
Comments on the Top Mass Relevant for ILC and LHC

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Outline

- General remarks
 - Field theoretic aspects: Short-distance vs. pole mass
 - Renormalon problem
 - Top mass in MC programs
- Top threshold at ILC
- Total cross section at LHC
- Jet reconstruction: status
- Outlook and Conclusions

General Remarks

Quantum Field Theory:

Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: m is the rest mass

No other mass concept exists at the classic level.

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{classic}} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}} \quad (p^2 - m^2) q(x) = 0$$

$$\mathcal{L}_{\text{classic}} = -\frac{1}{4} F_{\alpha\beta}^A F_A^{\alpha\beta} + \sum_{\text{flavors } q} \bar{q}_\alpha (iD - m_q)_{\alpha\beta} q_b \quad D^\mu = \partial^\mu + igT^C A^{\mu C}$$

$$\longrightarrow \quad i \frac{p + m}{p^2 - m^2 + i\epsilon}$$

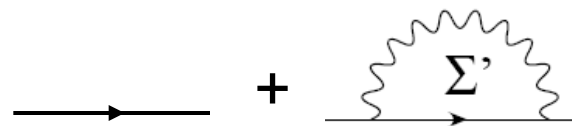
classic particle poles

$$\text{oooooo} \quad -i \frac{(g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} (\xi - 1))}{p^2 + i\epsilon}$$

Concept of a Quark Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance


$$\longrightarrow + \text{loop } \Sigma' = \not{p} - m^0 + \Sigma(p, m^0)$$

$m^0 \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$

Mass Renormalization Schemes you know:

Pole mass: mass = classic rest mass

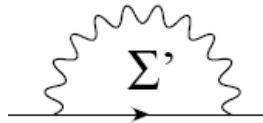
$$m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \quad \delta m^{\text{pole}} = \Sigma(m, m)$$

$\overline{\text{MS}}$ mass:

$$m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon}$$

Concept of a Quark Mass

So ... do we have to care?



$$\begin{aligned} \Sigma(m, m) &= -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \alpha_s \gamma^\mu \frac{q + k + m}{(q + k)^2 - m^2} \gamma_\mu \frac{1}{q^2} \\ &\stackrel{q \ll m}{=} \frac{2}{3} \int \frac{d^3 q}{(2\pi)^3} \frac{\alpha_s(q)}{\bar{q}^2} = -\frac{1}{2} \int \frac{d^3 q}{(2\pi)^3} V(\vec{q}^2) \end{aligned}$$

On-shell limit: Causes linear sensitivity to infrared momenta leads to factorially growing coefficients in perturbation theory.

$$\Sigma(m, m) \sim \sum_n \alpha_s^{n+1} (2\beta_0)^n n!$$

OK, we can absorb the bad correction into the mass

Recall:

$$\begin{aligned} \longrightarrow + \text{loop} &= p - m^0 + \Sigma(\not{p}, m^0) \\ &\sim p - m^{\text{pole}} \end{aligned}$$

What's the problem?

Concept of a Quark Mass

- The on-shell limit is intrinsic to the definition S-matrix elements involving external heavy quarks. (Cannot be avoided in perturbation theory)
- Linear infrared sensitivity for the **Quark self-energy AND the Interaction** in the on-shell limit.



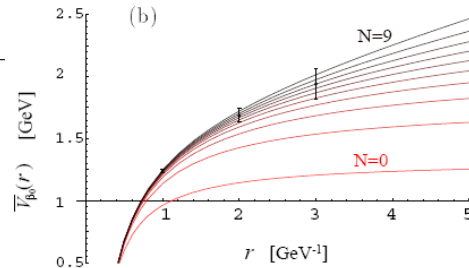
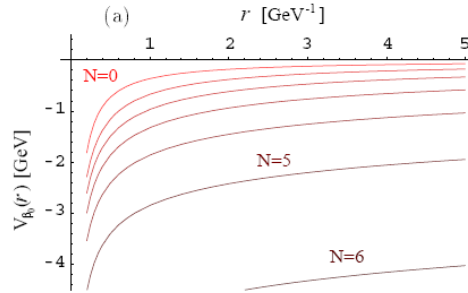
- The heavy quark on-shell limit is, however, artificial/unphysical and all linear infrared sensitivity cancels in a IR-safe process.
- Use of pole mass prohibits the cancellation to become manifest.
 - Pole mass: order-dependent concept
 - In practice: Relevant if one asks for precision $\delta m_t < 1 \text{ GeV}$

Examples

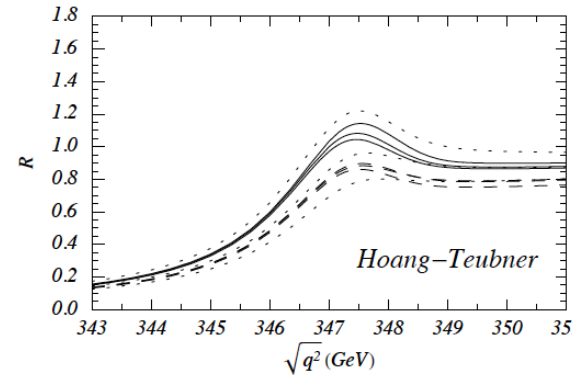
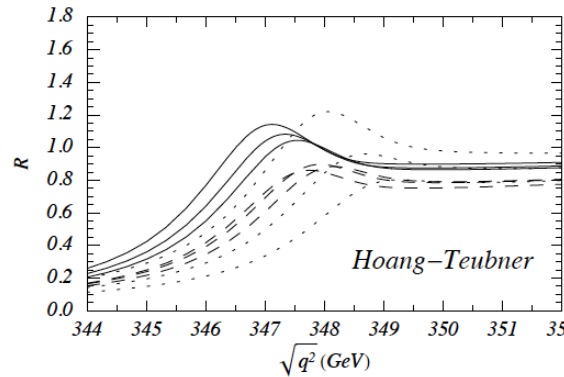
pole mass scheme

Short-distance mass scheme

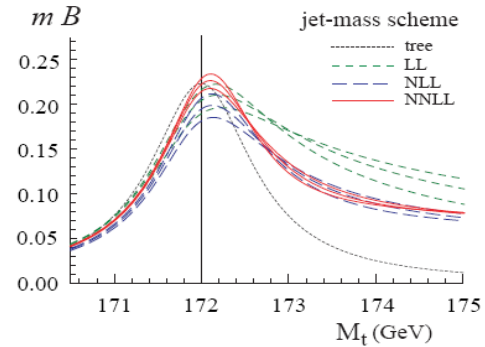
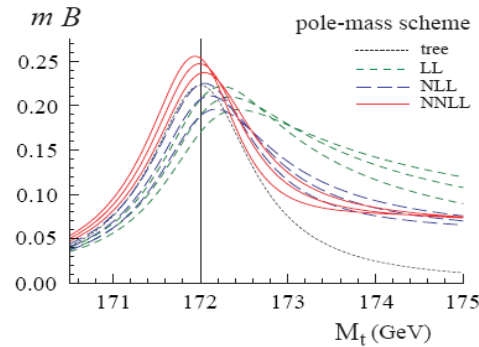
Static energy:



Top Threshold @ LC:



Reconstructed Top jets:



Fleming et al.

Concept of a Quark Mass

Short-distance mass schemes:

$$m^{\text{sd}}(R) = m^{\text{pole}} - R \left(a_1 \frac{\alpha_s}{4\pi} + a_2 \left(\frac{\alpha_s}{4\pi} \right)^2 + \dots \right)$$

Generic form of a short-distance mass scheme.

MS mass: $R = \bar{m}(\mu), \quad a_1 = \frac{16}{3} + 8 \ln \frac{\mu}{m}$

Processes where heavy quarks are off-shell and energetic.

Threshold masses (1S, PS, RS, kinetic masses)

$$R \sim m\alpha_s$$

Quarkonium bound states: heavy quarks are close to their mass-shell.

Jet masses (jet mass)

$$R \sim \Gamma_Q$$

Single quark resonance: heavy quarks are very close to their mass-shell.

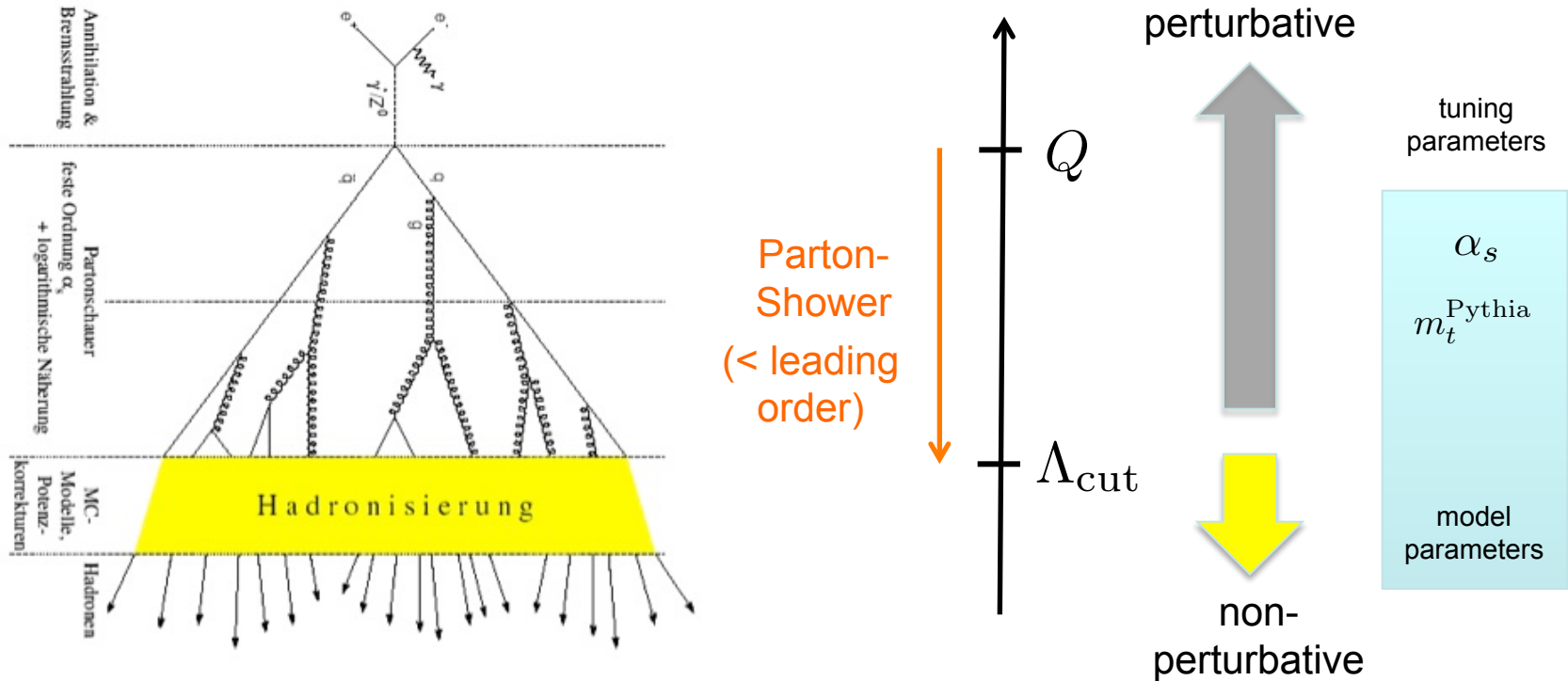
The a_i 's are chosen such that the renormalon is removed.

The scale R is of order the momentum scale relevant for the problem.

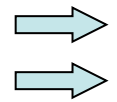
MC Mass

Universal instrument to describe hadronic final states.

- Hadronization model and α_s are “tuned” to experimental data.



Where is  ?

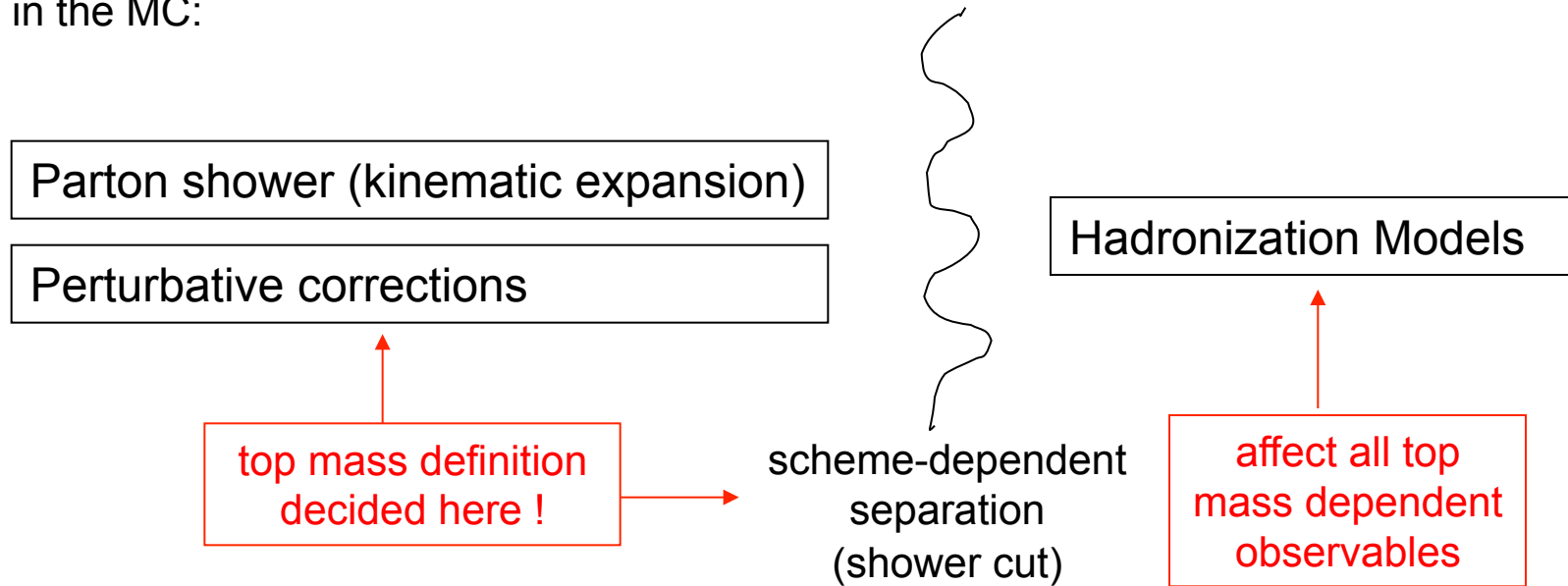


ME corrections (controllable in principle)
 Parton-shower: m_t^{Pythia} is a short-distance mass!
 Due to shower cut Λ_{cut} ?

Answer might be process- and observable-dependent !

MC Mass

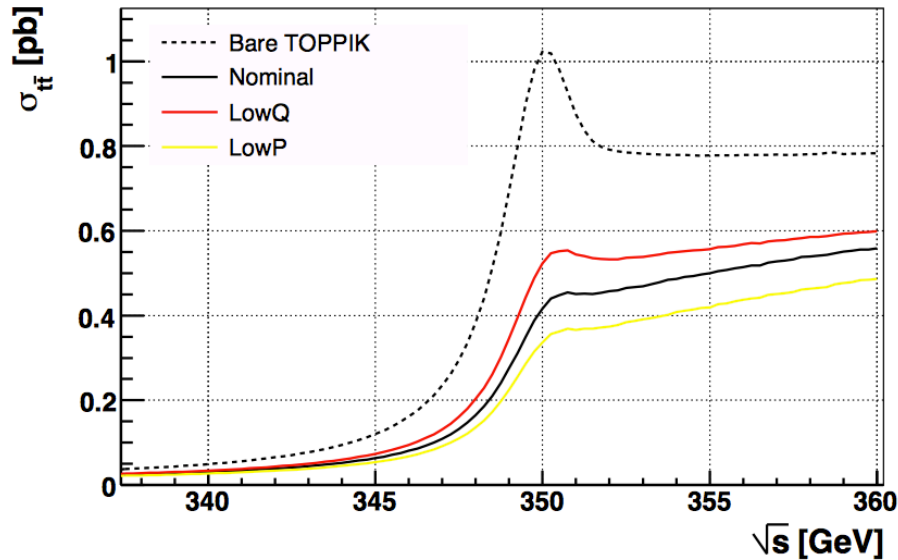
- Concept of mass in the MC depends on the **structure and reliability of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC:



- Assume that the MC is a good QCD box (LO of s.th. more precise): How can one pin down the relation between m_t^{Pythia} and the Lagrangian mass ?
- Is the MC really a good QCD box ? **Is the MC more a model or more QCD ?**

Answer for m_t^{Pythia} might be process- and observable-dependent if the MC is not a good QCD box !

Total $t\bar{t}$ Cross Section (ILC)



Principle: m_t from $\sigma_{t\bar{t}}(m_t)$

Advantages:

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood
(renormalons, summations)
- Top decay protects from non-pert effects

Much of the discriminating power of the approach related to the strong mass-dependence ($t\bar{t}$ resonance).

Peak position very stable in theory predictions (threshold mass scheme).

Typical results:

$$\rightarrow \delta m_t^{\text{exp}} \simeq 50 \text{ MeV}$$

$$\rightarrow \delta m_t^{\text{th}} \simeq 100 \text{ MeV}$$

What mass?

$$\sqrt{s}_{\text{rise}} \sim 2m_t^{\text{thr}} + \text{pert.series}$$

(short distance mass: $1S \leftrightarrow \overline{MS}$)

Total $t\bar{t}b\bar{b}$ Cross Section (ILC)

Theory issues (Pros) : Multi-scale problem ($m, mv, mv^2 \sim 1.5 \text{ GeV}$)

- NNNLO fixed-order approach (pQCD for total cross section) ... published shortly, I guess
- NNLL RG-improved approach (pQCD for total cross section) Hoang, Stahlhofen (2013)

Norm and shape of σ_{tot} much less precise than peak position: $d\sigma_{\text{tot}}/\sigma_{\text{tot}} \sim 5\%$

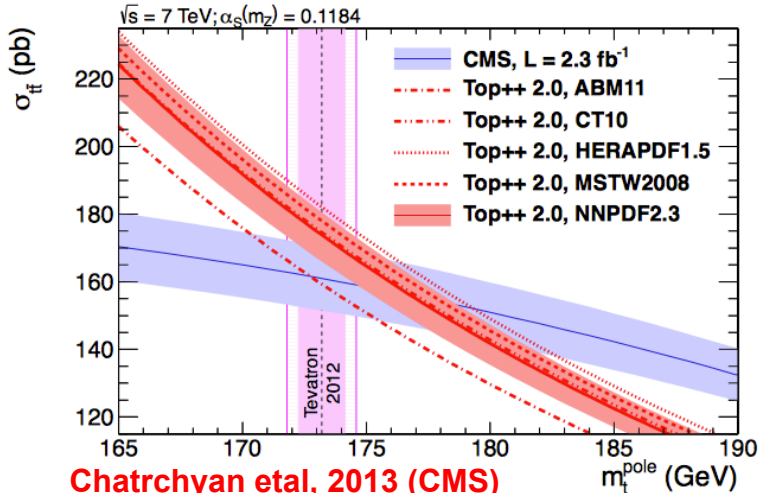
Theory issues (Cons) : Only little / no progress has been achieved in 10 years

- Electroweak/unstable particle theory for total cross section (w.i.p)
- Differential distributions (almost nope)
- Unstable particle effects in distributions (none)
- Monte-Carlo-Simulations for threshold (w.i.p.)
- PDF's for ILC (impact of luminosity spectrum, ISR, etc) (very little)

➡ Last 10 years: Most energy went into QCD corrections of total cross section.
Still many more problems to be addressed: **Status: "strong arms, thin legs"**

At this time: $\delta_{\text{theory}} \gg \delta_{\text{experiment}}$

Total $t\bar{t}$ Cross Section (LHC)



Chatrchyan et al, 2013 (CMS)

arXiv:1307.1907

Principle: m_t from $\sigma_{t\bar{t}}(m_t)$

Theory Progress:

- NNLO (qq channel)+NNLL available
- Pole and $\overline{\text{MS}}$ predictions available

Czakon, Mitov + other groups

- Theory issue: large sensitivity to gluon pdf $\leftrightarrow \alpha_s$
- Experimental issue: get σ_{tot} from $\sigma(\text{experiment})$
- Norm errors feed in the top mass errors

$$\Rightarrow m_t^{\text{pole}} = 176^{+3.8}_{-3.4} \text{ GeV}$$

Chatrchyan et al, 2013 (CMS)

arXiv:1307.1907

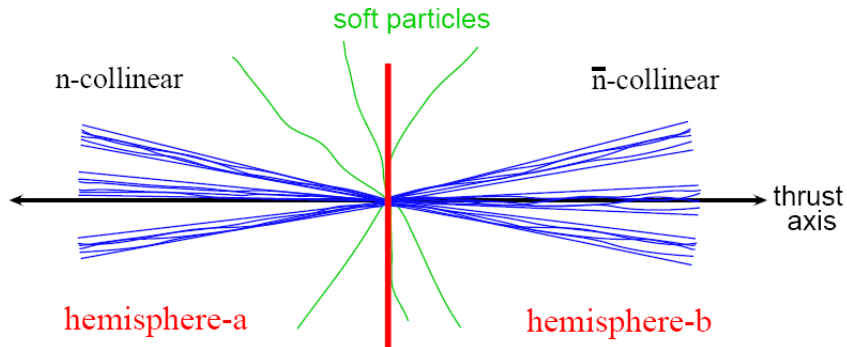
No apparent discrepancy at this time with assumption $m_t^{\text{pole}} = m_t^{\text{Pythia}}$.

Smaller errors hard, because many hard problems need to be resolved.

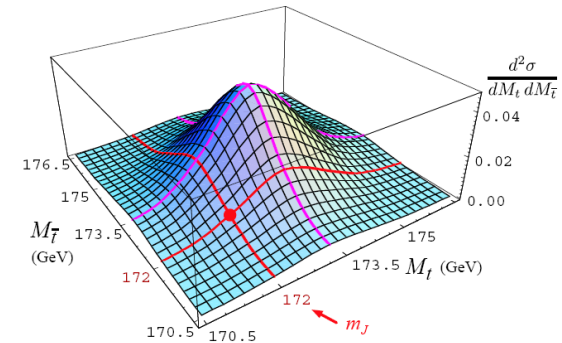
Reconstructed Top Jets (ILC)

Invariant mass distribution: (boosted tops)

Fleming, Mantry, Stewart, AH (2008)



- Hemisphere top jets
- Related to event-shapes



$$\left(\frac{d^2 \sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m \left(m, \frac{Q}{m}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} dl^+ dl^- B_+ \left(\hat{s}_t - \frac{Ql^+}{m}, \Gamma, \mu \right) B_- \left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m}, \Gamma, \mu \right) S_{\text{hemi}}(l^+, l^-, \mu)$$

JET

JET

SOFT

→ Differential strongly top mass-dependent observable.

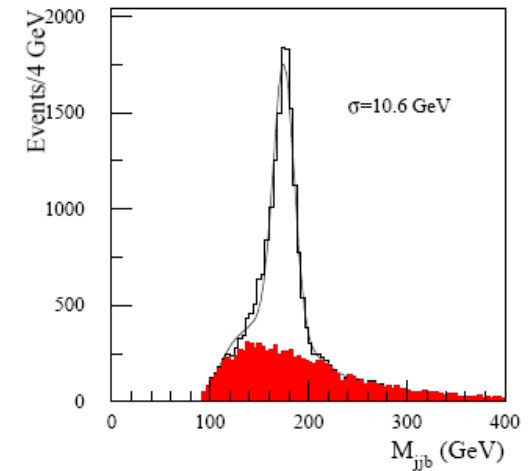
Top Mass dependent Distributions

Developments/w.i.p: (SCET: highly energetic top quarks)

- Variable flavor number scheme for final state jets (w.i.p.)
- Jet mass distribution at the LHC (w.i.p.)
- Heavy quark effects in pdf's (ACOT scheme)
- Jet substructure for top initiated jets
- Effects of the underlying event
- p_T distributions

Aims:

- Measure top mass directly without MC.
- Tests: How well does MC do QCD? / “Measure” the MC top mass ?



Outlook & Conclusion

Conclusion:

- MC most versatile tool to analyze data
QCD parameters in MC not a priori well understood: m_t^{Pythia} .
- Top Threshold: Lot of progress for total cross section. Still lots of open questions and subtleties (distributions missing!, electroweak, photons, finite lifetime effects) \Rightarrow more conceptual progress needed
- Top Jets (boosted Tops): progress/w.i.p for LC and LHC.
Direct top mass determination independent of MC.
Direct competition to top threshold (very slow Tops) is emerging.
- Measurement of MC top mass OR Test: MC = QCD box ?
Only feasible for distributions.