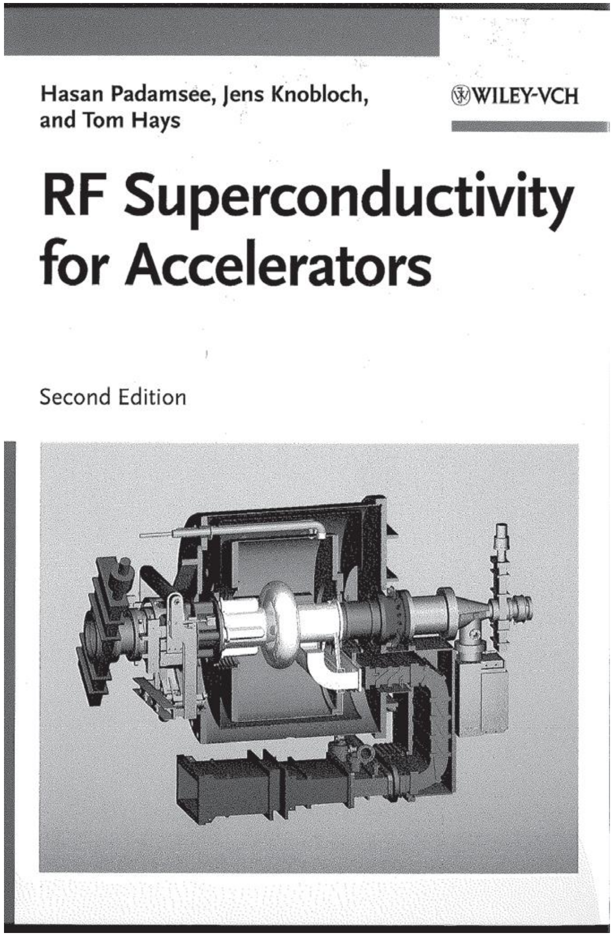
# Course B: Superconductive RF

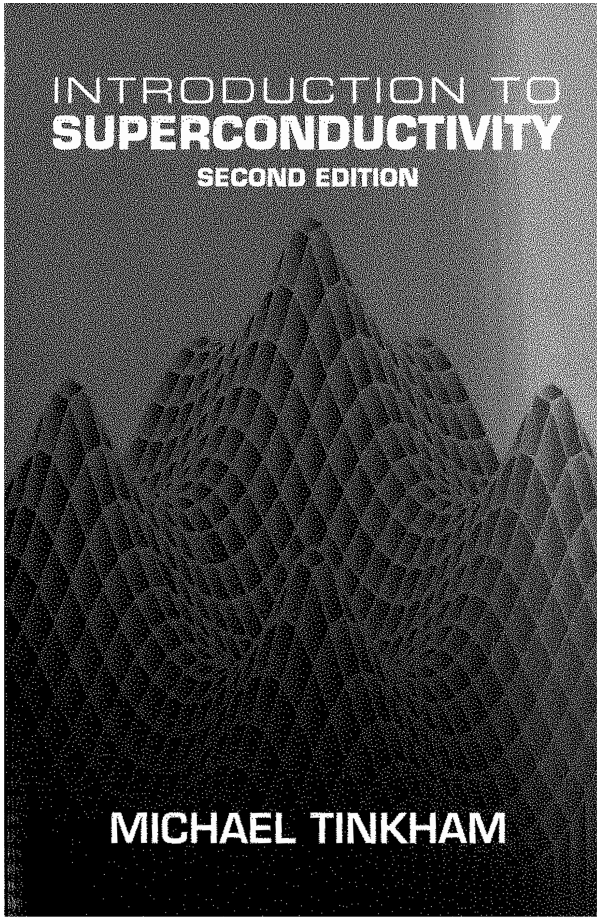
T. Saeki (KEK)

LC school 2013

5 - 15 Dec. 2013, Antalya, Turkey

# Reference books



# Course B: Superconductive RF

# RF Superconductivity Fundamental

T. Saeki (KEK)

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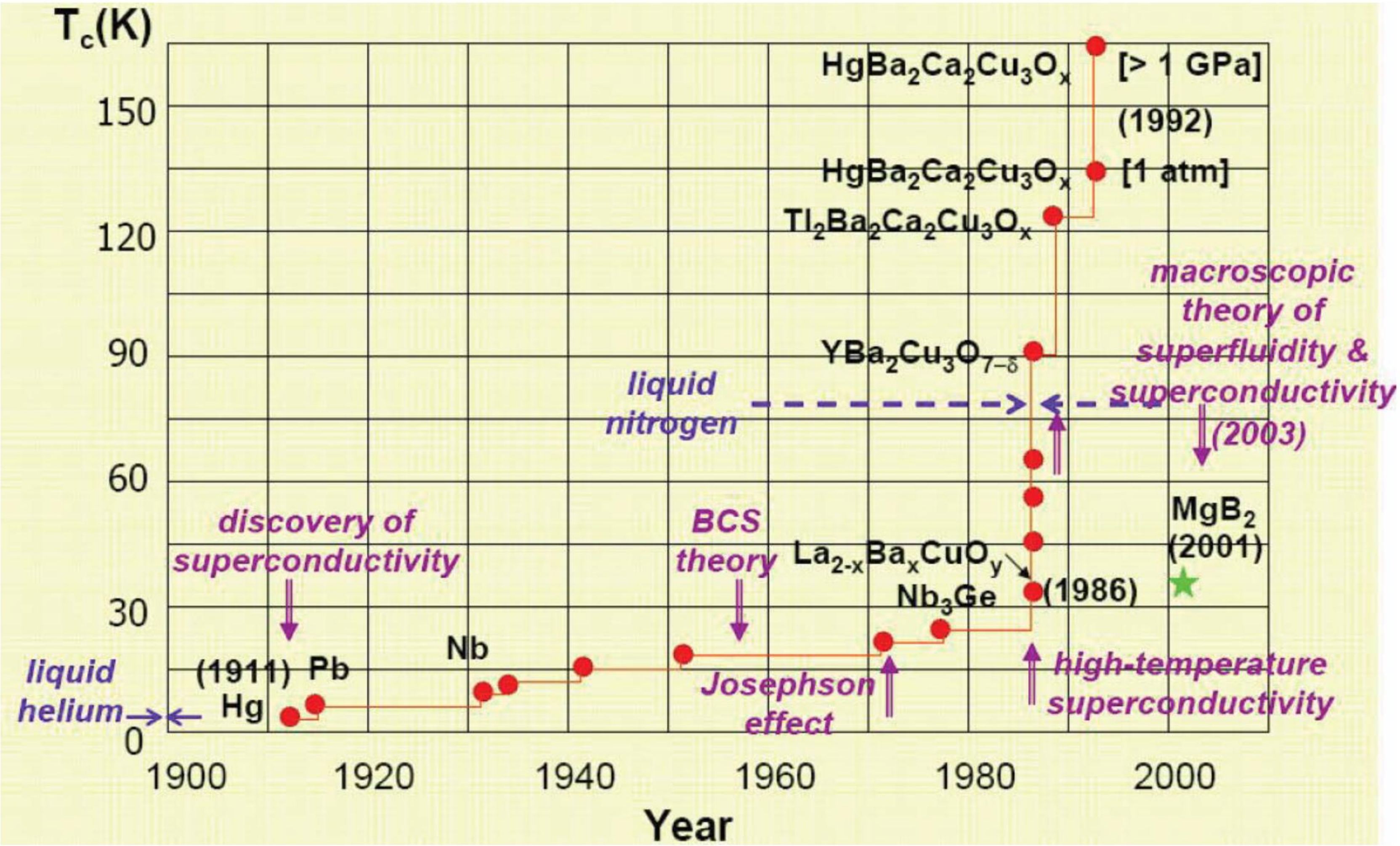
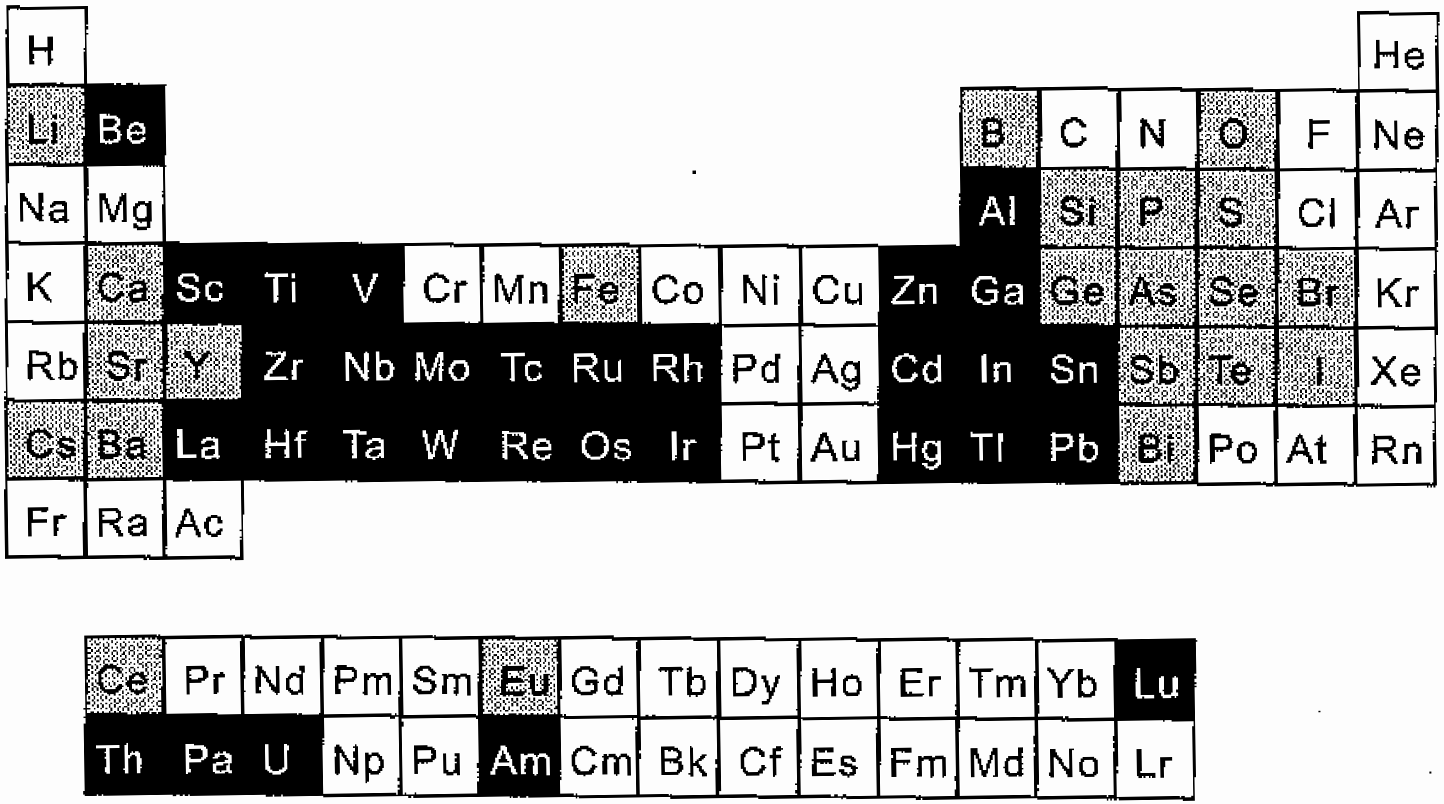
9 Dec. 2013, Antalya, Turkey

# History of Superconductivity 1



# Superconductive materials




- Al: SC materials in normal crystal condition
- Si: SC materials only in special conditions like high pressure, amorphous, etc.
- Cu: Materials for which SC phase has not been found yet.

Compound materials have also SC phase. ➡ High-temperature superconductivity materials.

# Non superconductive materials

- Even if they are metal, alkaline metals and good metals with high conductivity are non SC materials.
- Even if they are metal,  transition elements and Rare Earth Elements (REE) with magnetization are non SC materials.

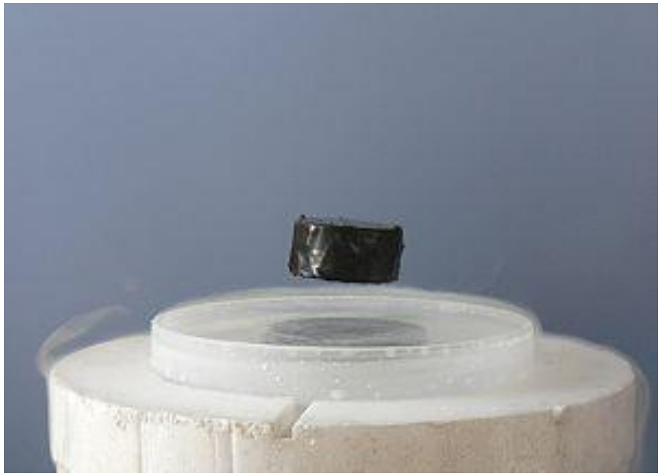
# History of Superconductivity 2 (1/2)

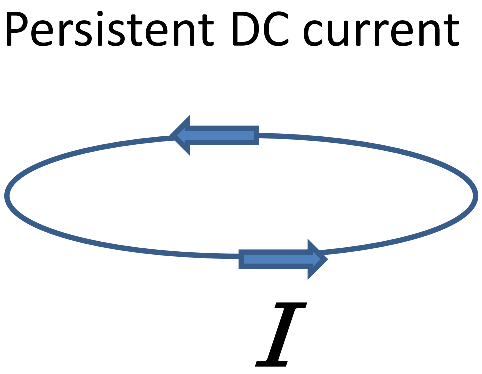
- 1908: H. K. Onnes and Van Der Waals liquefied He for the first time in Leiden.
- 1911: H. K. Onnes discovered superconductivity with Hg at 4.1 K in Leiden.
- 1914: H. K. Onnes discovered persistent current in loop superconductor.
- 1933: F. W. Meissner discovered Meissner effect.
- 1935: F. W. London and H. London established London equations which explained Meissner effect.
- 1935 – 37: L. V. Schvinikov, De Haas and Casimir-Jonker discovered two Hc's. Later this is called Hc1 and Hc2 of type-II superconductor.

# History of Superconductivity 2 (2/2)

- 1950: V. L. Ginzburg and L. D. Landau established GL theory, introducing thermo-dynamical states in superconductor.
- 1952: A. Abrikosov predicted Type-II superconductor.
- 1953: Pippard introduced Pippard coherent length.
- 1953: SC phase of Nb3Sn was found with Tc = 17 K (at rather high temperature).
- 1957: J. Bardeen, L. N. Cooper, and J. R. Schrieffer established BCS theory (microscopic understanding of superconductivity).
- 1961: Nb3Sn was confirmed to be Type-II superconductor.
- 1980's: High-temperature SC materials were found.

# Superconductivity in DC






Decay time of DC superconductive current > $10^5$ years

The perfect conductivity is the first hallmark of superconductivity.
But the superconductivity is not identical to the perfect conductivity.

# Perfect conductivity

Tc = Critical temperature

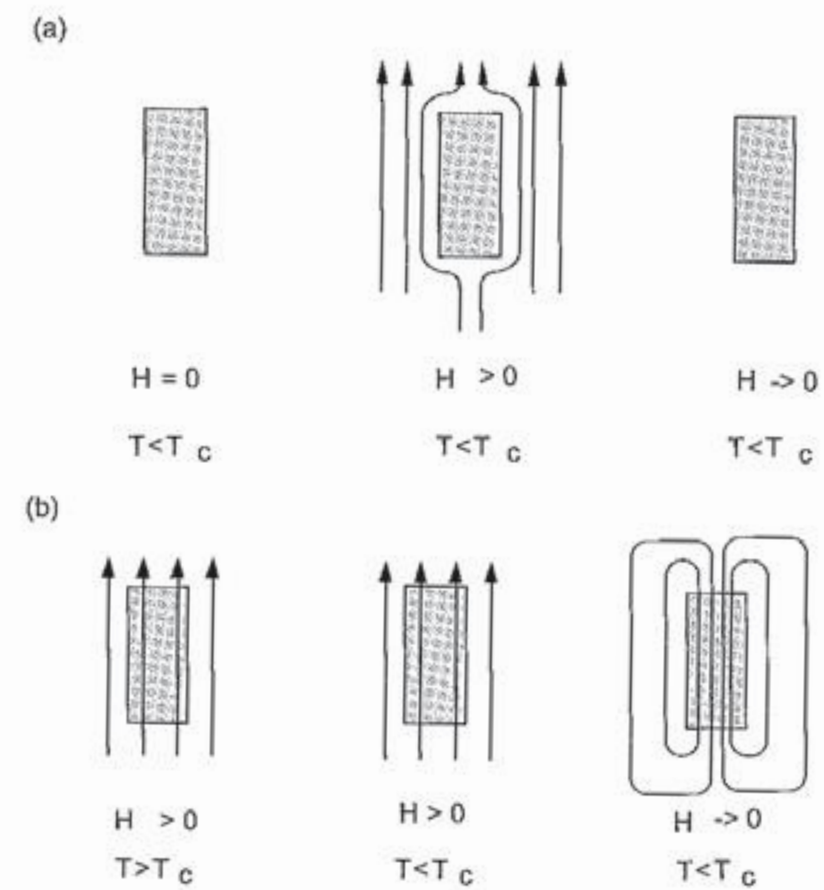
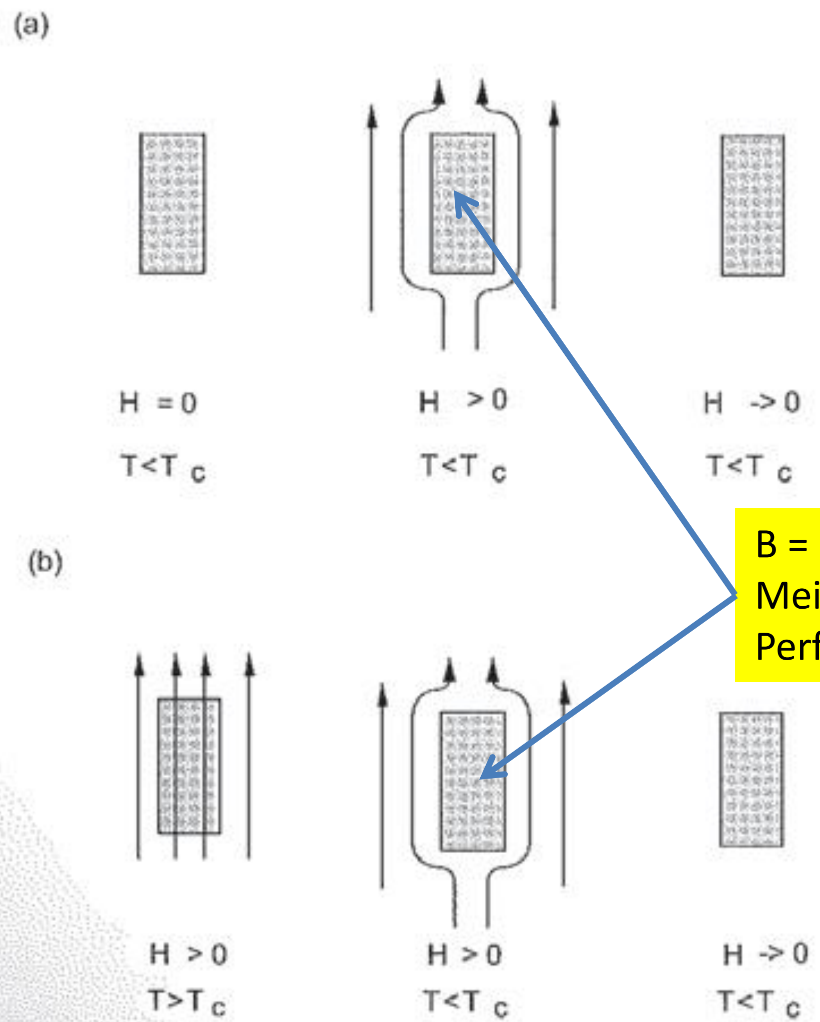



Figure 4.3: (a) Screening of external magnetic field by a perfect conductor. (b) Flux trapping in a perfect conductor.

# Superconductivity

Tc = Critical temperature

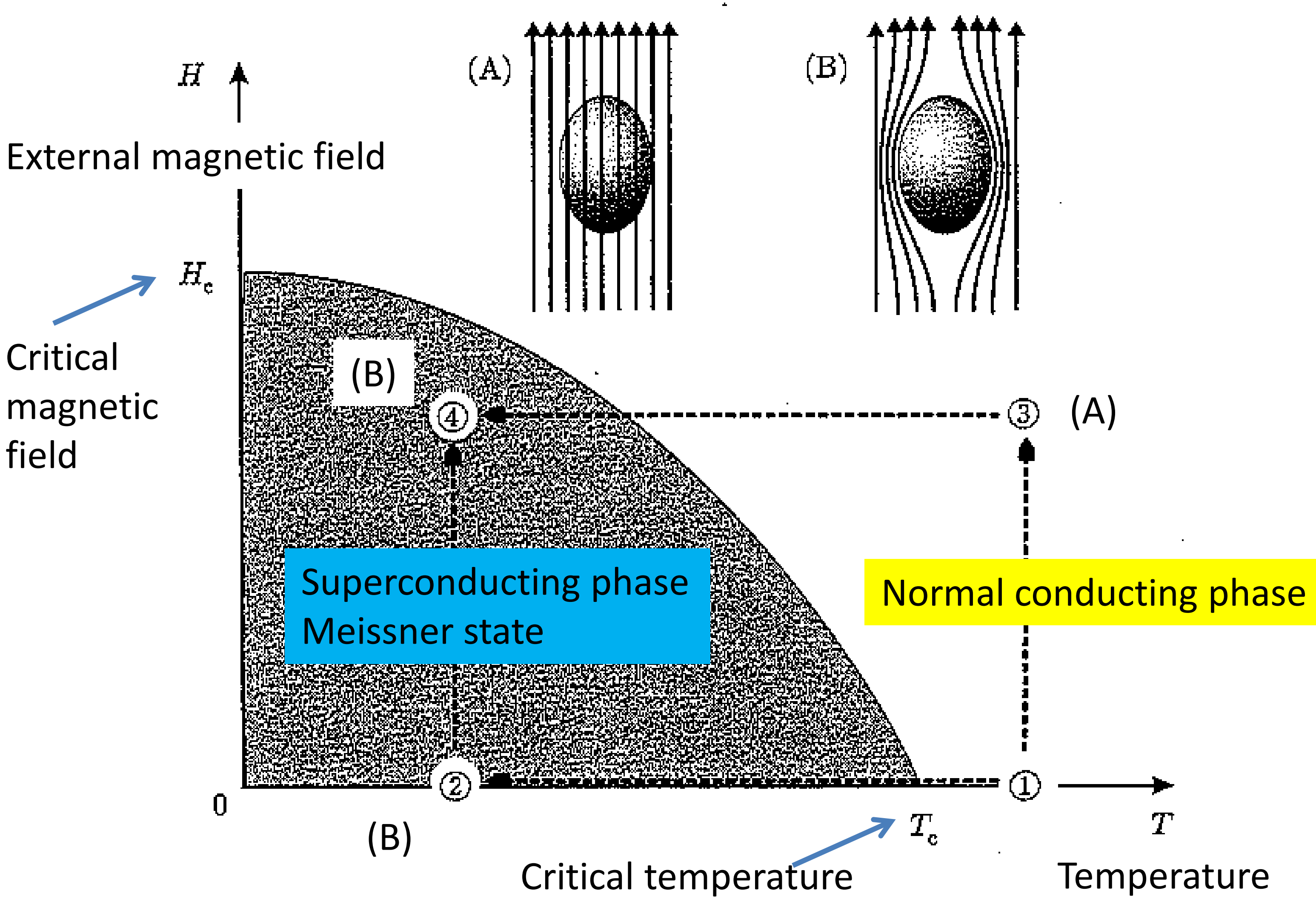



$B = \mu_0 H + \mu_0 M = 0$
Meissner effect
Perfect diamagnetism

Figure 4.4: (a) Screening of external magnetic field by a superconductor. (b) Meissner effect (flux expulsion) in a superconductor.

# Superconductivity

Superconductivity is the thermodynamic state.
If you give the thermodynamic parameters: T and H, the state is defined exactly.



# Thermodynamic properties

**Entropy** | **Specific Heat**

**Energy** | **Free Energy**

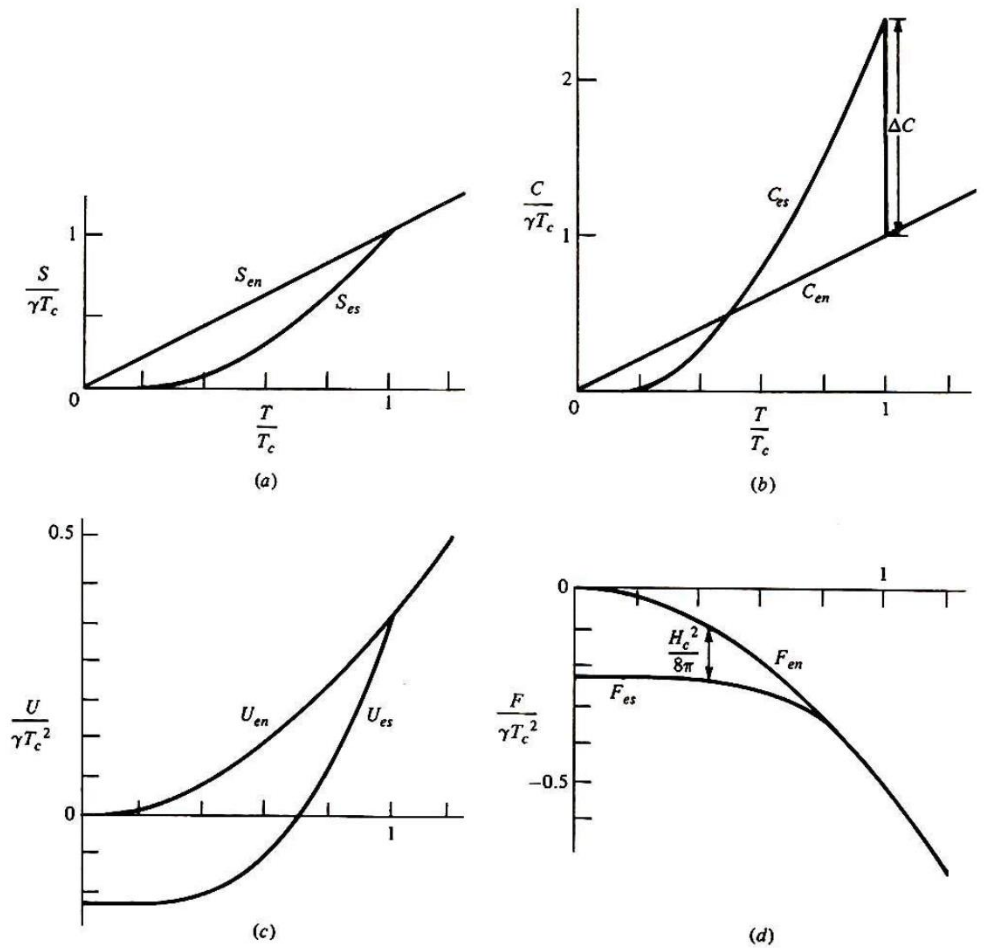
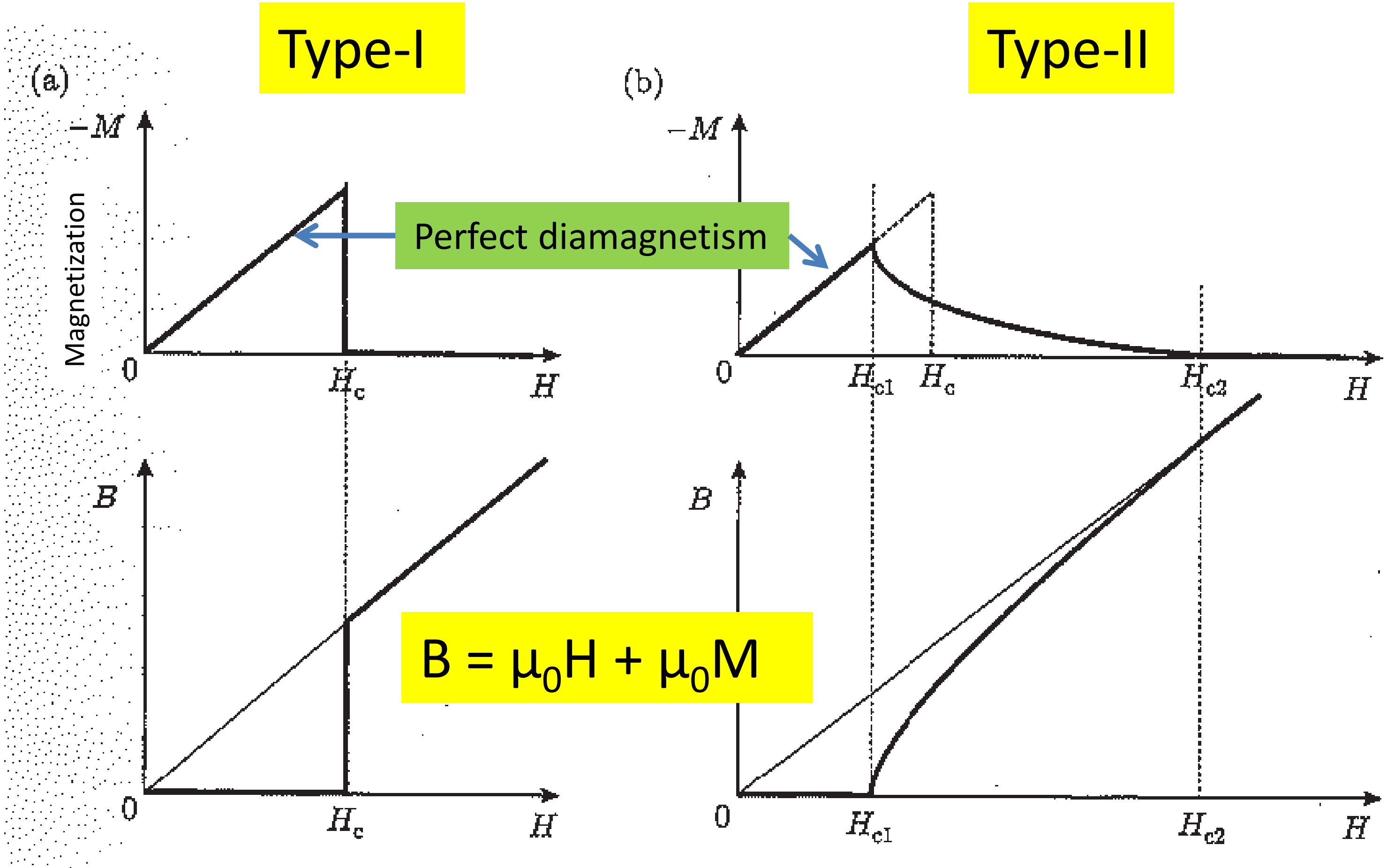



FIGURE 2-3
Comparison of thermodynamic quantities in superconducting and normal states. $U_{en}(0)$ is chosen as the zero of ordinates in ($c$) and ($d$). Because the transition is of second order, the quantities $S$, $U$, and $F$ are continuous at $T_c$. Moreover, the slope of $F_{es}$ joins continuously to that of $F_{en}$ at $T_c$, since $\partial F/\partial T = -S$.

# Type-I and Type-II superconductors

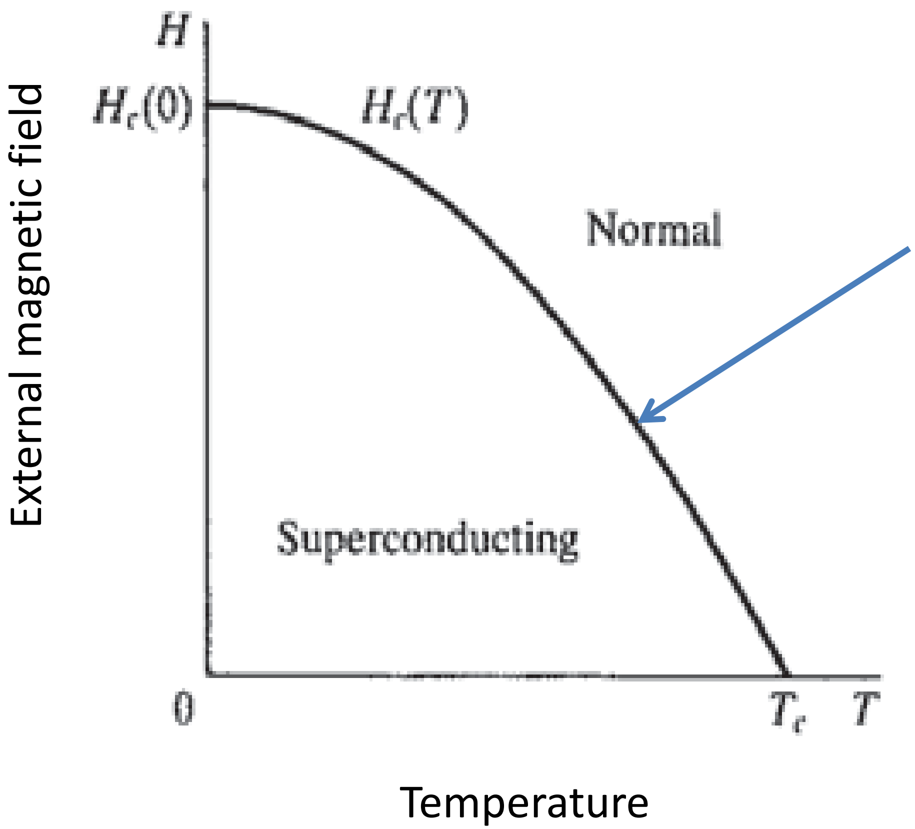



$B = \mu_0 H + \mu_0 M$

H = External magnetic field

# Empirical expression of Hc(T)

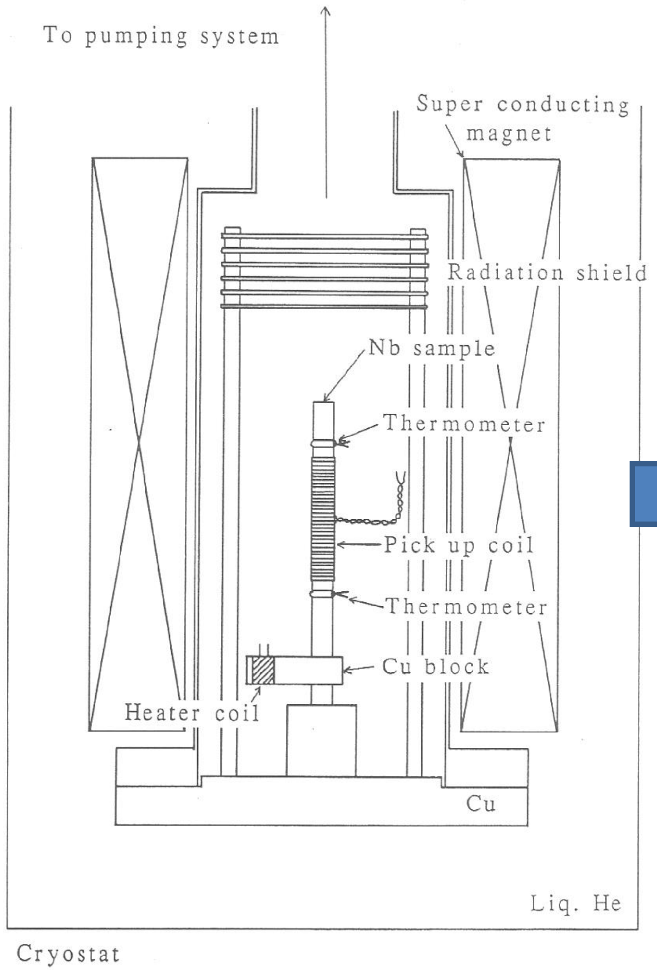
It was found empirically that Hc(T) is quite well approximated by a parabolic law.

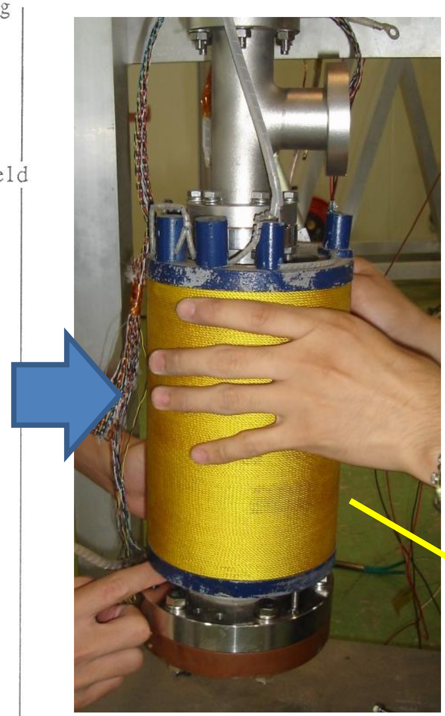


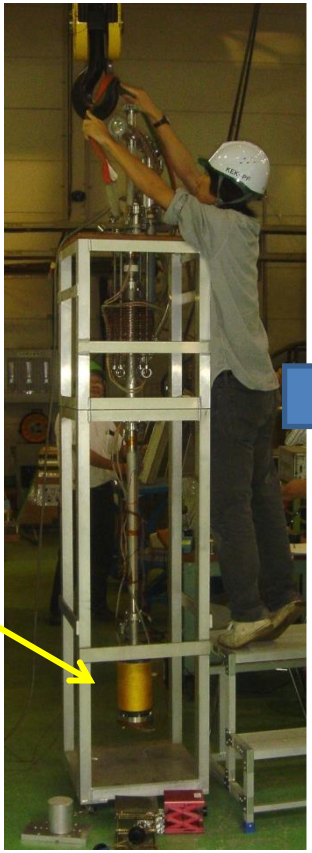

$$H_c(T) \approx H_c(0)[1 - (T/T_c)^2]$$

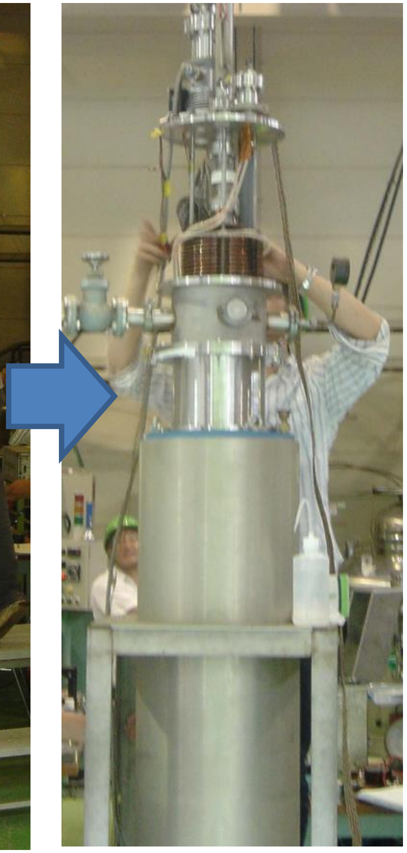
# Hc(T) measurements of Nb (Type-II)

Installed in cryostat



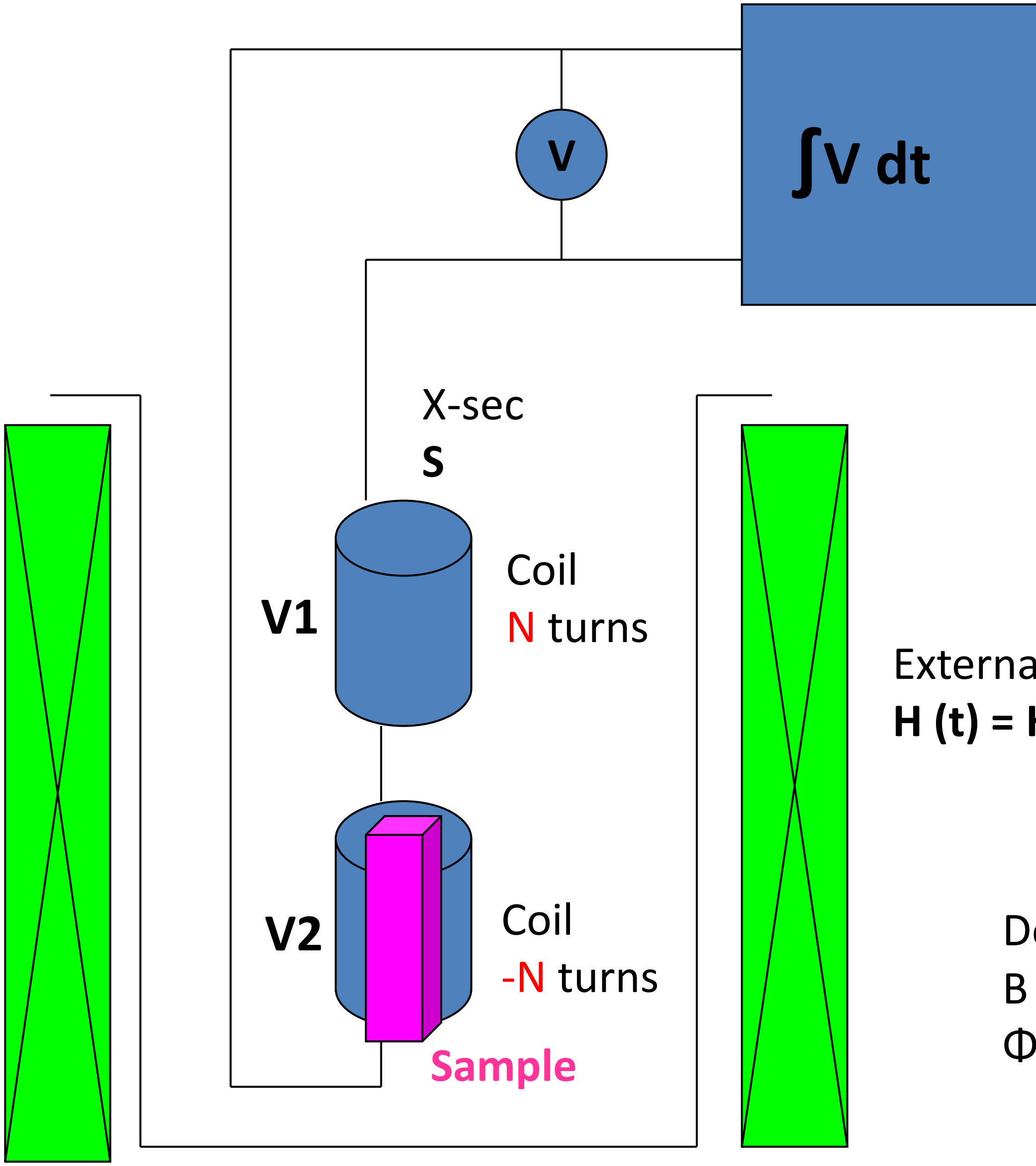







# Hc(T) measurement of Nb (Type-II)

**Setup**




External Magnetic Field

**H (t) = H0 x t (increase constantly)**

Definition (MKS)

B = $\mu_0$ H + $\mu_0$ M

Φ= N x B x S

# Hc(T) measurement of Nb (Type-II)

**$V = V1 + V2$**

**$V1 = -(d\phi / dt) = -(d\, nBS / dt) = -NS\,(dB / dt)$**
**$= -NS\,(d\,\mu o\, H / dt) = -\mu o\, NS\,(dH / dt)$**

**$V2 = (d\phi / dt) = (d\, NBS / dt) = NS\,(dB / dt)$**
**$= NS\,\{ d\,\{\mu o\, H + \mu o\, M\} / dt \}$**
**$= \mu o\, NS\,(dH / dt) + \mu o\, NS\,(dM / dt)$**

**$V = V1 + V2 = -\mu o\, NS\,(dH / dt) + \mu o\, NS\,(dH / dt) + \mu o\, NS\,(dM / dt)$**
**$= \mu o\, NS\,(dM / dt)$**

**$\int V\, dt = \mu o\, NS \int (dM / dt)\, dt = \mu o\, NS\,( M(t) - M(0) )$**
**$= \mu o\, NS\, M(t)$ ( because $M(0) = 0$ )**

# Hc(T) measurement of Nb (Type-II)

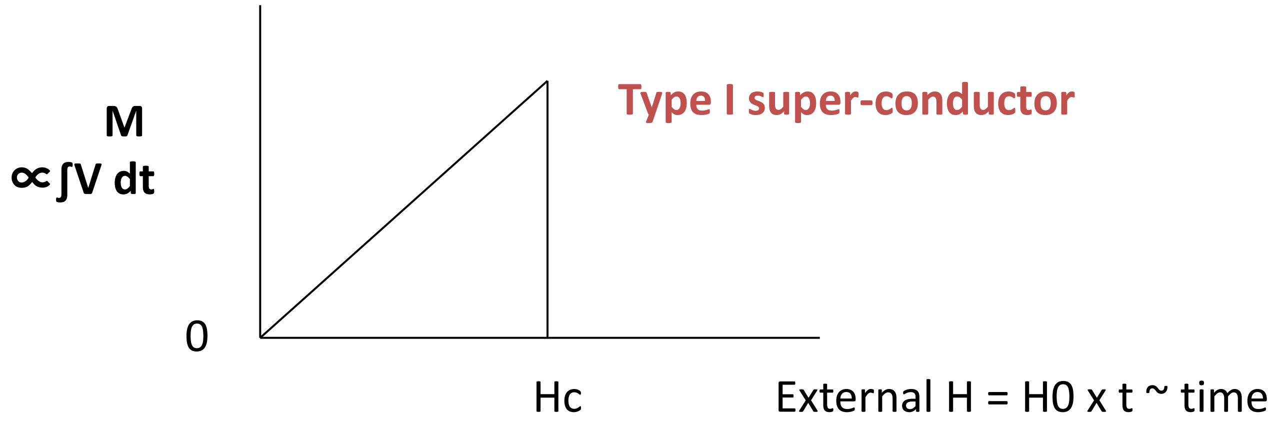
Pickup coil parameters
N = 250 turns
S = 4.65 mm x 2.8 mm = 13.02 x 10**-6 m**2
NS = 250 x 13.02 x 10**-6 = 3.23 x 10**-3 m**2

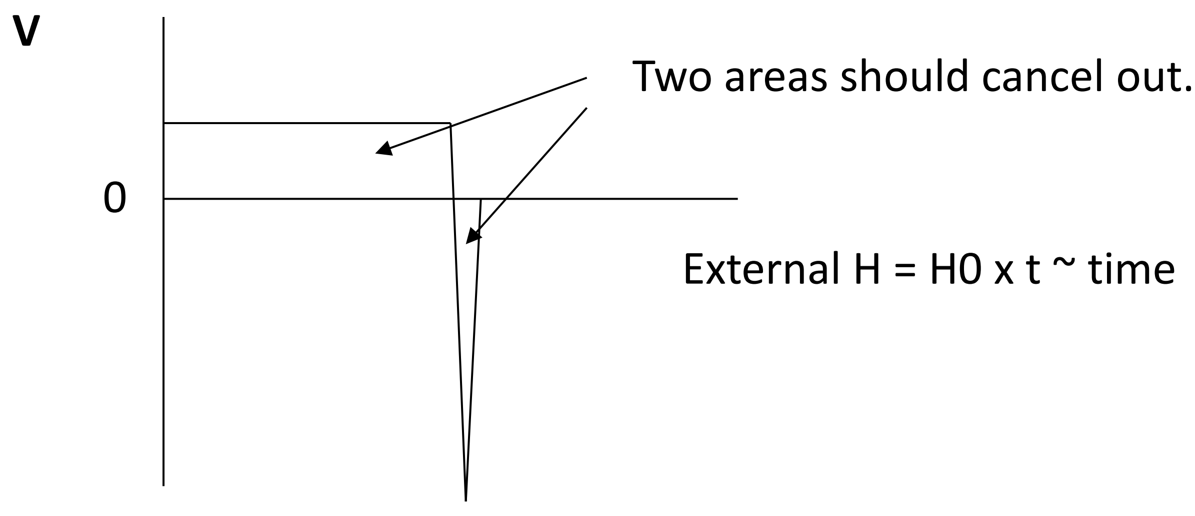
External B parameters :
    External Coil Current 1 A ⇔ B = 653.7 gauss
    = 0.06537 Wb/m**2
Ramping rate of B : 0.1 A / sec ⇔ 6.537 x 10**-3 Wb/m**2 / sec
    dB / dt = 6.537 x 10**-3 Wb/m**2 / sec

**V1 =- NS (dB / dt) =** 3.23 x 10**-3 x 6.537 x 10**-3
= 21 x 10**-6 Volt

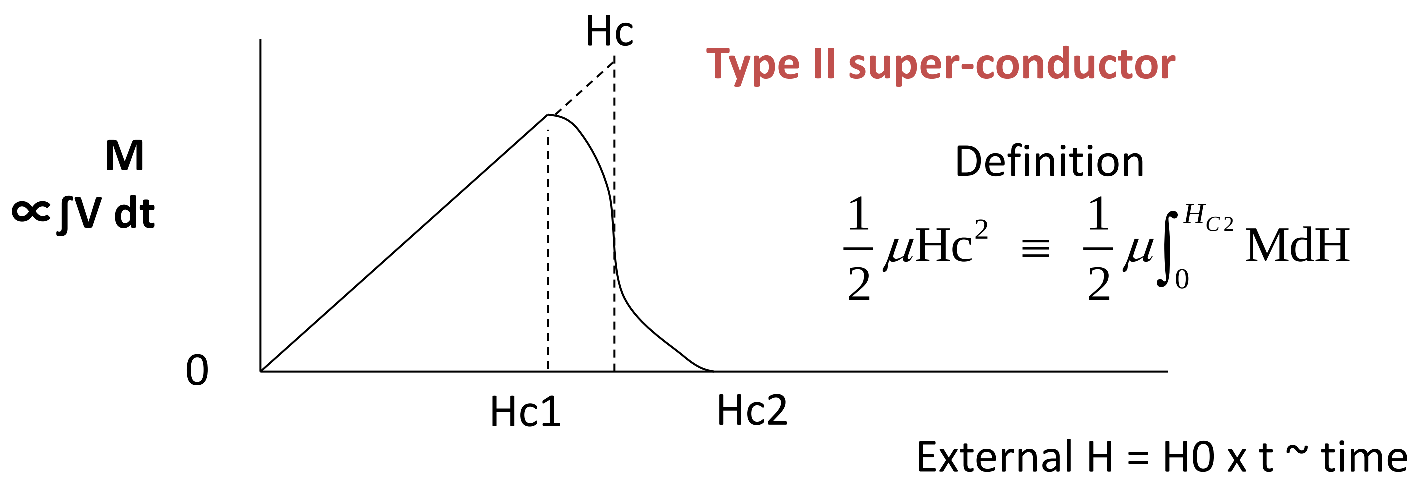
Output voltage from the pickup coil is about 20 micro-volt.

# Hc(T) measurement if Type-I



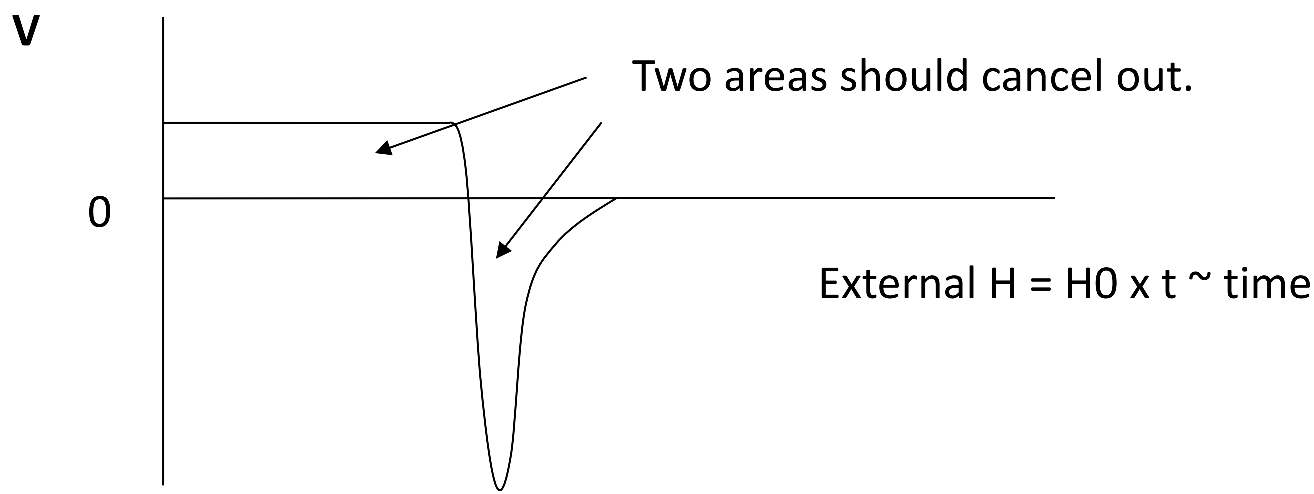
# Hc(T) measurement of Nb (Type-II)



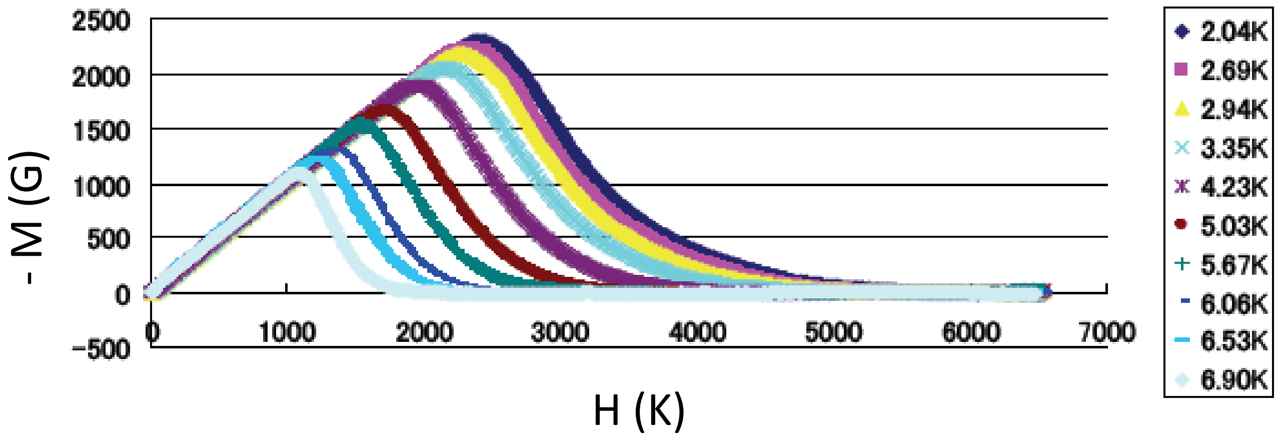

**Type II super-conductor**

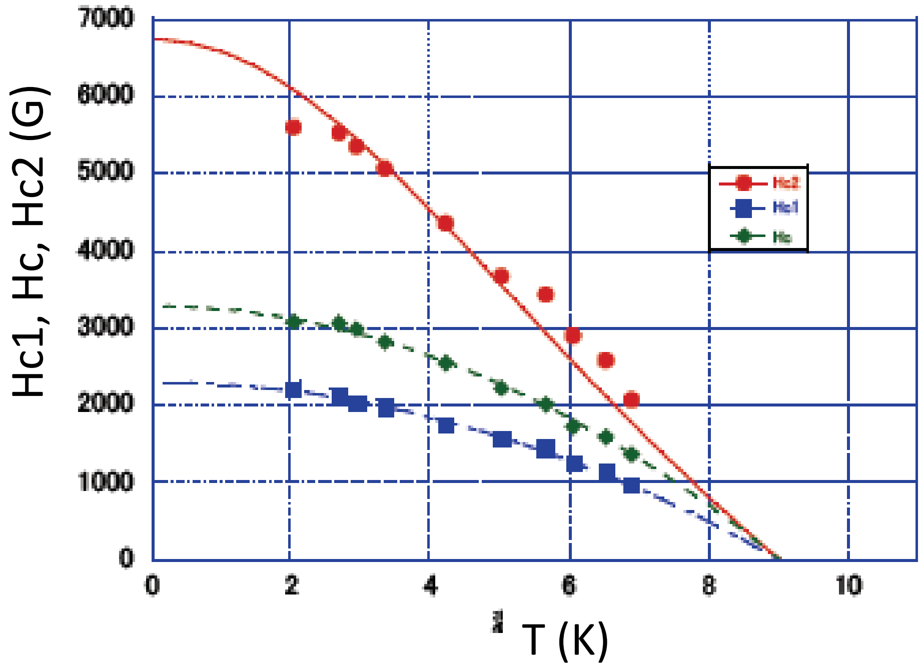
Definition

$$\frac{1}{2}\mu \mathrm{Hc}^2 \equiv \frac{1}{2}\mu\int_0^{H_{C2}} \mathrm{MdH}$$



# Hc(T) measurement of Nb (Type-II)




$$H_{C1}(T) = H_{C1}(0)\left\{1 - \left(\frac{T}{Tc}\right)^2\right\}$$

$$H_C(T) = H_C(0)\left\{1 - \left(\frac{T}{Tc}\right)^2\right\}$$

$$H_{C2}(T) = H_{C2}(0)\left\{\frac{1 - \left(\frac{T}{T_C}\right)^2}{1 + \left(\frac{T}{T_C}\right)^2}\right\}$$

# London equations

**Proposed a 2-fluid model with a normal fluid and superfluid components**

***$n_s$* : density of the superfluid component of velocity $v_s$**
***$n_n$* : density of the normal component of velocity $v_n$**

$$m\frac{\partial \vec{\upsilon}}{\partial t} = -e\vec{E}$$ superelectrons are accelerated by $E$

$$\vec{J}_s = -en_s\,\vec{\upsilon}$$

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m}\vec{E}$$ superelectrons

$$\vec{J}_n = \sigma_n \vec{E}$$ normal electrons

# London equations

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$

Maxwell: $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\left( \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} \right) = 0 \qquad \Rightarrow \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = \text{Constant}$$

F&H London postulated: $$\boxed{\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = 0}$$

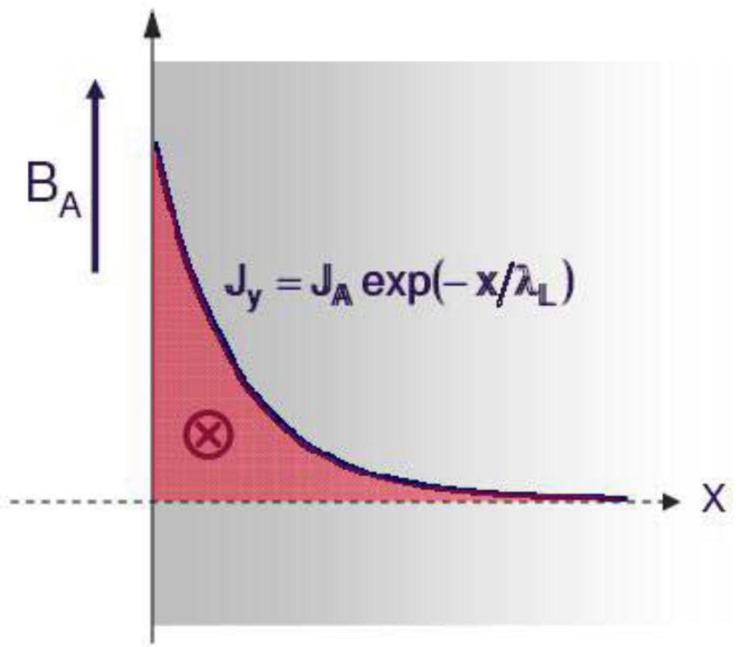
# London equations

**combine with** $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$

$$\boxed{\nabla^2 \vec{B} - \frac{\mu_0 n_s e^2}{m} \vec{B} = 0}$$

$$B(x) = B_o \exp\left[-x / \lambda_L\right]$$

$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$

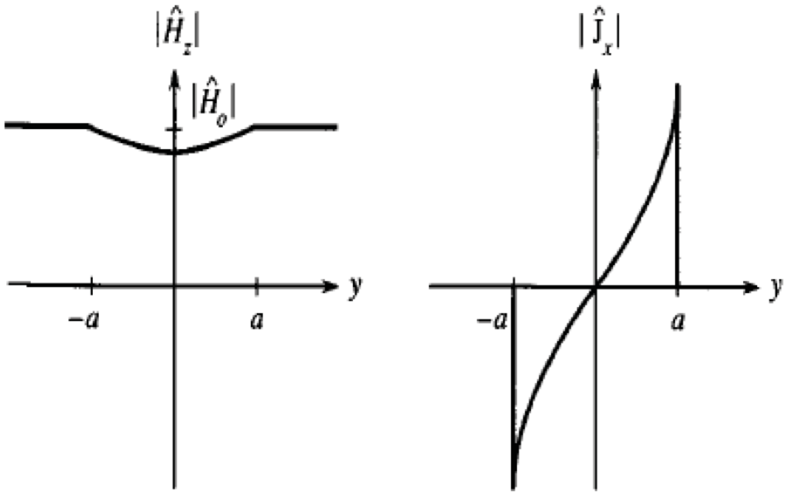



**The magnetic field, and the current, decay exponentially over a distance λ (a few 10s of nm)**

# Penetration depth in thin film

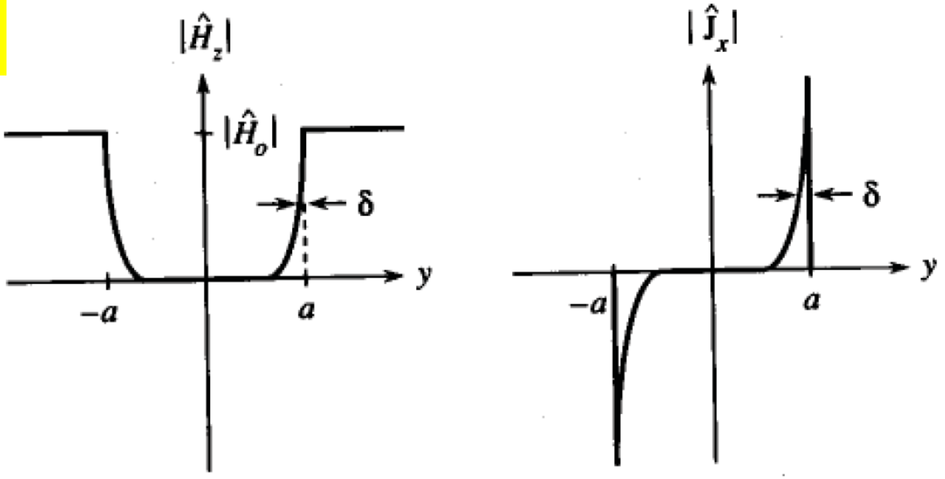
Thickness a < penetration depth

**Very thin films**




Thickness a > penetration depth

**Very thick films**



# London equations

$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$

From Gorter and Casimir two-fluid model

$$n_s \propto \left[1 - \left(\frac{T}{T_C}\right)^4\right]$$

$$\lambda_L(T) = \lambda_L(0)\frac{1}{\left[1 - \left(\frac{T}{T_C}\right)^4\right]^{\frac{1}{2}}}$$

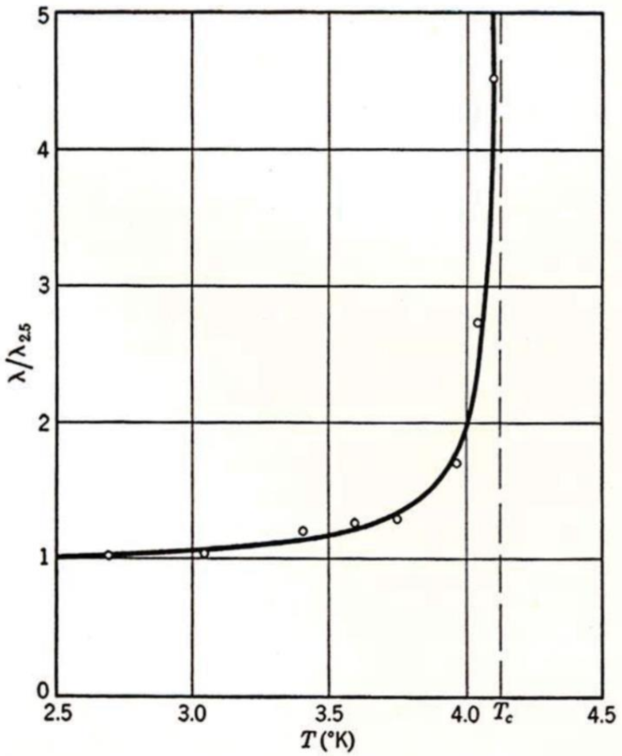



FIG. 21. Penetration depth as a function of temperature. (After Shoenberg, *Nature*, **43**, 433, 1939.)

# London equations

London Equation: $\lambda^2 \nabla \times \vec{J}_s = -\dfrac{\vec{B}}{\mu_0} = -\vec{H}$

$$\nabla \times \vec{A} = \vec{H}$$

choose $\nabla \bullet \vec{A} = 0,$ $A_n = 0$ on sample surface (London gauge)

$$\boxed{\vec{J}_s = -\frac{1}{\lambda^2}\vec{A}}$$

Note: Local relationship between $\vec{J}_s$ and $\vec{A}$

# Pippard coherent length

Pippard introduced the concept of coherent length for the first time.

Analogy to the chambers's nonlocal generation of Ohm's law : $\mathbf{J}(\mathbf{r}) = \sigma \mathbf{E}(\mathbf{r})$

$$\mathbf{J}(\mathbf{r}) = \frac{3\sigma}{4\pi\ell}\int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{E}(\mathbf{r}')]e^{-R/\ell}}{R^4}d\mathbf{r}'$$

where R = r – r'

throughout a volume of radius l (mean free path) about r

Pippard extended

$$\mathbf{J}_s = n_s e\langle \mathbf{v}_s\rangle = \frac{-n_s e^2 \mathbf{A}}{mc} = \frac{-\mathbf{A}}{\Lambda c}$$

To

$$\mathbf{J}_s(\mathbf{r}) = -\frac{3}{4\pi\xi_0\Lambda c}\int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}(\mathbf{r}')]}{\mathbf{R}^4}e^{-R/\xi}d\mathbf{r}'$$

where R = r – r'

Coherent length

$$\xi_0 = a\frac{\hbar v_F}{kT_c} \qquad \frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}$$

If choosing a = 0.15, Pippard could fit the experimental data on both tin and aluminum. This form was later confirmed by BCS theory.

# Ginzburg-Landau theory

- Ginzburg-Landau theory is a particular case of Landau's theory of second order phase transition
- Formulated in 1950, before BCS
- Masterpiece of physical intuition
- Grounded in thermodynamics
- Even after BCS it still is very fruitful in analyzing the behavior of superconductors and is still one of the most widely used theory of superconductivity

# Ginzburg-Landau theory

- Theory of second order phase transition is based on an order parameter which is zero above the transition temperature and non-zero below

- For superconductors, GL use a complex order parameter $\Psi(r)$ such that $|\Psi(r)|^2$ represents the density of superelectrons

- The Ginzburg-Landau theory is valid close to $T_c$

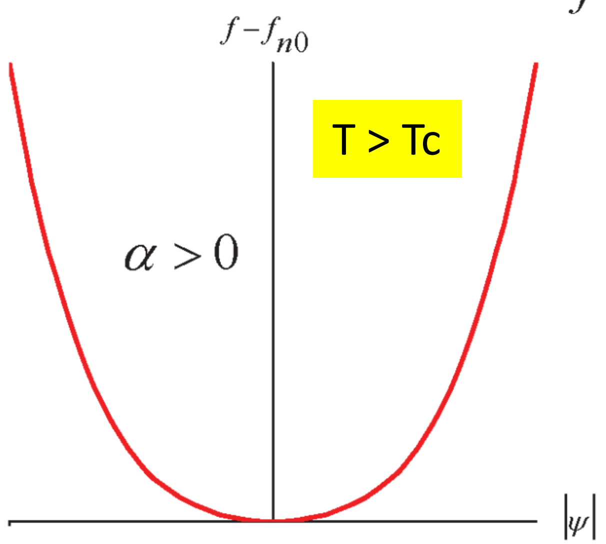
# Ginzburg-Landau theory

- Assume that Ψ(r) is small and varies slowly in space
- Expand the free energy in powers of Ψ(r) and its derivative

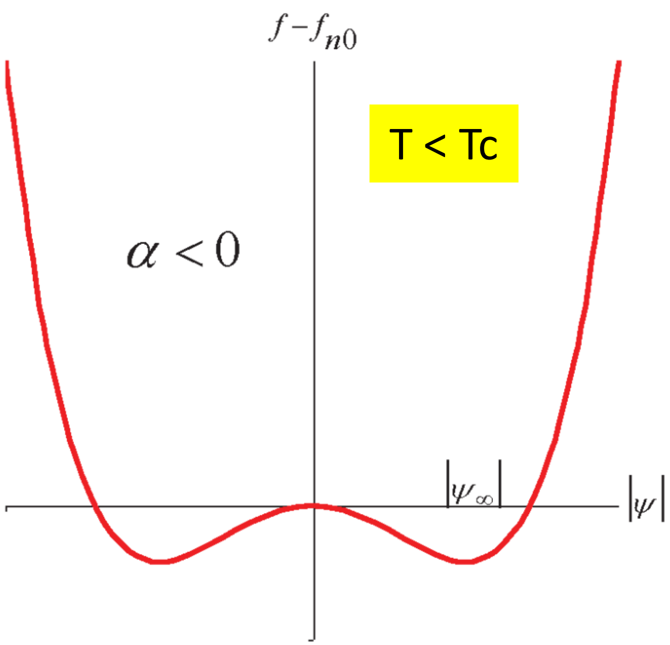
$$f = f_{n0} + \alpha |\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

# Ginzburg-Landau theory

$$f - f_{n0} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$




$$|\psi_\infty|^2 = -\frac{\alpha}{\beta}$$




Near $T_c$ we must have $\beta > 0$ $\alpha(t) = \alpha'(t-1)$

At the minimum $f - f_{n0} = -\frac{H_c^2}{8\pi} = -\frac{\alpha^2}{2\beta} \Rightarrow |\psi|^2$ and $H_c \propto (1-t)$

# Ginzburg-Landau theory

**London penetration depth: length over which magnetic field decay**

$$\lambda_L(T) = \left(\frac{m^* \beta}{2e^2 \alpha'}\right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$

**Coherence length: scale of spatial variation of the order parameter (superconducting electron density)**

$$\xi(T) = \frac{\hbar}{|2m^* \alpha(T)|^{1/2}}$$

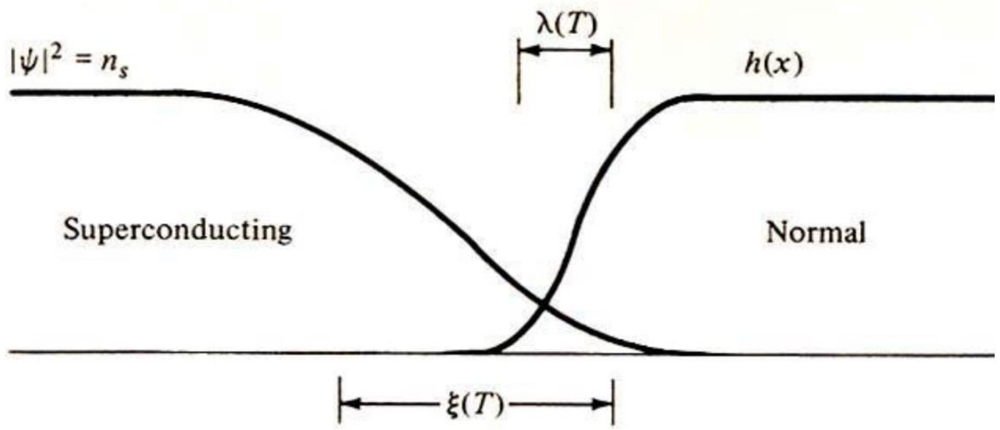
**The critical field is directly related to those 2 parameters**

$$H_c(T) = \frac{\phi_0}{2\sqrt{2}\,\xi(T)\,\lambda_L(T)}$$

$$\phi_0 \equiv \frac{h}{2e} = 2.07 \times 10^{-15}\,\mathrm{Wb}$$

The ratio of the two characteristic lengths defines the GL parameter: $\kappa = \frac{\lambda}{\xi}$

# Ginzburg-Landau theory




$$\sigma \simeq \frac{1}{8\pi}\left[H_c^2\xi - H^2\lambda\right]$$

$\frac{H^2\lambda}{8\pi}$: Energy that can be gained by letting the fields penetrate

$\frac{H_c^2\xi}{8\pi}$: Energy lost by "damaging" superconductor

# Ginzburg-Landau theory

Interface is stable if $\sigma$>0

If $\xi >> \lambda$ $\sigma$>0

Superconducting up to $H_c$ where superconductivity is destroyed globally

If $\lambda$ >>$\xi$ $\sigma$<0 for $H^2 > H_c^2 \frac{\xi}{\lambda}$

Advantageous to create small areas of normal state with large area to volume ratio

$\rightarrow$ quantized fluxoids

More exact calculation (from Ginzburg-Landau):

$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}}$ : Type I

$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$ : Type II

# Ginzburg-Landau theory

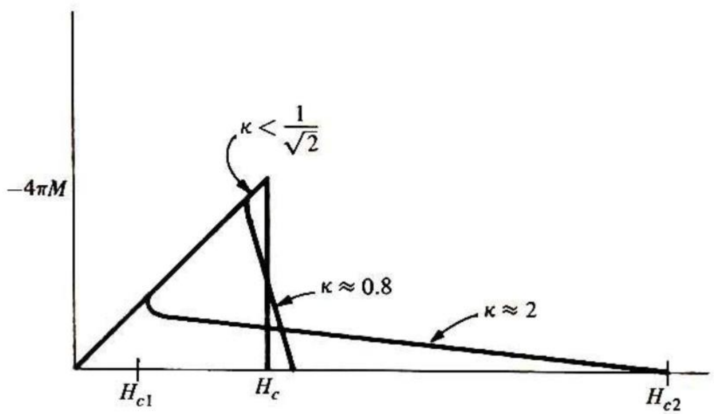



FIGURE 5-2
Comparison of magnetization curves for three superconductors with the same value of thermodynamic critical field $H_c$, but different values of $\kappa$. For $\kappa < 1/\sqrt{2}$, the superconductor is of type I and exhibits a first-order transition at $H_c$. For $\kappa > 1/\sqrt{2}$, the superconductor is type II and shows second-order transitions at $H_{c1}$ and $H_{c2}$ (for clarity, marked only for the highest $\kappa$ case). In all cases, the area under the curve is the condensation energy $H_c^2/8\pi$.

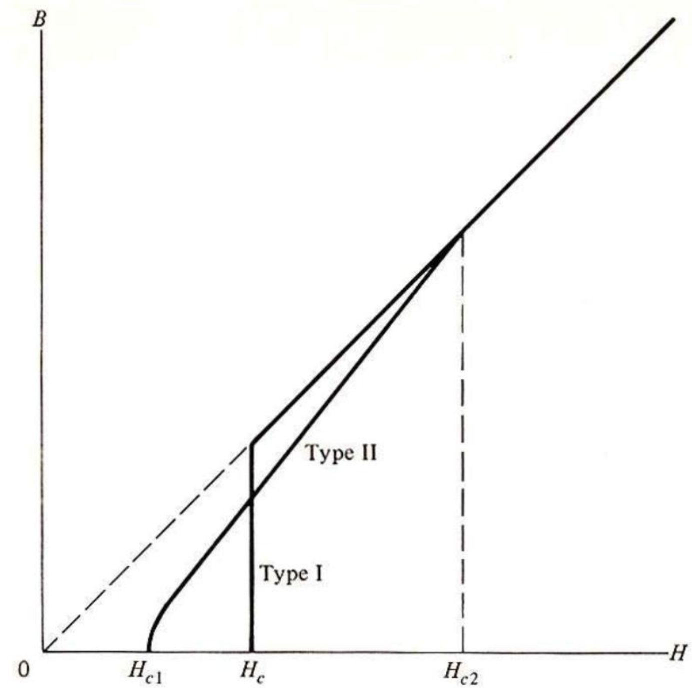
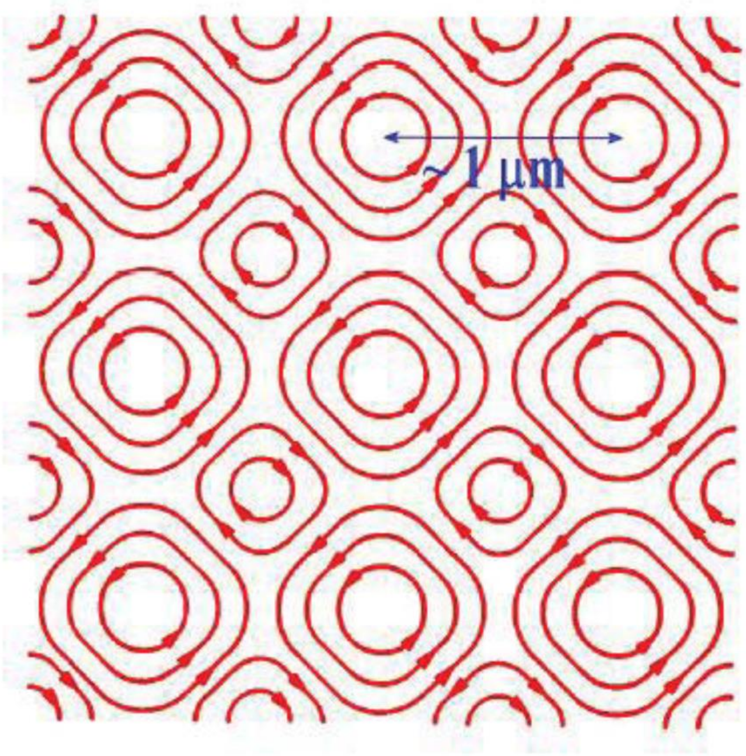
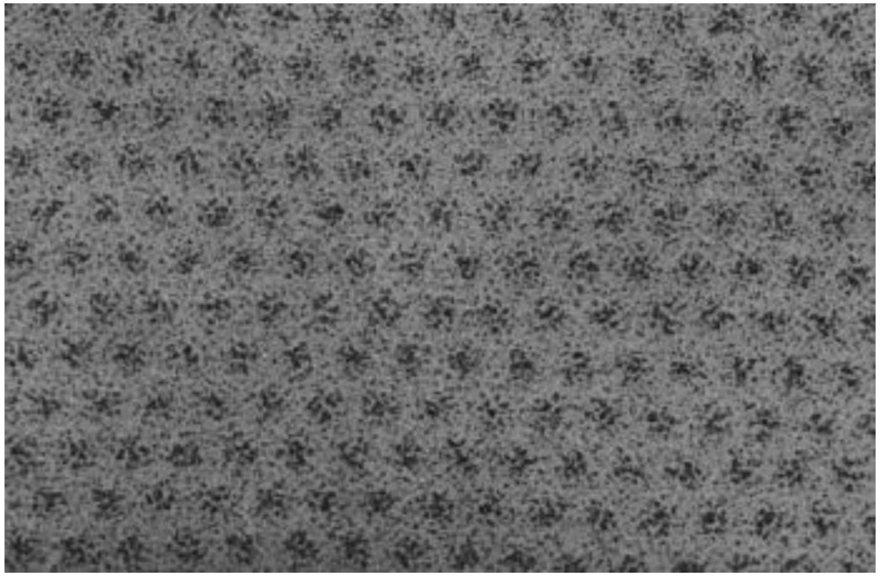



FIGURE 1-5
Comparison of flux penetration behavior of type I and type II superconductors with the same thermodynamic critical field $H_c$. $H_{c2} = \sqrt{2}\,\kappa H_c$. The ratio of $B/H_{c2}$ from this plot also gives the approximate variation of $R/R_n$, where $R$ is the electrical resistance for the case of negligible pinning, and $R_n$ is the normal-state resistance.

# Ginzburg-Landau theory




At the center of each vortex is a normal region of flux h/2e



Vortex lines in $Pb_{.98}In_{.02}$

The energy is low to make vortex lines which have the boundary of SC / NC.

# Ginzburg-Landau theory

**Even though it is more energetically favorable for a type I superconductor to revert to the normal state at $H_c$, the surface energy is still positive up to a superheating field $H_{sh}>H_c$ → metastable superheating region in which the material may remain superconducting for short times.**

Type I $\quad H_c \quad$ Thermodynamic critical field

$$H_{sh} \simeq \frac{H_c}{\sqrt{\kappa}}$$ Superheating critical field

Type II $\quad H_c \quad$ Thermodynamic critical field

$$H_{c2} = \sqrt{2}\ \kappa\ H_c$$

$$H_{c1} \simeq \frac{H_c^2}{H_{c2}}$$

$$\simeq \frac{1}{2\kappa}(\ln\kappa + .008)H_c \qquad \text{(for } \kappa \gg 1\text{)}$$

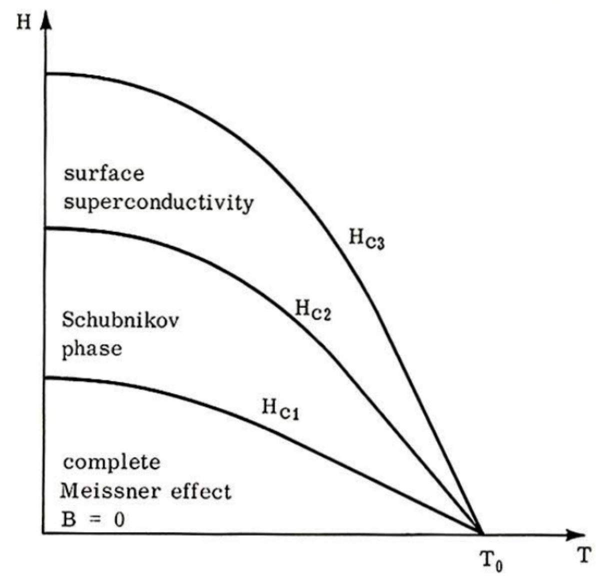



**Figure 3-1**
Phase diagram for a long cylinder of a Type II superconductor.

# Ginzburg-Landau theory

Ginsburg-Landau:

$$H_{sh} \sim \frac{0.9H_c}{\sqrt{\kappa}} \quad \text{for } \kappa << 1$$

$$\sim 1.2 \; H_c \quad \text{for } \kappa \sim 1$$

$$\sim 0.75 \; H_c \quad \text{for } \kappa >> 1$$

**The exact nature of the rf critical field of superconductors is still an open question**

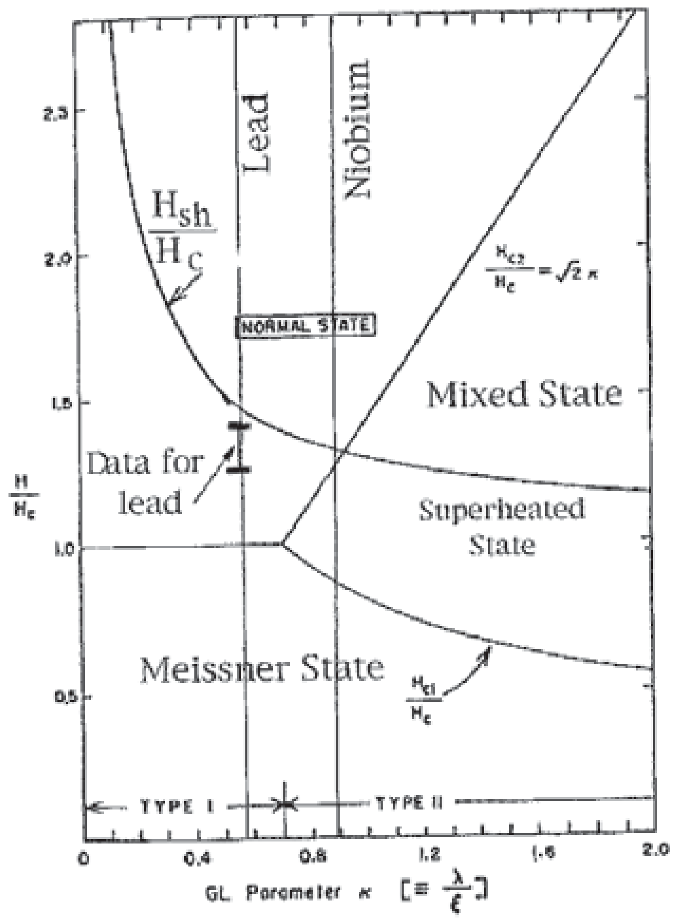



Figure 5.9: Phase diagram showing the Meissner, normal, mixed, and superheated states. The normalized critical fields $H_{c1}$, $H_c$, $H_{c2}$ and $H_{sh}$ are shown as functions of the GL parameter. The range of data for the rf critical field of lead is also shown.

# Ginzburg-Landau theory

Material parameter for various superconductors

| Superconductor | $\lambda_L$ (0) (nm) | $\xi_0$ (nm) | $\kappa$ | $2\Delta(0)/kT_c$ | $T_c$(K) |
|---|---|---|---|---|---|
| Al | 16 | 1500 | 0.011 | 3.40 | 1.18 |
| In | 25 | 400 | 0.062 | 3.50 | 3.3 |
| Sn | 28 | 300 | 0.093 | 3.55 | 3.7 |
| Pb | 28 | 110 | 0.255 | 4.10 | 7.2 |
| Nb | 32 | 39 | 0.82 | 3.5-3.85 | 8.95-9.2 |
| Ta | 35 | 93 | 0.38 | 3.55 | 4.46 |
| $Nb_3Sn$ | 50 | 6 | 8.3 | 4.4 | 18 |
| NbN | 50 | 6 | 8.3 | 4.3 | ≤17 |
| $Yba_2Cu_3o_x$ | 140 | 1.5 | 93 | 4.5 | 90 |

# BCS theory

- What needed to be explained and what were the clues?
  - Energy gap (exponential dependence of specific heat)
  - Isotope effect (the lattice is involved)
  - Meissner effect

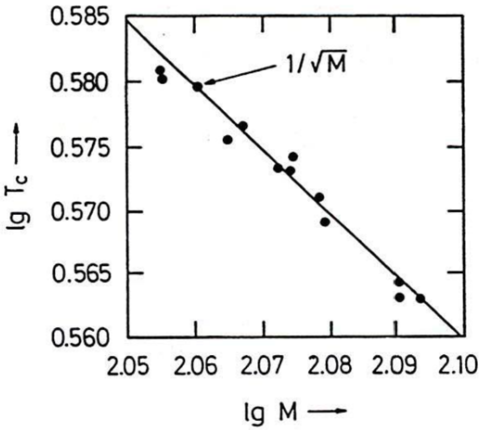
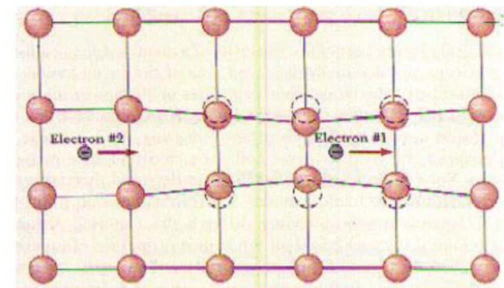



Figure 26: The critical temperature of various tin isotopes.

# BCS theory

Assumption: Phonon-mediated attraction between electron of equal and opposite momenta located within $\hbar\omega_D$ of Fermi surface




Moving electron distorts lattice and leaves behind a trail of positive charge that attracts another electron moving in opposite direction

Fermi ground state is unstable

Electron pairs can form bound states of lower energy

Bose condensation of overlapping Cooper pairs into a coherent Superconducting state

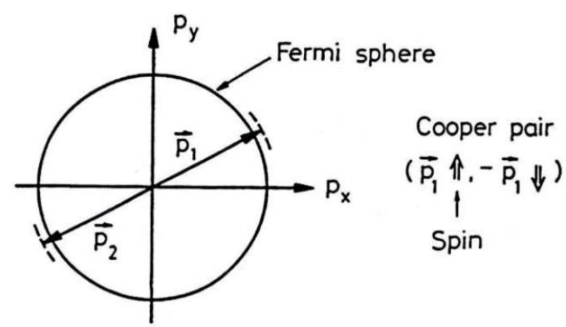



Figure 20: A pair of electrons of opposite momenta added to the full Fermi sphere.

# BCS theory

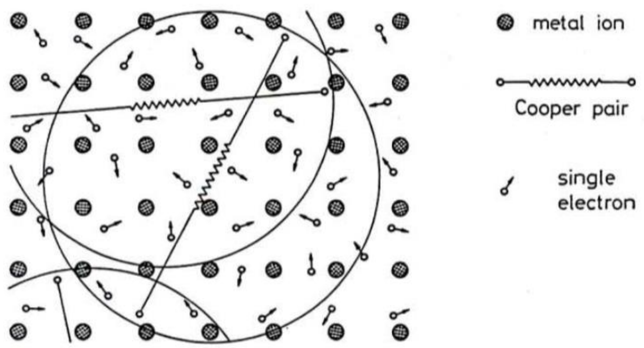



Figure 22: Cooper pairs and single electrons in the crystal lattice of a superconductor. (After Essmann and Träuble [12]).

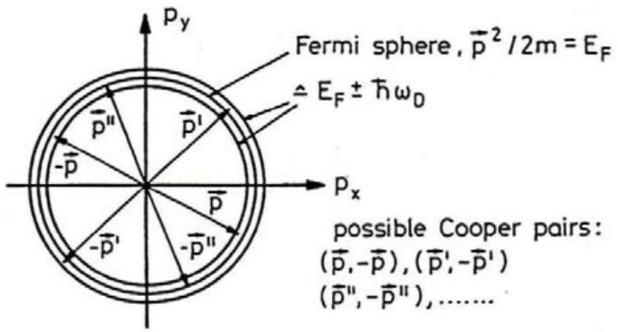



Figure 23: Various Cooper pairs $(\vec{p}, -\vec{p})$, $(\vec{p}', -\vec{p}')$, $(\vec{p}'', -\vec{p}'')$, ... in momentum space.

The size of the Cooper pairs is much larger than their spacing

They form a coherent state

# BCS theory

- The BCS model is an extremely simplified model of reality
  - The Coulomb interaction between single electrons is ignored
  - Only the term representing the scattering of pairs is retained
  - The interaction term is assumed to be constant over a thin layer at the Fermi surface and 0 everywhere else
  - The Fermi surface is assumed to be spherical

- Nevertheless, the BCS results (which include only a very few adjustable parameters) are amazingly close to the real world

# BCS theory

**At finite temperature:**

**Implicit equation for the temperature dependence of the gap:**

$$\frac{1}{V\rho(0)} = \int_0^{\hbar\omega_D} \frac{\tanh\left[(\varepsilon^2+\Delta^2)^{1/2}/2kT\right]}{(\varepsilon^2+\Delta^2)^{1/2}} d\varepsilon$$

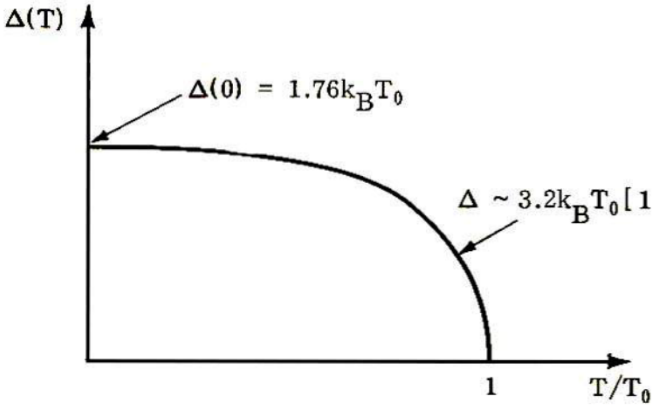
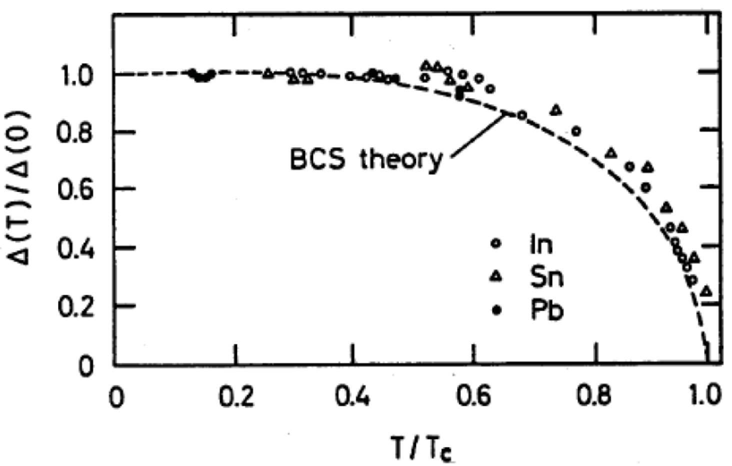



**Figure 4-4**
Variation of the order parameter Δ with temperature in the BCS approximation.



# BCS theory

Specific heat

$$C_{es} \simeq \quad \exp\left(-\frac{\Delta}{kT}\right) \text{ for } T < \frac{T_c}{10}$$

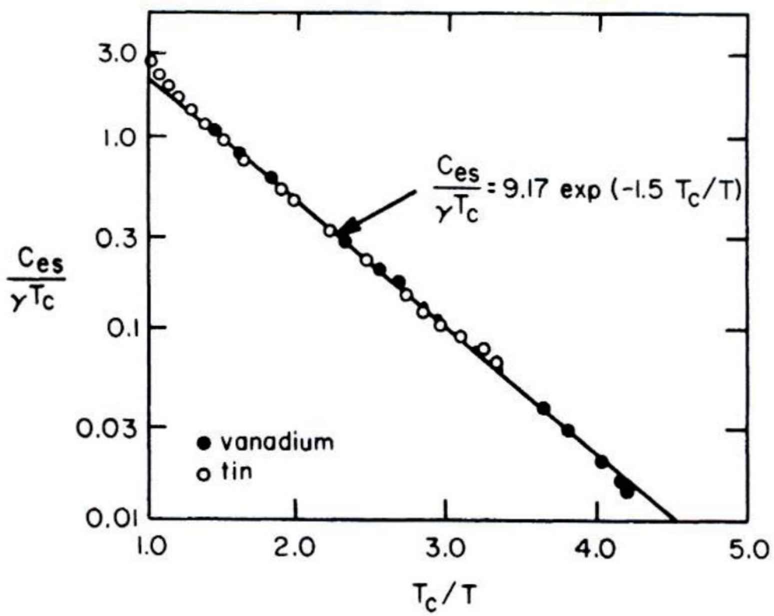



**Fig. 22.** Reduced electronic specific heat in superconducting vanadium and tin. [From Biondi et al., (*150*).]

# BCS theory

$$H_0\phi + H_{ex}\ \phi = i\hbar\frac{\partial\phi}{\partial t}$$

$$H_{ex} = \frac{e}{mc}\sum A(r_i,t)p_i$$

$H_{ex}$ is treated as a small perturbation

$$H_{rf} << H_c$$

There is, at present, no model for superconducting surface resistance at high rf field

$$J \propto \int \frac{R[R\cdot A]I(\omega,R,T)e^{-\frac{R}{l}}}{R^4}dr$$

similar to Pippard's model

$$J(k) = -\frac{c}{4\pi}K(k)A(k)$$

$K(0)\neq 0$: Meissner effect

# BCS theory

Represented accurately by $\lambda \sim \dfrac{1}{\sqrt{1-\left(\dfrac{T_c}{T}\right)^4}}$ near $T_c$

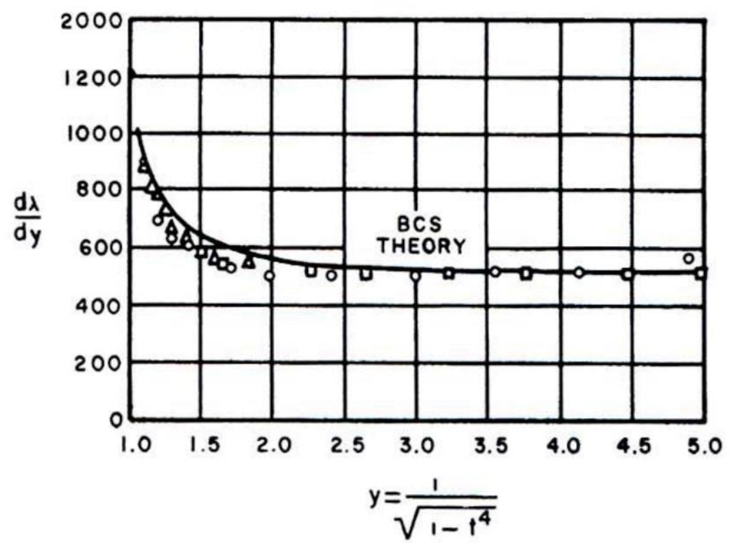



**Fig. 30.** Temperature dependence of $d\lambda/dy$ for tin obtained by Schawlow and Devlin (*207*) compared with the theoretical curve obtained from the BCS theory.

# BCS theory / Surface resistance

Temperature dependence

–close to $T_c$ : dominated by change in $\lambda(t)$ $\quad \frac{t^4}{(1-t^2)^{3/2}}$

–for $T < \frac{T_c}{2}$ : dominated by density of excited states $\sim e^{-\Delta/kT}$

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right)$$

Frequency dependence

$\omega^2$ is a good approximation

# BCS theory / Surface resistance

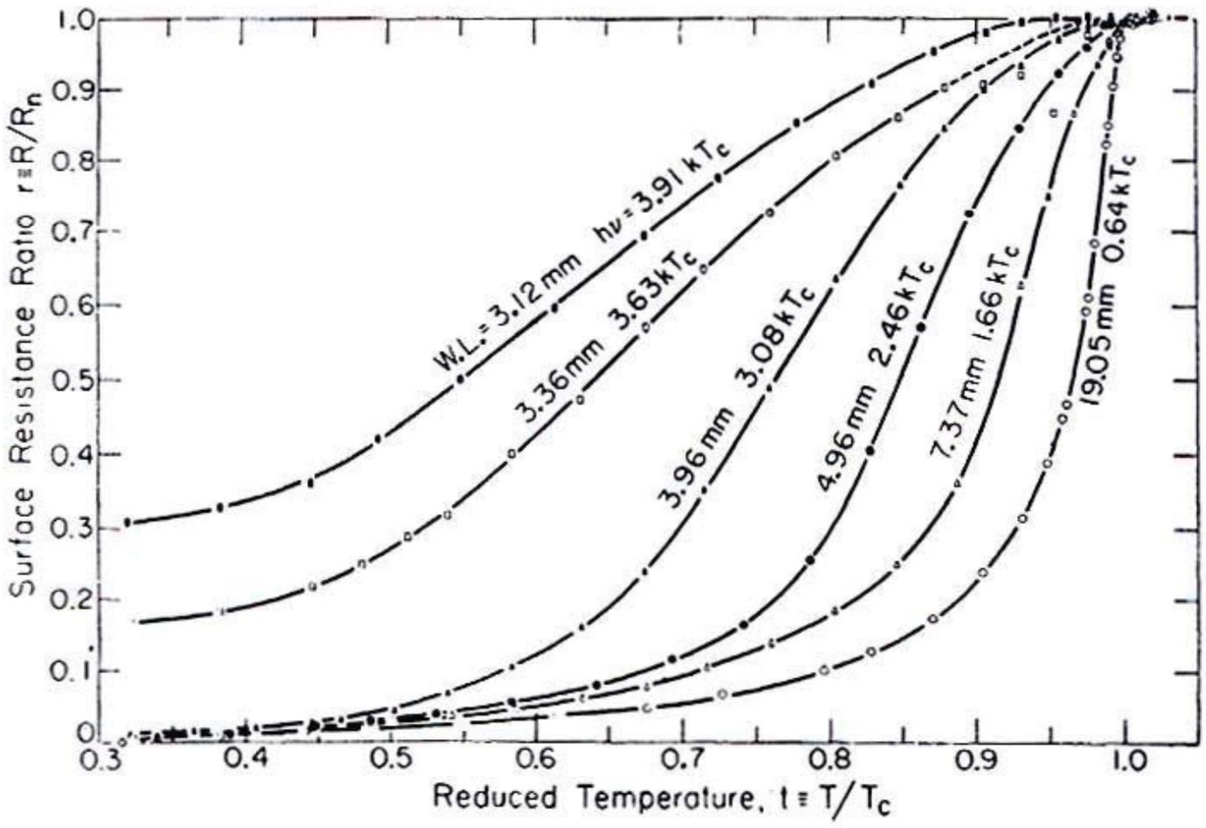



**Fig. 1.** Measured values of the surface resistance ratio $r$ of superconducting aluminum as a function of the reduced temperature $t$ at several representative wavelengths. The wavelengths and corresponding photon energies are indicated on the curves [After Biondi and Garfunkel (*15*).]

# BCS theory / Surface resistance

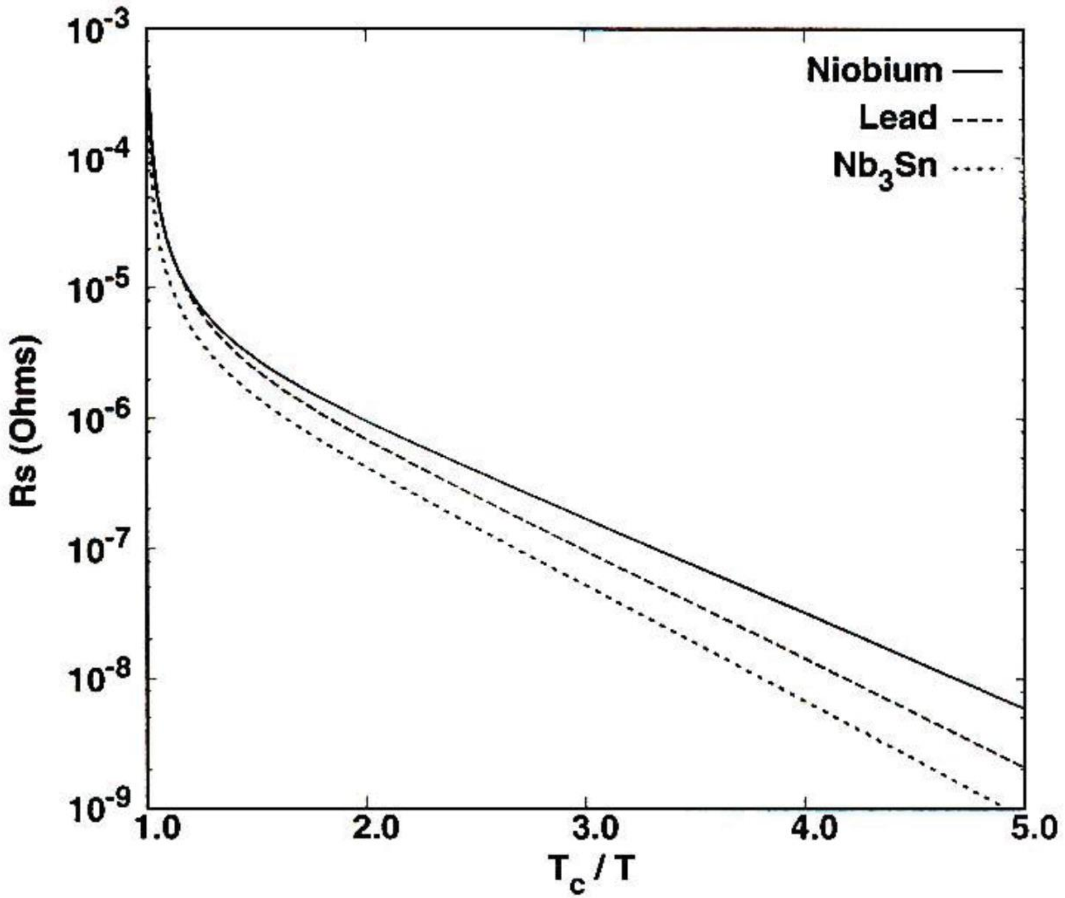



Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and $Nb_3Sn$ as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

# BCS theory / Surface resistance

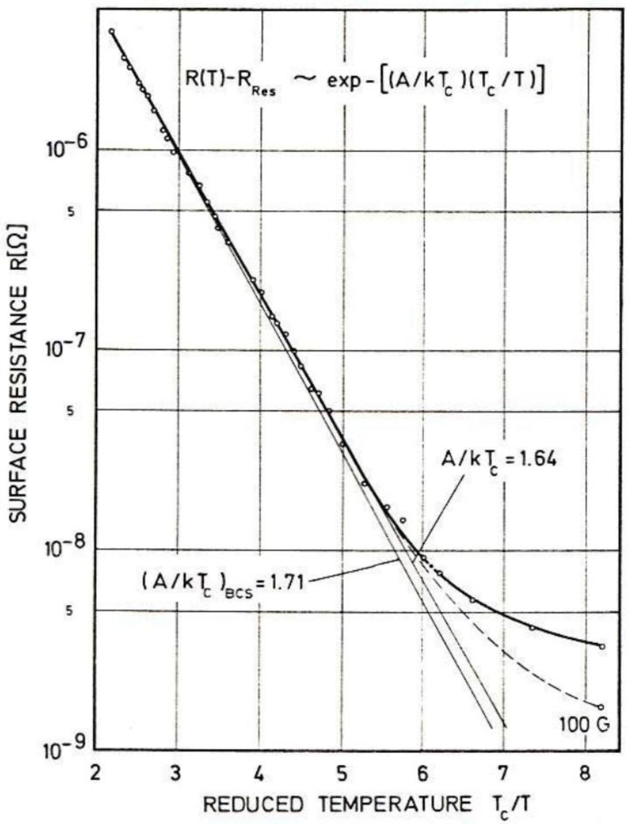



Fig. 2. Temperature dependence of surface resistance of niobium at 3.7 GHz measured in the $TE_{011}$ mode at $H_{rf} \simeq 10$ G. The values computed with the BCS theory used the following material parameters:

$T_c = 9.25$ K; $\lambda_L(T=0, l=\infty) = 320$ Å;

$\Delta(0)/kT = 1.85$; $\xi_F(T=0, l=\infty) = 620$ Å; $l = 1000$ Å or 80 Å.

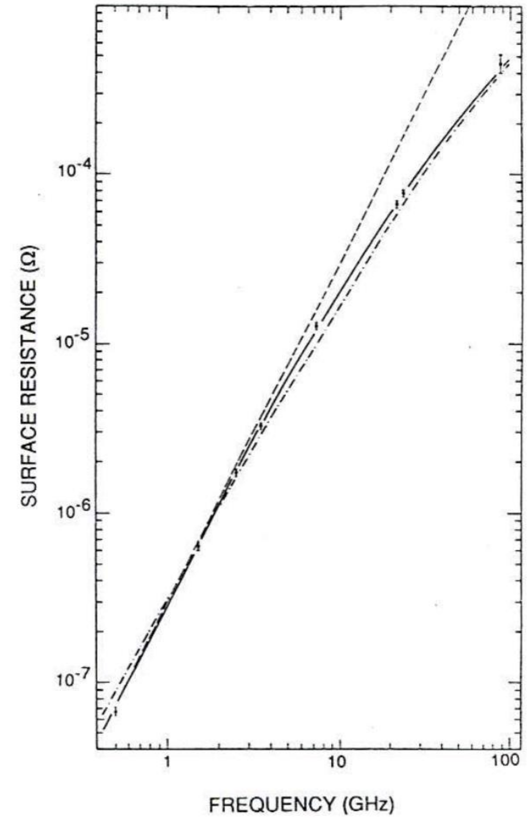



**Fig. 5.** The surface resistance of Nb at 4.2 K as a function of frequency [62,63]. Whereas the isotropic BCS surface resistance (– · – · ) resulted in $R \propto \omega^{1.8}$ around 1 GHz, the measurements fit better to $\omega^2$ (– – –). The solid curve, which fits the data over the entire range, is a calculation based on the smearing of the BCS density-of-states singularity by the energy gap anisotropy in the presence of impurity scattering [61]. The authors thank G. Müller for providing this figure.

# BCS theory / Surface resistance

Temperature dependence

–close to $T_c$ :

dominated by change in $\lambda(t)$ $\dfrac{t^4}{\left(1-t^2\right)^{3/2}}$

–for $T < \dfrac{T_c}{2}$ :

dominated by density of excited states $\sim e^{-\Delta/kT}$

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right)$$

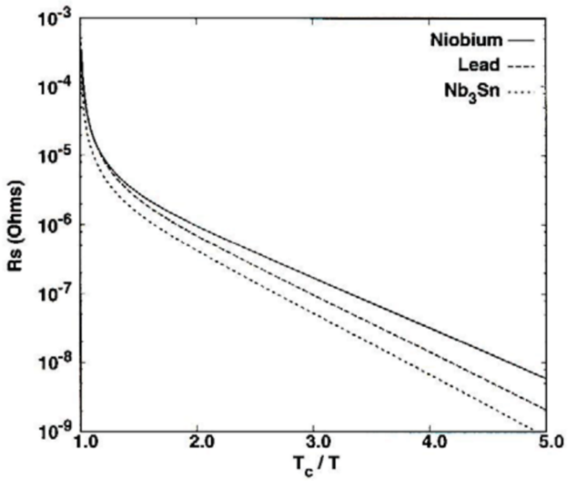



Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and $Nb_3Sn$ as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

Frequency dependence

$\omega^2$ is a good approximation

A reasonable formula for the BCS surface resistance of niobium is

$$R_{BCS} = 9\times 10^{-5}\,\frac{f^2\,(\text{GHz})}{T}\exp\left(-1.83\frac{T_c}{T}\right)$$

# BCS theory / Surface resistance

- The surface resistance of superconductors depends on the frequency, the temperature, and a few material parameters
  - Transition temperature
  - Energy gap
  - Coherence length
  - Penetration depth
  - Mean free path

- A good approximation for $T<T_c/2$ and $\omega<<\Delta/h$ is

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

# BCS theory / Surface resistance

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

In the dirty limit $l \ll \xi_0$ $R_{BCS} \propto l^{-1/2}$

In the clean limit $l \gg \xi_0$ $R_{BCS} \propto l$

$R_{res}$:

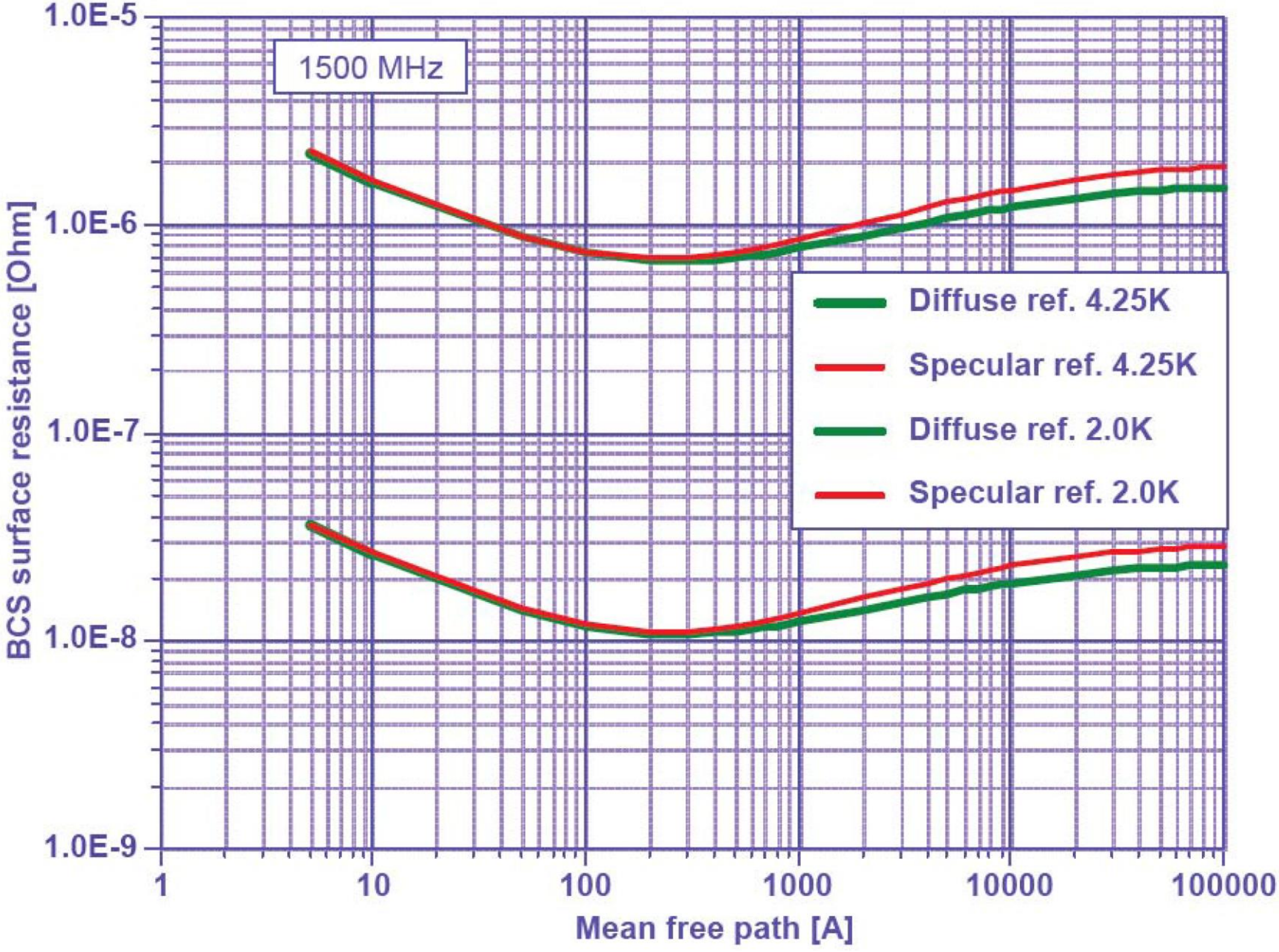
Residual surface resistance

No clear temperature dependence

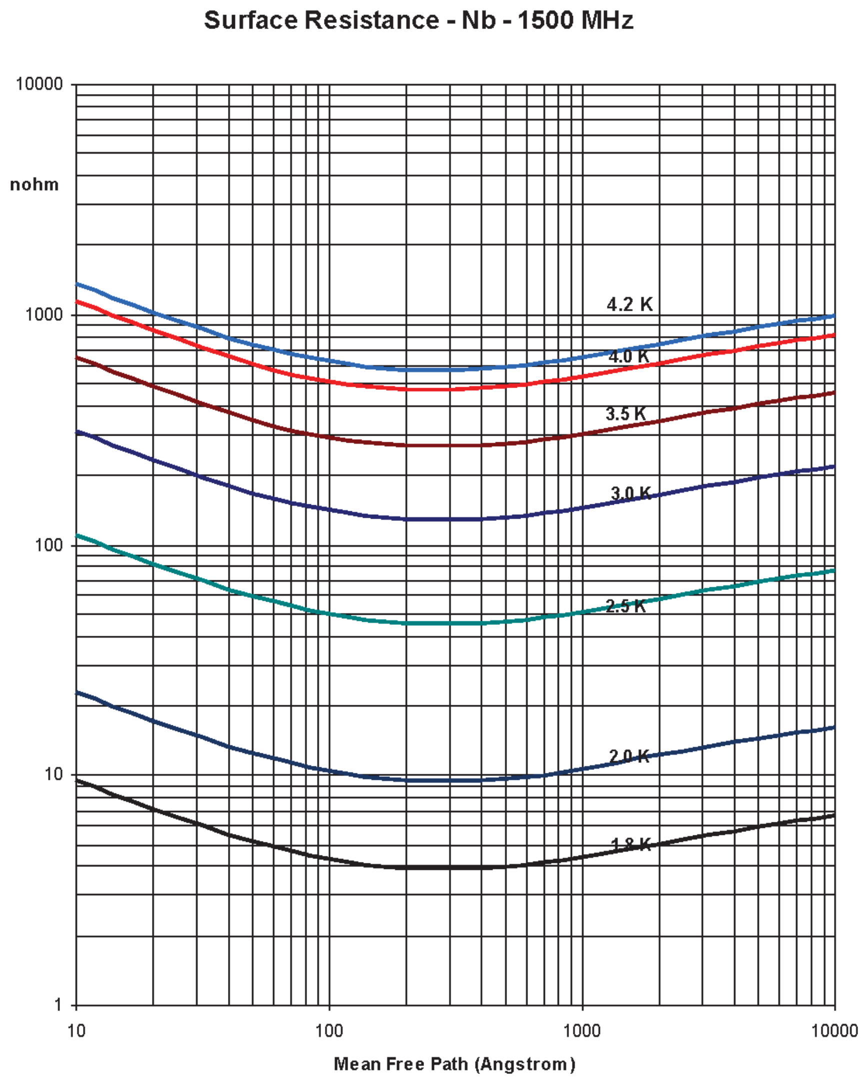
No clear frequency dependence

Depends on trapped flux, impurities, grain boundaries, …

# BCS theory / Surface resistance



# BCS theory / Surface resistance



# BCS theory / Surface resistance

- Normal Conductors
  - Skin depth proportional to $\omega^{-1/2}$
  - Surface resistance proportional to $\omega^{1/2 \rightarrow 2/3}$
  - Surface resistance independent of temperature (at low T)
  - For Cu at 300K and 1 GHz, $R_s$=8.3 mΩ

- Superconductors
  - Penetration depth independent of ω
  - Surface resistance proportional to $\omega^2$
  - Surface resistance strongly dependent of temperature
  - For Nb at 2 K and 1 GHz, $R_s \approx 7$ nΩ