

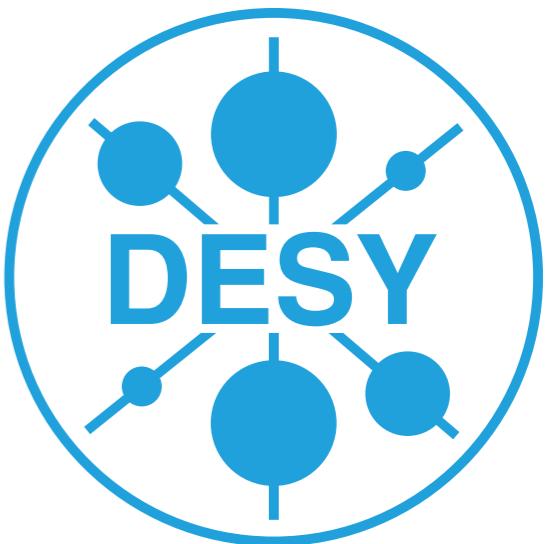
The W boson mass prediction in the SM, MSSM and NMSSM

based on: S. Heinemeyer, W. Hollik, G. Weiglein, L. Z., [arXiv:1311.1663],
O. Stål, G. Weiglein, L. Z., in preparation

Lisa Zeune

Helmholtz Alliance Linear Collider Forum

Bonn, 30 April 2014



W boson mass

- Electroweak precision observables $M_W, \sin^2 \theta_{\text{eff}}, a_\mu \dots$

→ Highly sensitive to quantum corrections of new physics

→ Precise measurement and precise theoretical calculation needed

Experimental measurement

- Precise experimental measurement

$$M_W^{\text{exp}} = 80.385 \pm 0.015 \text{ GeV}$$

Tevatron Electroweak Working Group, '12

- Improvement at the LHC very challenging

- Largest uncertainty in M_W prediction:
experimental error of top mass measurement

- Recent combination of LHC and Tevatron: $m_t^{\text{exp}} = 173.34 \pm 0.76 \text{ GeV}$

ATLAS, CDF, CMS, DØ, '14

- Significant improvement of M_W^{exp} and m_t^{exp} possible at a linear collider

W boson mass measurement at a linear collider

- Three uncorrelated methods to measure the W boson mass at the ILC
 - Run at WW threshold
 - Kinematic reconstruction using semi-leptonic channels
 - Direct reconstruction of the hadronic mass
- Each of these methods has experimental precision of ~ 5 MeV

Run at higher energies

ΔM_W [MeV]	LEP2	ILC	ILC
\sqrt{s} [GeV]	161	161	161
\mathcal{L} [fb^{-1}]	0.040	100	480
$P(e^-)$ [%]	0	90	90
$P(e^+)$ [%]	0	60	60
total	210	4.1-4.5	2.3-2.9

Wilson et al, Snowmass Study '13

ΔM_W [MeV]	ILC	ILC	ILC	ILC
\sqrt{s} [GeV]	250	350	500	1000
\mathcal{L} [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

ΔM_W [MeV]	LEP2	ILC	ILC	ILC
\sqrt{s} [GeV]	172-209	250	350	500
\mathcal{L} [fb^{-1}]	3.0	500	350	1000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
total systematics	21	2.4	2.9	3.5
statistical	30	1.5	2.1	1.8
total	36	2.8	3.6	3.9

Determination of the W boson mass

- Comparison of muon decay amplitude in Fermi model and SM:

$$\frac{G_\mu}{\sqrt{2}} = \frac{M_Z^2 e^2}{8 M_W^2 (M_Z^2 - M_W^2)} (1 + \Delta r(M_W, M_Z, m_t, \dots, X))$$

↑
model dependent

- G_μ, e, M_Z known with high precision

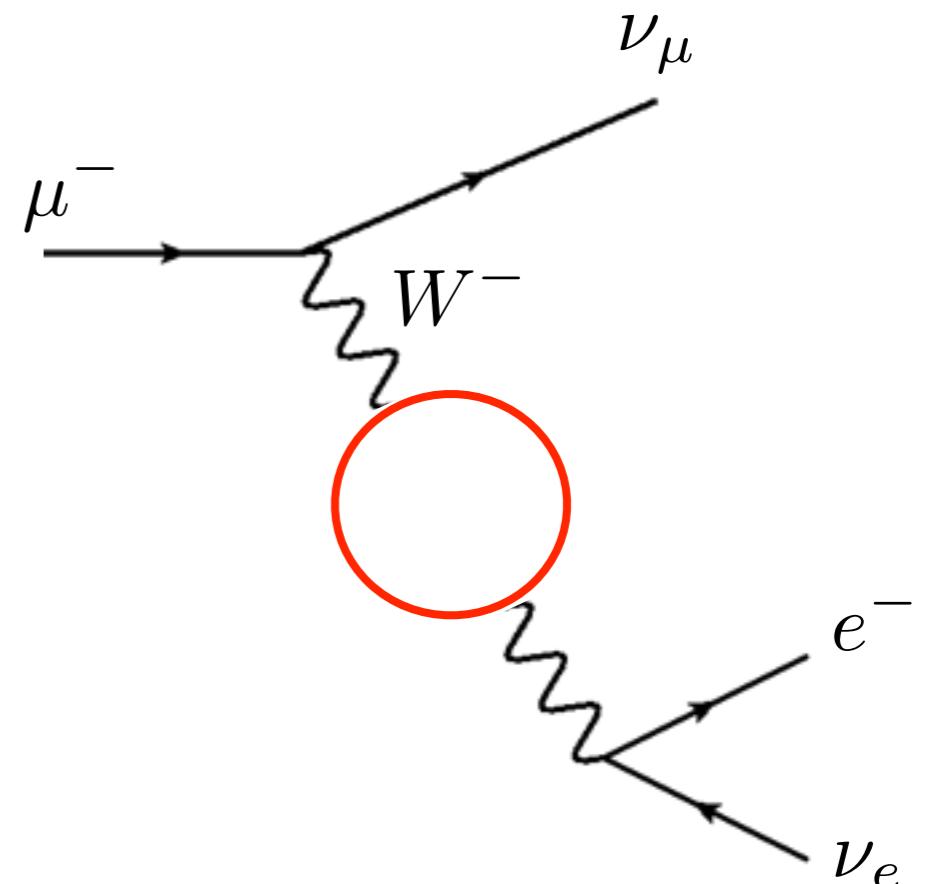
- Rearranging this formula

→ Model dependent prediction for the
W boson mass

- Precise calculation of Δr

→ Test models

→ Constrain model parameters



Δr in the MSSM and NMSSM

- 1-loop calculation of Δr in the complex MSSM and the NMSSM
- Incorporation of the relevant available SM and SUSY higher order corrections

$$\Delta r^{(N)\text{MSSM}} = \Delta r^{\text{SM}} + \Delta r^{\text{SUSY}}$$

SM part

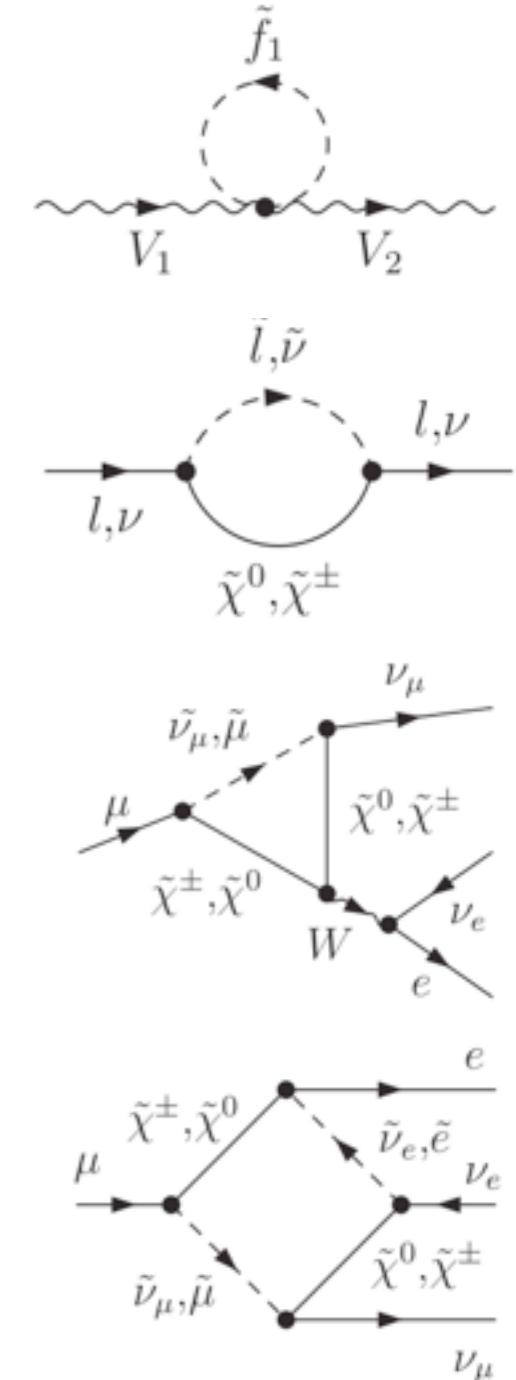
- Complete 2-loop result
- Leading 3- and 4-loop contributions

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas, Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong, ...

SUSY part

- 1-loop contributions from SUSY Higgs bosons, sfermions, charginos and neutralinos
- Available leading SUSY 2-loop corrections

Djouadi, Haestier, Heinemeyer, Stoeckinger, Weiglein, Consoli, Hollik, Jegenlehner, ...



- Most precise prediction for M_W in the MSSM and the NMSSM

Uncertainties in the M_w calculation

- Current parametric uncertainties

Freitas et al '13, Snowmass Study

	$\Delta m_t = 0.9 \text{ GeV}$	$\Delta(\Delta\alpha_{\text{had}}^{(5)}) = 1.38(1.0) \times 10^{-4}$	$\Delta M_Z = 2.1 \text{ MeV}$
$\Delta M_W \text{ [MeV]}$	5.4	2.5 (1.8)	2.6

- Parametric uncertainties with improved measurements

	$\Delta m_t = 0.5(0.1) \text{ GeV}$	$\Delta(\Delta\alpha_{\text{had}}^{(5)}) = 5 \times 10^{-5}$	$\Delta M_Z = 2.1 \text{ MeV}$
$\Delta M_W \text{ [MeV]}$	3.0 (0.6)	1.0	2.6

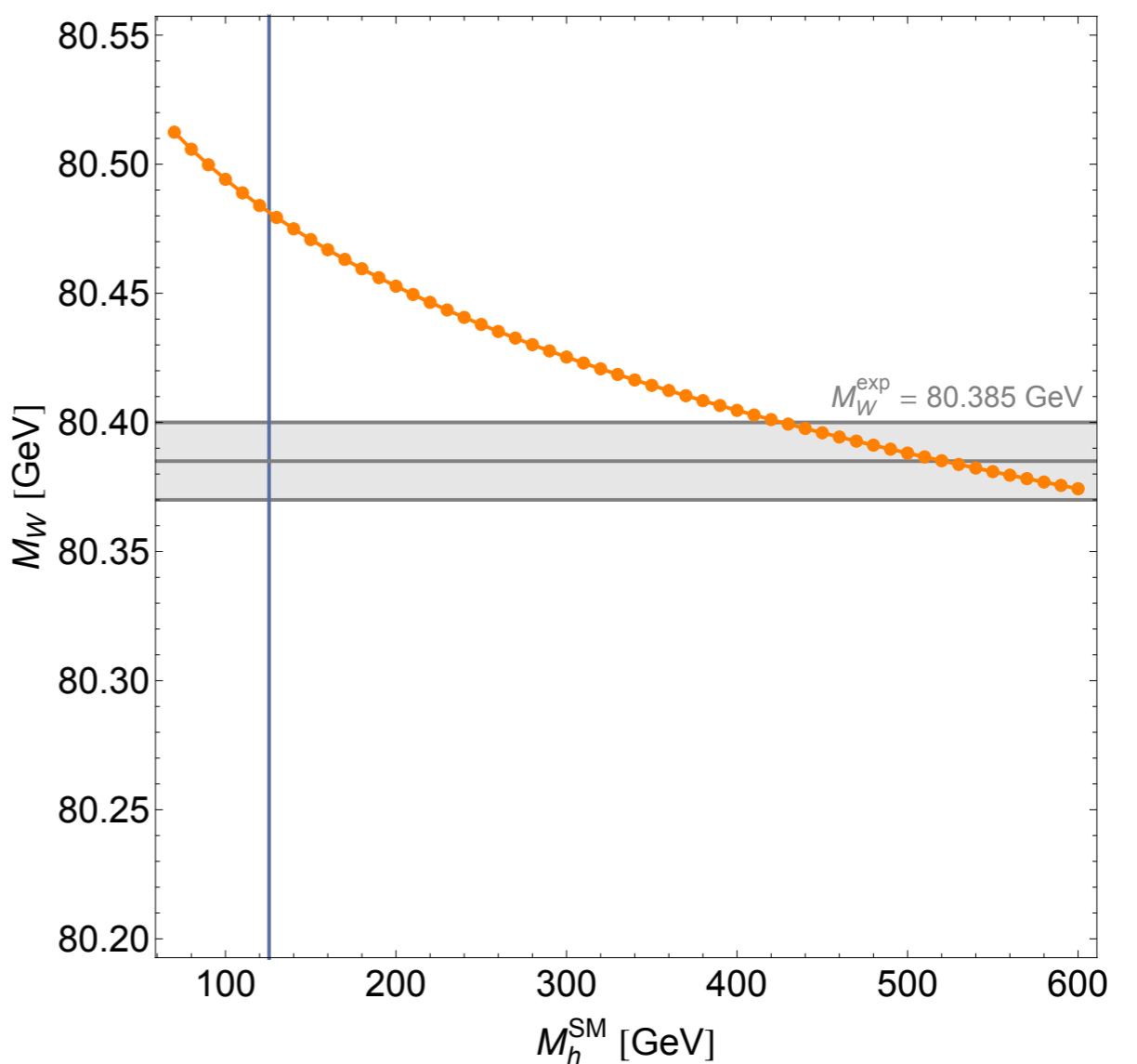
- Uncertainties from missing higher order corrections

SM: 4 MeV M. Awramik, M. Czakon, A. Freitas and G. Weiglein '03

MSSM: 4 - 9 MeV J. Haestier, S. Heinemeyer, D. Stöckinger, G. Weiglein '05

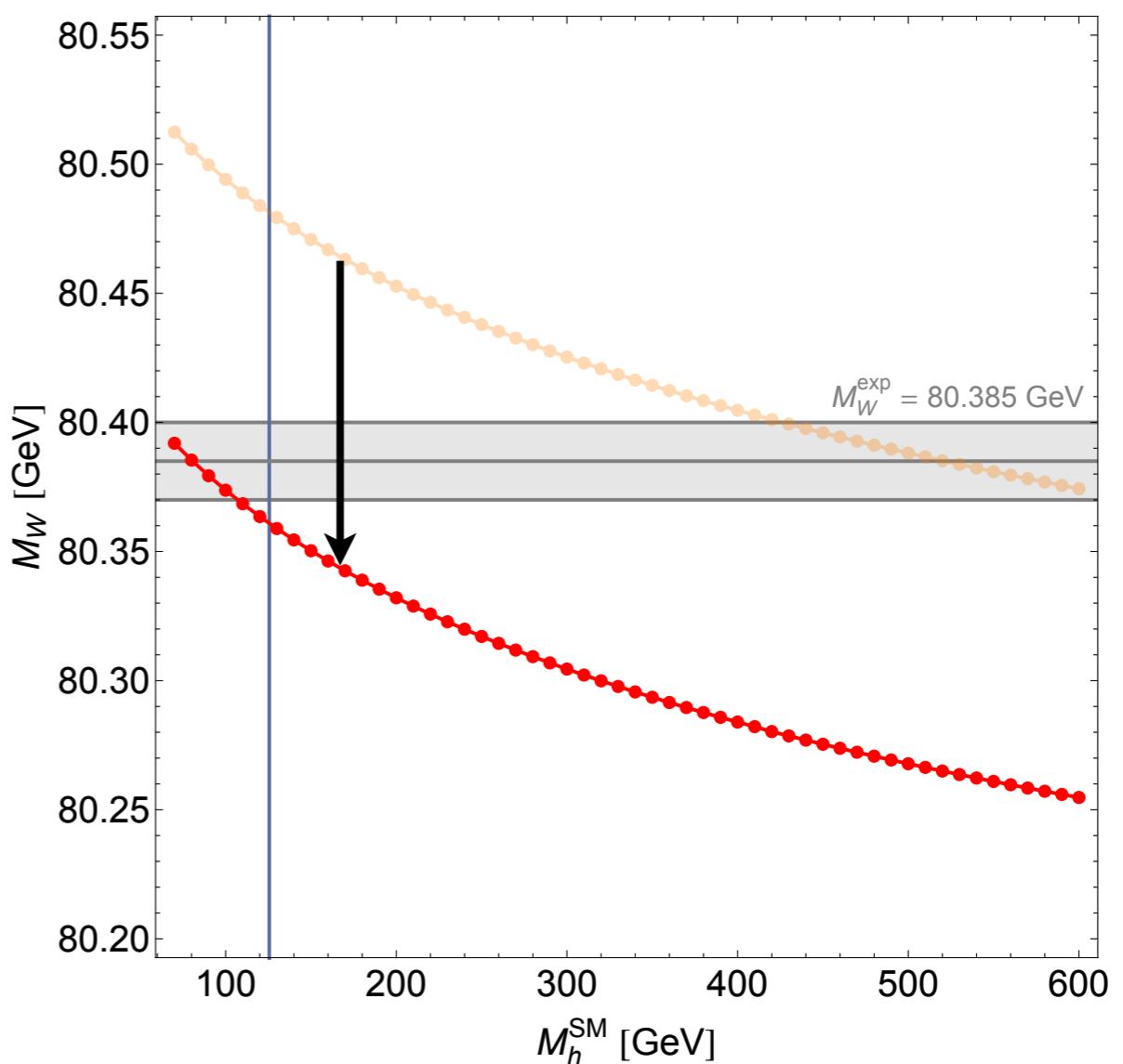
W boson mass in the SM

- Tree-level prediction differs from measurement by more than 30σ
- Including 1-loop contributions:

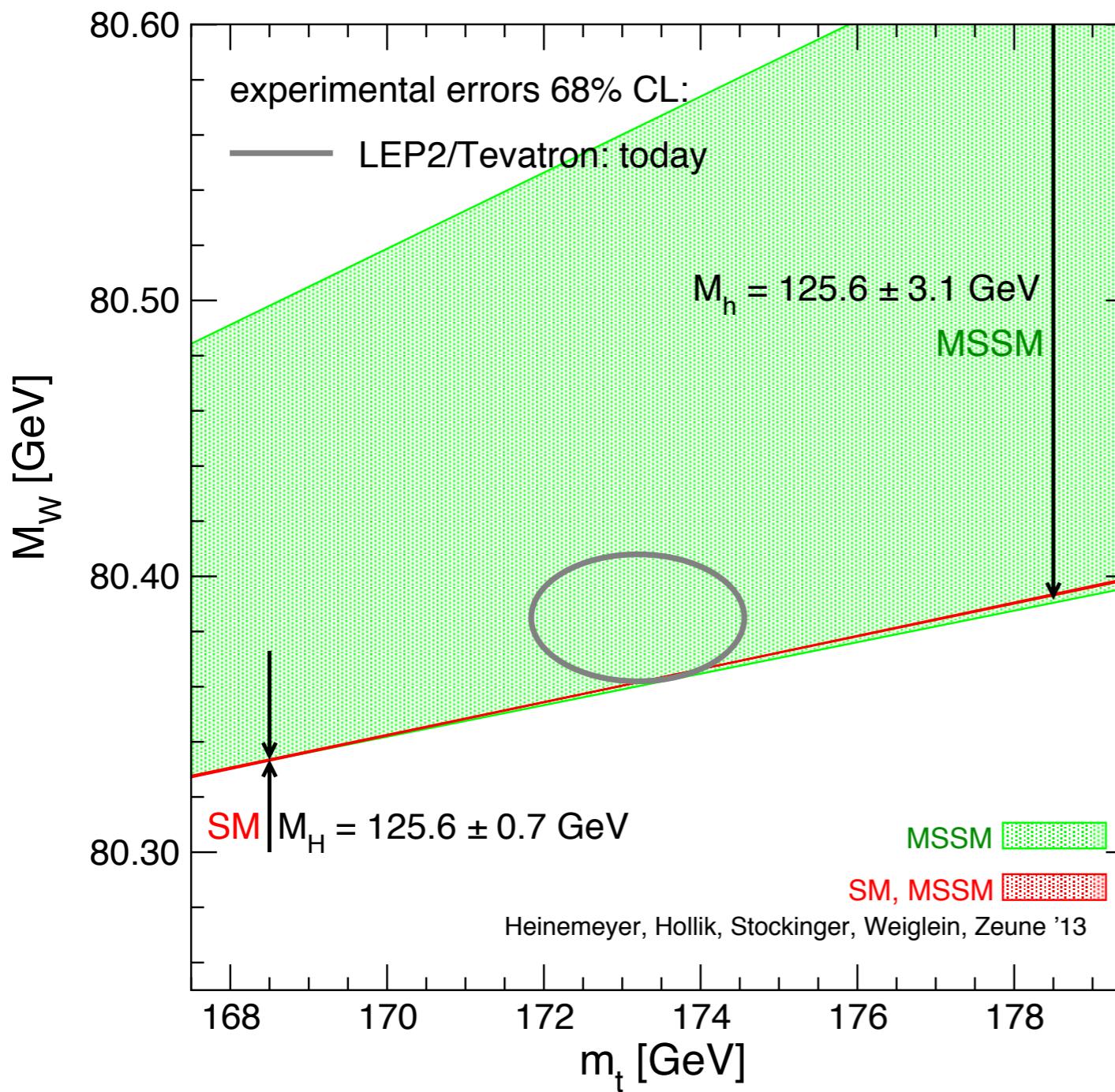


W boson mass in the SM

- Tree-level prediction differs from measurement by more than 30σ
- Including all higher order corrections:
- Corrections beyond 1-loop cause downward shift by more than 100 MeV
 - Crucial to include to get a reliable prediction also in SUSY models
- SM result
$$M_W^{\text{SM}}(m_t = 173.34 \text{ GeV}, M_H^{\text{SM}} = 125.6 \text{ GeV}) = 80.362 \text{ GeV}$$
- Differs from the measurement by 1.5σ

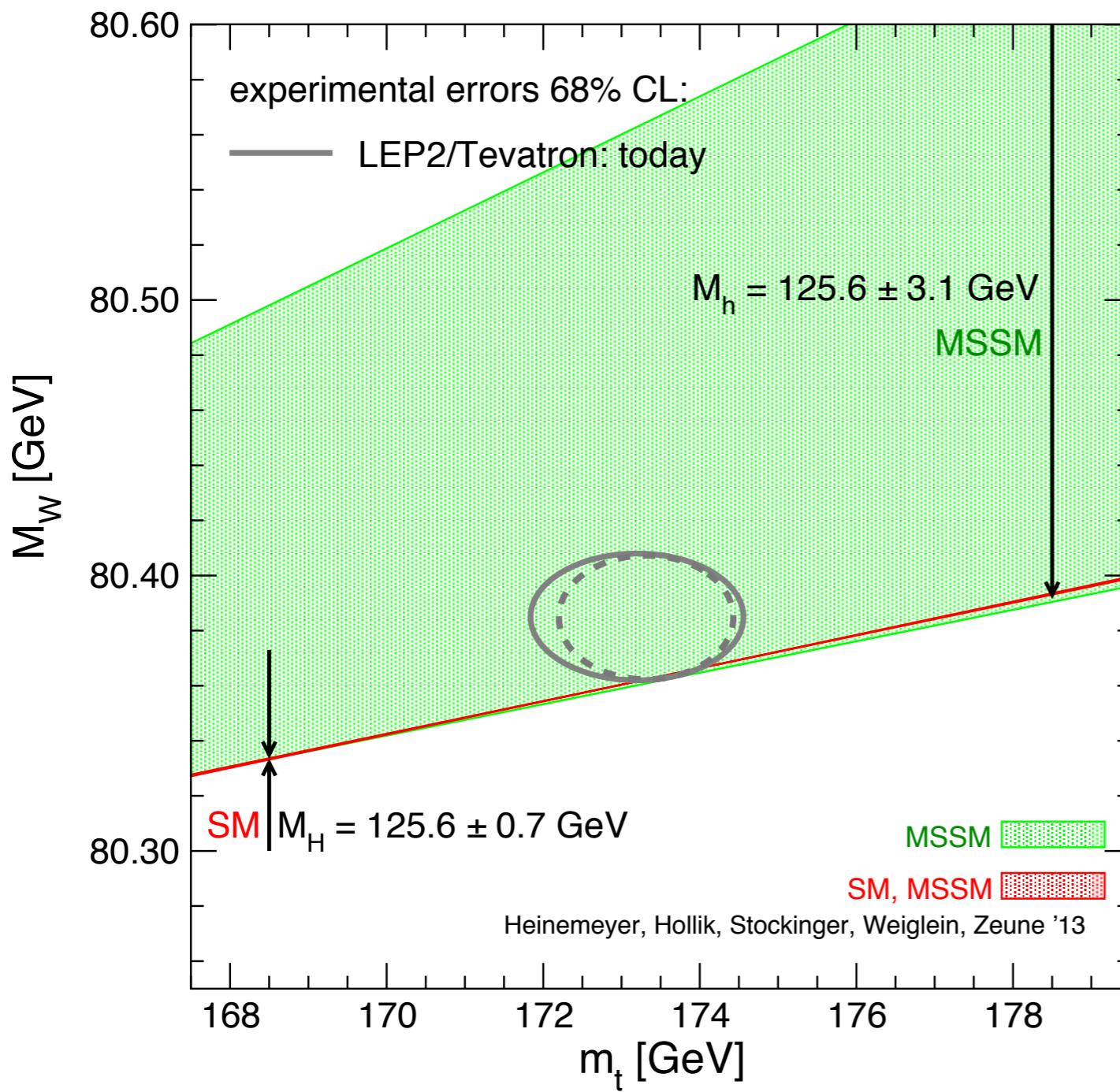


W boson mass in the MSSM



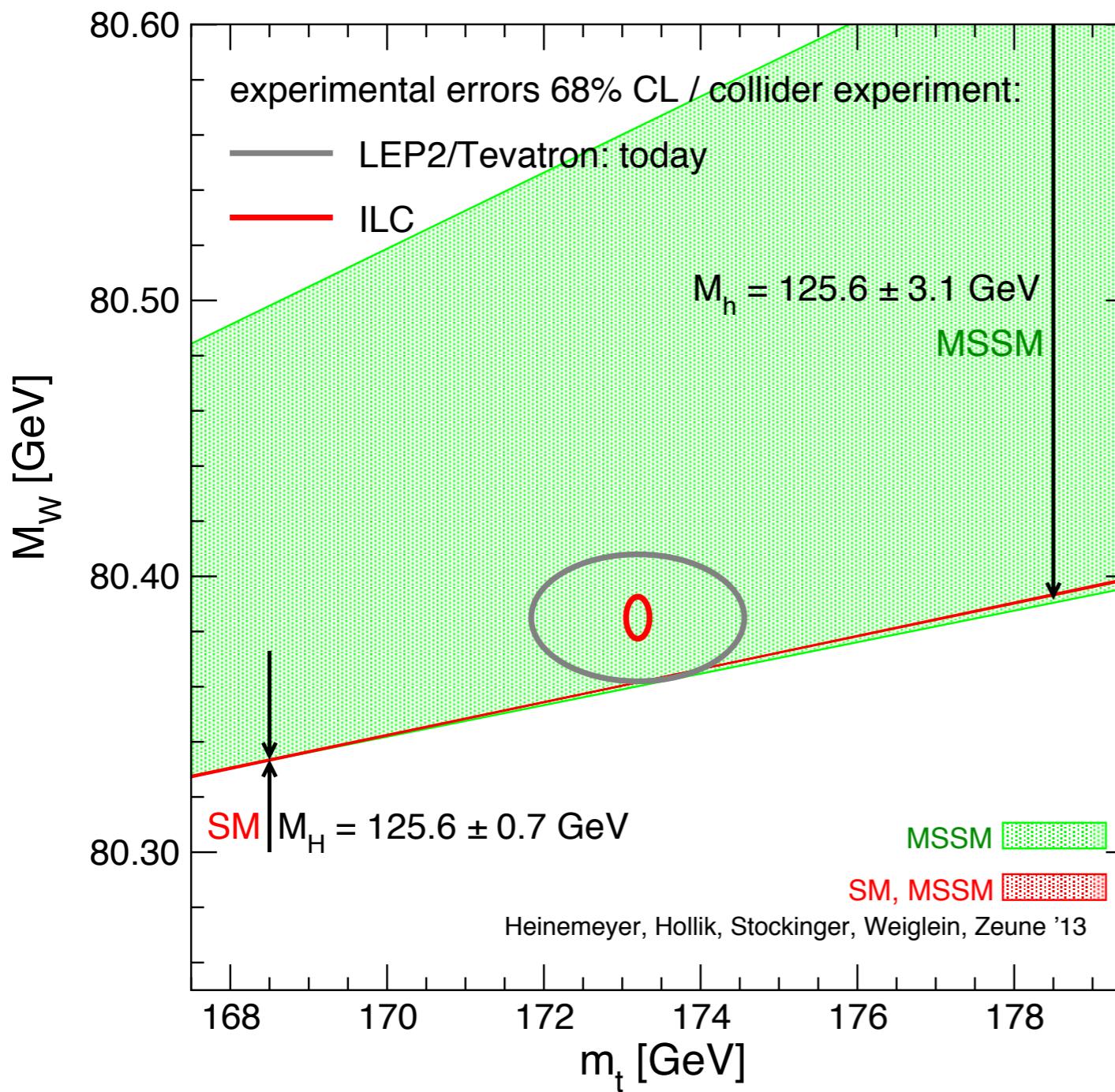
- Allowed MSSM region:
 - Discovered Higgs interpreted as lightest CP - even MSSM Higgs (3 GeV theory uncertainty added)
 - Applied constraints: Higgs searches, LEP SUSY limits, neutralino LSP
- Overlap region, SM and MSSM:
 - Observed signal interpreted as SM Higgs
- Non-zero SUSY contribution slightly favoured

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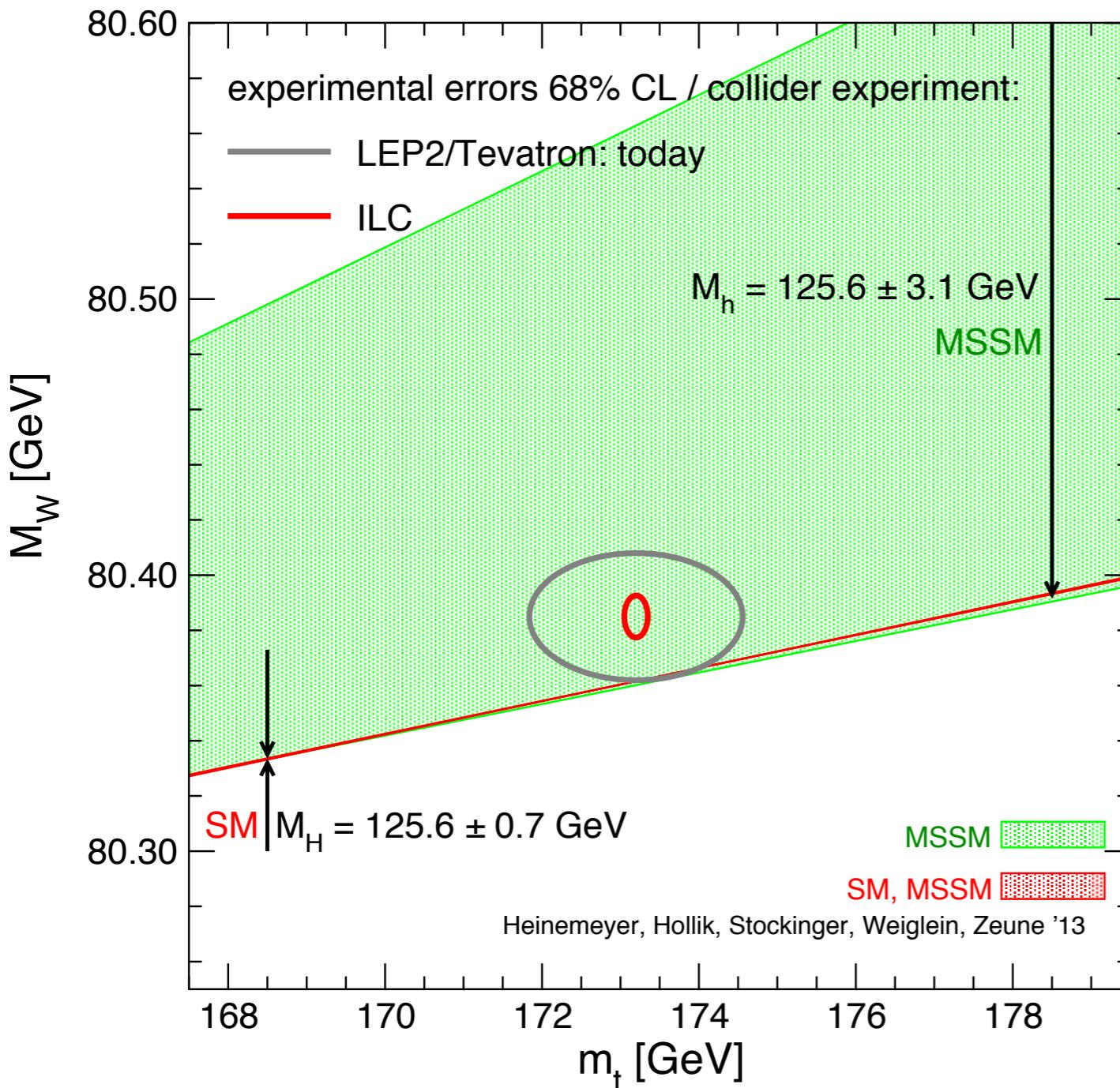
W boson mass in the MSSM



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- Overlap region, SM and MSSM:
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- With linear collider precision it might be possible to discriminate between SM and MSSM

SUSY contributions to M_W



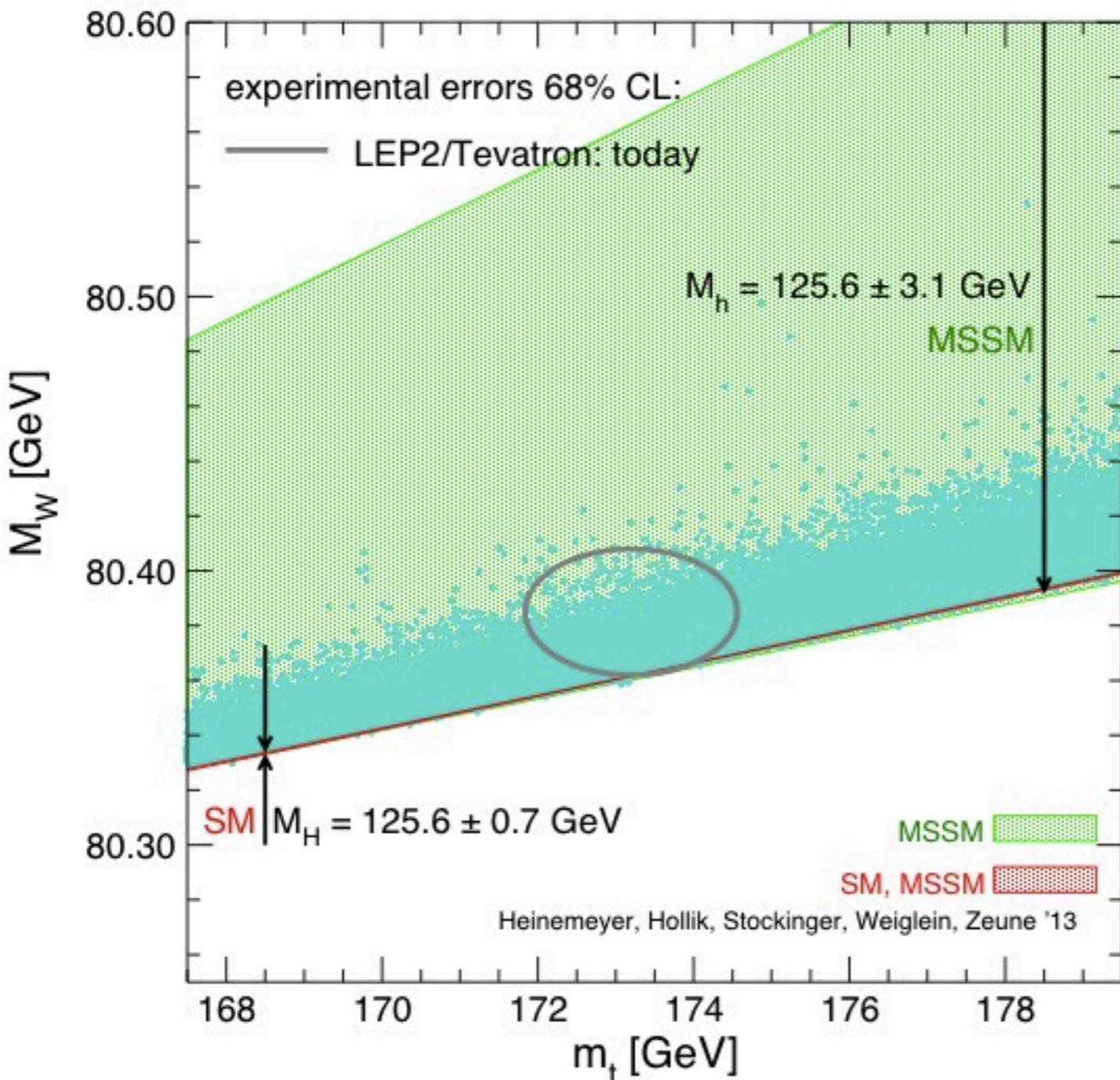
$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho + \Delta r_{\text{rem}}$$

- Largest SUSY contribution:
 \tilde{t}/\tilde{b} contributions to

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

Sensitive to mass splitting
between isospin partners

SUSY contributions to M_W



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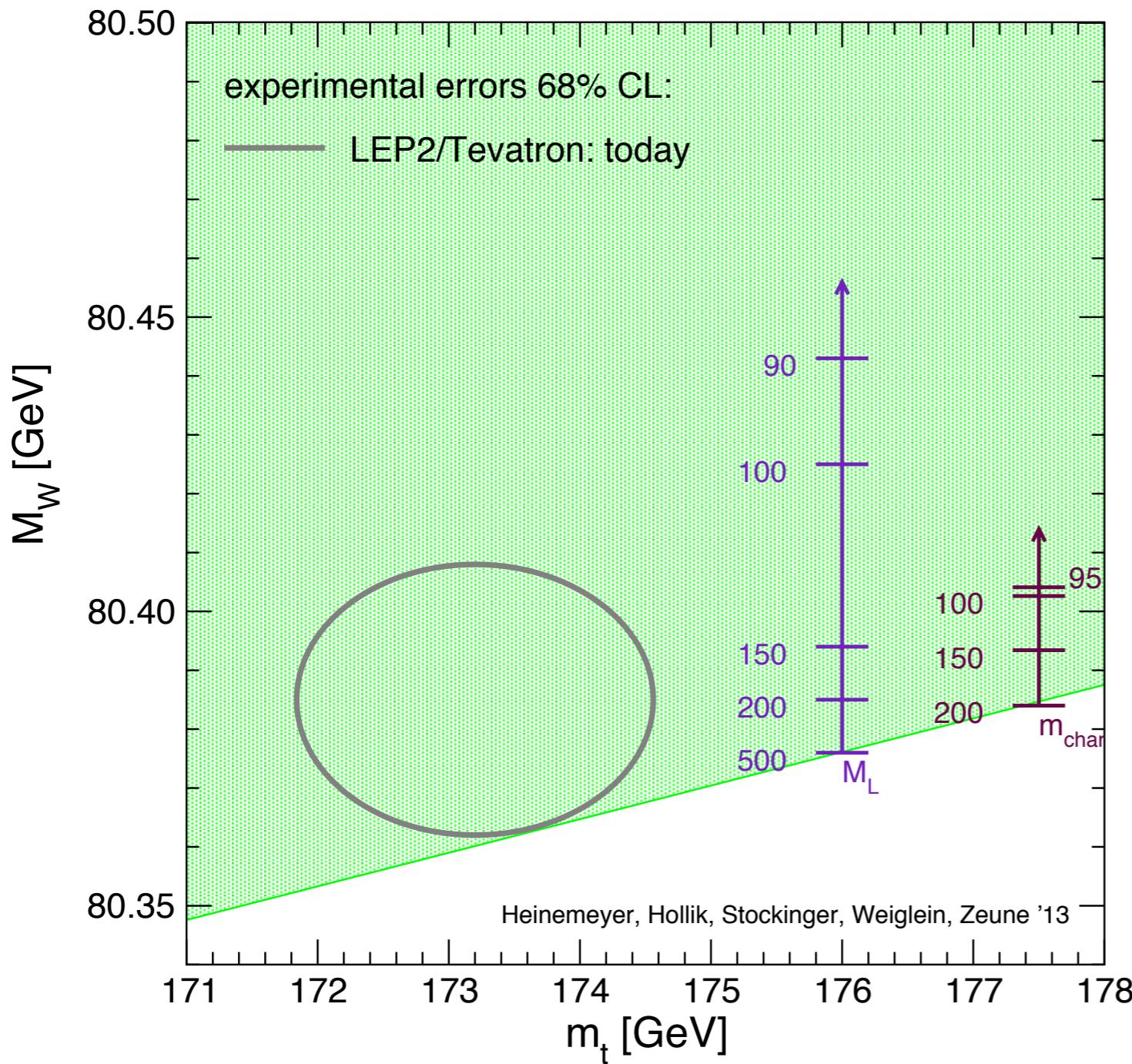
- Largest SUSY contribution:
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$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

Sensitive to mass splitting
between isospin partners

Stops and sbottoms heavier
than 500 GeV, squarks and
gluino heavier than 1200 GeV

SUSY contributions to M_W



- Sizable SUSY corrections possible from sleptons, charginos and neutralinos

Sleptons: up to 60 MeV

Charginos/neutralinos:
up to 20 MeV

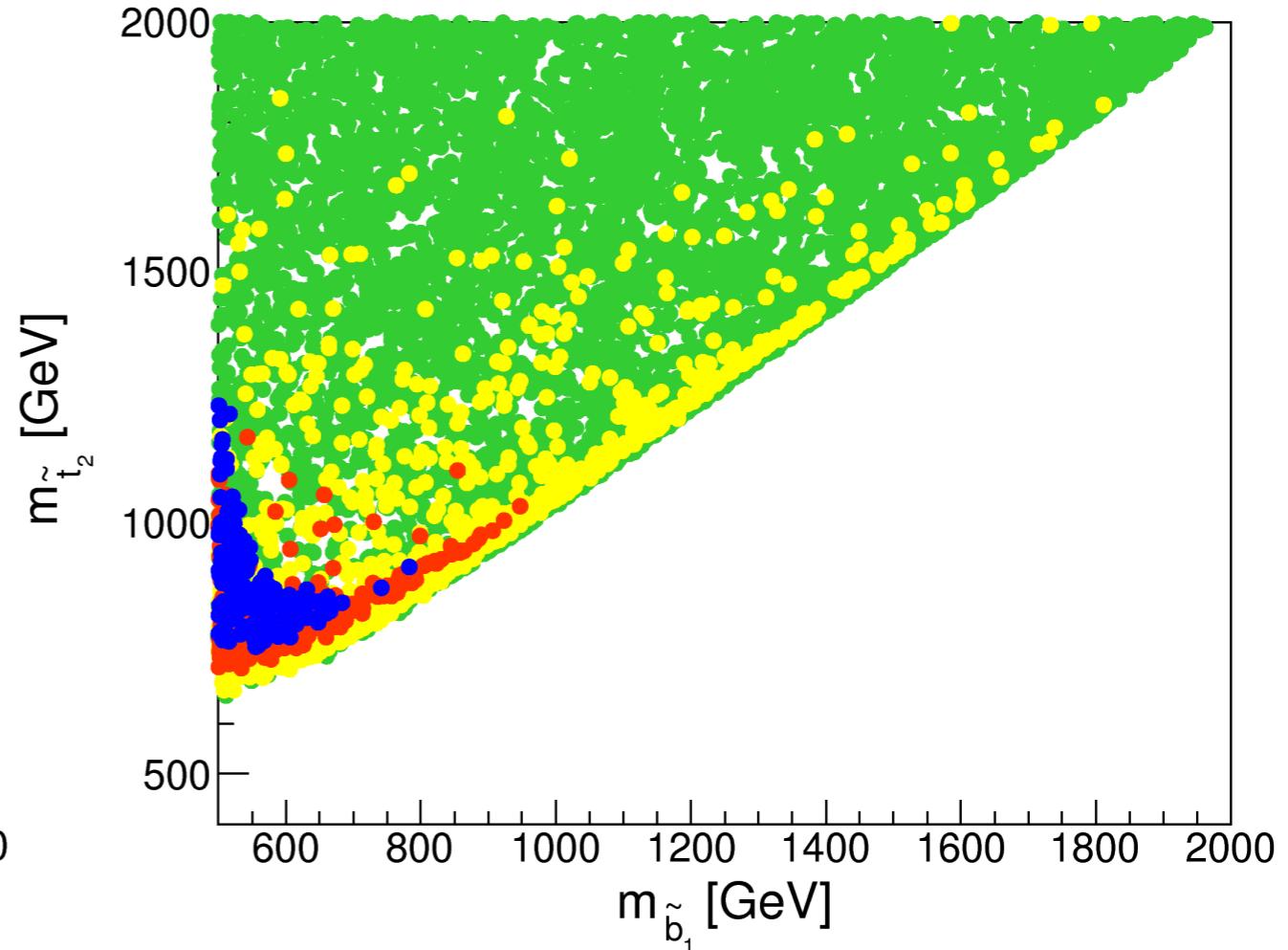
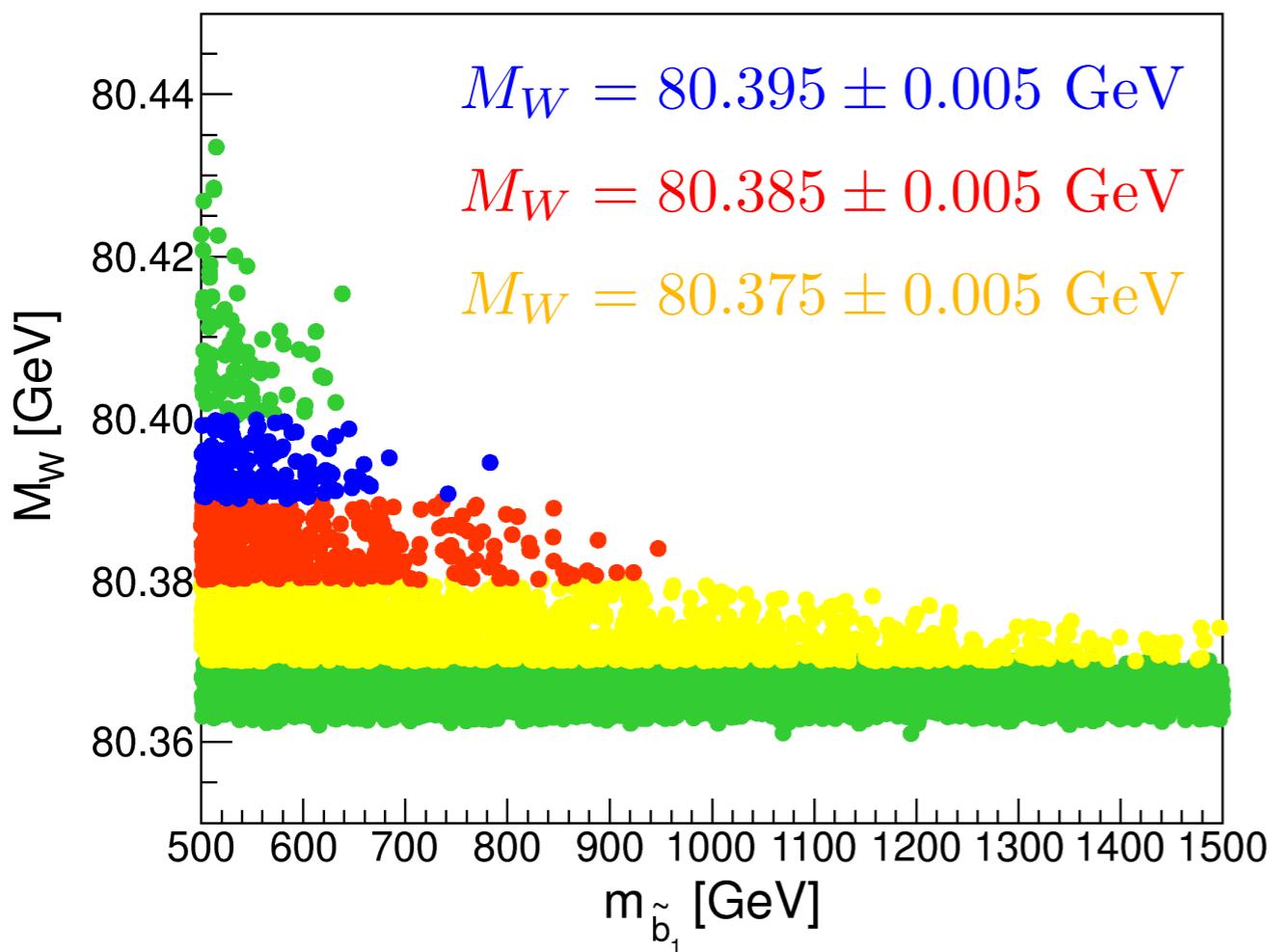
Possible future scenario

Assumptions:

- LHC discovers stop at 400 GeV
- No other SUSY particles discovered
 - Lower mass limits:
sleptons 300 GeV, charginos 300 GeV, other third generation squarks
500 GeV, remaining colored particles 1200 GeV
- W boson mass can be measured with 5 MeV precision

Possible future scenario

All points: Stop at 400 GeV, all other SUSY particles heavy, Higgs at 126 GeV

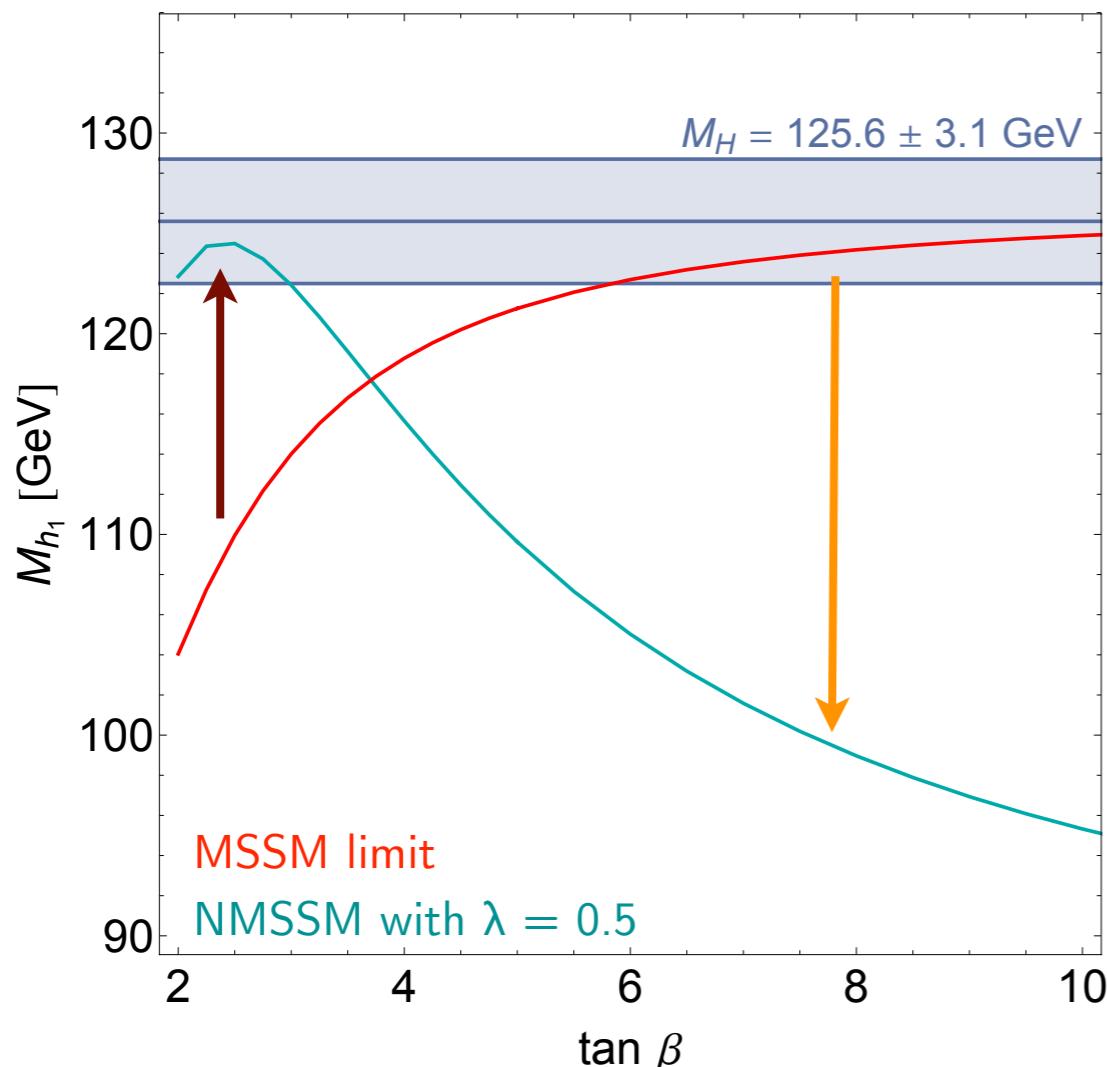


- If M_W stays at its **current value** or **goes up**:
 - Precise measurement of the W boson mass restricts light sbottom and heavy stop mass ranges to small regions
 - Indication where to search for additional SUSY particles

NMSSM Higgs sector contributions to M_W

- Additional Higgs singlet (and singlino)
 - Modified Higgs and neutralino sector
- The lightest CP-even Higgs is SM-like, the singlet-like Higgs is heavier, then:

$$M_{h_1}^2 = M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \Delta_{\text{mix}} + \Delta_{\text{rad}}$$

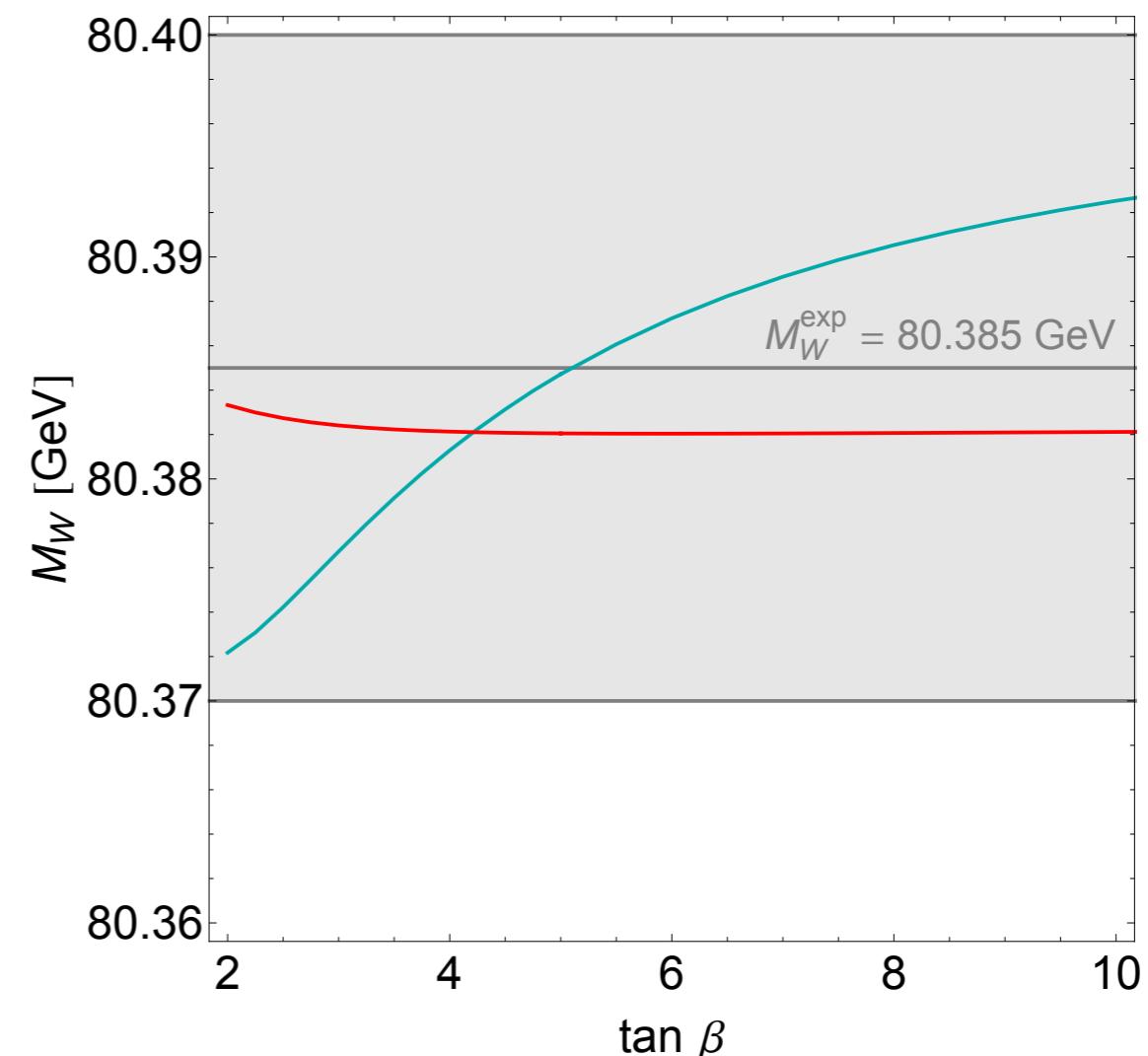
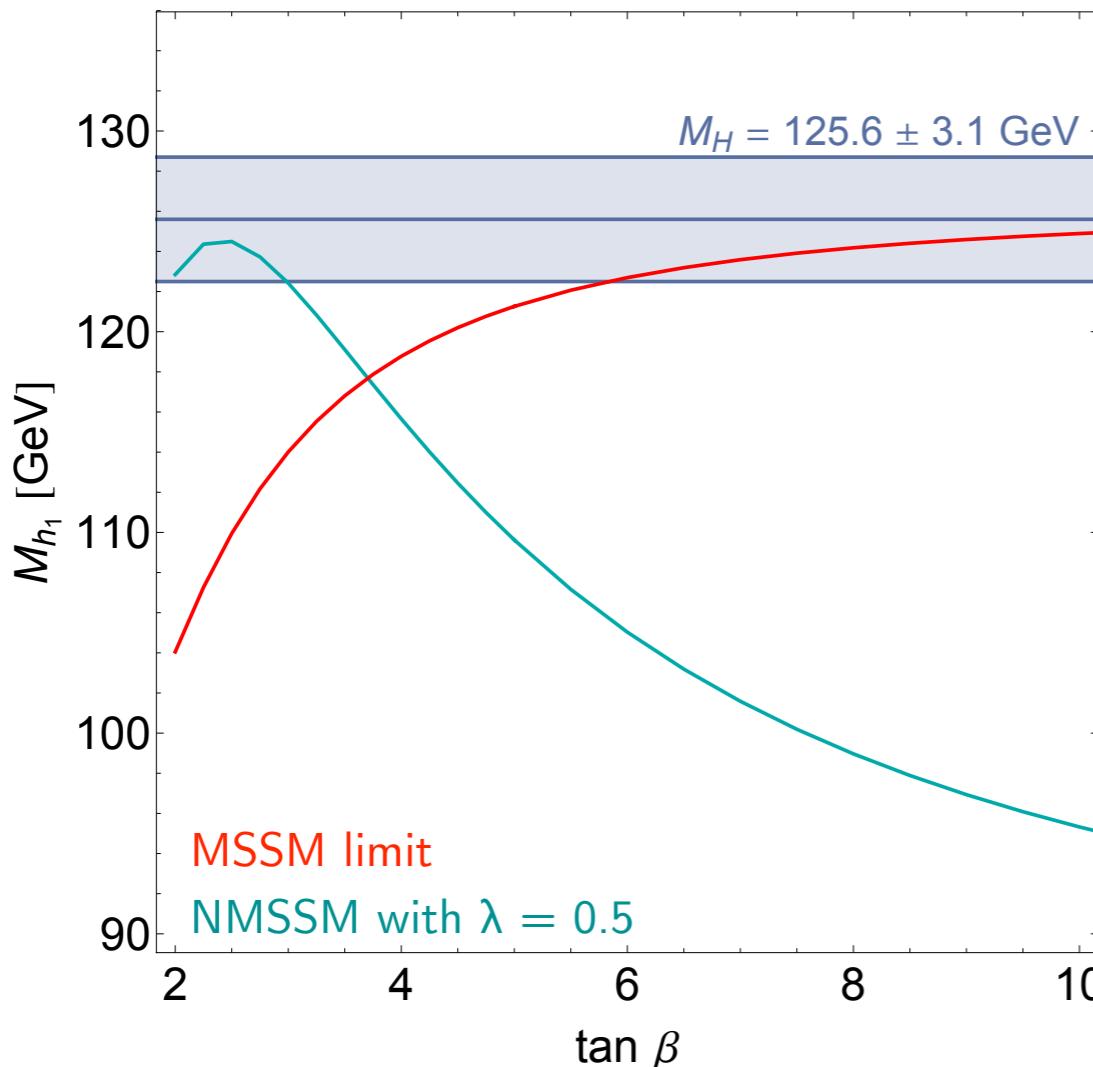


Can lead to significant upwards shift for small $\tan \beta$ and large $\lambda \rightarrow$ tree-level mass above Z boson mass possible

Describes mixing with singlet

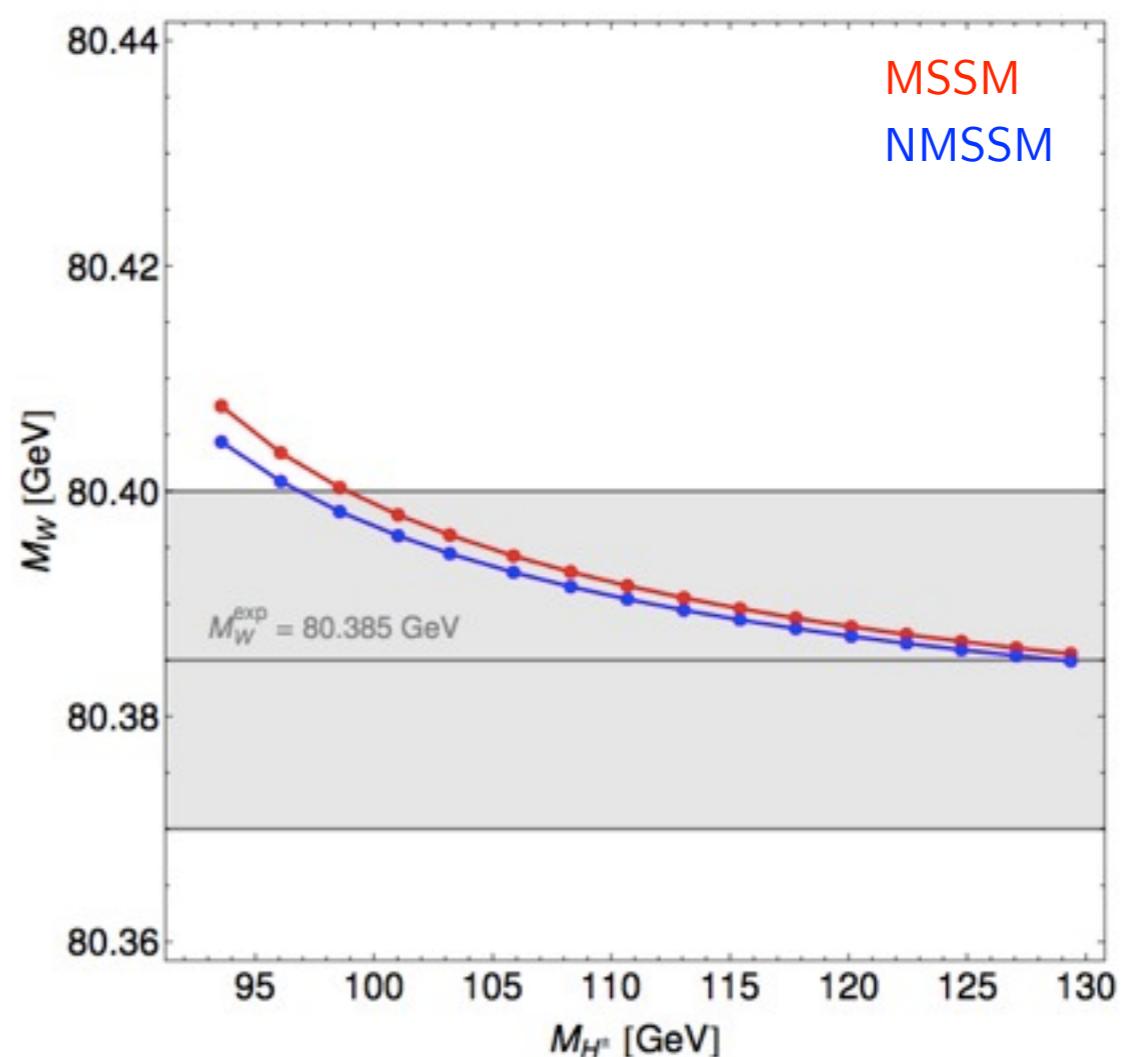
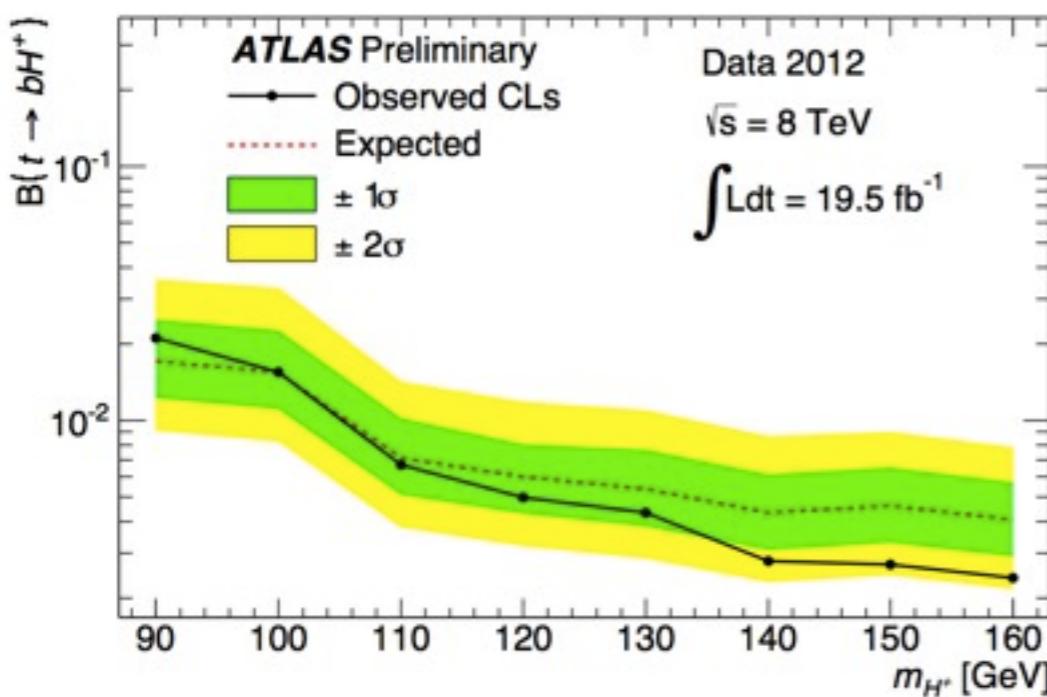
NMSSM Higgs sector contributions to M_W

- NMSSM Higgs sector contribution:
 - Predominantly SM type contribution with $M_{h_{\text{SM}}} = M_{h_1}$ (in the absence of light charged Higgs)
 - Slightly reduced compared to SM due to singlet component



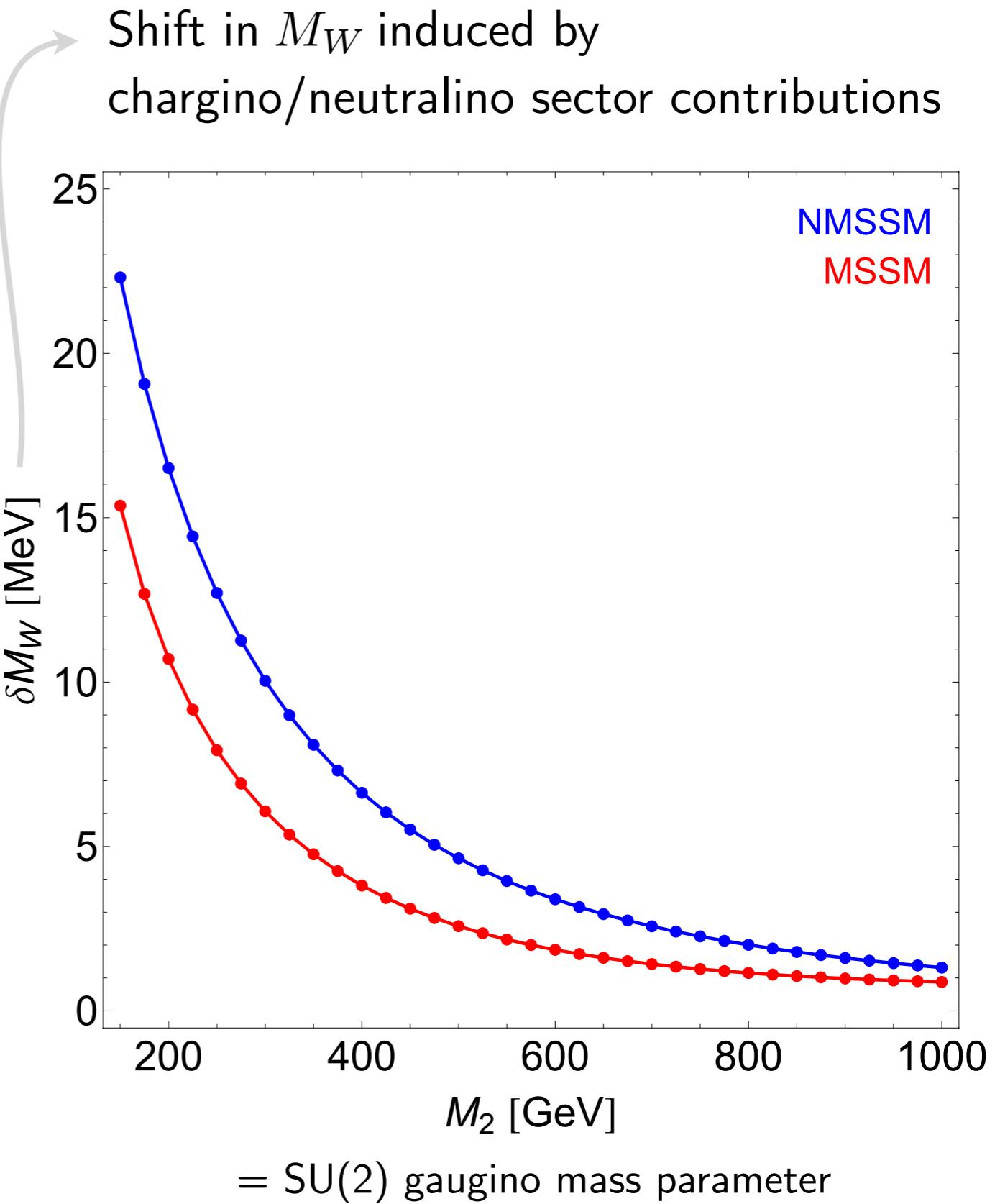
Light charged Higgs contributions

- Light charged Higgs bosons (both in the MSSM and the NMSSM) can give significant contributions to the W boson mass
- Very constrained by ATLAS bounds on light charged Higgs



NMSSM neutralino sector contributions to M_W

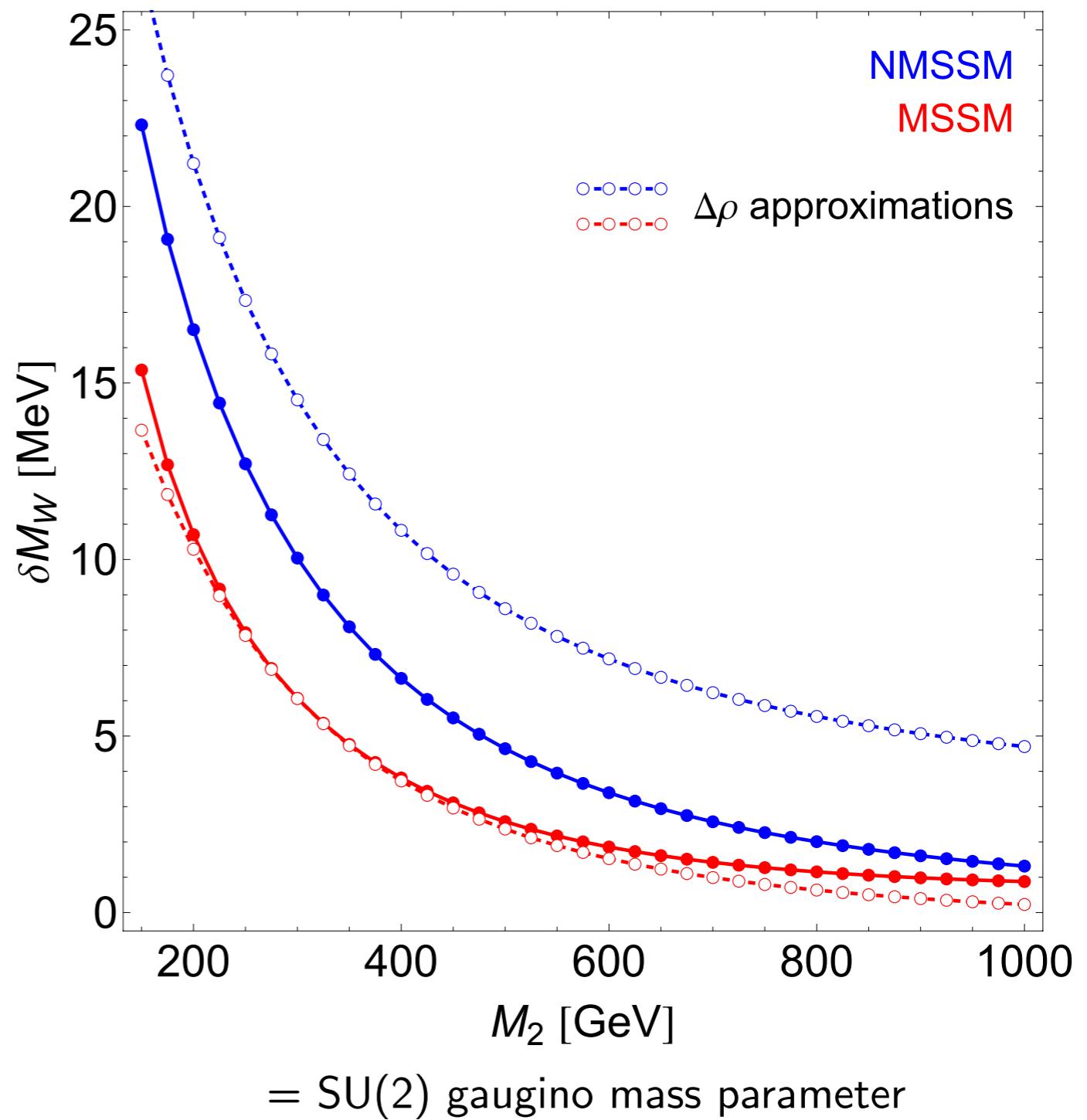
- Parameters chosen such that soft mass parameters agree and $\mu = \mu_{\text{eff}} = 200$ GeV
- Sizable NMSSM W boson mass contributions from neutralino sector



NMSSM neutralino sector contributions to M_W

- Parameters chosen such that soft mass parameters agree and $\mu = \mu_{\text{eff}} = 200$ GeV
- Sizable NMSSM W boson mass contributions from neutralino sector
- Sizable difference between the full chargino/neutralino contributions and the approximation by the $\Delta\rho$ term

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho + \Delta r_{\text{rem}}$$



Conclusions

- Precise prediction for the W boson mass in the SM, the complex MSSM and the NMSSM
- SUSY contributions can be large
 - NMSSM: Sizeable difference to MSSM from modified neutralino sector
- Measurement favours non-zero SUSY contribution
 - Linear collider measurement would provide sensitivity for discriminating between SM and SUSY models
- Future scenarios
 - I.: Increased limits on stop and sbottoms would have large impact of W boson mass prediction
 - II.: LHC discovers light stop but nothing else: Precise W boson mass measurement could give indications where to search for additional particles

Back-up material

W boson mass in the MSSM - Scan ranges

Parameter	Minimum	Maximum
μ	-2000	2000
$M_{\tilde{E}_{1,2,3}} = M_{\tilde{L}_{1,2,3}}$	100	2000
$M_{\tilde{Q}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}}$	500	2000
$M_{\tilde{Q}_3}$	100	2000
$M_{\tilde{U}_3}$	100	2000
$M_{\tilde{D}_3}$	100	2000
$A_e = A_\mu = A_\tau$	$-3 M_{\tilde{E}}$	$3 M_{\tilde{E}}$
$A_u = A_d = A_c = A_s$	$-3 M_{\tilde{Q}_{12}}$	$3 M_{\tilde{Q}_{12}}$
A_b	$-3 \max(M_{\tilde{Q}_3}, M_{\tilde{D}_3})$	$3 \max(M_{\tilde{Q}_3}, M_{\tilde{D}_3})$
A_t	$-3 \max(M_{\tilde{Q}_3}, M_{\tilde{U}_3})$	$3 \max(M_{\tilde{Q}_3}, M_{\tilde{U}_3})$
$\tan \beta$	1	60
M_3	500	2000
M_A	90	1000
$M_2 (\sim 2 M_1)$	100	1000

Applied constraints:

- Neutralino LSP
- LEP limits on SUSY particle masses
- HiggsBounds (version 4.0.0)
- Limit on maximal splitting:

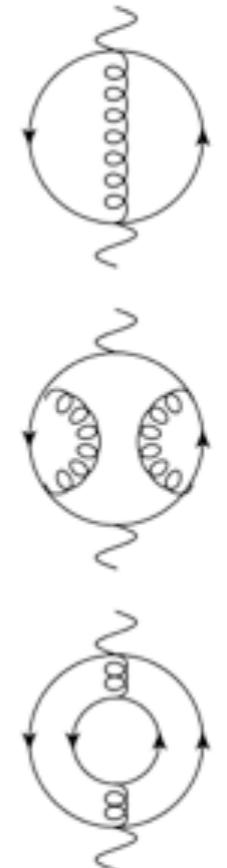
$$m_{\tilde{t}_2}/m_{\tilde{t}_1} < 2.5 \text{ and}$$

$$m_{\tilde{b}_2}/m_{\tilde{b}_1} < 2.5$$

SM model contributions to Δr

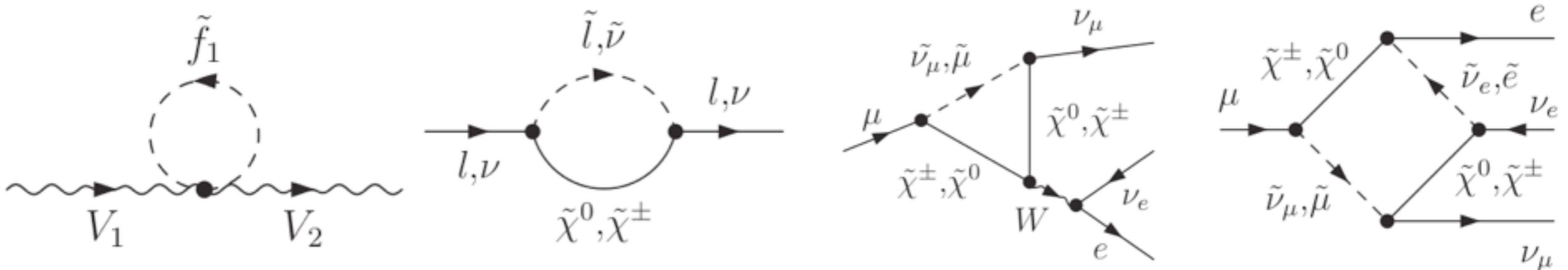
$$\begin{aligned}\Delta r^{\text{SM}} = & \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r_{\text{ferm}}^{(\alpha^2)} + \Delta r_{\text{bos}}^{(\alpha^2)} \\ & + \Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)} + \Delta r^{(G_\mu m_t^2 \alpha_s^3)}\end{aligned}$$

- $\Delta r^{(\alpha)}$: 1-loop contribution
- $\Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$: 2- and 3-loop QCD corrections
Chetyrkin, Kuhn, Steinhauser '95, Djouadi, Verzegnassi '88, ...
- $\Delta r_{\text{ferm}}^{(\alpha^2)} + \Delta r_{\text{bos}}^{(\alpha^2)}$: fermionic and bosonic electroweak 2-loop corrections (fitting formula)
Awramik, Czakon, Freitas '06,
Awramik, Czakon, Freitas, Weiglein '03
- $\Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}$: 3-loop top quark contribution
Faisst, Kuhn, Seidensticker, Veretin '03
- $\Delta r^{(G_\mu m_t^2 \alpha_s^3)}$: 4-loop QCD correction
Boughezal '06



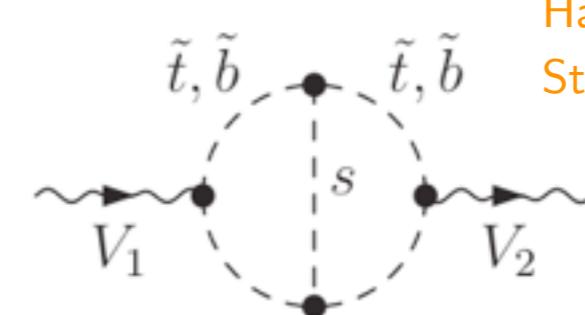
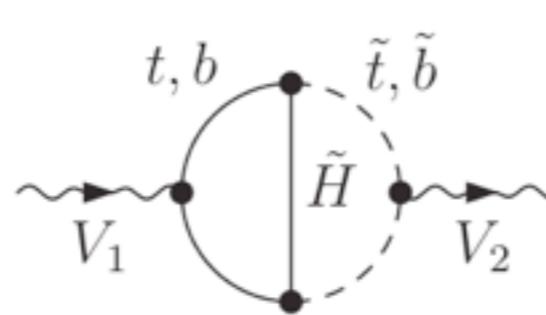
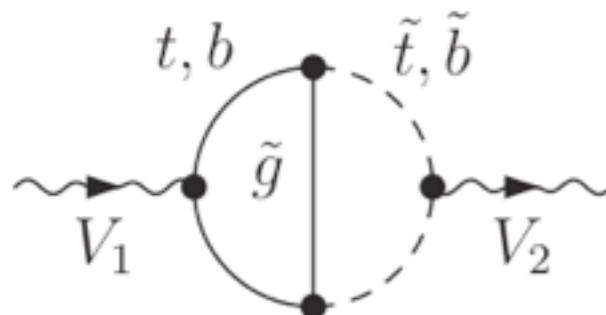
SUSY contributions to Δr

- 1-loop contributions from MSSM Higgs bosons, sfermions, charginos and neutralinos



- Supersymmetric 2-loop contributions

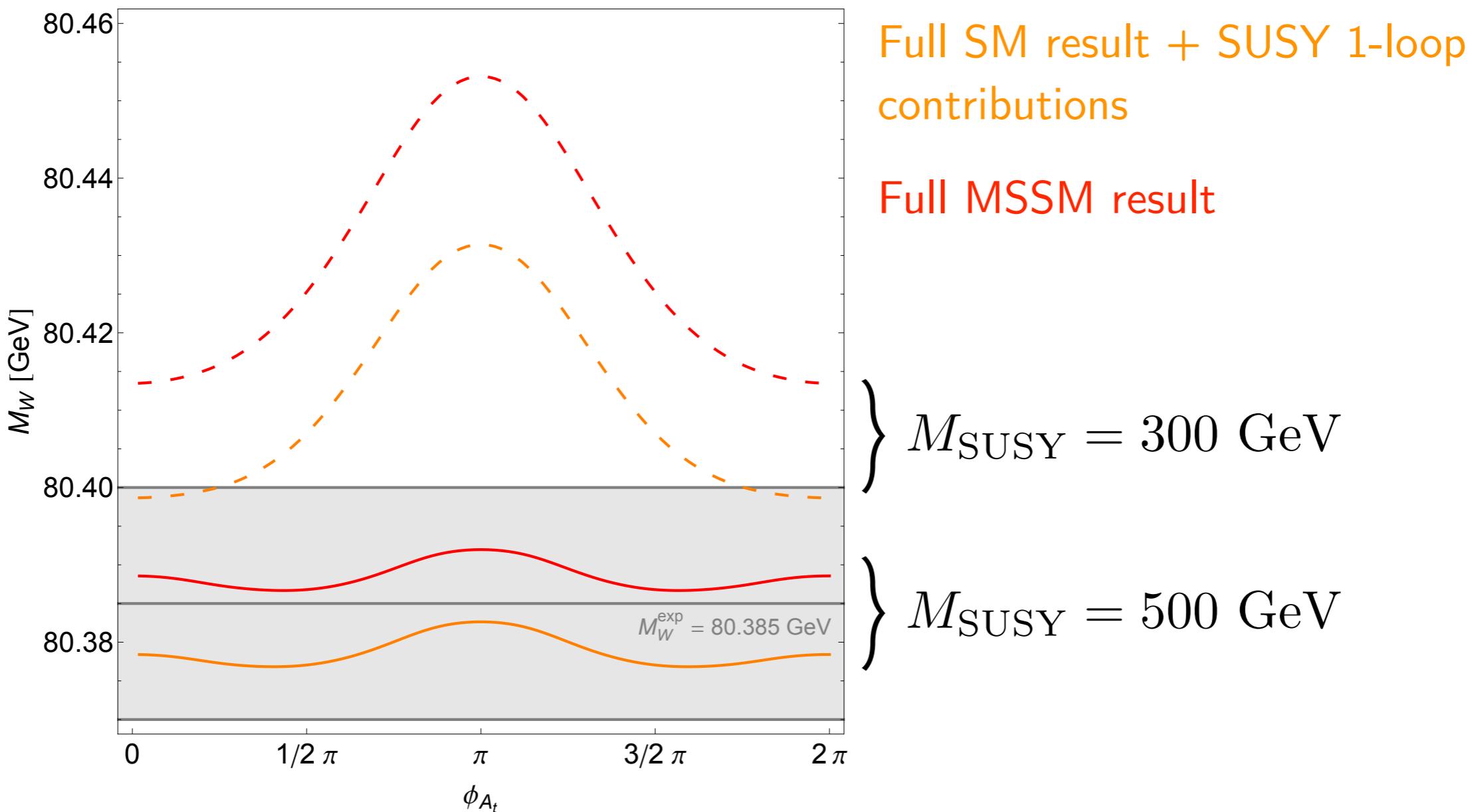
- SUSY QCD corrections: (S)quark loops with gluon and gluino exchange
Djouadi et. al '98
- Yukawa contributions: (S)quark loops with Higgs and Higgsino exchange



Haestier, Heinemeyer,
Stoeckinger, Weiglein '05

- Leading reducible 2-loop corrections Consoli, Hollik, Jegenlehner '89

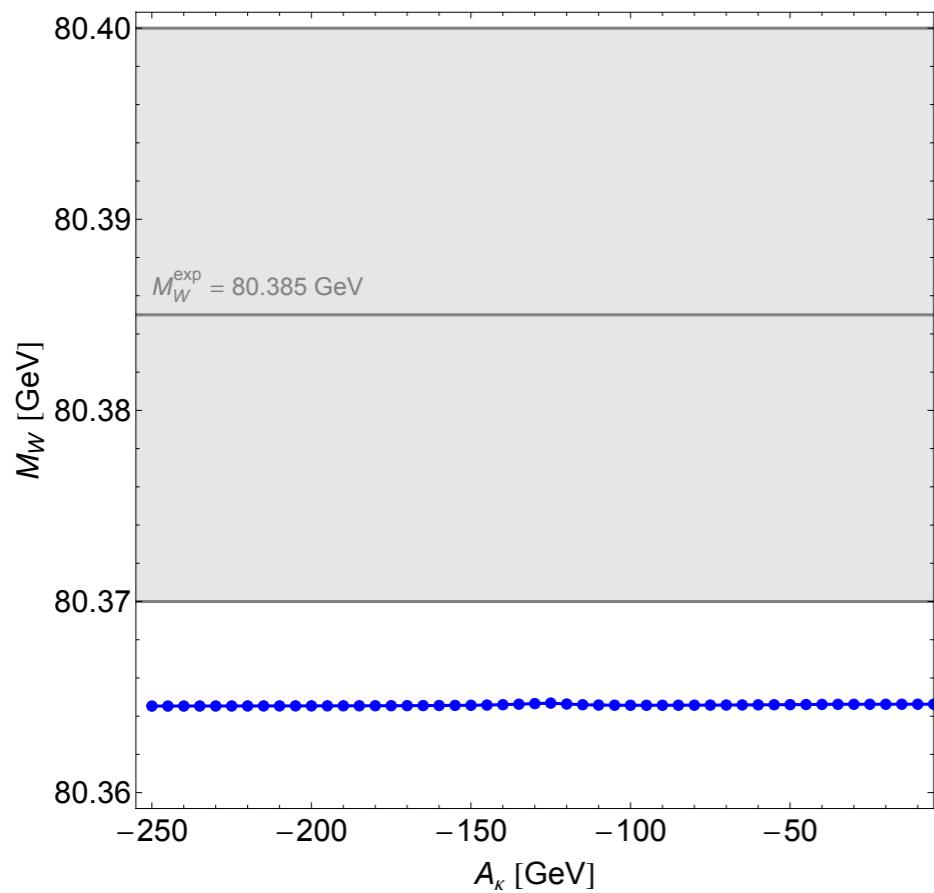
Dependence on complex phases



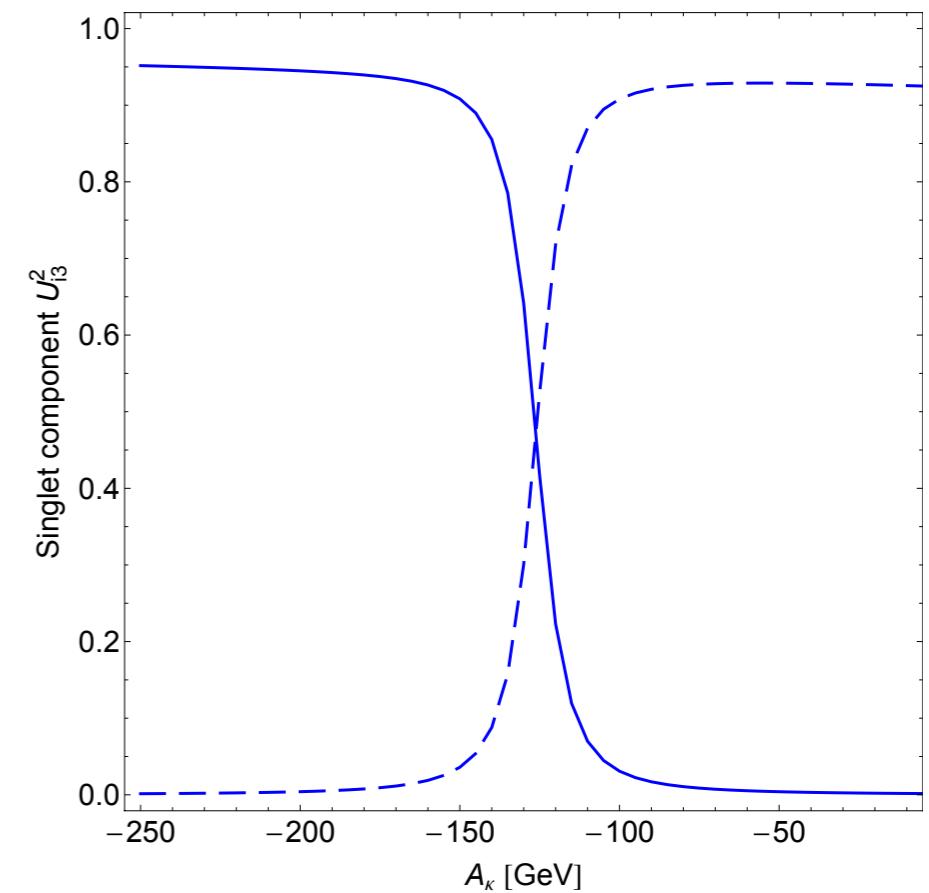
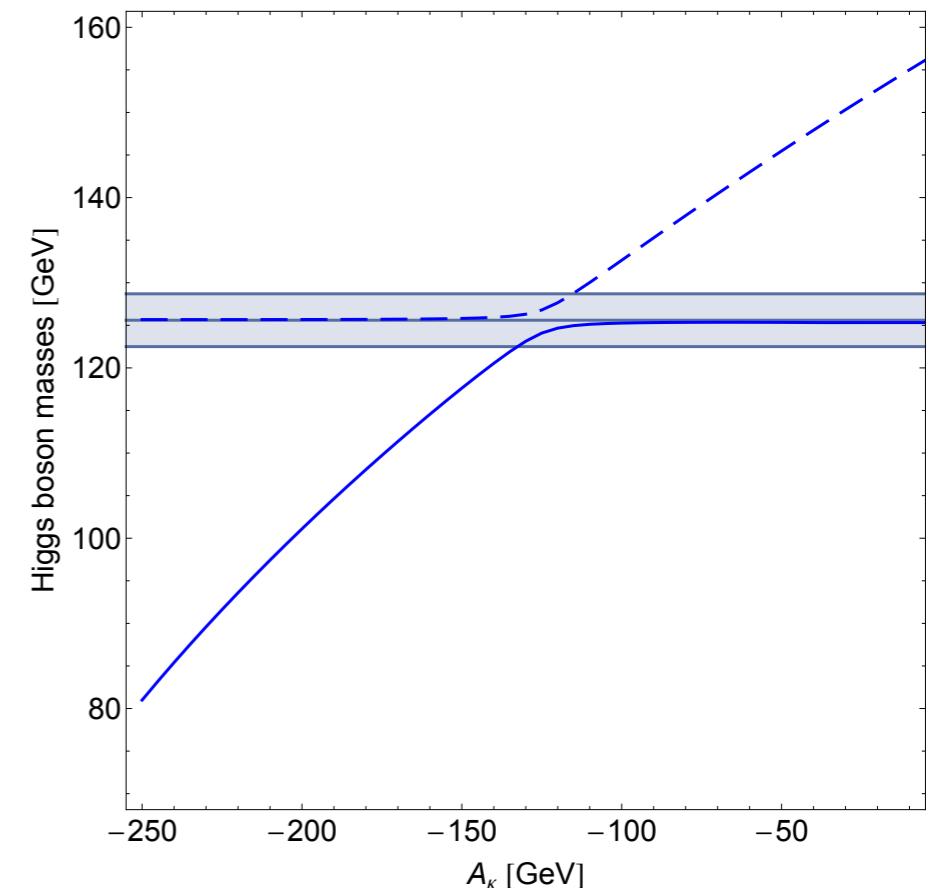
- Complex phase in the stop sector has sizeable effect on the sfermion 1-loop contributions to M_W

Singlet doublet mixing

- NMSSM scenario:
 - Lightest two states ‘share’ the singlet component
 - For : $-150 \text{ GeV} \lesssim A_\kappa \lesssim -100 \text{ GeV}$ strong singlet doublet mixing between the two Higgs states (which are close in mass)



Negligible
effect on
W boson
mass
prediction



Effective weak mixing angle: status

- Current most precise single measurements:

$$\begin{aligned}\sin^2 \theta_{\text{eff}}^l &= (1 - M_W^2/M_Z^2)(1 + \Delta\kappa) \\ &= 1/4 (1 + \text{Re } g_V/g_A)\end{aligned}$$

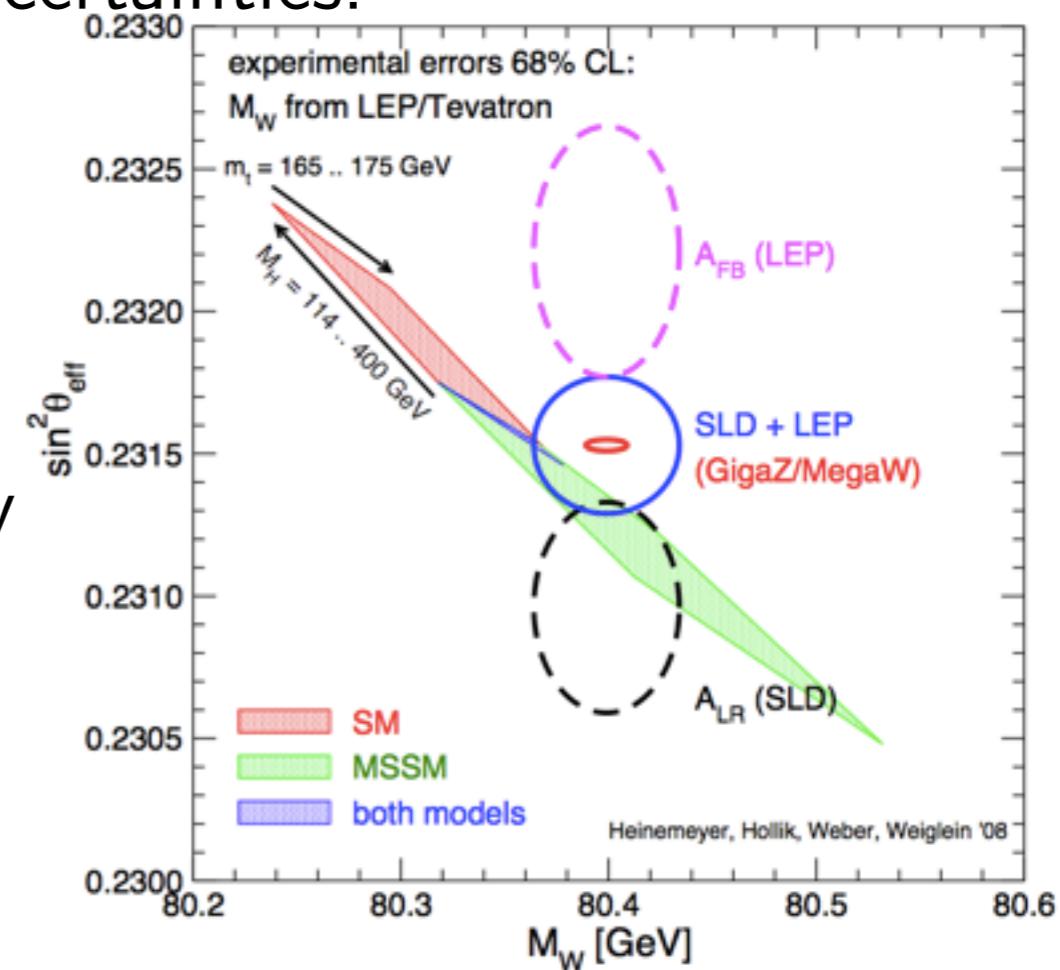
$A_{\text{FB}}^b(\text{LEP}) : \sin^2 \theta_{\text{eff}}^{\ell \text{ exp, LEP}} = 0.23221 \pm 0.00029$ ← via forward-backward
 $A_{\text{LR}}^e(\text{SLD}) : \sin^2 \theta_{\text{eff}}^{\ell \text{ exp, SLD}} = 0.23098 \pm 0.00026$ ← asymmetry of b quarks
 difference → via left-right asymmetry of electrons

- Precise theoretical calculation, current uncertainties:

$$\Delta \sin^2 \theta_{\text{eff}}^l \text{ MSSM, theo} = (5 - 7) \times 10^{-5}$$

$$\Delta \sin^2 \theta_{\text{eff}}^l \text{ SM, theo} = 4.5 \times 10^{-5}$$

- ILC (with Giga-Z option) can significantly improve this measurement



W boson mass measurement - hadron colliders

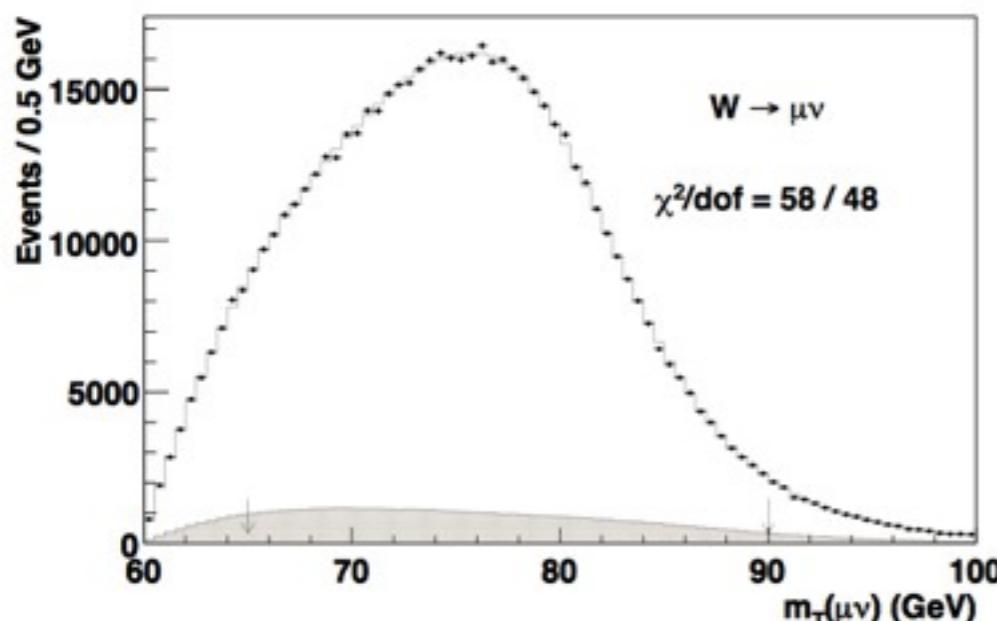
- Process: $q\bar{q} \rightarrow W \rightarrow l\nu$ ($l = \mu, e$)
- Most important observables: p_T, M_T
- Template likelihood fit:

$$\mathcal{L} = \prod_{i=1}^N \frac{e^{-m_i} n_i^{m_i}}{n_i!}.$$

Transverse mass:

$$M_T = \sqrt{2 p_T^l \not{p}_T (1 - \cos \Delta\phi)}$$

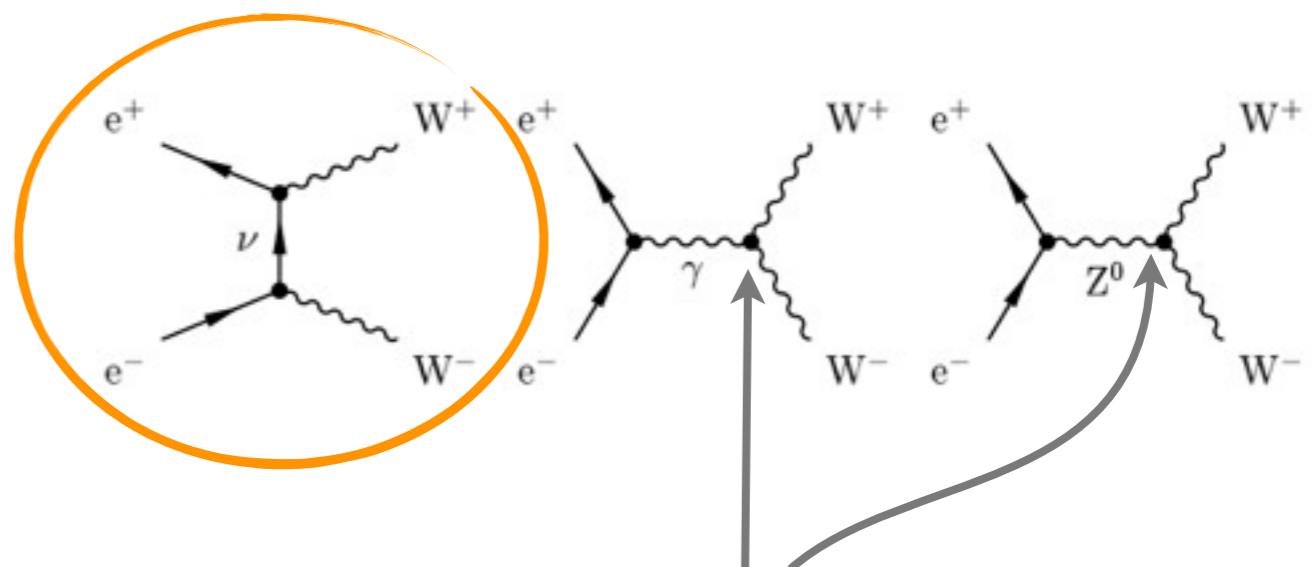
CDF measurement:



Source	Uncertainty (MeV)
Lepton energy scale and resolution	7
Recoil energy scale and resolution	6
Lepton removal	2
Backgrounds	3
$p_T(W)$ model	5
Parton distributions	10
QED radiation	4
W -boson statistics	12
Total	19

W boson mass measurement at a linear collider

- Three uncorrelated methods to measure the W boson mass at the ILC
 - Run at WW threshold
 - Kinematic reconstruction using semi-leptonic channels
 - Direct reconstruction of the hadronic mass
- Each of these methods has experimental precision of ~ 5 MeV
- T-channel used for clean measurement



Possible new physics contributions may modify the triple gauge vertex