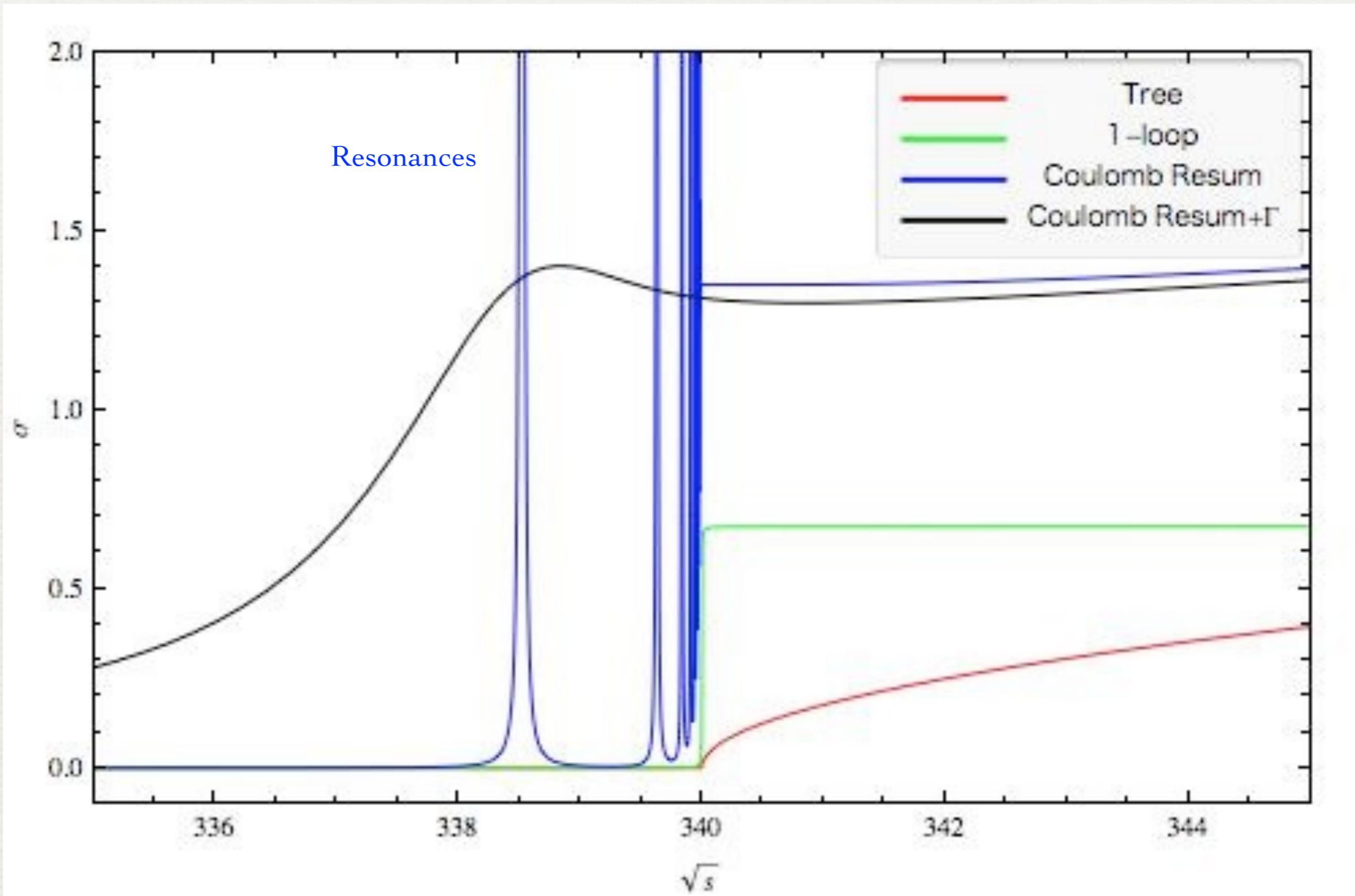


Top threshold at ILC

Yuichiro Kiyo (Juntendo Univ.),
Top Physics at LC , 2014 @LPNHE

TTbarXSection: Bneke,YK, Schuller :
Part I, arXiv:1312.4791[hep-ph],
Part II in preparation,
In collaboration with Karlsruhe group;

$e^+e^- \rightarrow tt$ near threshold



Top threshold

- ♦ m_t and y_t measurement is important to test SM/BSM
- ♦ $(\Delta m_t)_{\text{exp}} \leq 50 \text{ MeV} \rightarrow$ theory goal $\delta\sigma \leq 3\%$
- ♦ NNLO result @2001

Scale uncertainty about 20 %

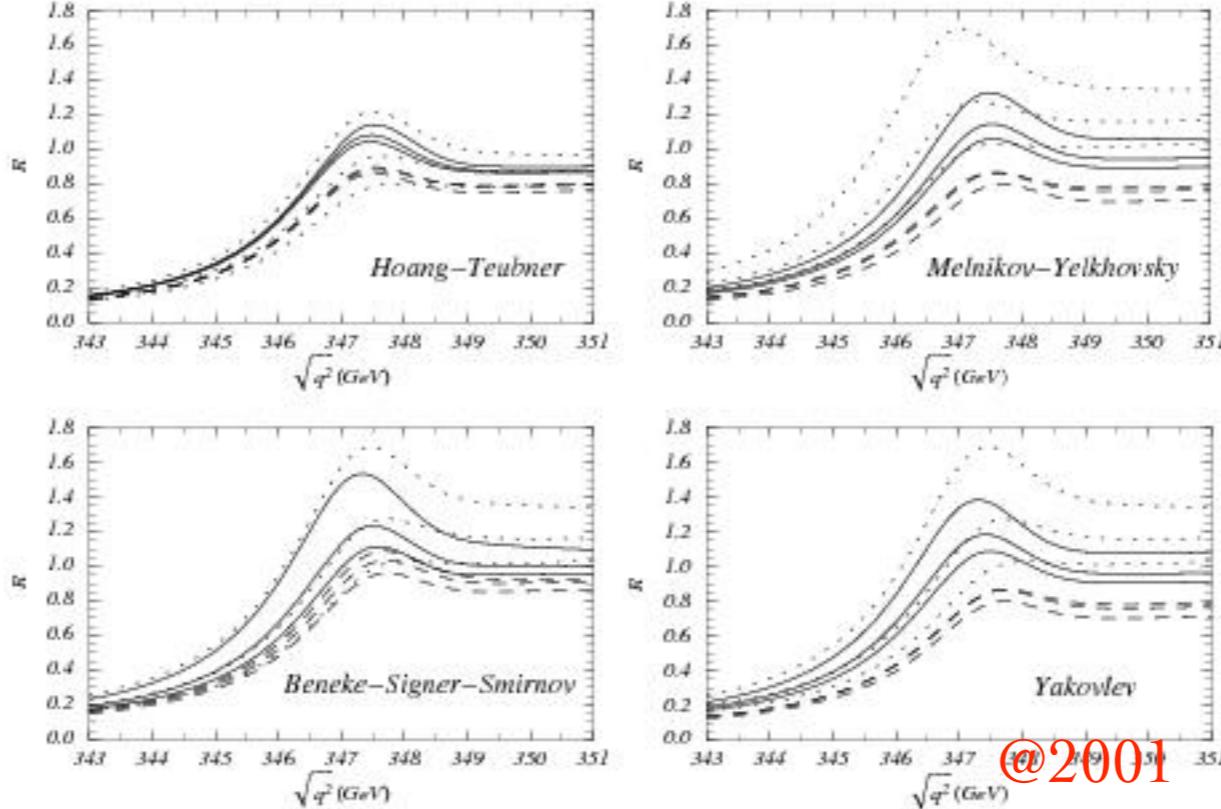


Figure 1: The total normalised photon-induced $t\bar{t}$ cross section

- ♦ We will advance the theory calculation to NNNLO

Effective Field Theory

- Integrate out Hard (Caswell-Lepage('86))

$$\mathcal{L}_{\text{NRQCD}} = \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m_t} \right) \psi + [\psi \rightarrow \chi] + \dots$$

- Integrate out Soft/Potential gluons

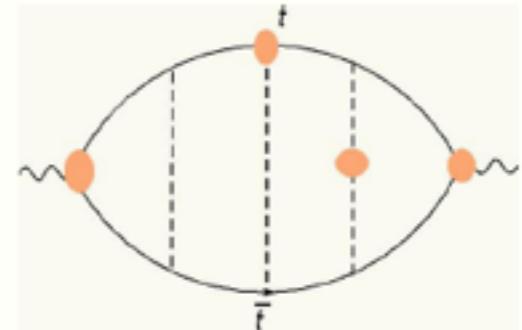
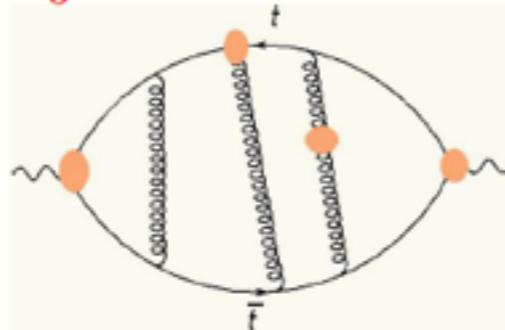
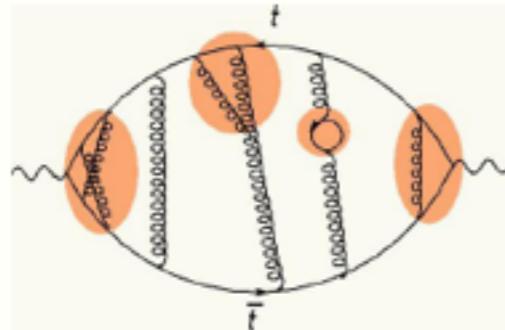
(Pineda-Soto('98); Luke-Manohar-Rothstein('99))

$$\begin{aligned} \mathcal{L}_{\text{pNRQCD}} = & \psi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_t} \right) \psi + \int d\vec{r} [\psi^\dagger \chi] V_{pot}(r) [\chi^\dagger \psi] \\ & + ig \psi^\dagger [A_{0,us} + \frac{\nabla \vec{A}_{us}}{m}] \psi - \frac{1}{4} \vec{F}_{us}^2 + \dots \end{aligned}$$

- Remaining Mode is Ultra Soft gluon: $k \sim m(v^2, \vec{v}^2)$

Threshold XS in EFT

Principal quantity is $\Pi(q) = i \int d^4x e^{iqx} \langle 0 | J^\mu(x) J_\mu(0) | 0 \rangle$



- Integrating hard mode \rightarrow NRQCD (Caswell-Lepage '86):

$$J^i(x) = [\bar{t}\gamma^i t] \rightarrow c_v [\psi^\dagger \sigma^i \chi]$$

- Integrating soft/potential modes \rightarrow PNRQCD

(Pineda-Soto '97/Luke - Manohar-Rothstein'99):

$$\begin{aligned} \mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left[i\partial_0 + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} - g_s \mathbf{x} \mathbf{E}(t, \mathbf{0}) \right] \psi + (\psi \leftrightarrow \chi) \\ & + \int d\mathbf{r} [\psi^\dagger \psi] (x + \mathbf{r}) V_{pot}(\mathbf{r}) [\chi^\dagger \chi] (x) + \dots \end{aligned}$$

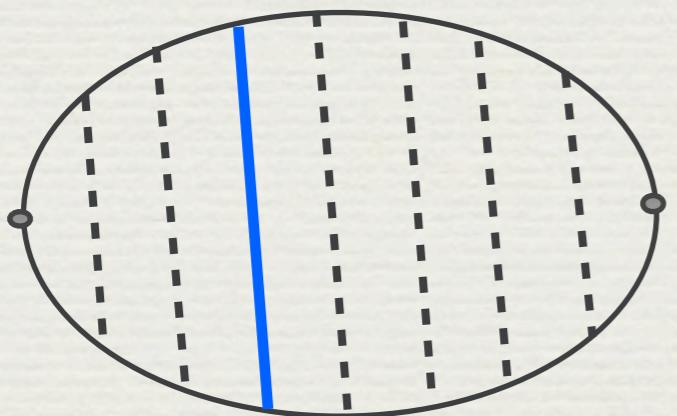
$$\Pi(q) = i \int d^4x e^{iqx} c_v^2 \langle 0 | [\psi^\dagger \sigma^i \chi](x) [\chi^\dagger \sigma_i \psi](0) | 0 \rangle$$

Potential Perturbation

$$V_{pot} = -\frac{C_F \alpha_s}{r} + \frac{C_2}{r^2} + C_3 \delta(\mathbf{r}) + \dots$$

- Higher order corr to the potential (case of Coulomb pot)

$$\tilde{V}_C = -\frac{4\pi C_F \alpha_s(\mathbf{q})}{\mathbf{q}^2} \times \left[1 + \frac{\alpha_s(\mathbf{q})}{4\pi} a_1 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi} \right)^2 a_2 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi} \right)^3 [a_3 + 8\pi^2 C_A^3 \left(\frac{1}{3\epsilon} + \ln \frac{\mu_{US}^2}{\mathbf{q}^2} \right)] \right]$$

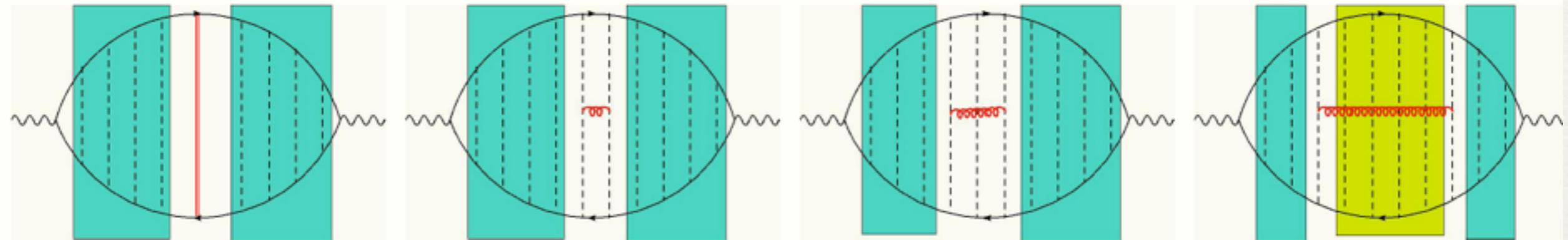


All order Coulomb ladder

Single insertion of a_1, a_2, a_3 is NLO,
NNLO and NNNLO, respectively

We treat higher order potentials as perturbation to
the all order resummation of LO Coulomb exchange.
→ We call Potential Insertion

Ultrasoft Correction

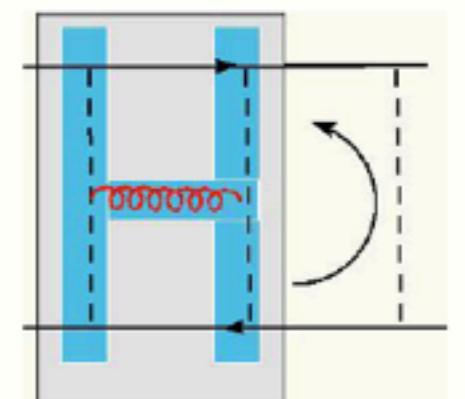


- quark-gluon vertex is $1/m$ suppressed; $\psi^\dagger (iD_0 + \frac{D^2}{2m}) \psi$
- n_g , number of potential exchange $\sim \Delta t$; $n_g > 1 \Leftrightarrow$ UV finite
- ADM $1/\epsilon$ of the Coulomb pot is a counter term for us corr

$$\delta V_C^{\text{ADM}} = \frac{1}{\epsilon} \frac{C_A^3 \alpha_s^4}{\mathbf{q}^2}$$

$$H_0 = -\frac{\alpha_s^3 C_A^2}{\epsilon} \left(\frac{1}{mq} + \frac{2(p^2/m - E)}{\mathbf{q}^3} + \epsilon L_{\text{Bethe}} \right) + O(\epsilon)$$

$$H_1 = -\frac{1}{\epsilon} \frac{C_A^2 (C_A - 2C_F) \alpha_s^4}{\mathbf{q}^2} + L_{\text{Bethe}} + O(\epsilon)$$



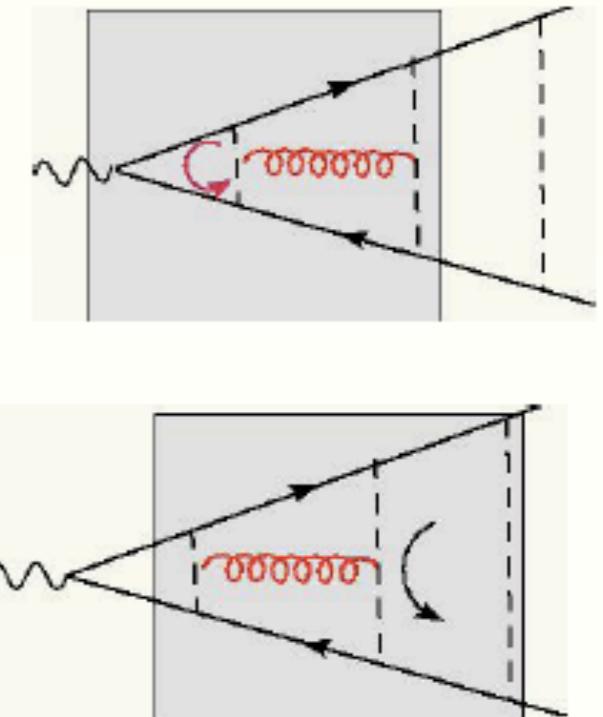
- UV cancelation happens tricky way:
 $\frac{(p^2/m - E)}{\mathbf{q}^3} \Rightarrow \frac{C_F \alpha_s}{\mathbf{q}^2}$ (Eq. of motion)

US renormalization II: Vertex

$$\delta V_C^{\text{ADM}} = \frac{1}{\epsilon} \frac{C_A^3 \alpha_s^4}{q^2}$$

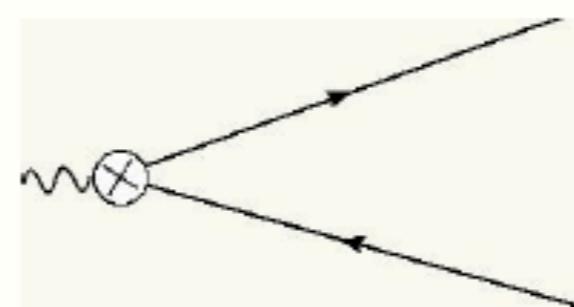
$$H_0 = -\frac{\alpha_s^3 C_A^2}{\epsilon} \left(\frac{1}{mq} + \frac{2(p^2/m-E)}{q^3} + \epsilon L_{\text{Bethe}} \right) + O(\epsilon)$$

$$H_1 = -\frac{1}{\epsilon} \frac{C_A^2 (C_A - 2C_F) \alpha_s^4}{q^2} + L_{\text{Bethe}} + O(\epsilon)$$



- Loop near photon vertices are more singular
↔ Vertex Renormalization
- $1/\epsilon$ cancelation does not happen exactly anymore
 - due to vertex divergence, (3+1) dimensional Eq. of motion is invalid
 - diagrams with different loop order get mis-match of Eq. of motion

Needs external current renormalization



- ♦ all the logarithmic parts obtained analytically
- ♦ there are uncanceled $i\Gamma/\epsilon$, which in turn produces scale dependence \rightarrow unstable top EFT

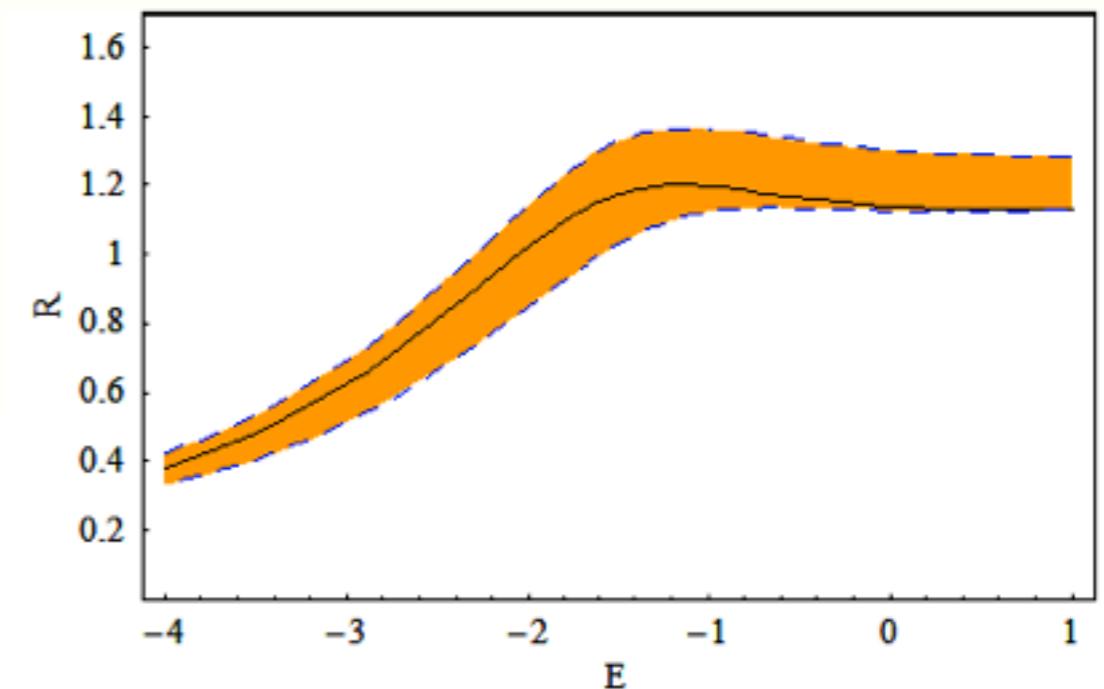
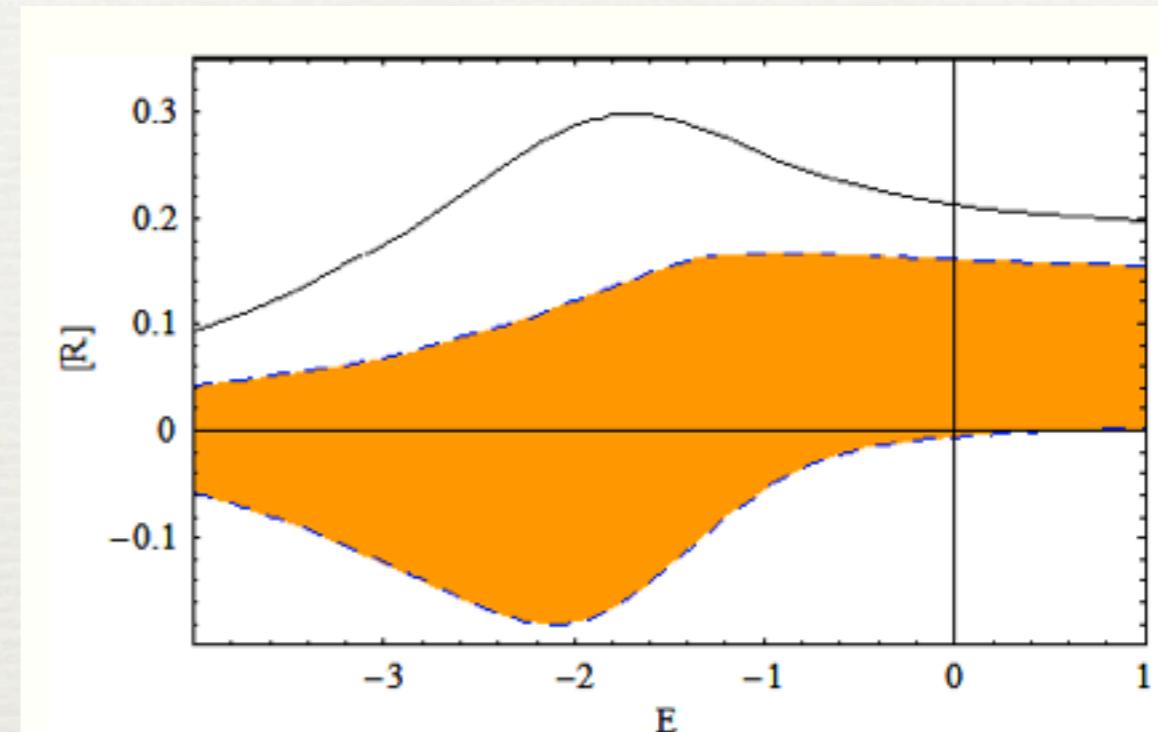
$$\begin{aligned}
 \delta^{us} G(E) = & \frac{2m^2 \alpha_s^4}{9\pi^2} \left\{ \left[\frac{17 i\hat{\Gamma}_t}{24} + \frac{527 \hat{G}_C}{72} \right] \frac{1}{\epsilon^2} + \left[\frac{17 i\hat{\Gamma}_t}{12} + \frac{221 \hat{G}_C}{36} \right] \frac{L_\mu}{\epsilon} + \left[\left(\frac{19}{12} \ln 2 - \frac{91}{72} \right) i\hat{\Gamma}_t \right. \right. \\
 & + \left(-\frac{119}{12} \ln 2 + \frac{2059}{108} \right) \hat{G}_C \Big] \frac{1}{\epsilon} + \left[-\frac{34 i\hat{\Gamma}_t}{3} - \frac{595 \hat{G}_C}{9} \right] L_{\alpha_s}^2 + \left[-\frac{17 i\hat{\Gamma}_t}{12} - \frac{833 \hat{G}_C}{36} \right] L_\mu^2 \\
 & + \left[\frac{34 i\hat{\Gamma}_t}{3} + \frac{748 \hat{G}_C}{9} \right] L_{\alpha_s} L_\mu + \left[\frac{2380 \mathcal{P}^2}{27} + \left(\frac{272 \ln 2}{9} - \frac{23483}{162} + \frac{2380}{27\lambda} + \frac{272}{27\lambda^2} \right) \mathcal{P} \right. \\
 & + \left(\frac{27\lambda}{2} - \frac{16}{3\lambda} \right) \psi' + \frac{64}{27\lambda^3} + \frac{4(-1331 + 306 \ln 2)}{81\lambda} + \frac{4(-199 + 114 \ln 2)}{81\lambda^2} \Big] L_{\alpha_s} \\
 & + \left[-\frac{1496 \mathcal{P}^2}{27} + \left(-\frac{34 \ln 2}{3} + \frac{5065}{72} - \frac{1496}{27\lambda} - \frac{136}{27\lambda^2} \right) \mathcal{P} + \left(\frac{8}{3\lambda} - \frac{81\lambda}{8} \right) \psi' \right. \\
 & \left. \left. - \frac{32}{27\lambda^3} + \frac{163 - 114 \ln 2}{27\lambda^2} + \frac{271 - 51 \ln 2}{9\lambda} \right] L_\mu + \delta^{us}(\hat{E}) \right\}, \\
 L_\mu &= \ln \frac{\mu}{m_t}, \quad L_{\alpha_s} = \ln \alpha_s, \quad \lambda = \frac{C_F}{2\sqrt{-\hat{E}}}, \quad \mathcal{P} = \ln \left(\frac{C_F}{\lambda} \right) + \gamma_E + \psi(1 - \lambda),
 \end{aligned}$$

Beneke-YK 08

US corr

- ◆ Ultrasoft correction alone; constant(solid), log +const. orange band
- ◆ $R_{LO} + R_{us}$

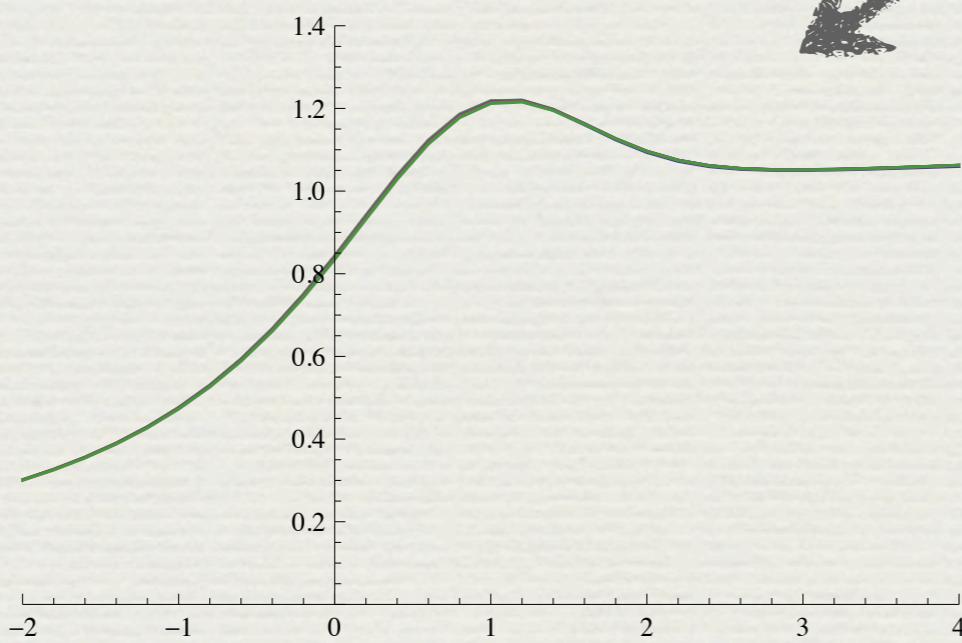
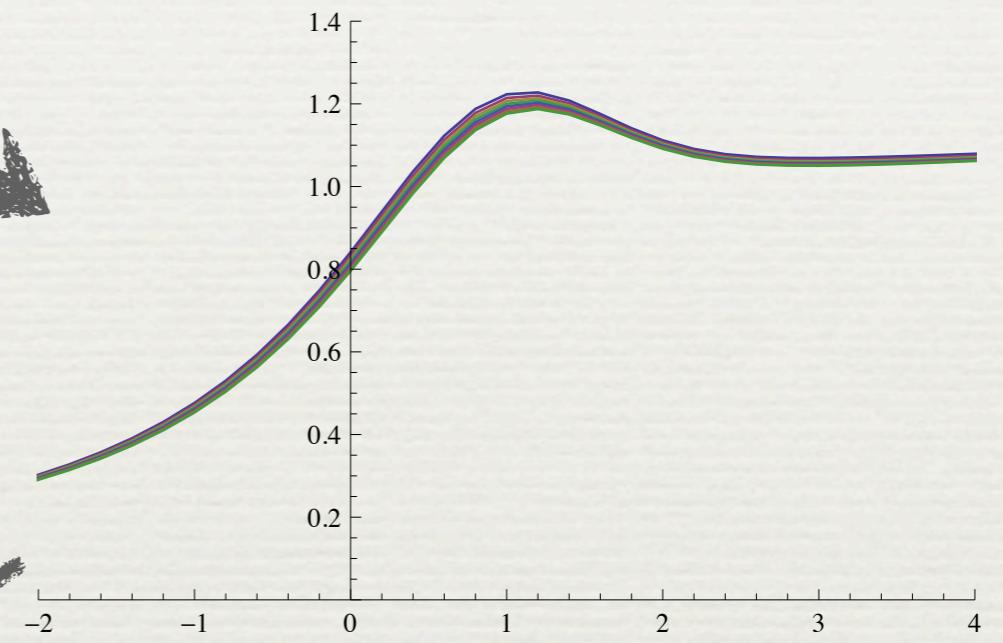
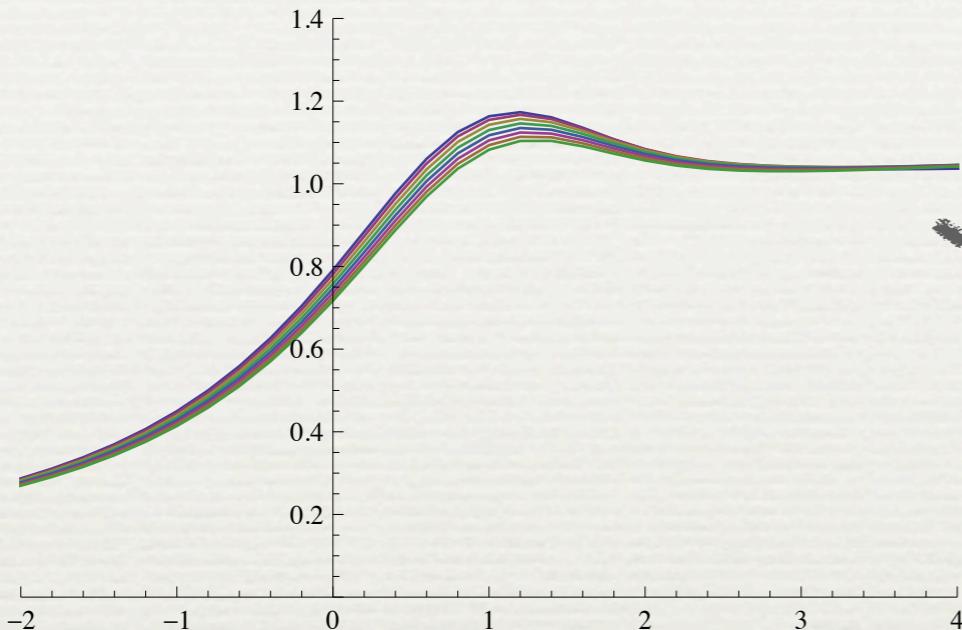
Ultrasoft correction is scale dependent and not physical



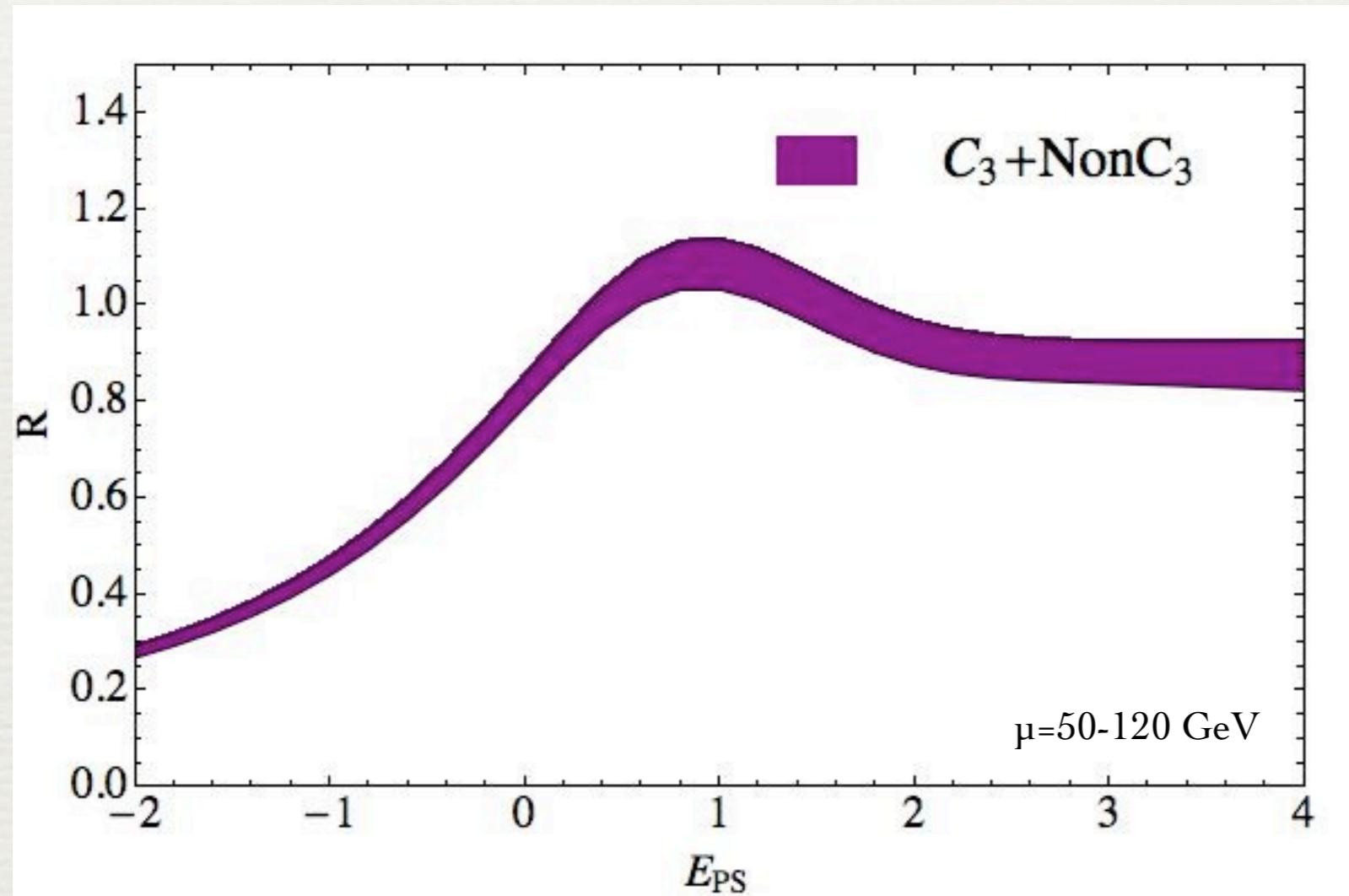
Bulding blocks (2014)

$J^i = c_v \psi^\dagger \sigma^i \chi + d_v \frac{1}{6m_t^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi$	$\mathcal{L}_{\text{QCD}} \Leftrightarrow \mathcal{L}_{\text{PNRQCD}}$	
$c^{(2)}$: Beneke-Signer-Smirnov('97), Czanecki-Melnikov('97)	a_2 : Schröder('98)	Anzai-YK-Sumino, Smirnov-Smirnov-Steinhauser (2010)
$c_{\cancel{n_f}}^{(3)}$: Marquard-Piclum- Seidel-Steinhauser(06)	$a_3, \cancel{\text{pade}}$: Chishtie-Elias (01) (New: a_3, n_f Smirnov-Smirnov-Steinhauser (Sep.08))	
$d_v^{(1)}$: Luke-Savage('97)	$\delta \mathcal{L}^{(1)}$: Manohar('97), Beneke-Signer-Smirnov('99), Wüster-Schuller('03)	
Marquard-Piclum-Seidel-Steinhauser arXiv:1401.3004[hep-ph]	$\delta \mathcal{L}^{(2)}$: Kniehl-Penin- Smirnov-Steinhauser(02) ($\delta \mathcal{L}^{(2)} = \mathcal{O}(\epsilon)$ not known)	
	$\delta \mathcal{L}^{(us)}$: Brambilla-Pineda-Soto-Vairo('99), Kniehl-Penin- Smirnov-Steinhauser(02)	
		Beneke-YK-Marquard-Penin-Piclum-Seidel-Steinhauser arXiv:1401.3004[hep-ph]

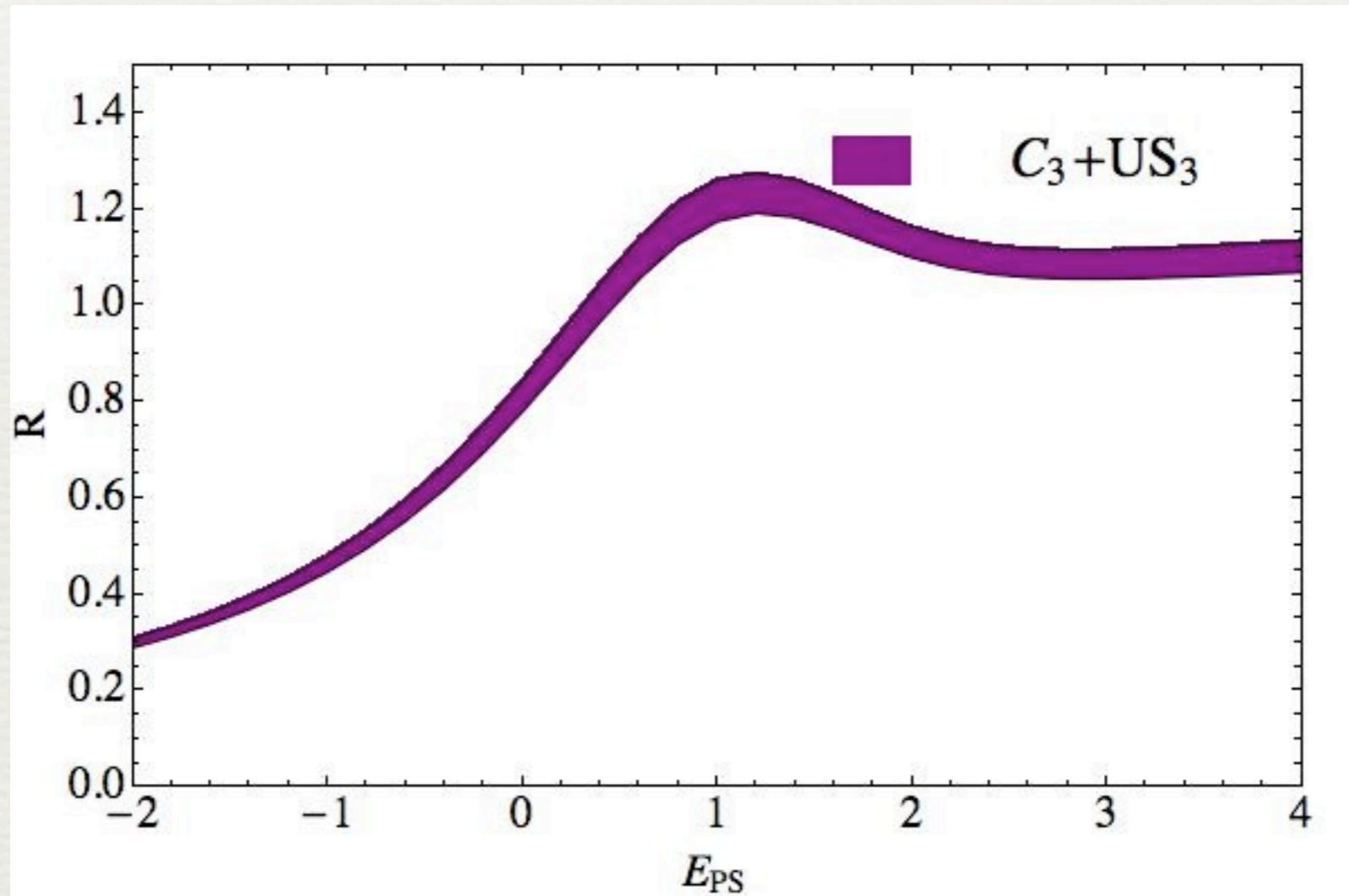
Coulomb Corr



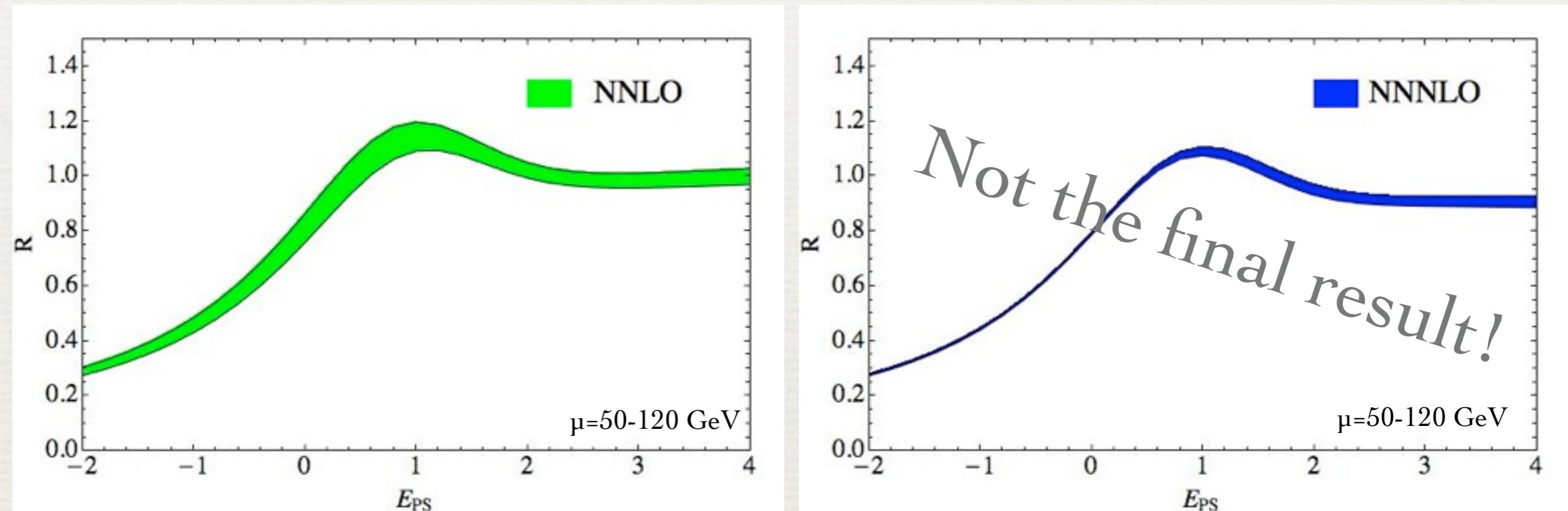
NonCoulomb Corr



Ultrasoft Corr



N^2LO and N^3LO



Beneke-YK-Schullet(TTbarXSection)
+Marquard-Piclum-Seidel-Steinhauser, Penin(c3, b2ep)

Inputs:

$m_{PS}(20\text{GeV})=170\text{ GeV}$,

$\Gamma_t=1.4\text{ GeV}$,

$\alpha_s(M_Z)=0.1180$

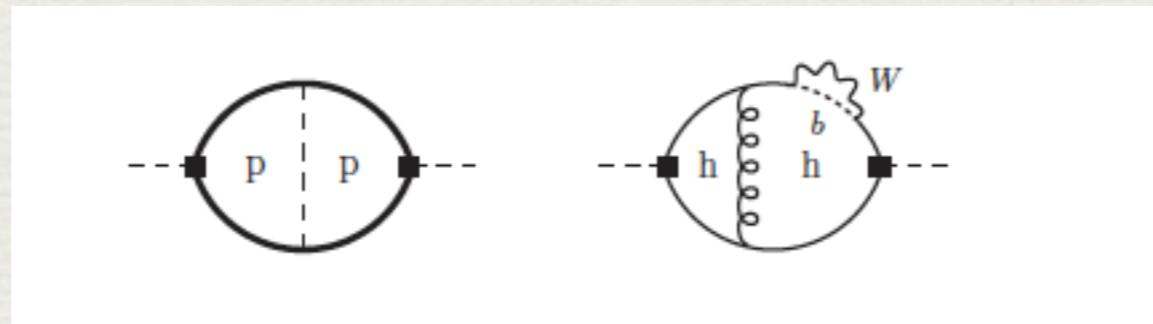
Issue of unstable top

- ◆ There are several different effects concerning to unstable nature of top quarks.
→ talk by A. Hoang, and P. Ruiz
- ◆ We have taken into account one of unstable top effect by $E \rightarrow E + i\Gamma_t$, We have introduced some uncanceled scale dependence.

$$\delta^{(2)}G_{div} = \frac{(2\pi)^2}{3\epsilon\alpha_s} C_F (2C_F + 3C_A) G_C(E) + \frac{\Gamma}{\epsilon} \frac{2\pi C_F m}{\alpha_s}$$

multiplicative Z-factor

need to renormalized counter term by unstable top EFT



It was started to seriously think this issue few years ago
[Hoang-Reisser-Ruiz, Beneke-Falgari-Schwinn-Signer-Zanderighi, Jatzen-Ruiz, Penin-Piclum,...].
We have started to implement a prescription in our code, to fix the scale dependence, but not yet completed.

Summary

- ◆ $e^+e^- \rightarrow tt$ cross section near threshold
computed at NNNLO \rightarrow sizable correction,
reduction of scale dep.
- ◆ Still few missing parts $\rightarrow c_3, \dots$
Maquard-Piclum-Seidel-Steinhauser(2014)
- ◆ c_3 (singlet) still missing
- ◆ Implementation of some of unstable effects
started Jantzen-Femenia(2013), Beneke-Jantzen-Femenia(2010), Hoang-Reisser-Femenia(2010)