Problems Linac Basics

1) Calculate the relative longitudinal motion of two electrons with an energy of about 9 GeV and a difference of 3% over a distance of 21 km.

2) A superconducting linac consists of cavities with a length of L = 1.1 m and an external coupling of $Q_{ext} = 10^5$. It is operated in matched conditions (no reflected power) with a gradient $G_0 = 20 \text{ MV/m}$ and a beam current of $I_0 = 10 \text{ mA}$.

a) What is the input power per cavity *P* required?

b) The management wants to double the beam current but keep the gradient the same. In order to stay matched, which input power P_{new} does one need? Which other parameter needs to be changed and how?

3) A harmonic oscillator is a solution to Hill's equation for $K(s) = K_0 > 0$. Show that this fullfils the differential equation describing the development of $\beta(s)$. Which value has $\beta(s)$?

4) How much energy is roughly stored in one ILC cavity at nominal gradient?

5) Show that the beta-function in a drift (i.e. K(s) = 0) around a waist (i.e. $\beta(s = 0) = \beta_0$ and $\beta'(s = 0) = 0$) is a parabola. Hint: use this as an Ansatz. (Optional)