# SPIN TRACKING IN THE ILC DAMPING RING

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DESY

- Why do we need DR?
- Model of Polarization
- Theoretical model of spin dynamics
  - Spin precession
  - Spin motion in the curvilinear coordinate system
  - Suitable coordinate frame for spin motion
  - Equation of spin-orbit motion
  - Synchrotron radiation and radiation damping
  - Sokolov-Ternov effect
- Beam Sizes Simulation
- Spin-Orbit Resonances
- Spin Dynamics
- Summary

# DAMPING RING

- The positron source produces trains of bunches *with large emittances*.
- The beam is stored for the time between machine pulses which is 1/10Hz = 0.1sec=100 ms
- Number of turns in DR: N=0.1·fc =0.1·3e8/3200=9256

| | Injected emittance | Extracted emittance |
|---|---|---|
| Horizontal $e^+$ | 1 $\mu$m | 0.8 nm |
| Vertical $e^+$ | 1 $\mu$m | 0.002 nm |
| Longitudinal $e^+$ | > 30 $\mu$m | 10 $\mu$m |

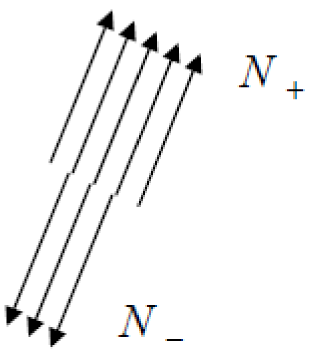
- **Luminosity:** $L \sim 1/\left(\sigma_x \cdot \sigma_y\right)$

# DEFINITION OF POLARIZATION

Average of normalized expectation values – polarization:

$$\vec{P} = \frac{1}{|\langle \vec{S} \rangle|} \frac{\sum_{i=1}^{1=N} \langle \vec{S}_i \rangle}{N}$$

Spin of fermions is a statistical mixture of two basic pure states: spins up and spins down




$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

# SPIN PRECESSION

Spin precession in EM guide fields → T-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}(r,p,t) \times \vec{S},$$

$$\vec{\Omega} = -\frac{q}{m\gamma}\left[(1+a\gamma)\,\vec{B} - \frac{a\vec{p}\cdot\vec{B}}{(1+\gamma)\,m^2c^2}\vec{p} - \frac{1}{mc^2}\left(a+\frac{1}{1+\gamma}\right)\vec{p}\times\vec{\mathcal{E}}\right]$$

$a = (g-2)/2$ is the anomalous gyromagnetic factor (for electron,positron = 0.00115965212 )

# SPIN PRECESSION

In a purely magnetic field there is some similarities between T-BMT and Lorentz force equation

$$\frac{d\vec{p}}{dt} = -\frac{e}{\gamma m}\vec{B}_{\perp} \times \vec{p}$$
$$\frac{d\vec{S}}{dt} = -\frac{e}{\gamma m}\left(\left(1 + a\gamma\right)\vec{B}_{\perp} + \left(1 + a\right)\vec{B}_{\parallel}\right) \times \vec{S}$$

– $\vec{B}_{\parallel} = 0$ (no solenoids) → $\delta\theta_{spin} = \left(1 + a\gamma\right)\delta\theta_{orbit}$

– if the orbit is deflected by an angle *φ* in a transverse magnetic field, then the spin is rotated by an angle *aγφ* relative to the orbit.

– in the coordinate system that rotates with the particle's momentum:
$\frac{d\vec{S}}{dt} = -\frac{q}{m}a\vec{B}_{\perp} \times \vec{S}$ → for particle with velocity *v* $\int B dl = 2\pi\frac{mcv}{qa} \approx 9.24\, Tm$
results on spin rotation angle of 2π

# SPIN MOTION IN THE CURVILENEAR COORDINATE SYSTEM

In DR the EM fields produced by the elements which are fixed in the space,
$\longrightarrow$ instead of time (t) the distance around the ring (s) can be used.

After transformation to curvilinear coordinates, the T-BMT equation becomes

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\vec{u}, s) \times \vec{S}$$

Contribution due to synchro-betatron motion

$$\vec{\Omega}(\vec{u}, s) = \vec{\Omega}^{c.o} + \vec{\omega}^{s.b}$$

Contains fields along periodic closed orbit

Contains designed fields

$$\vec{\Omega}^{c.o} = \vec{\Omega}^{d.o} + \vec{\omega}^{imp}$$

Represents effects of magnet misalignments, correction fields, etc.

$$\vec{\Omega}^{c.o}(s + C) = \vec{\Omega}^{c.o}(s)$$

# SPIN MOTION IN THE CURVILENEAR COORDINATE SYSTEM

In DR with circumference C all fields are 2π periodic.
→ convenient to use the azimuth Θ= 2π/C as an independent variable

T-BMT equation becomes $$\frac{d\vec{S}}{d\theta} = \vec{\Omega}(\vec{u},\theta) \times \vec{S}, \;\; \vec{\Omega}(\vec{u},\theta+2\pi) = \vec{\Omega}(\vec{u},\theta)$$

Equation of particle motion $$\frac{d\vec{u}}{d\theta} = \vec{v}(\vec{u},\theta), \;\; \vec{v}(\vec{u},\theta+2\pi) = \vec{v}(\vec{u},\theta)$$

By introducing anti-symmetric matrix

$$\mathbf{\Omega} = \begin{pmatrix} 0 & -\Omega_s & \Omega_z \\ \Omega_s & 0 & -\Omega_x \\ -\Omega_z & \Omega_x & 0 \end{pmatrix} \longrightarrow \frac{d\vec{S}}{d\theta} = \mathbf{\Omega}(\vec{u},\theta)\vec{S}$$

$$\theta_0 \longrightarrow \theta_1 \quad \vec{u}(\theta_1) = M_{6x6}(\theta_1,\theta_0)\vec{u}(\theta_0) \quad \vec{S}(\theta_1) = \mathbf{R}_{3x3}(\theta_1,\theta_0)\vec{S}(\theta_0)$$

$$\frac{d\mathbf{R}_{3x3}(\theta_1,\theta_0)}{d\theta} = \mathbf{\Omega}\mathbf{R}_{3x3}(\theta_1,\theta_0), \;\; \mathbf{R}_{3x3}(\theta_0,\theta_0) = \mathbf{I}$$

# COORDINATE FRAME FOR SPIN MOTION

Suitable coordinate frame is necessary for description of spin motion.

Solving eigenvalue problem for the one turn rotation matrix $\mathbf{R}^{c.o}_{3\times 3}$

$$R^T = R^{-1} \Rightarrow R^T R = I$$

The eigenvalues of an orthogonal matrix $1,\ e^{2\pi i\nu_0},\ e^{-2\pi i\nu_0}$

There is always at least one real eigenvector $\hat{n}_0(s_0)$ :

$$\hat{n}_0(s_0 + C) = R(s_0 + C, s_0)\hat{n}_0(s_0) = \hat{n}_0(s_0)$$

The other two eigenvectors are complex and conjugate pairs $\hat{m}_0 \pm i\hat{l}_0$ for $e^{\pm 2\pi i\nu_0}$

Right-handed coordinate system: $\hat{n}_0(s) = \hat{m}_0(s) \times \hat{l}_0(s)$

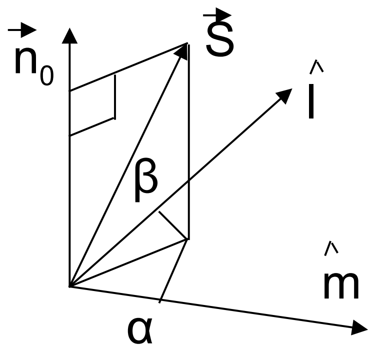
$$l_0 \perp \hat{n}_0,\ \hat{m}_0 \perp \hat{n}_0,\ \hat{l}_0 \perp \hat{m}_0$$

$\hat{m}_0(\theta + 2\pi) + i\hat{l}_0(\theta + 2\pi) = e^{i2\pi\nu_0}\left[\hat{m}_0(\theta) + i\hat{l}_0(\theta)\right]$ – not periodic.

By applying a further rotation by an angle $\psi_{spin}$ $\quad \psi_{spin}(\theta + 2\pi) = \psi_{spin}(\theta) + 2\pi\nu_0$

→ periodic coordinate system $\hat{m}(\theta) + i\hat{l}(\theta) = e^{-i\psi_{spin}(\theta)}\left[\hat{m}_0(\theta) + i\hat{l}_0(\theta)\right]$

# EQUATION OF SPIN-ORBIT MOTION




General solution of T-BMT

$$\vec{S} = \sqrt{1-\alpha^2-\beta^2}\,\vec{n}_0(\theta) + \alpha\,\hat{m}(\theta) + \beta\,\hat{l}(\theta)$$

The relative reduction of polarization

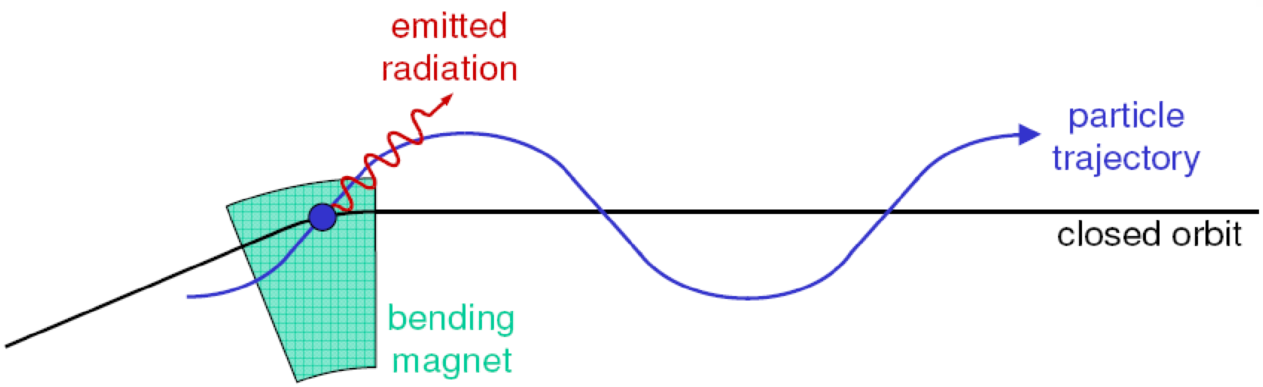
$$\frac{dP}{dt} \approx \frac{1}{2}\frac{d}{dt}\left\langle \alpha^2 + \beta^2 \right\rangle = \frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2 + \sigma_\beta^2)$$

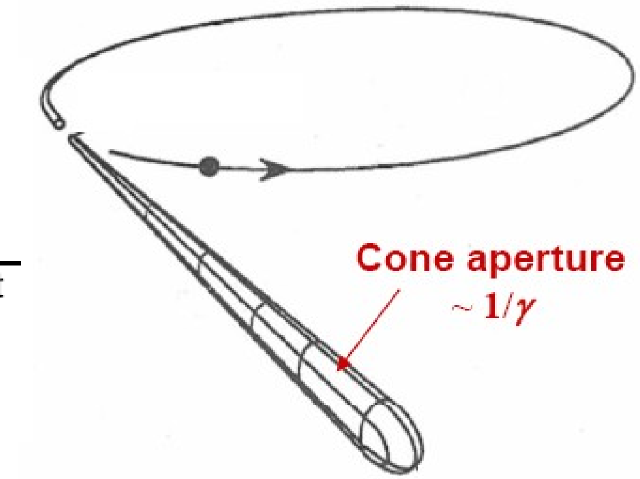
Equation of spin-orbital motion

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ \sigma \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_1) = \begin{bmatrix} M_{6\times 6} & 0_{6\times 2} \\ G_{2\times 6} & D_{2\times 2} \end{bmatrix}(s_1, s_0)\begin{pmatrix} x \\ x' \\ y \\ y' \\ \sigma \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_0)$$

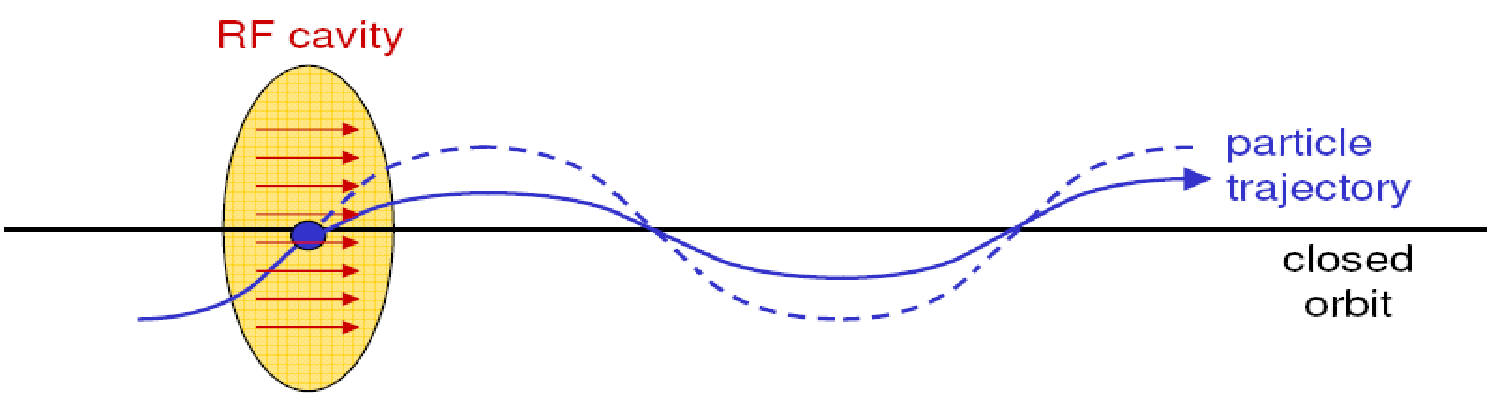
# SYNCHROTRON RADIATION AND RADIATION DAMPING

Particles lose longitudinal and transverse momentum in bending magnets.




In an RF cavity, the particle sees an accelerating electric field parallel to the closed orbit: the RF cavities in a damping ring restore the energy lost by synchrotron radiation.



# SOKOLOV-TERNOV EFFECT

Relativistic particles emit synchrotron radiation and small fraction of radiated photons can cause spin flip from up to down and vice versa.
Flip rates for electrons:

$$W_{\uparrow\downarrow} = \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m_e^2c^2\left|\rho\right|^3}\left(1+\frac{8}{5\sqrt{3}}\right) \quad W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m_e^2c^2\left|\rho\right|^3}\left(1-\frac{8}{5\sqrt{3}}\right)$$

Initially unpolarized stored -/+ beam gradually becomes polarized following:

$$P(t) = P_{ST}(1-\exp(-t/\tau_0))$$

Maximum attainable (equilibrium) polarization: $P_{ST} = \frac{W_{\uparrow\downarrow}-W_{\downarrow\uparrow}}{W_{\uparrow\downarrow}+W_{\downarrow\uparrow}} = \frac{8}{5\sqrt{3}} \simeq 0.9238$

And build-up rate is $\tau_0^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_e|\rho|^3}$

For ILC DR ~ 2000 sec.

# COMPUTER SIMULATION

- SLICKTRACK (by D.P. Barber): linearized orbit and non-linear spin motion.

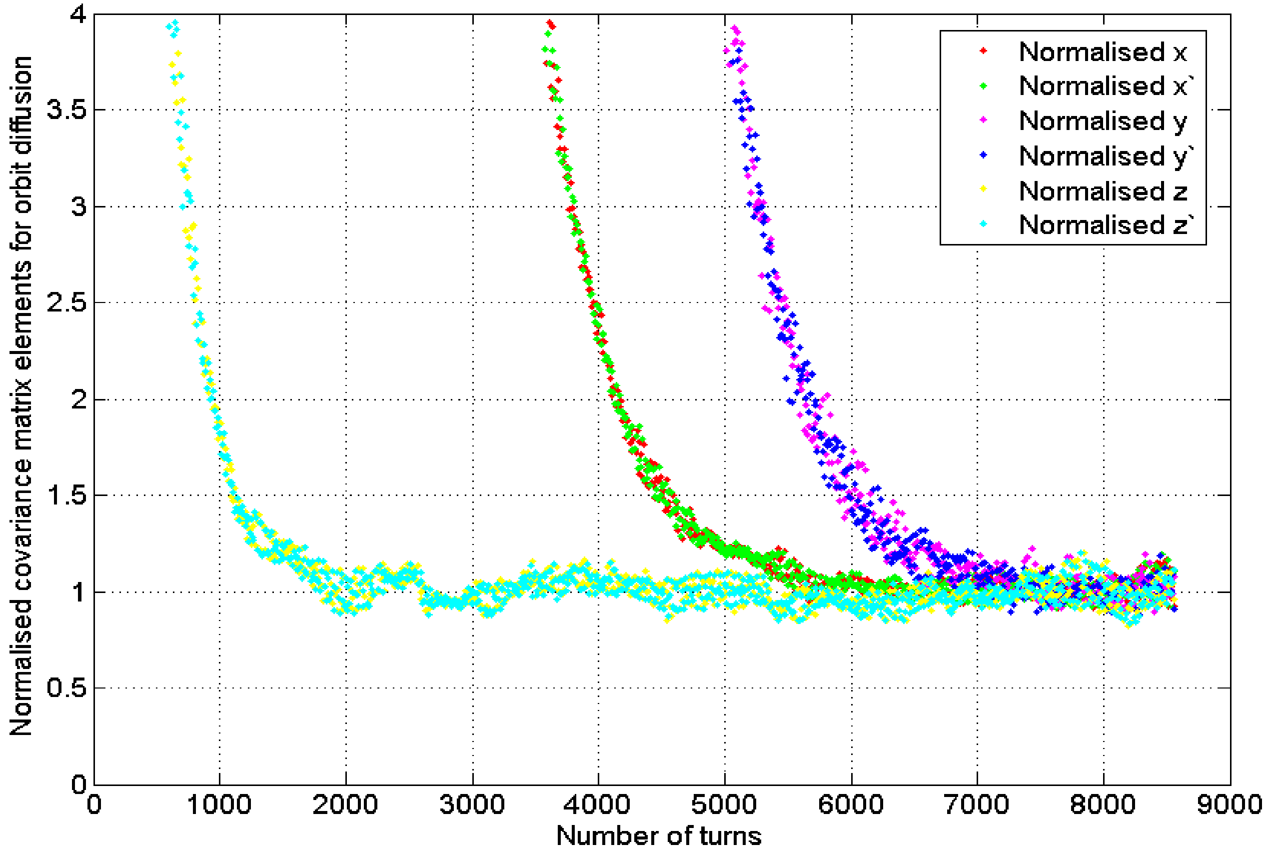
- It allows to estimate level of depolarization including synchrotron radiation in the DR using full 3-D spin motion.

- The orbit damping effects are included as the damping terms in the transport matrices.

- The effects of stochastic photon emission are modeled by means of «big photon» emission in the middle of each dipole.

# BEAM SIZES SIMULATION

| | Injected parameters | Extracted parameters |
|---|---|---|
| Normalised horizontal emittance | 0.01 m rad | 5.5 µm rad |
| Normalised vertical emittance | 0.01 m rad | 20 nm rad |
| Bunch length | 34.0 mm | 6 mm |
| Energy spread | 1.5 % | 0.11 % |



# SPIN-ORBIT RESONANCES

$$\nu_0 = k_0 + k_I \nu_I + k_{II} \nu_{II} + k_{III} \nu_{III}$$

tune of horizontal betatron motion

tune of vertical betatron motion

tune of synchrotron motion

Order of resonance: $|k_I| + |k_{II}| + |k_{III}|$

- Imperfection resonances or integer resonances: $|k_I| + |k_{II}| + |k_{III}| = 0$

  $\nu_0 = k_0 \rightarrow$ one turn spin rotation matrix is unit matrix and $\hat{n}_0$ is not unique

  $\hat{n}_0$ is tilted from the vertical and spin precess around a new value of $\hat{n}_0$

- Intrinsic resonances: $|k_I| + |k_{II}| + |k_{III}| = 1$

  Consider vertical betatron oscillations: all spins rotate around $\hat{n}_0$ by $2\pi\nu_0$

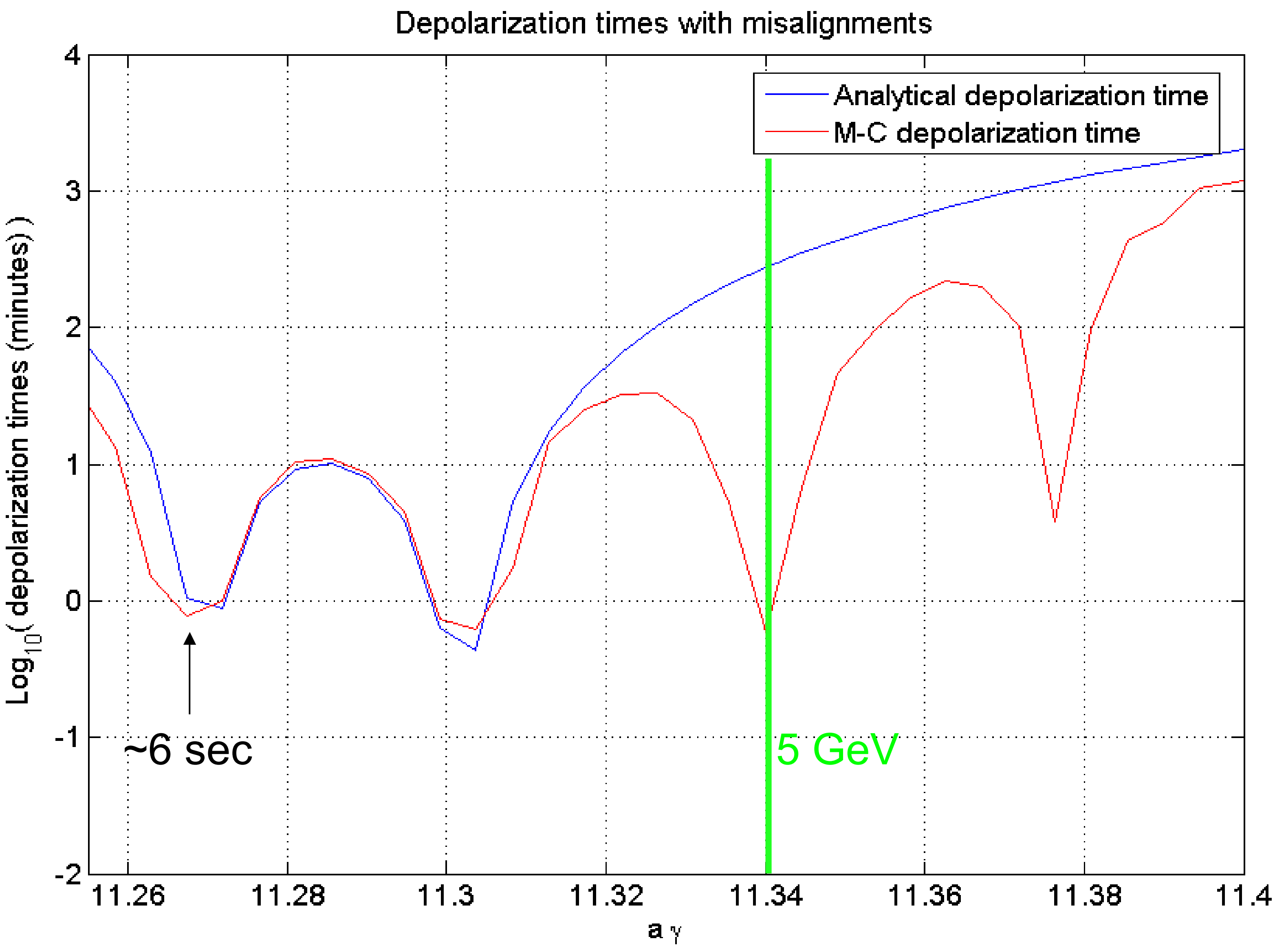
  $(\hat{n}_0, \hat{m}, \hat{l})$ rotates by $2\pi k_{II}$ and spins are rotated by $2\pi(\nu_0 - k_{II})$

  $\rightarrow$ at $\nu_0 = k_{II}$ net spin rotation due to main guide felds vanishes $\rightarrow$ spin motion is affected only by the extra fields “picked-up” along the oscillating trajectory away from closed orbit

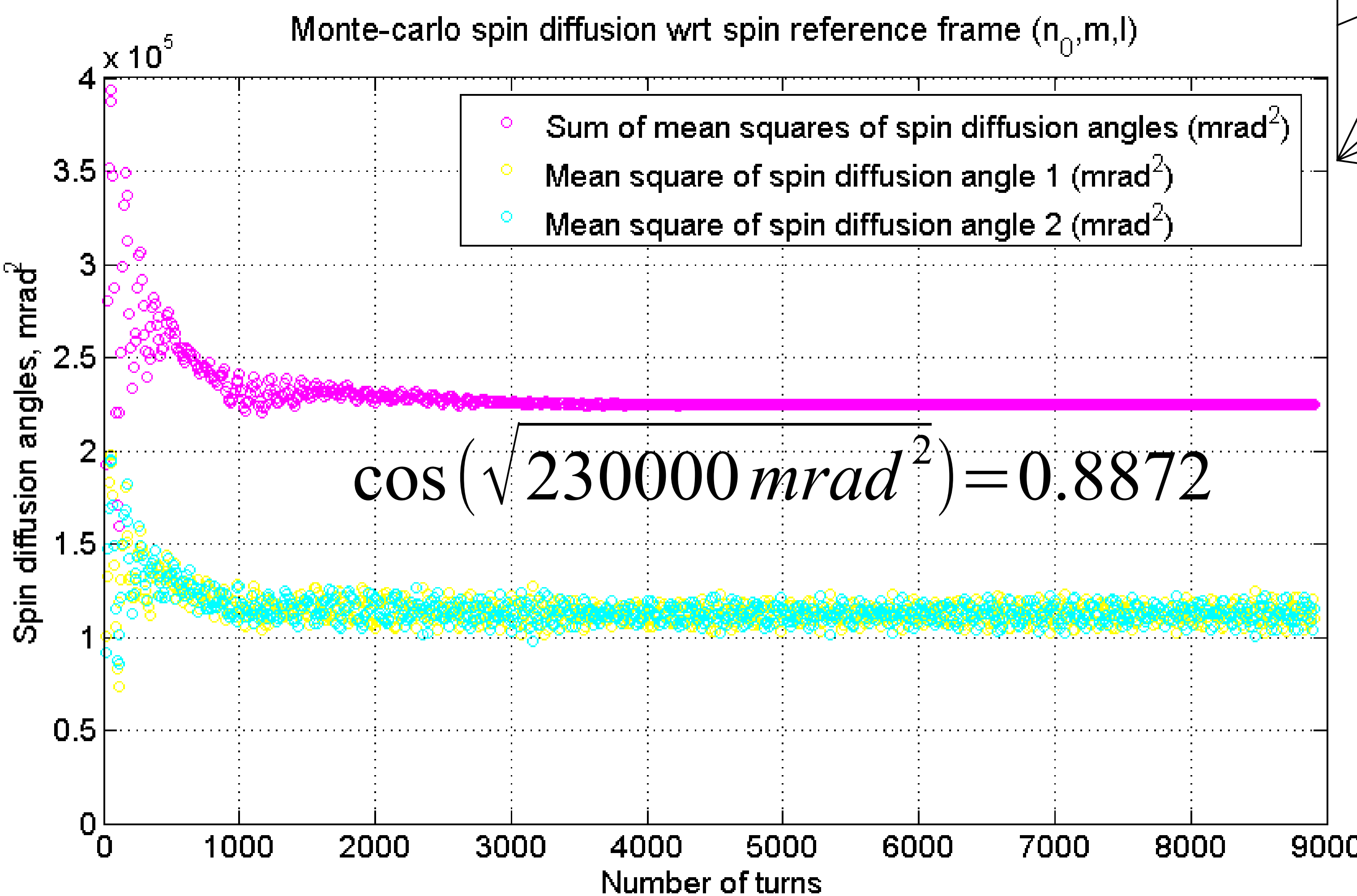
- Synchrotron sideband resonances: $\nu_0 = k_0 \pm \nu_{x,y} + k_s \nu_s$

  spin precession rate is affected by modulation of energy oscillations

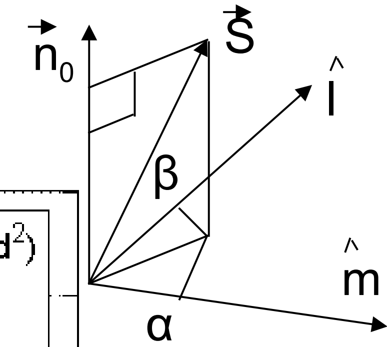
# SPIN-ORBIT RESONANCES IN THE ILC DR



# SPIN DIFFUSION IN THE ILC DR




$$\cos\left(\sqrt{230000\,mrad^2}\right)=0.8872$$



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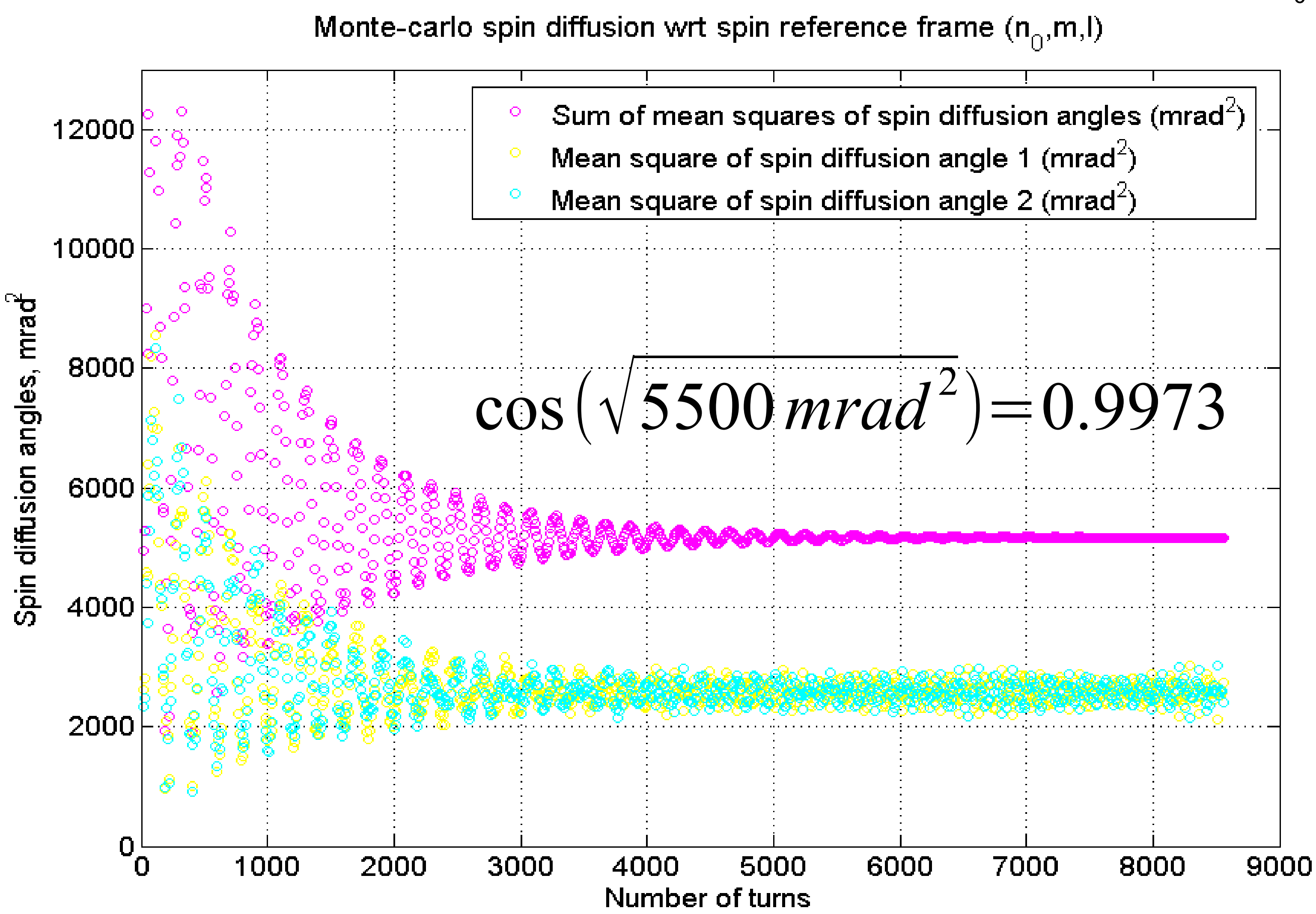
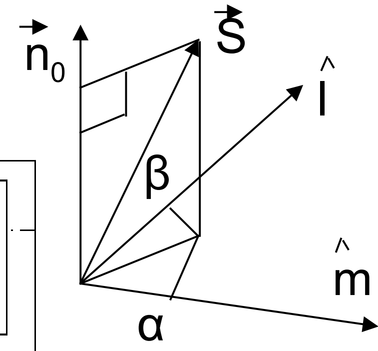



$$\cos\left(\sqrt{5500\,mrad^{2}}\right)=0.9973$$



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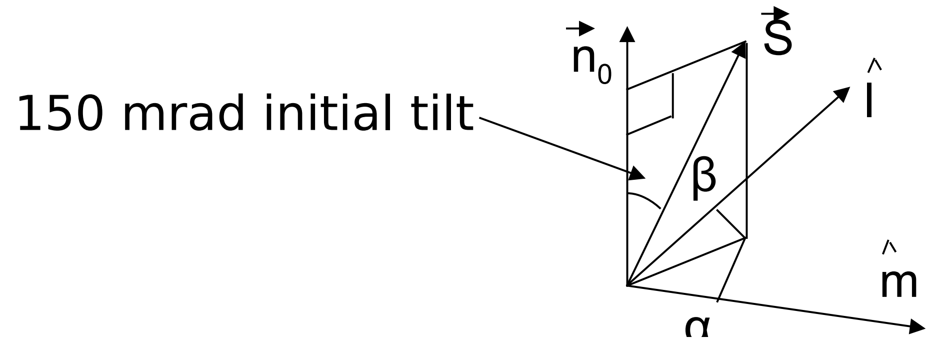


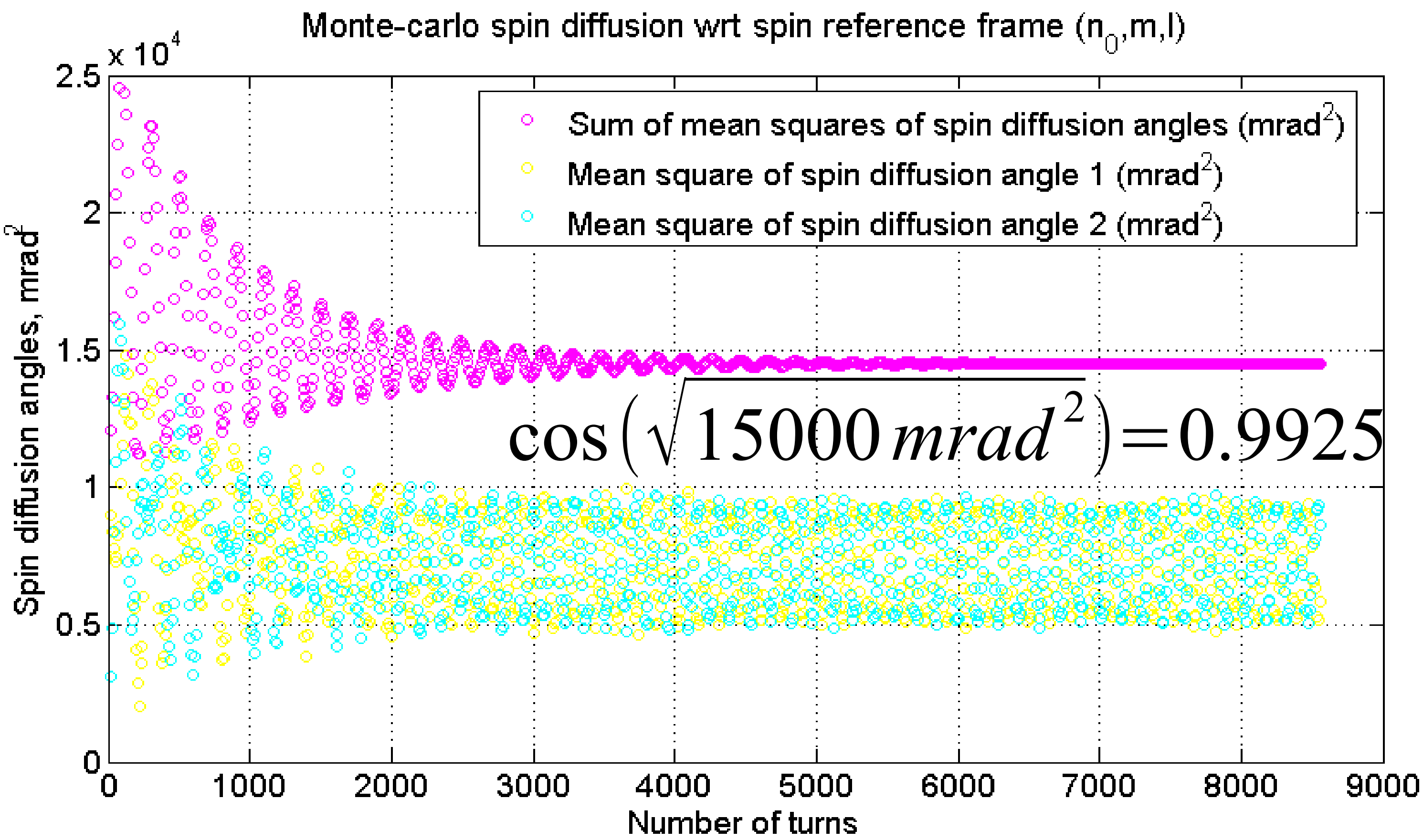


$$\cos\left(\sqrt{15000\,mrad^2}\right) = 0.9925$$

# SUMMARY

- Spin tracking in the ILC DR has been performed.
- Spin-orbit resonance at 5 GeV that has been found for DTC04 3.2 km damping ring lattice leads to depolarization about 11%.
- At 5.066 GeV energy away from resonances depolarization is 0.27%
- Depolarization due to initial tilt of spins from vertical direction is 0.75%.