cm}$   
 $12 < z < 13\text{cm}$   
 $10 < z < 11\text{cm}$   
 $8 < z < 9\text{cm}$   
 $6 < z < 7\text{cm}$   
 $4 < z < 5\text{cm}$   
 $2 < z < 3\text{cm}$   
 $0 < z < 1\text{cm}$



# LCTPC Transverse Resolution

2011 data  
Single module

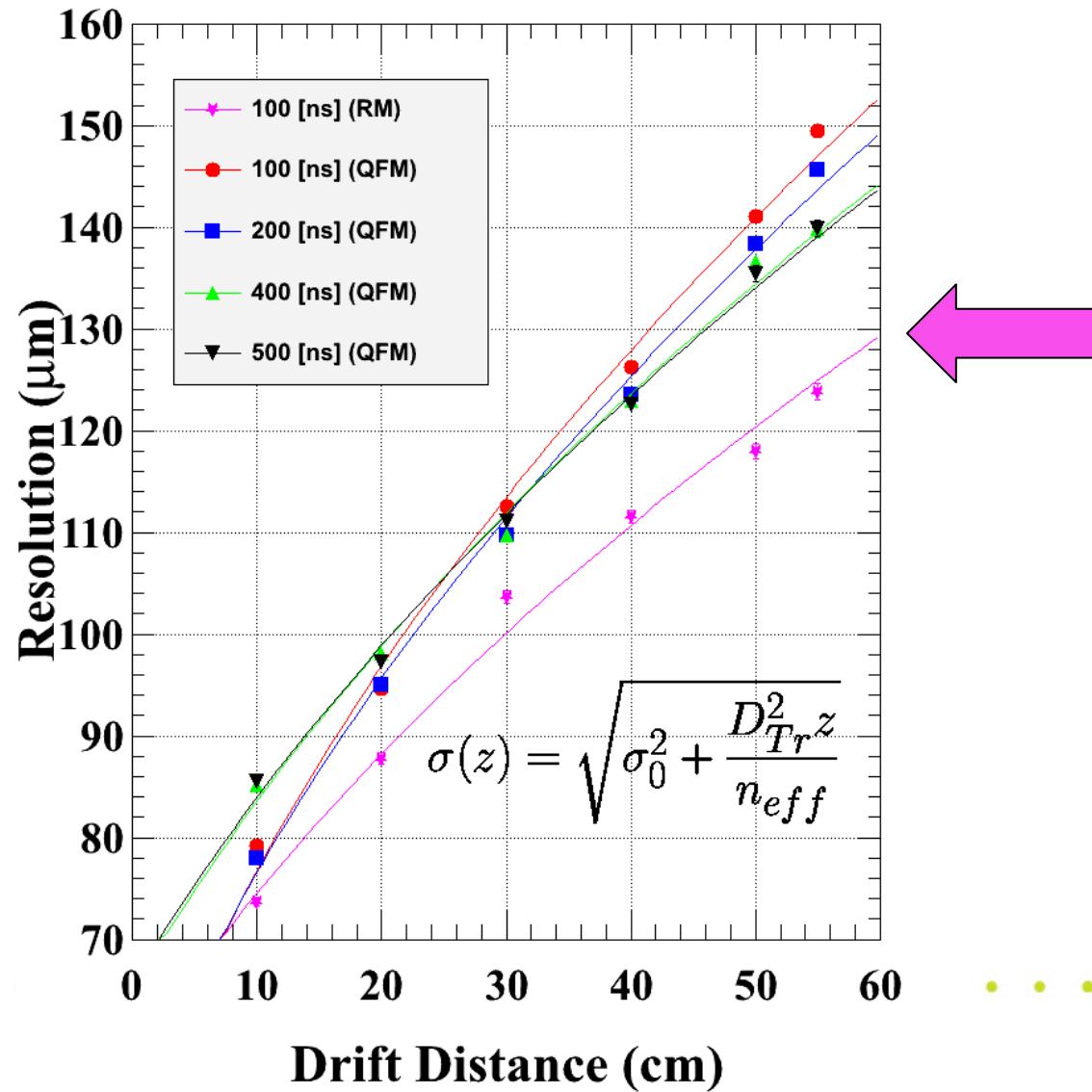


# Transverse Resolution

2011 data  
Single module

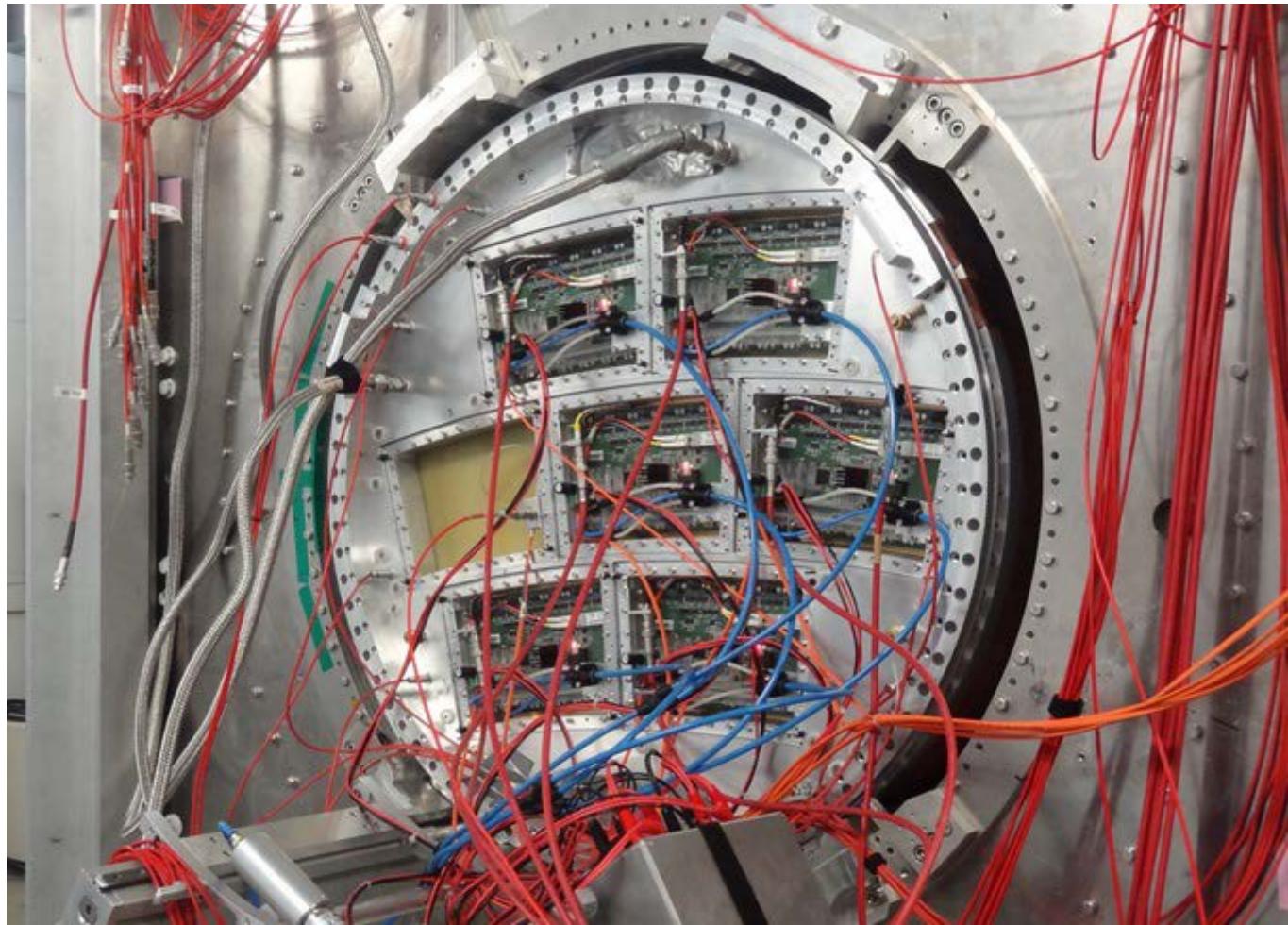
Source:  
Nicholi Shiell  
M.Sc. Thesis  
Carleton University

Resolution v. Drift Distance (All Scans)



# 7-module LCTPC

2012 data  
7-module



- MarlinTPC is the global effort to develop a single analysis code package for all the different prototype TPCs being developed.
- It is far from complete, but it has a solid foundation
- Furthermore, now seems to be the optimal time to no longer rely on stand alone code with hardcoded geometry, stand alone track-fit algorithm, calibration constants, etc....
- Goal to have processors: calibration for PRF determination, bias corrections and resolution determination (transverse and longitudinal)

## – NativeToLCIO

- Converts data from the native file format of the detector hardware to the LCIO standard

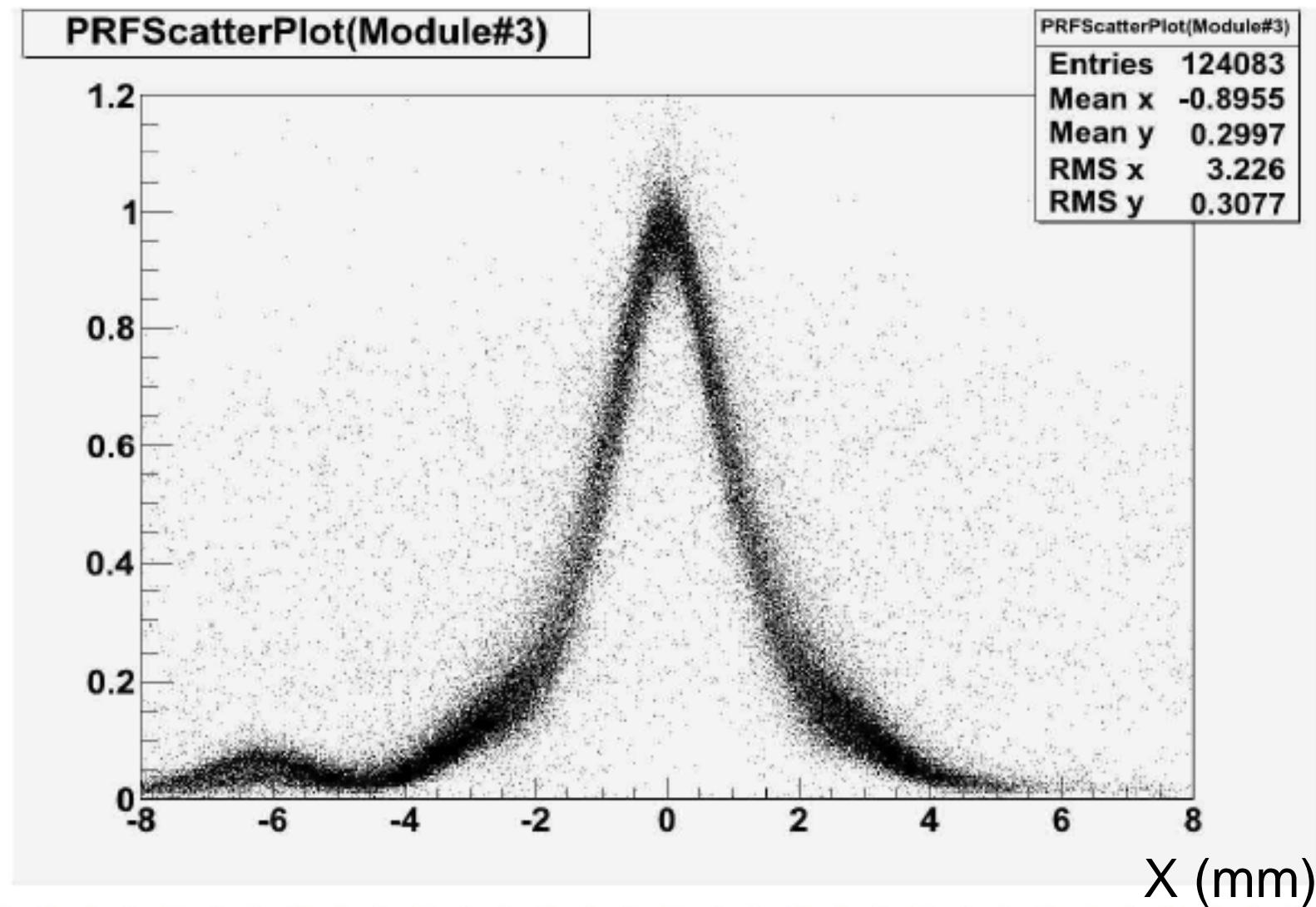
## – Main Code (i.e. Processor)

- DD: creates dense data files from LCIO
- Need a seed track
- PRF: determines track parameters and/or pad response function (PRF)
- BIAS: calculates and saves values used for bias and reso ROOT scripts

## – ROOT Scripts:

- BIAS: calculates and corrects for signal bias inherent to the detector
- RESO: calculates the resolution

# PRF first look



$$\chi^2 = \sum \left[ \frac{A_i - k \text{PRF}(x_i, r, w_i)}{A_i} \right]^2$$

$$\text{PRF}(x; r, w) = \frac{\exp(-4\ln(2)(1-r)x^2/w^2)}{1 + 4rx^2/w}$$

$x_i$  = position of track on row i

r = fraction of Lorentzian (r=1 pure lorentzian  
r=0 pure gaussian)

w = width

where we assumed  $\sigma^2(A_i) = A_i$

Seed track (initial position “x” of the track) is obtained via a weighted mean of amplitudes on a row

Remark: Need to implement “eta” correction for determination of the weighted mean per row for avoiding small bias.



# Amplitude Determination

$$k f_i = A_i$$

$$k = \frac{A_i}{f_i}$$

K – Constant coefficient of the PRF

A<sub>i</sub> – Pad Amplitude

f<sub>i</sub> - PRF evaluated at position of i<sup>th</sup> pad

Define PRF coefficient as:

$$k = \frac{\sum A_i}{\sum f_i}$$

Remark: In the old method “k” computed by setting  $\frac{d\chi^2}{dk} = 0$

$$k = \frac{\sum_j^m \frac{A_j \text{PRF}(x_j)}{\sigma_{A_j}^2}}{\sum_j^m \frac{\text{PRF}^2(x_j)}{\sigma_{A_j}^2}} = \frac{\sum_j^m \frac{f_j}{A_j}}{\sum_j^m \frac{(f_j)^2}{(A_j)^2}} \approx \frac{\sum A_j}{\sum f_j} \text{ since } \sum f_i \approx \int f dx = \text{constant}$$

# 3 Pad Example (e.g.)

$$kf_1 = A_1 \qquad kf_1 - A_1 = 0$$

$$kf_2 = A_2 \quad \longrightarrow \quad kf_2 - A_2 = 0$$

$$kf_3 = A_3 \qquad kf_3 - A_3 = 0$$

Since all equations equal 0 adding them together should also give zero:

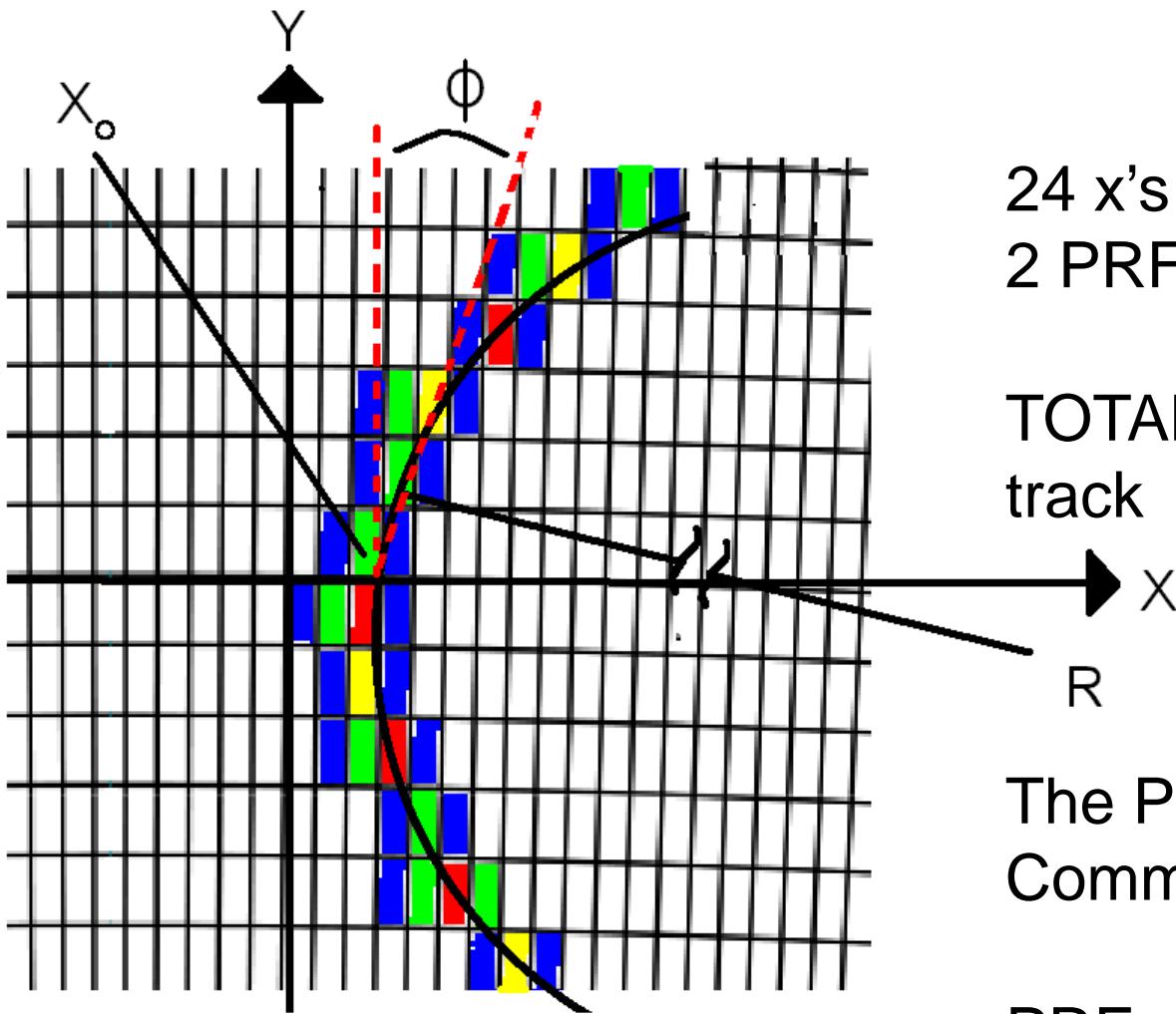
$$kf_1 - A_1 + kf_2 - A_2 + kf_3 - A_3 = 0$$

$$kf_1 + kf_2 + kf_3 = A_1 + A_2 + A_3$$

$$k(f_1 + f_2 + f_3) = A_1 + A_2 + A_3$$

$$k = \frac{A_1 + A_2 + A_3}{f_1 + f_2 + f_3}$$

# PRFBasedHitFinderProcessor



24 x's [one position per row]  
2 PRF parameters

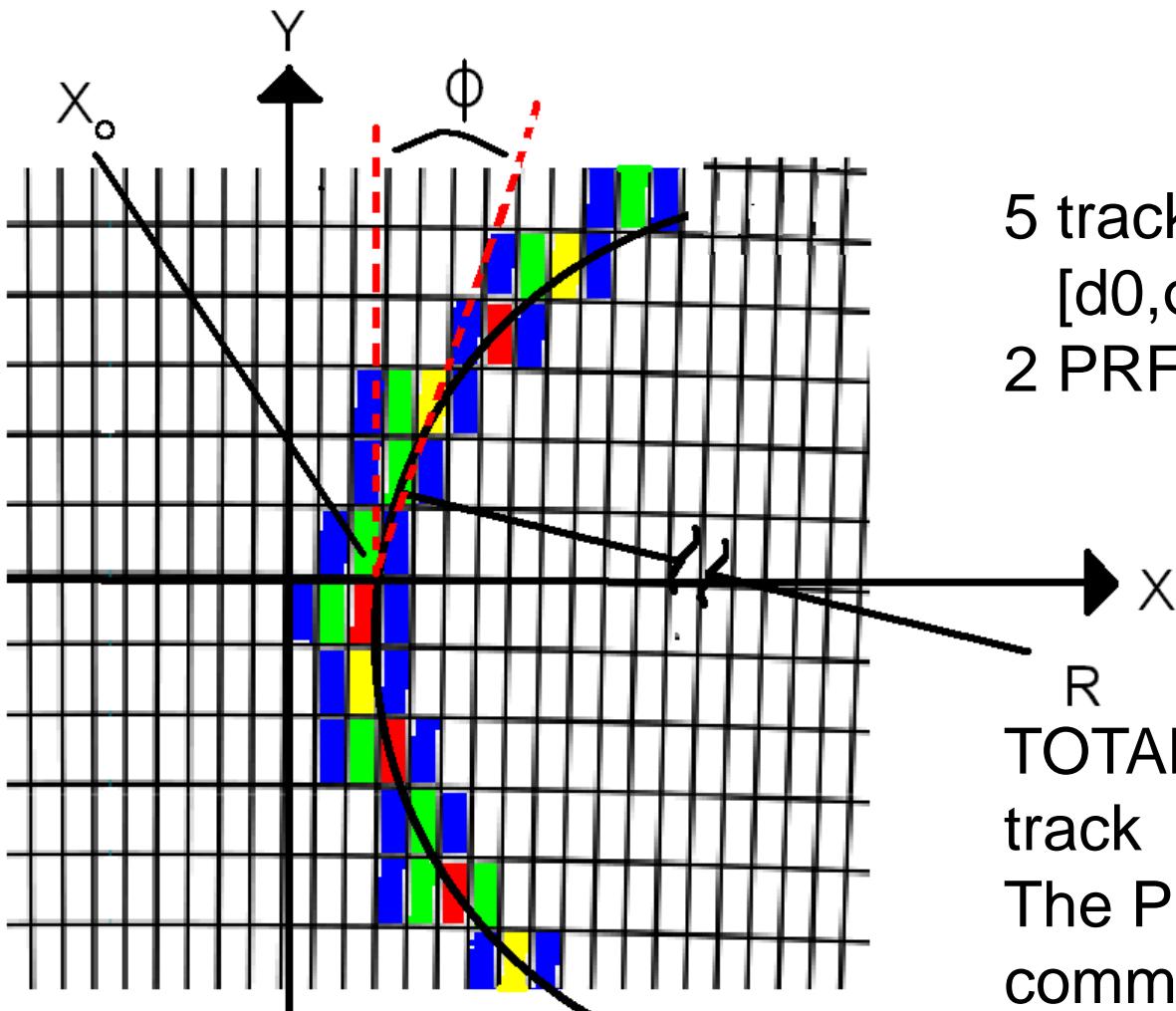
TOTAL of 26 parameters per track

X  
R

The PRF parameters r and w  
Common to all track

PRF weakly constrained

# PRFBasedTrackFinderProcessor



5 track parameters  
[ $d_0, \phi, CU \lambda, Z$ ]  
2 PRF parameters

X  
R

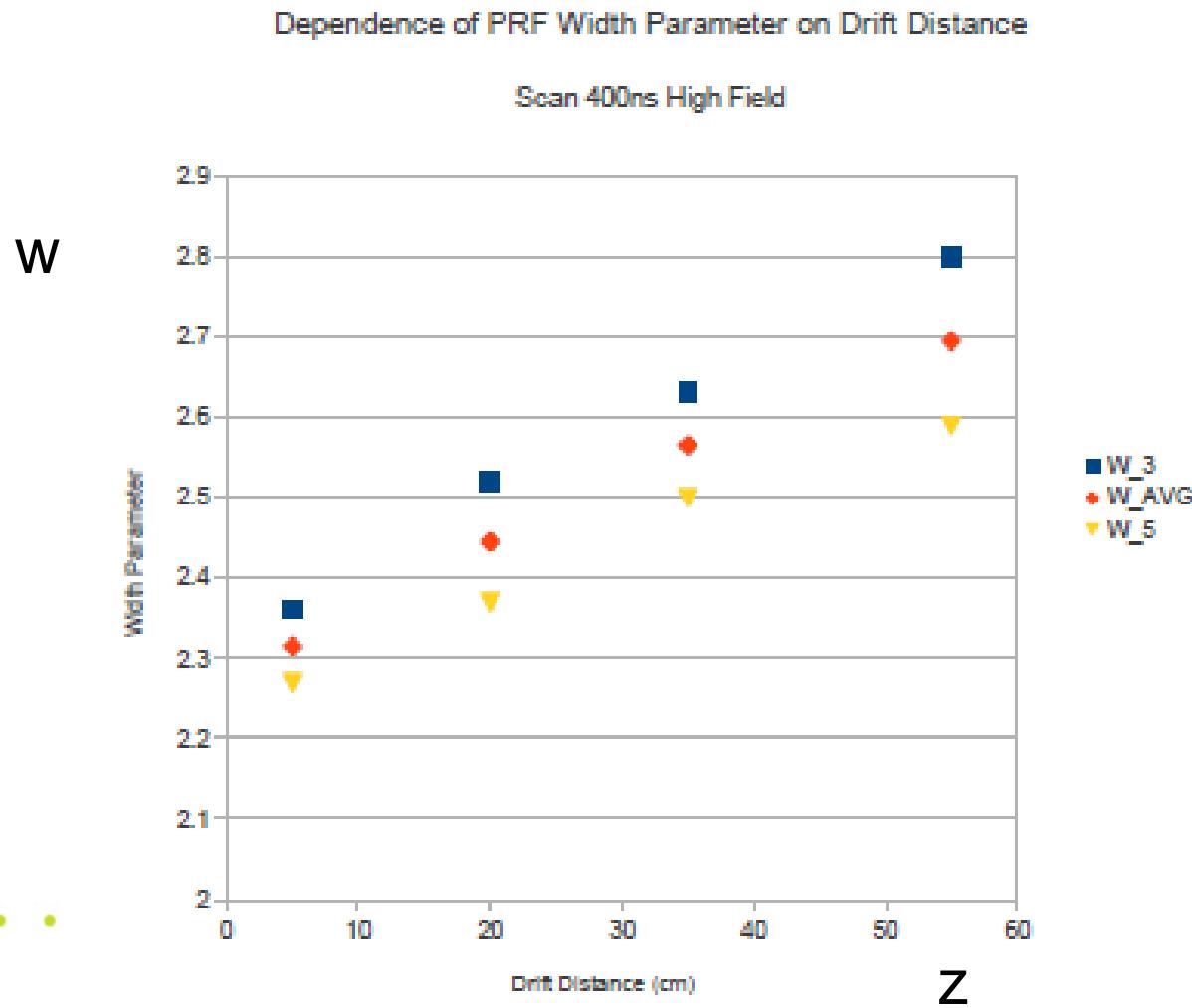
TOTAL of 7 parameters per track  
The PRF parameters  $r$  and  $w$  common to all track

PRF very well constrained

# PRF width parameters

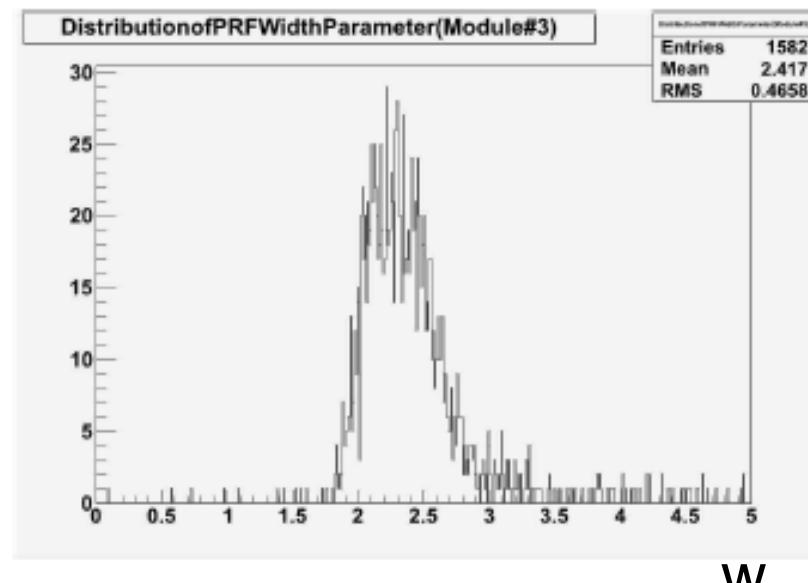
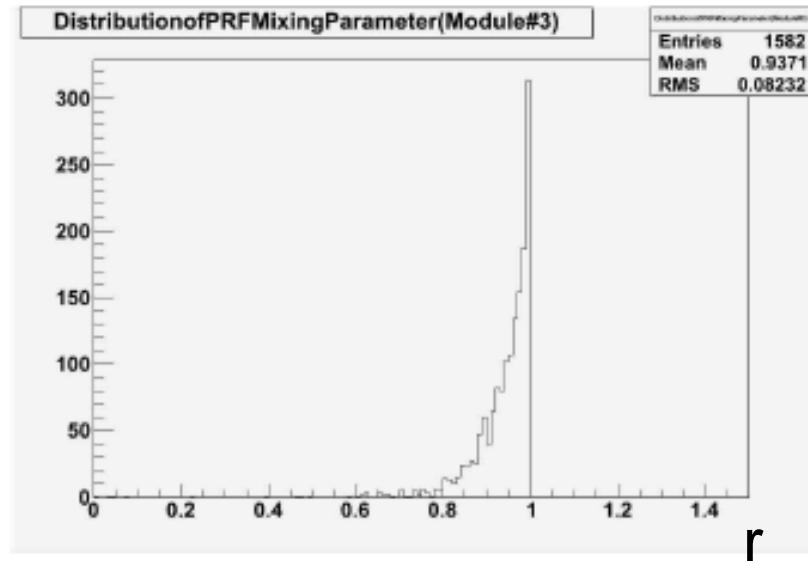
Run #	Drift Distance (cm)	W_3	W_5	W_AVG
2180	5	2.36	2.27	2.315
2182	20	2.52	2.37	2.445
2186	35	2.63	2.5	2.565
2188	55	2.799	2.59	2.6945

$$r \approx 0.93$$

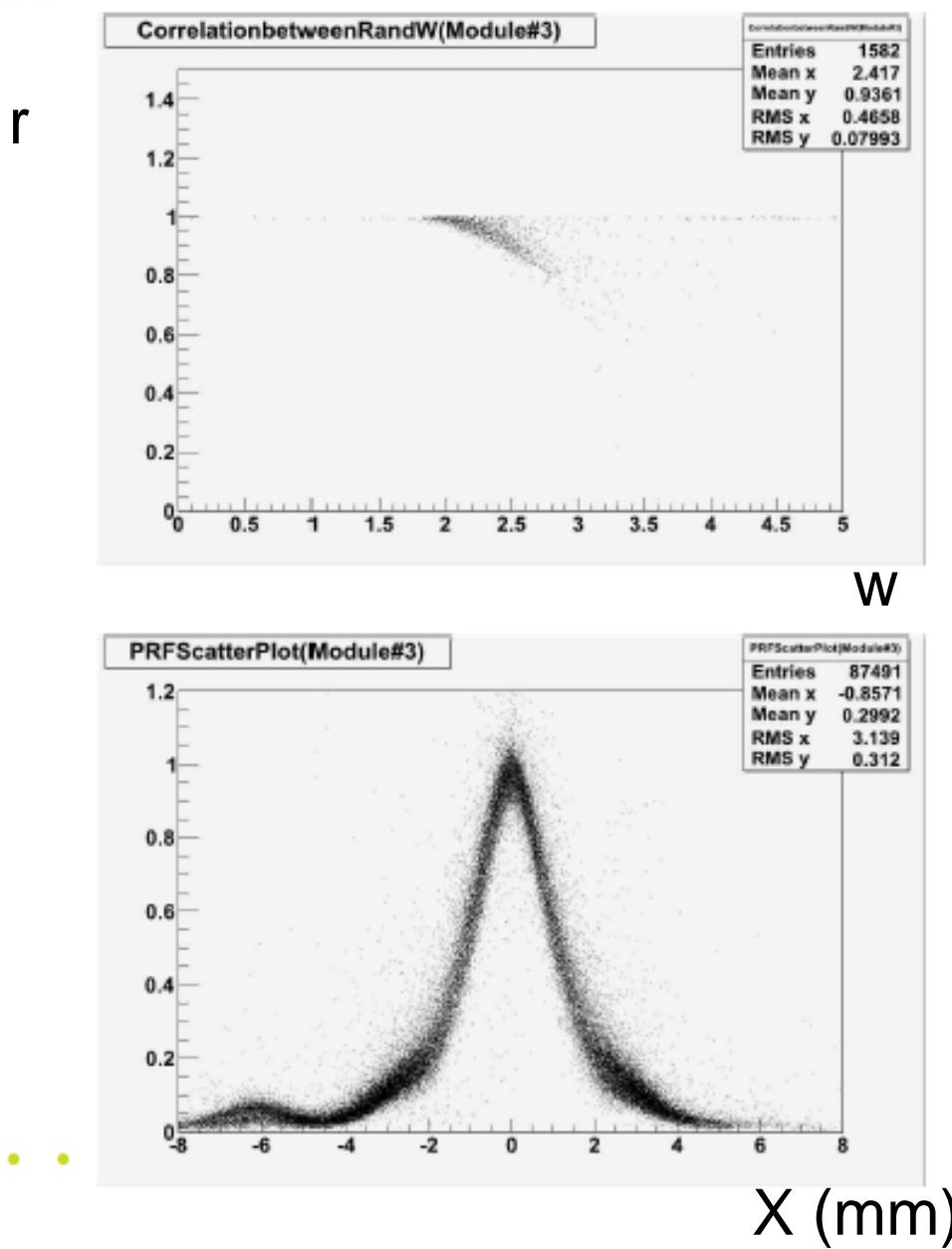


# PRF “new”

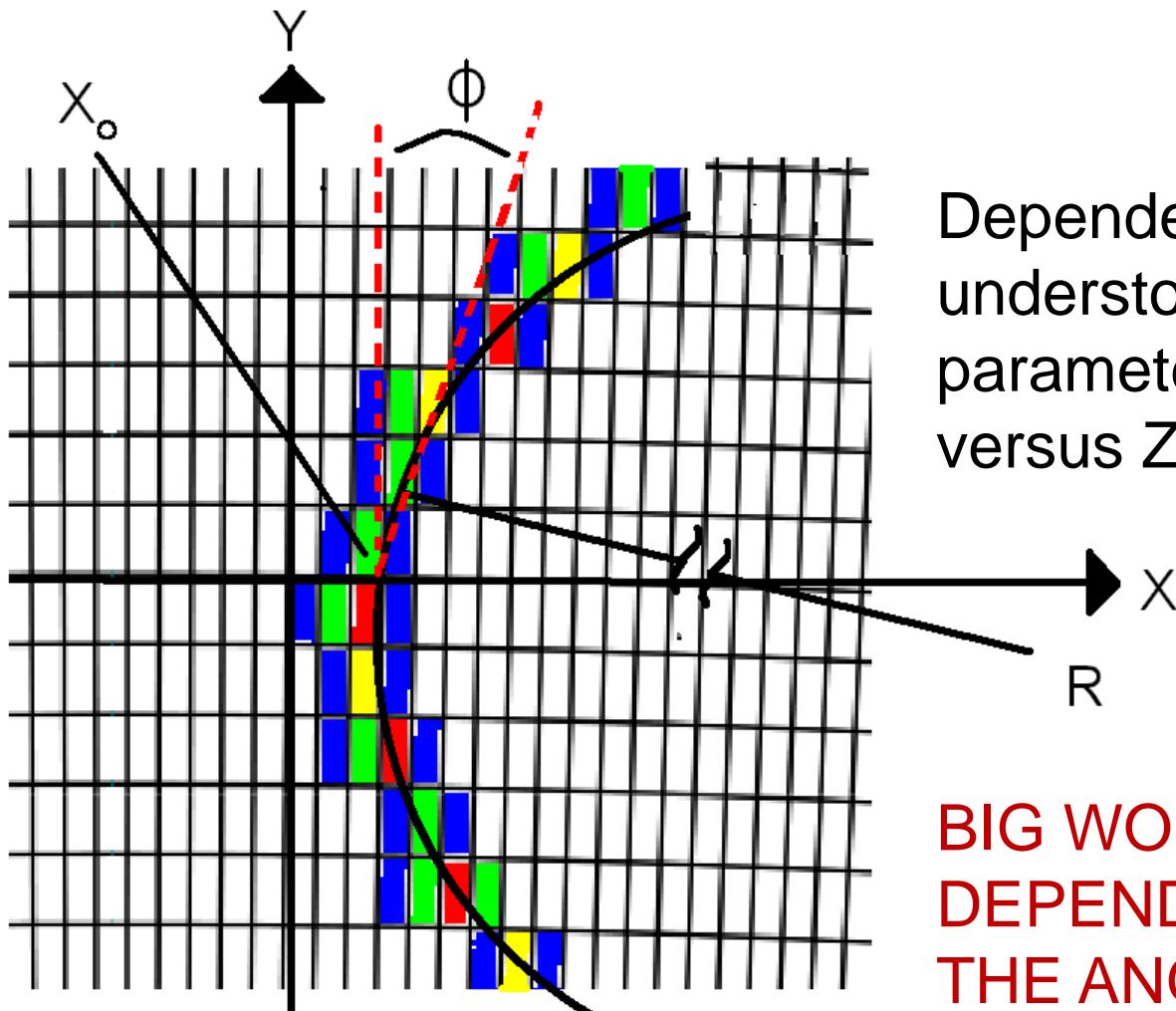
Run 2180 (Peaking = 400ns Drift = 5cm R < 1.0)



## PRF “new”

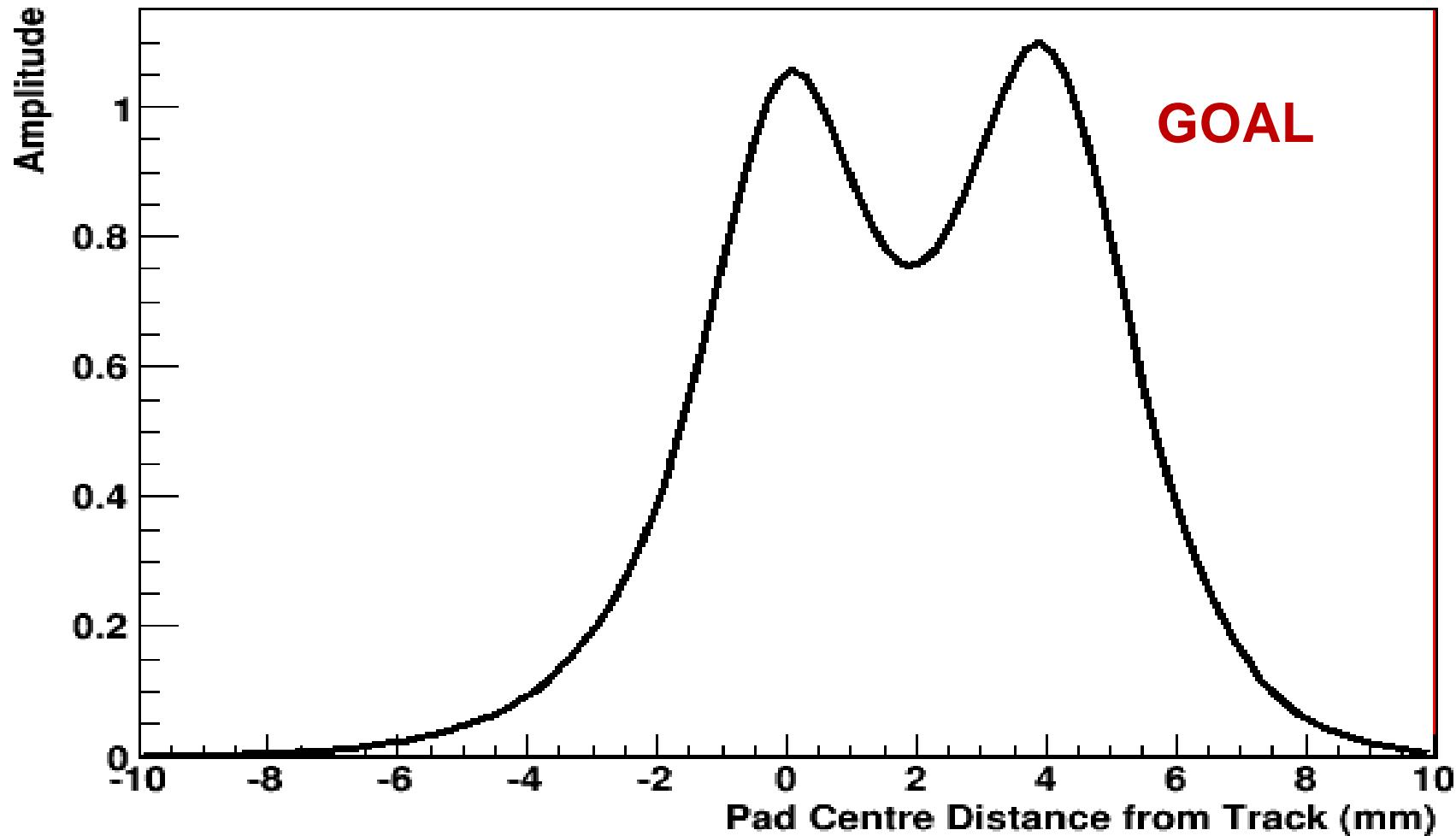


# PRF dependency in $\phi$



# Example: Two-track finding per row

PRF for Two Tracks (Centred on Left Track)



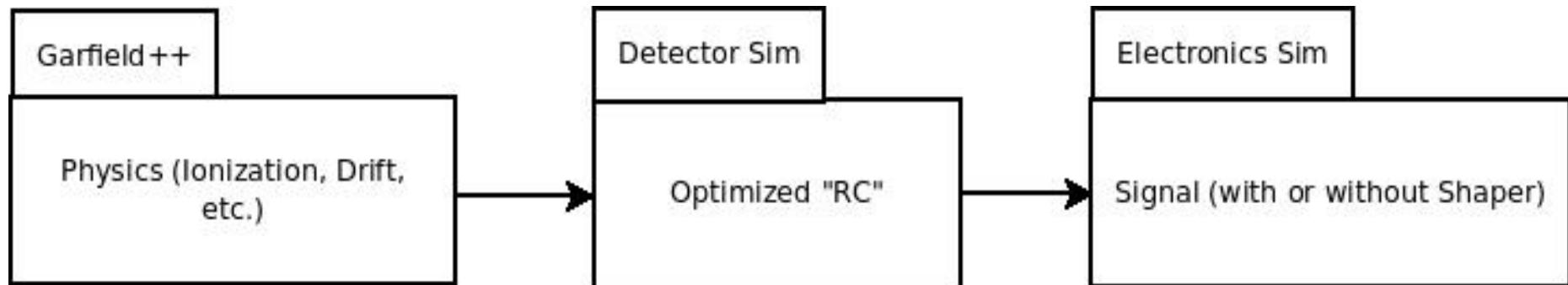
- Implement PRF Parameterization with Errors
  - Reconstruction in 3D (x,y,z) and properly account for errors when calibrating the PRF, such that the PRF can be used to find 3D hits and their errors in Marlin
- Implement PRF Calibration in Marlin
  - The calibration process formerly done in the FTPC code now ported to Marlin. This will allow direct calibration with the multi-module prototype, which could potentially return better parameterizations from previous prototypes
- Simulation Signals:  $N_{\text{electron/ion}} \rightarrow (A,t) \rightarrow \text{PRF}$ 
  - Full understanding of ionization, transport, geometry, and electronics response for 3D tracks

- Concurrently developing simulation of Micromegas detector
- The procedure for the analysis is, basically



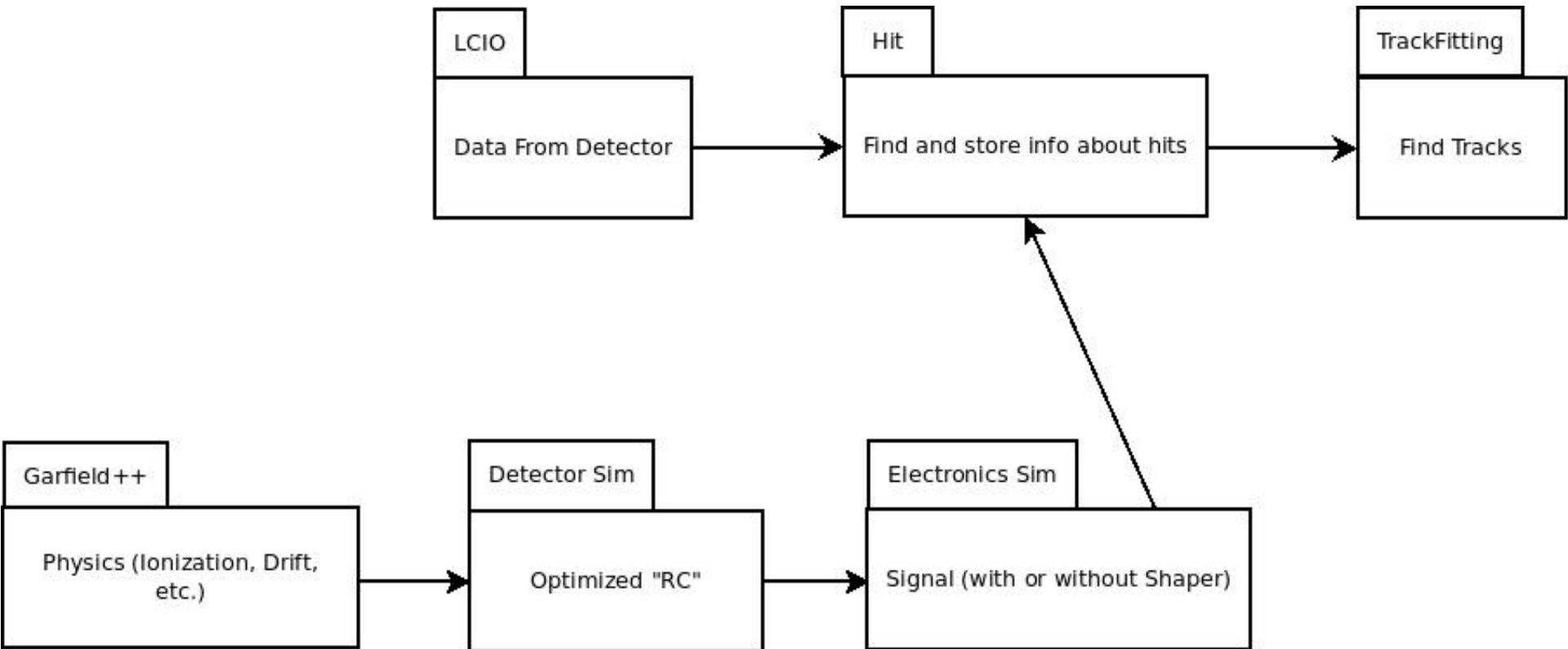
# Simulation

The simulation will perform the following calculations,

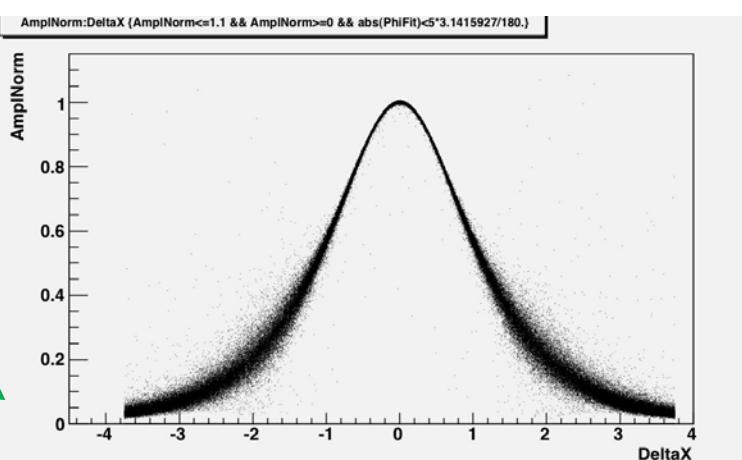


And this will fit in with the analysis work, by simply replacing the detector data with the simulated data in the analysis procedure.

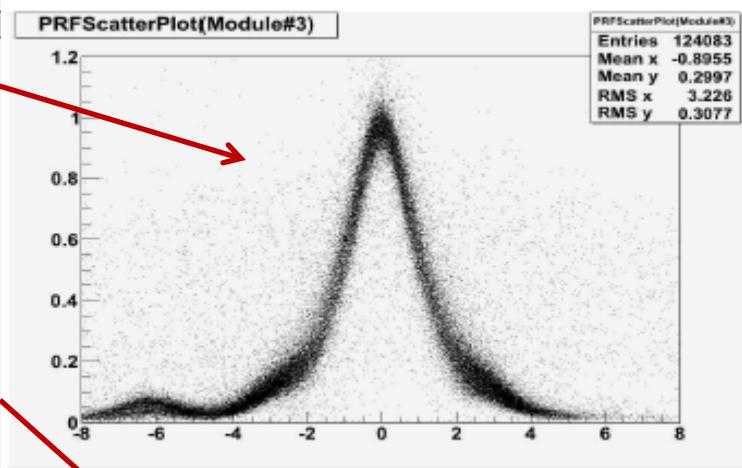
# Simulation



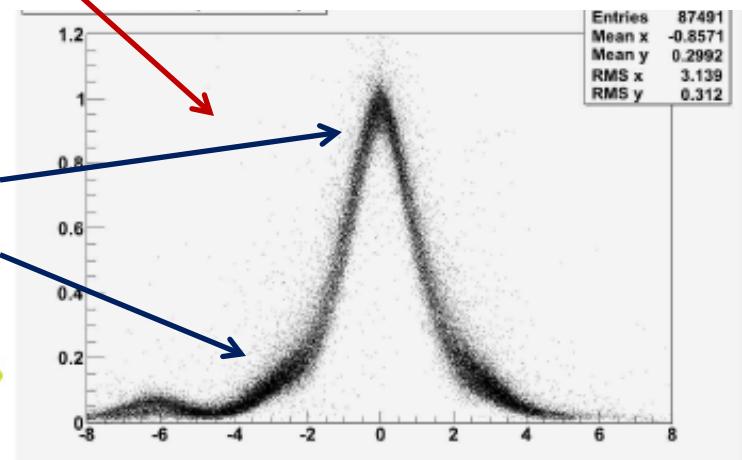
( $x=-6$ ) ?



More “triangular” looking PRF in 2012 due to less pad hits per row



Small bias of track position to be on pads.



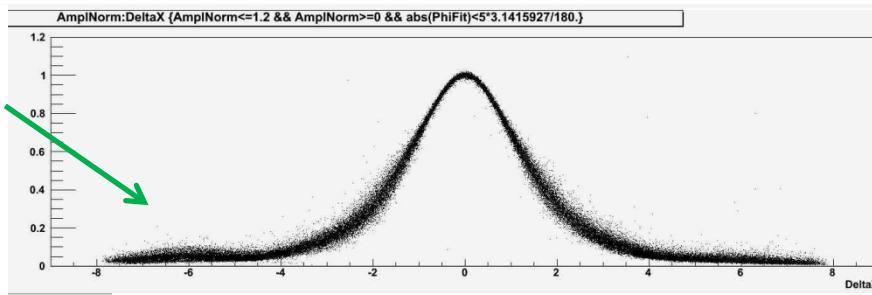
# Summary

2011 (single module)  
Average number of pad per hit per row is **4.5**

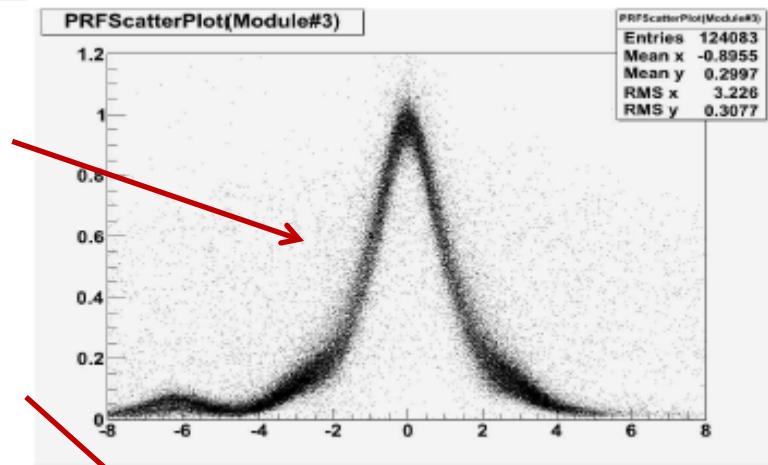
2012 (7-module)  
First look hit based  
Average number of pad per hit per row is **2.1**  
leads to artificial narrower PRF [less stat]

2012 (7-module)  
New MarlinTPC  
Track/PRF fit

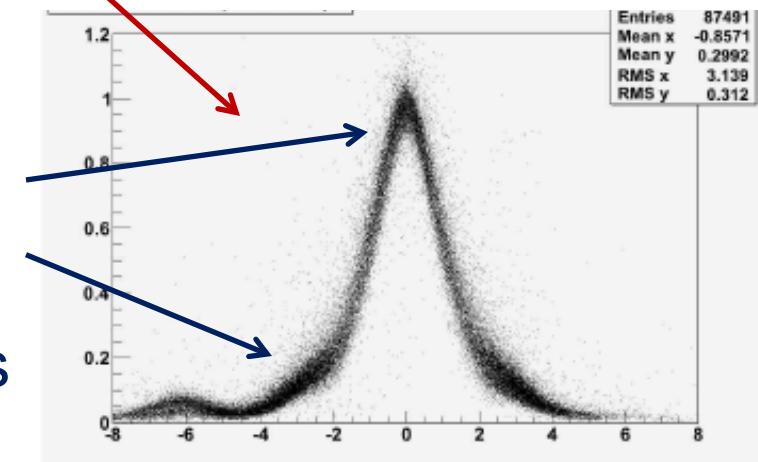
Is was  
there  
smaller ?



More  
“triangular”  
looking PRF  
in 2012 due  
to less pad  
hits per row



Small bias  
of track  
position to  
be on pads



# Summary

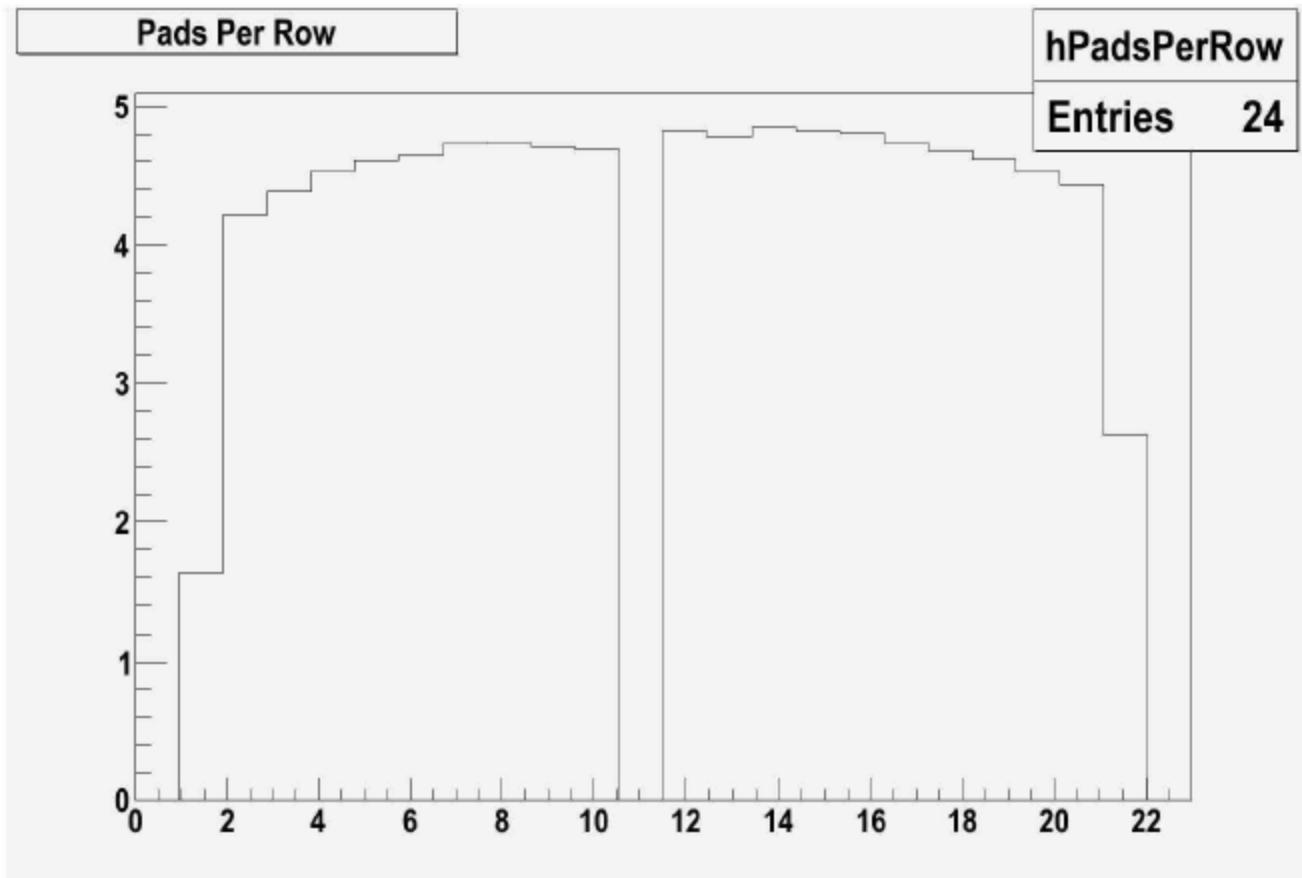
2011 (single module)  
Average number of pad  
per hit per row is **4.5**

2012 (7-module)  
First look hit based  
Average number of pad  
per hit per row is **2.1**  
leads to artificial  
narrower PRF [**less stat**]

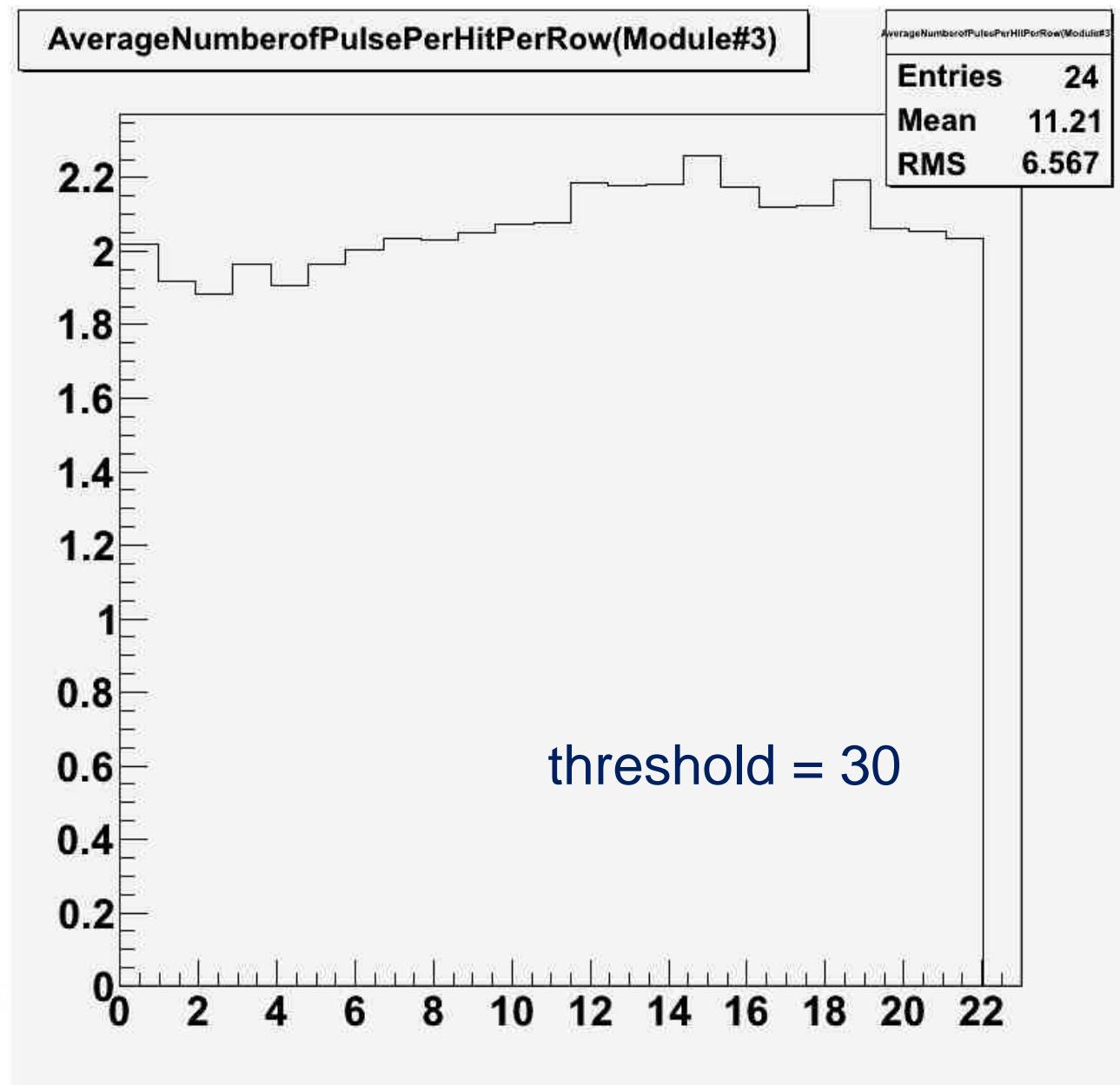
2012 (7-module)  
New MarlinTPC  
Track/PRF fit  
**Pad cross-talk**

# Average hit per row (2011) = **4.5**

2011 Run# 1229 No Cut

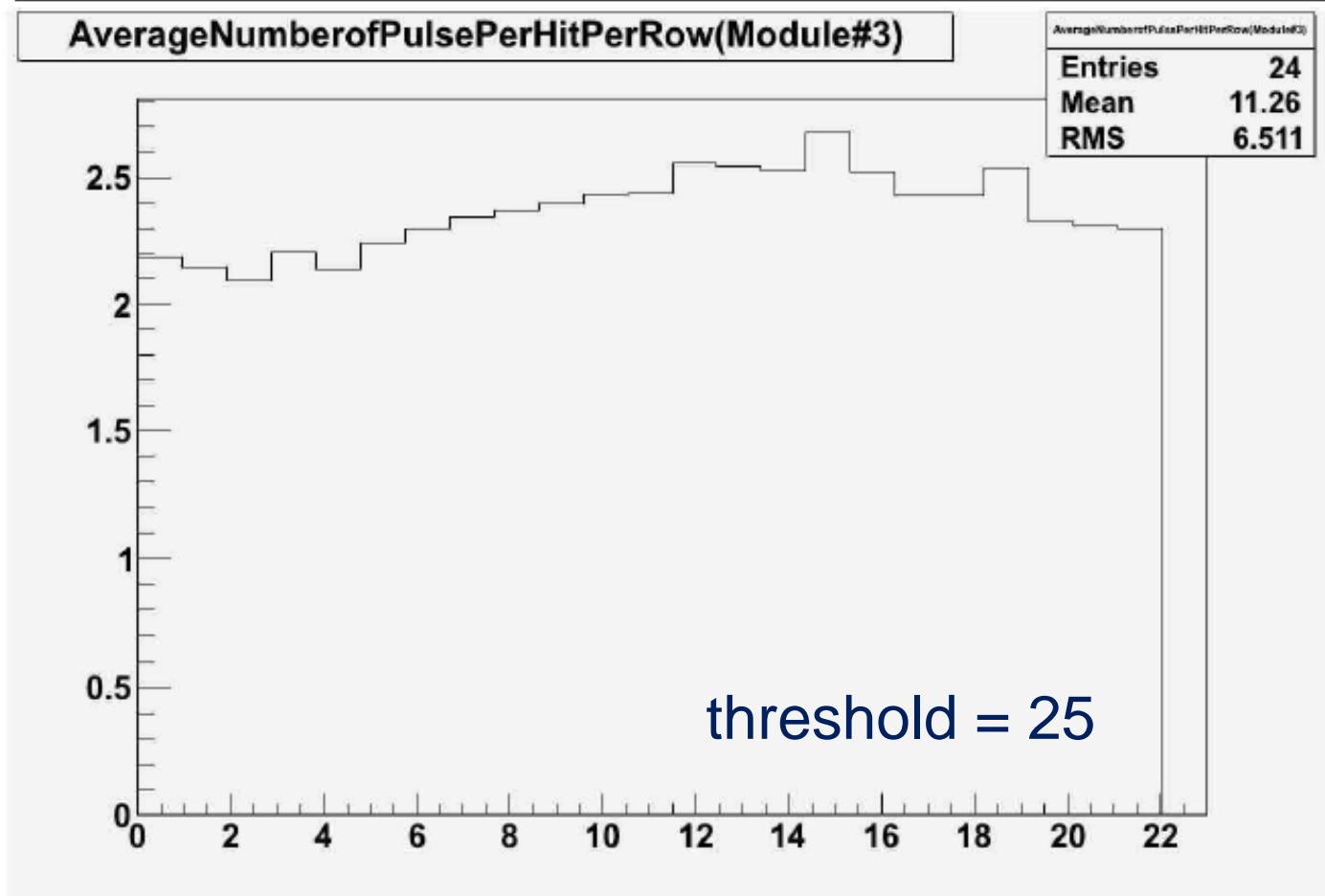


# Hit per row (2012) = 2.1



# Hit per row (2012) = 2.4

2012 Run# 2185 Threshold 25 Min Pulses 1



- Look at 2012 data
  - Resolution acceptable (see Wenxin's talk)
  - MarlinTPC used for data analysis of 7-module data
  - Now can fit PRF and/or tracks
- Diagnostics:
  - Need to measure the characteristics of the resistive layer and induction gap (need feedback from engineers and design electronic experts)
  - More noise so higher threshold
  - It leads to less hit per row ( $4.5 \rightarrow 2.1$ )
  - Narrower PRF
  - Other effects to be investigated and corrected
    - Cross talk
    - Bias estimators (position on row)

# Still to do...

- Future...
  - Alignment is a easy extension of (extra d.o.f.)  
**PRFBasedTrackFinderProcessor**
  - Two tracks resolution
  - Synthetic data (test full fit)
  - $dE/dx$
  - etc...

- Progress toward **PRFBasedTrackFinderProcessor**
  - Second implementation in MarlinTPC soon completed
  - Investigation of error on amplitude and time (A,t)
  - Pad Response Function (PRF) to define a hit in 3D
  - Investigate  $\varphi$  dependency
  - Transverse resolution versus Z ( $\sigma_0$  and  $N_{\text{eff}}$ ) of a hit as well as longitudinal resolution (time resolution) to be used for later "track fitting" (PRF-track is chicken-egg)
  - Ready for 9-module testbeam (end of January 2013)
  - Expect improvement for new modules
- Long Term:
  - Handling Error from (A,t)  $\rightarrow$  (PRF,t)  $\rightarrow$  (x,y,z) for a Hit to find unbiased track estimators and their uncertainties
  - Simulation of amplitude and time (A,t) to close the loop