

Matrix Element Method for ILC Physics Analysis

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<http://ilcphys.kek.jp/meeting/physics/archives/2009-05-19/GGGuide.pdf>

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MEM: maximal use of event kinematics

(approach the true likelihood of each event)

- one of the Multivariate methods
- first used for precision top mass measurement at D0
- recently used for the $H \rightarrow ZZ^* \rightarrow 4l$ analysis in Higgs discovery by CMS
- not being widely used comparing to BDT, MVA, but to me MEM is even more interesting

What is Matrix Element (Amplitude)

(squared \sim differential cross section)

Cross Section Formula

$$e^+ e^- \rightarrow X_1 + \cdots + X_f + \cdots + X_n$$

$\begin{array}{ccc} \vdots & & \vdots \\ (p^+, s^+) & & (p^-, s^-) \\ \vdots & & \vdots \\ & & (p_f, s_f) \end{array}$

$$d\sigma = \frac{1}{2s\beta_e} \sum_{s^+, s^-, s_f} w_{s^+} w_{s^-} |\mathcal{T}_{fi}|^2 d\Phi_n$$

$\begin{array}{c} \vdots \\ \text{spin weight for } e^- \\ \vdots \\ \text{spin weight for } e^+ \end{array}$

$$w_{s=\pm} = \frac{1 \pm P_s}{2} \quad \left(-1 \leq P_s = \frac{N_+ - N_-}{N_+ + N_-} \leq +1 \right)$$

$$\mathcal{T}_{fi} = \langle p_f, s_f | \hat{T} | p^+, s^+; p^-, s^- \rangle$$

(technically, ME² is the weight of each phase space point)

tools to calculate ME (thank Fujii-san!)

Helicity Amplitudes: HELAS

External Lines

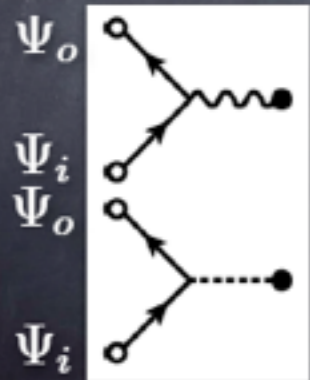


Ψ_i
 Ψ_o

4-momentum
helicity
particle
spinor
mass
anti-particle

$IXXXXX(p, m, \lambda, \pm 1, \Psi_i)$
 $OXXXXX(p, m, \lambda, \pm 1, \Psi_o)$

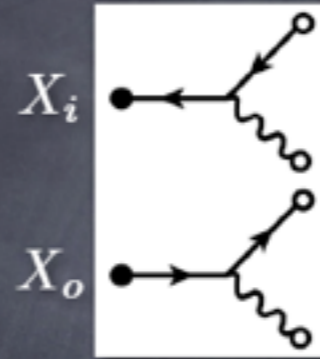
Currents



incoming spinor
outgoing spinor
width
mass
wave fun.
 $G_V(1)$: left
 $G_V(2)$: right

$JIOXXX(\Psi_i, \Psi_o, G_V, m_V, \Gamma_V, V)$
 $HIOXXX(\Psi_i, \Psi_o, G_S, m_S, \Gamma_S, S)$

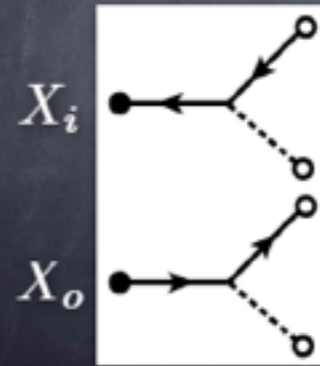
Virtual Fermions



X_i
 V
 Ψ_o
 V

incoming spinor
vector
mass
width
incoming virtual spinor
 $G_V(1)$: left
 $G_V(2)$: right

$FVIXXX(\Psi_i, V, G_V, m_X, \Gamma_X, X_i)$
 $FVOXXX(\Psi_o, V, G_V, m_X, \Gamma_X, X_o)$

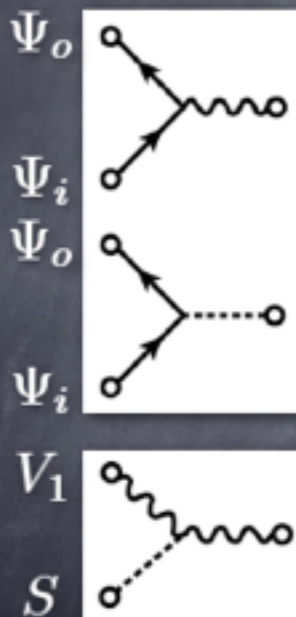


X_i
 S
 Ψ_o
 S

outgoing spinor
outgoing virtual spinor
scalar

$FSIXXX(\Psi_i, S, G_S, m_X, \Gamma_X, X_i)$
 $FSOXXX(\Psi_o, S, G_S, m_X, \Gamma_X, X_o)$

Vertices

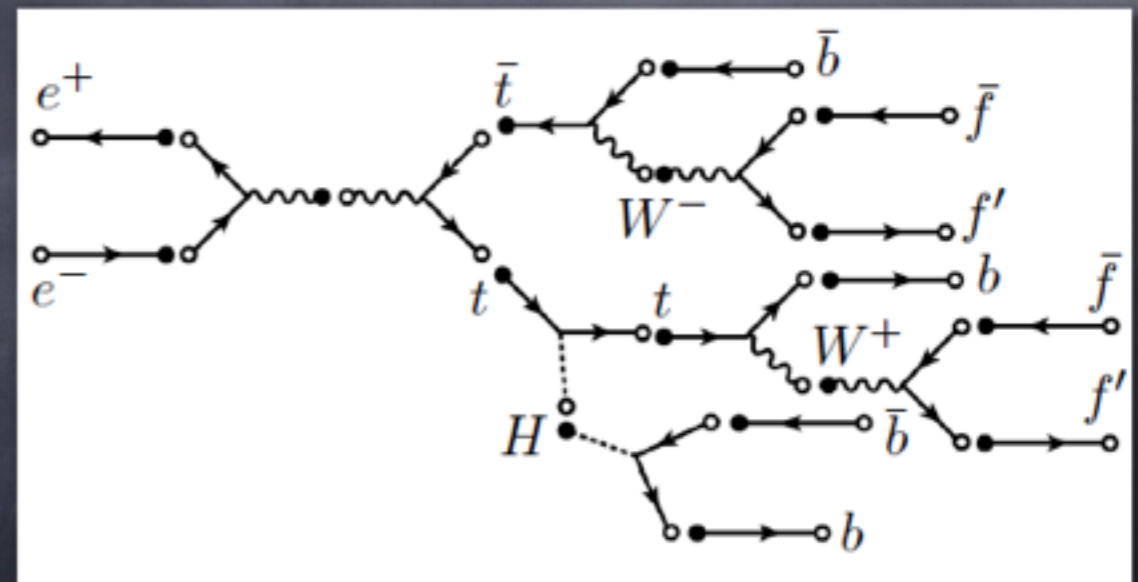


incoming spinor
outgoing spinor
vector
amplitude
 $G_V(1)$: left
 $G_V(2)$: right
scalar

$IOVXXX(\Psi_i, \Psi_o, V, G_V, A)$
 $IOSXXX(\Psi_i, \Psi_o, S, G_S, A)$
 $VVSXXX(V_1, V_2, S, G_{VVS}, A)$

Composition of Full Amplitude

$$e^+ e^- \rightarrow t \bar{t} H$$



Note: there are some other diagrams
See [physimm/top/TTHStudy](#)

Note: there are some more subroutines in HELAS (see manual)

how to calculate ME in our processors

- export physsim to ilcsoft (only HELLib, GENLib needed), done.
- simplify generator source code to provide ME, example of ZHHBases ready.
- the above libraries will be common as an independent package of ilcsoft (will include as many different processes as we need).
- in analysis processor, each event we only need provide four momentum of final states (parton level) to XXXBases class.

`login.cc.kek.jp:/home/ilc/tianjp/analysis/PostDBD/MEM/lib/libPhyssim.so`

`/home/ilc/tianjp/analysis/PostDBD/MEM/example/src/MEMExampleProcessor.cc`

(preliminary)

Detector Effect

- unfortunately, the four momentum we measured have resolution \rightarrow we need **detector transfer function** (jet-energy resolution, momentum resolution, etc.) and integrate all possible truth four momentum.
- and even worse, some four momentum can not be measured (missing neutrinos) \rightarrow integrate all possible truth four momentum.

$$L(\mathbf{p}_i^{\text{vis}} | \mathbf{a}) = \frac{1}{\sigma_{\mathbf{a}}} \left[\prod_{j \in \text{inv.}} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} \right] \left[\prod_{k \in \text{vis.}} \int \frac{d^3 p_k}{(2\pi)^3 2E_k} W_i(\mathbf{p}_i^{\text{vis}} | p_k, \mathbf{a}) \right] |M(p_j, p_k; \mathbf{a})|^2$$

ongoing, lots of study needed, anyone interested in this study is very welcome to join the effort