Phenomenology with SUSY models with extended Higgs sector

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DESY

Outline

- Introduction: minimal extensions of MSSM.
- Distinguishing NMSSM and MSSM through the neutralino/chargino sector.
- Footprints of gauge extended MSSM: Higgs couplings.
- Conclusions and outlook.

Minimal extensions of the MSSM

The 125.5 GeV Higgs looks like the SM Higgs: is it true?

Which classes of BSM models would be consistent with it?

Motivated by Naturalness, several extensions of MSSM have been introduced:

F-term extensions

- MSSM + singlet: NMSSM
- MSSM + $SU(2)_L$-doublets
- MSSM + $SU(2)_L$-triplets
- ...

D-term extensions

- Quiver models:
  - Vector Higgs case
  - Chiral Higgs case
- ...

NMSSM vs MSSM: neutralino/chargino sector

G. Moortgat-Pick, SP, K. Rolbiecki: 1404.1053, 1405.xxxx

μ-problem and NMSSM

MSSM

1 spartner $\forall$ SM particle, 2 Higgs Doublets.

$$W_{h,\,\mathrm{MSSM}} = \mu\,\hat{H}_u \cdot \hat{H}_d$$

$\longrightarrow$ "μ-problem": why μ should be at the SUSY-breaking scale?

NMSSM

MSSM $+$ gauge singlet superfield $\hat{S} = (S, \tilde{S})$.

$$W_{h,\,(\mathbb{Z}_3\text{-})\mathrm{NMSSM}} = \lambda\,\hat{S}\,\hat{H}_u \cdot \hat{H}_d + \tfrac{\kappa}{3}\hat{S}^3$$

$$\longrightarrow \mu_{\mathrm{eff}} = \lambda\langle S\rangle = \lambda\,x.$$

How to distinguish between NMSSM and MSSM scenarios?

MSSM vs NMSSM?

MSSM	($\mathbb{Z}_3$-)NMSSM
$h,\ H,\ A,\ H^{\pm}$: $\tan\beta, m_A$	$S_{1,2,3},\ P_{1,2},\ H^{\pm}$: $\tan\beta,\ \lambda,\ x,\ \kappa,\ A_\lambda,\ A_\kappa$
$\tilde{\chi}_1^{\pm},\ \tilde{\chi}_2^{\pm}$: $M_2,\ \mu,\ \tan\beta$	$\tilde{\chi}_1^{\pm},\ \tilde{\chi}_2^{\pm}$: $M_2,\ \lambda\cdot x,\ \tan\beta$
$\tilde{\chi}^0_{1,2,3,4}$: $M_1,\ M_2,\ \mu,\ \tan\beta$	$\tilde{\chi}^0_{1,2,3,4,5}$: $M_1,\ M_2,\ \lambda,\ x,\ \kappa, \tan\beta$

To pinpoint the underlying model, one would usually look only at the Higgs scalar sector. [Benbrik et al., 1207.1096]

What if, given a MSSM and NMSSM scenarios:

- Higgs spectra are not distinguishable at the LHC and/or not reachable at the LC?
- Very similar chargino/neutralino spectra?
- Close $\sigma(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0)$, $\sigma(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-)$?

$\}\ \Rightarrow$ **Focus on this**

These conditions are possible for unconstrained scenarios [hep-ph/0502036].

Strategy: chargino/neutralino sectors for model distinction

We assume:

- We measure at LHC/LC only the light SUSY masses: $m_{\tilde{\chi}^0_{1,2}}$, $m_{\tilde{\chi}^\pm_1}$ ($m_{\tilde{\nu}}$, $m_{\tilde{e}_{R,L}}$); squarks $\sim \mathcal{O}(1$ TeV$)$.

- Experimental uncertainties: $\delta m_{\tilde{\chi}^\pm_1}$, $\delta m_{\tilde{\chi}^0_1}$, $\delta m_{\tilde{\chi}^0_2} \sim 0.1\%$.

- At the LC:
  - We exploit polarized beams: $P_{e^-} \in [-0.9, +0.9]$, $P_{e^+} \in [-0.6, +0.6]$.
  - We measure $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2)$ and $\sigma(e^+e^- \to \tilde{\chi}^+_1\tilde{\chi}^-_1)$ at $\sqrt{s} = 350$ ($t\bar{t}$ threshold), 500 GeV.

The strategy is to:

- χ^2-fit with `Minuit` the measured values to the MSSM parameters M_1, M_2, μ, $\tan\beta$. [Desch et al '03]

- Check the compatibility of the fitted (tree-level)-parameters with the MSSM.

- From the reconstructed parameters, derive MSSM neutralinos, as $m_{\tilde{\chi}^0_3}$, and cross-check at LHC/LC.

For $M_1 > M_2$, contempled also in AMSB, one can get (also [hep-ph/0502036]):

	M_1 [GeV]	M_2 [GeV]	$\mu, \mu_{eff} = \lambda \cdot x$ [GeV]	$\tan\beta$	κ	λ
MSSM	411	115.7	358.5	8		
NMSSM	365	111	484	9.5	0.16	0.06

Leading to $m_h = 125$ GeV and, and the tree-level masses [GeV]:

	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$m_{\tilde{\chi}_5^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$
MSSM	105.0	354.8	364.6	431.5		105.2	379.2
NMSSM	104.9	354.8	364.7	489.7	504	105.1	498.5

We also take $m_{\tilde{e}_L}$ =303.5, $m_{\tilde{e}_R}$ =303, $m_{\tilde{\nu}_e}$ =293.3 GeV.

MSSM	$\tilde{B}$	$\tilde{W}$	$\tilde{H}_a$	$\tilde{H}_b$
$\tilde{\chi}_1^0$	0.0%	93.0%	1.7%	5.4%
$\tilde{\chi}_2^0$	25.4%	4.9%	43.2%	26.6%
$\tilde{\chi}_3^0$	0.1%	1.1%	38.3%	60.5%

NMSSM	$\tilde{B}$	$\tilde{W}$	$\tilde{H}_a$	$\tilde{H}_b$	$\tilde{S}$
$\tilde{\chi}_1^0$	0.0%	96.6%	0.6%	2.8%	0.0%
$\tilde{\chi}_2^0$	63.6%	0.4%	3.6%	2.7%	29.8%
$\tilde{\chi}_3^0$	31.0%	0.0%	0.0%	0.3%	68.8%

Example: Light singlino scenario - fit

$\sigma_{\rm LO}(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ [fb]

$\sqrt{s}$ =350 GeV	MSSM	NMSSM
P=(-0.9,0.6)	2496.66±4.19	2578.73±4.31
P=(0.9,-0.6)	39.64±0.75	42.48±0.77
$\sqrt{s}$ =500 GeV	MSSM	NMSSM
P=(-0.9,0.6)	1167.64±2.16	1213.41±2.22
P=(0.9,-0.6)	18.33±0.38	18.77±0.39

$\sigma_{\rm LO}(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ [fb]

$\sqrt{s}$ =500 GeV	MSSM	NMSSM
P=(-0.9,0.6)	20.68±0.32	18.80±0.30
P=(0.9,-0.6)	.38±0.03	.29±0.02

- $\delta m/m = 0.1\%$; $\delta Pol/Pol = 0.5\%$; Statistic error: 1 σ at $\int \mathcal{L} = 500$ fb^{-1}.

χ^2-fit with NMSSM $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$, $m_{\tilde{\chi}_1^\pm}$, $\sigma_{L,R}(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ and $\sigma_{L,R}(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ to MSSM parameters:

M_1 [GeV]	M_2 [GeV]	μ [GeV]	$\tan\beta$
362.7 ±0.4	108.3±0.1	519.6±8.7	unconstrained $\gtrsim 8$

Fit result excludes that the "data" are consistent with the MSSM (χ^2/d.o.f. = 220.8/11).

Classes of scenarios

Looking at the NMSSM **chargino/neutralino** sector, we can distinguish two classes:

- High $\tilde{S}$ admixture in $\tilde{\chi}_1^0$ or $\tilde{\chi}_2^0$ [hep-ph/0502036].

  Easier to distinguish from MSSM looking at higgsino/gaugino features of neutralino from decay channels.

- $\tilde{S}$, mainly in the heavier states $\tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0$:

  - $\mu < M1, M2$
  - $\mu > M1, M2$

  Trickier scenario to be distinguished from MSSM, due to similar admixture in the lighter neutralinos and MSSM-like signatures.

  G. Moortgat-Pick, SP, K. Rolbiecki: 1404.1053, 1405.xxxx

Heavy singlino, case 1: $\mu < M_1 < M_2$

	M_1 [GeV]	M_2 [GeV]	$\mu, \mu_{eff} = \lambda \cdot x$ [GeV]	$\tan\beta$	A_λ	A_{kappa}
MSSM/NMSSM	450	1600	120	27	3000	-30

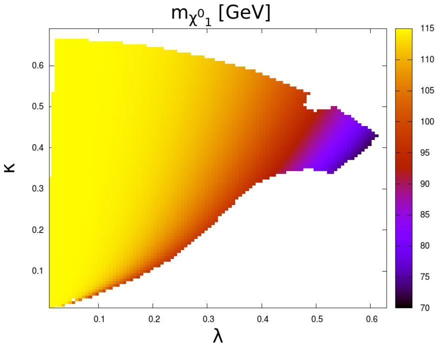
MSSM neutralino/chargino tree-level spectrum in [GeV]:

$m_{\tilde{\chi}^0_1}$	$m_{\tilde{\chi}^0_2}$	$m_{\tilde{\chi}^0_3}$	$m_{\tilde{\chi}^0_4}$	$m_{\tilde{\chi}^\pm_1}$	$m_{\tilde{\chi}^\pm_2}$
114.80	123.28	454.41	1604.08	119.40	1604.08

NMSSM, scanning the $\lambda - \kappa$ plane with:

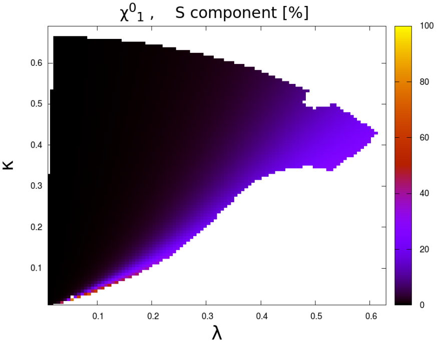
- `NMSSMTools-4.2.1` and `micrOMEGAs-3.0` for pheno and DM constraints.
  [Ellwanger et Al. '05], [Das et Al '11], [Belanger et Al. '05]
- `HiggsBounds-4.0.0` and `HiggsSignals-1.0.0` to check the Higgs sector.
  [Bechtle et Al. '05, '13]

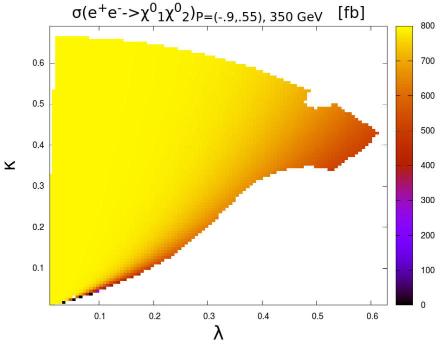
White areas correspond to excluded points.



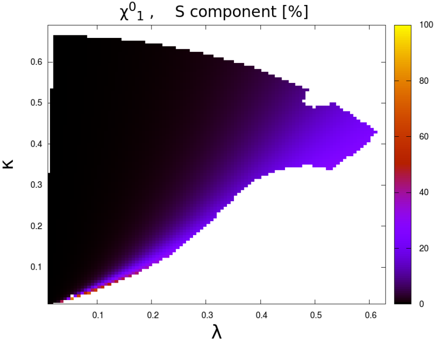
MSSM, $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2)$ [fb]	$\sqrt{s}$ =350 GeV	$\sqrt{s}$ =500 GeV
P=(-0.9,0.55)	791.7	391.4
P=(0.9,-0.55)	526.7	261.7

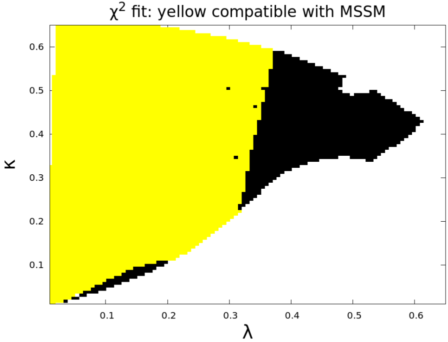
NMSSM:





Heavy singlino, case 1: $\mu < M_1 < M_2$





Assuming $\delta m/m = 0.5\%$ and $\delta\sigma/\sigma = 1\%$, the χ^2-fit finds regions with that are not compatible with the **MSSM**.

Search for heavier resonance $m_{\tilde{\chi}_3^0}$ at the ILC/LHC can point to the NMSSM.

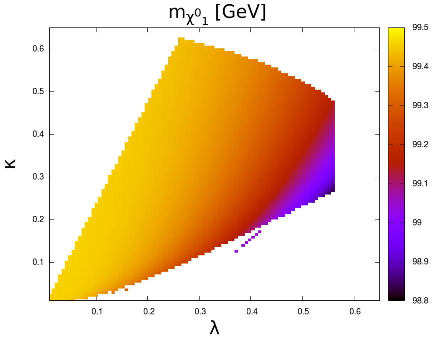
Heavy singlino, case 2: $M_2 < M_1 < \mu$

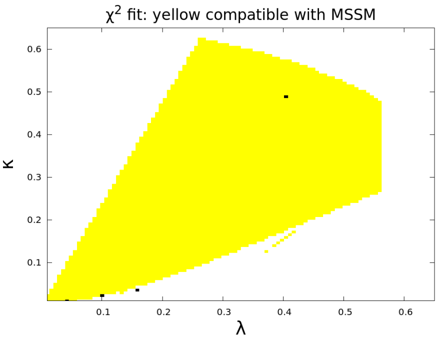
	M_1 [GeV]	M_2 [GeV]	$\mu, \mu_{eff} = \lambda \cdot x$ [GeV]	$\tan\beta$	A_λ	A_{kappa}
MSSM/NMSSM	240	105	505	9.2	3700	-50

MSSM neutralino/chargino tree-level spectrum in [GeV]:

$m_{\tilde{\chi}^0_1}$	$m_{\tilde{\chi}^0_2}$	$m_{\tilde{\chi}^0_3}$	$m_{\tilde{\chi}^0_4}$	$m_{\tilde{\chi}^\pm_1}$	$m_{\tilde{\chi}^\pm_2}$
99.46	237.03	510.13	518.65	99.55	518.71

NMSSM:



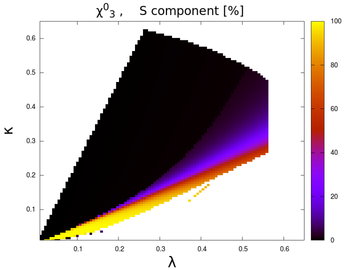


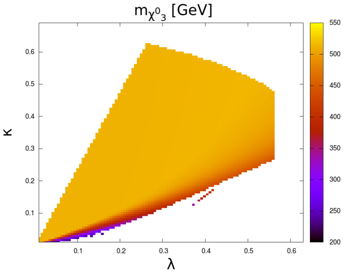
Assuming $\delta m/m = 0.5\%$ and $\delta\sigma/\sigma = 1\%$, the χ^2-fit is not sufficient to distinguish from **MSSM**.

Heavy singlino, case 2: $M_2 < M_1 < \mu$

A possibility is to look for the heavier neutralino and Higgs resonances at the LHC or TeV-LC.

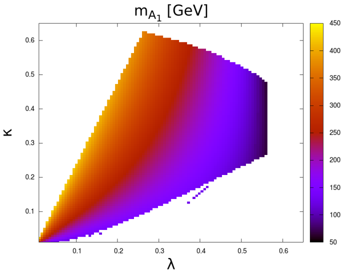
Detecting $\tilde{\chi}_3^0$ can be the key issue; while in the MSSM $m_{\tilde{\chi}_3^0}$=510 GeV, in the NMSSM:





One may also look for the lightest CP-odd Higgs, A_1, $\sim 100\%$ S-like, studying its decays.

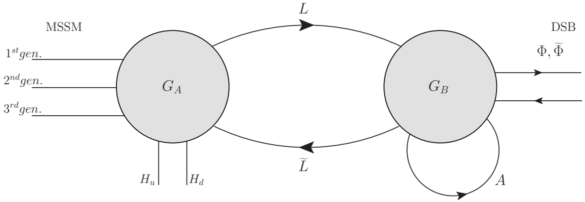
$\Longrightarrow$



Higgs couplings & non-decoupling D-terms

M. McGarrie, G. Moortgat-Pick, SP, 1405.xxxx

Quiver models and non-decoupling D-terms



P. Batra, A. Delgado, D. E. Kaplan, and T. M. Tait, [hep-ph/0404251].

$G_A, G_B, \ldots$ copies of $SU(2) \times U(1)$

L, $\tilde{L}$ get vevs at scale $\gtrsim$TeV $\Longrightarrow$ $G_A \times G_B$ breaks to $SU(2)_L \times U(1)_Y$

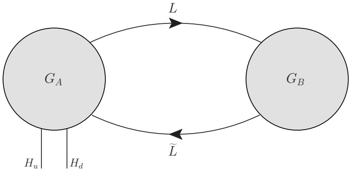
Additional non-decoupling D-terms in the Higgs potential

Features

- Higgs mass lifted at the tree-level, relaxing naturalness.
- Almost vanishing contributions to EW observables.
- GUT can be recovered ...

Gauge extended MSSM: quiver models

Vector Higgs case

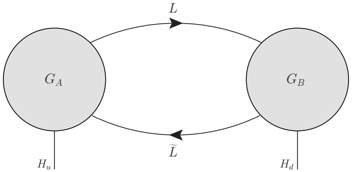


$$\delta\mathcal{L} = -\frac{3}{5}\frac{g_1^2\Delta_1}{8}(H_u^\dagger H_u - H_d^\dagger H_d)^2 - \frac{g_2^2\Delta_2}{8}\sum_a (H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d)^2$$

$$m_{h,0}^2 = \left[m_Z^2 + \left(\frac{\frac{3}{5}g_1^2\Delta_1 + g_2^2\Delta_2}{4}\right) v^2\right]\cos^2 2\beta$$

$$\Delta_i = \left(\frac{g_{A_i}^2}{g_{B_i}^2}\right)\frac{m_L^2}{m_{v_i}^2 + m_L^2}$$

Chiral Higgs case

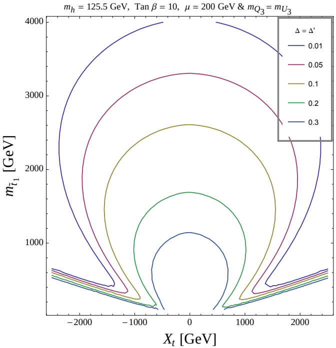


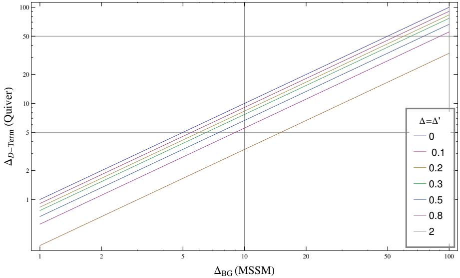
$$\delta\mathcal{L} = -\frac{3}{5}\frac{g_1^2\Omega_1}{8}\left(\xi_1 H_u^\dagger H_u + \frac{1}{\xi_1} H_d^\dagger H_d\right)^2 - \frac{g_2^2\Omega_2}{8}\sum_a \left(\xi_2 H_u^\dagger \sigma^a H_u - \frac{1}{\xi_2} H_d^\dagger \sigma^a H_d\right)^2 + \dots$$

$$m_{h,0}^2 = \left[m_Z^2 + \left(\frac{\frac{3}{5}g_1^2\xi_i\Omega_1 + g_2^2\xi_2\Omega_2}{4}\right) v^2\right] + \mathcal{O}\left(\frac{1}{\tan^2\beta}, \xi_i\right)$$

$$\xi_i = \frac{g_{Ai}}{g_{Bi}} \quad , \quad \Omega_i = \frac{m_L^2}{m_{v_i}^2 + m_L^2}$$

Example: vector Higgs case and Naturalness





$$\Delta_{BG} = \left| \frac{2\delta m^2_{H_u}}{m^2_Z} \right| \quad \Longrightarrow \quad \Delta_{D-Term} = \left| \frac{2\delta m^2_{H_u}}{m^2_{h,0}/\cos^2(2\beta)} \right| = \left| \frac{2\delta m^2_{H_u}}{m^2_Z + m^2_\Delta} \right| .$$

Quiver models: coupling enhancement

General renormalizable 2HDM scalar Higgs potential: J. F. Gunion and H. E. Haber, [hep-ph/0207010]

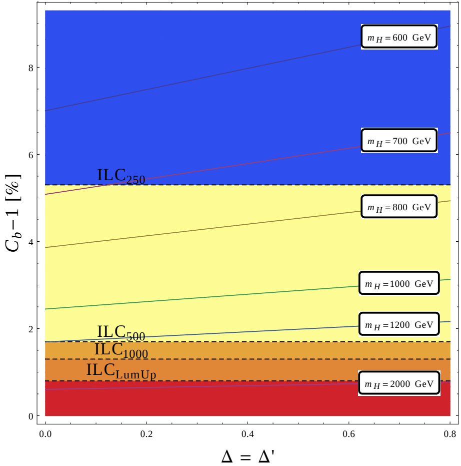
$$\begin{aligned}\mathcal{V} &= m_1^2|H_d|^2 + m_2^2|H_u|^2 + m_{12}^2(H_uH_d + H_u^\dagger H_d^\dagger) \\ &+ \frac{\lambda_1}{2}|H_d|^2 + \frac{\lambda_2}{2}|H_u|^2 + \lambda_3|H_u|^2|H_d|^2 + \lambda_4|H_d^\dagger H_u|^2 + \frac{\lambda_5}{2}[(H_uH_d)^2 + (H_u^\dagger H_d^\dagger)^2] \\ &+ \lambda_6|H_d|^2[(H_uH_d) + (H_u^\dagger H_d^\dagger)] + \lambda_7|H_u|^2[(H_uH_d) + (H_u^\dagger H_d^\dagger)]\end{aligned}$$

For $\tan\beta \gtrsim 3$ and heavy H_d one gets: K. Blum and R. T. D'Agnolo, [1202.2364]

$$c_b \equiv \frac{y_b}{y_b^{SM}} = \left(1 - \frac{m_h^2}{m_H^2}\right)^{-1}\left(1 - \frac{[\lambda_3 + \lambda_5]\, v^2}{m_H^2} + \frac{\lambda_7 v^2}{m_H^2}\right) \times \{1 + \mathcal{O}(\frac{1}{\tan^2\beta})\} + \ldots$$

$$c_t \equiv \frac{y_t}{y_t^{SM}} = 1 + \frac{\lambda_7 v^2}{2m_H^4}(1 - c_b^2) \qquad\qquad c_V \equiv \frac{y_V}{y_V^{SM}} = c_t + \frac{\lambda_7 v^2}{m_H^4}(c_b - 1)$$

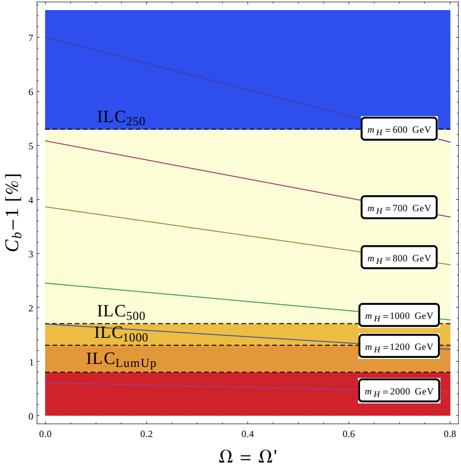
Coupling enhancement: vector Higgs case



Model indepedent determination of c_b at linear colliders is considered.

$$c_b^{\text{vector}} \simeq \left(1 - \frac{m_h^2}{m_H^2}\right)^{-1} \left(1 + \frac{[g_2^2(1 + \Delta_2) + \frac{3}{5} g_1^2 (1 + \Delta_1)] v^2}{4 m_H^2}\right)$$

Coupling enhancement: chiral Higgs case



Model indepedent determination of c_b at linear colliders is considered.

$$c_b^{\text{chiral}} \simeq \left(1 - \frac{m_h^2}{m_H^2}\right)^{-1} \left(1 + \frac{[g_2^2(1 - \Omega_2) + \frac{3}{5} g_1^2 (1 - \Omega_1)] v^2}{4 m_H^2}\right)$$

Conclusions and outlook

Minimal extensions of MSSM, as NMSSM or quiver models

- Relax naturalness raising m_h^{tree} through additional contributions to Higgs quartic couplings.
- Can answer the μ-problem and preserve gauge coupling unification.
- Less constrained by experimental bounds.

At future linear colliders

- Deviations from the SM and the MSSM can be detected looking at the
  - Higgs sector and couplings.
  - Neutralino/chargino sector and production cross sections.
- Polarised beams play a crucial role.

To do:

- Include quantum level precision.
- Include in the NMSSM analysis the production of heavier resonances at LC/LHC; add Higgs sector observables.

Thank you for your attention!

Backup: Higgs sector in the light singlino scenario

[GeV]	MSSM	NMSSM
M_1	411	365
M_2	115.7	111
M_3	600	$3M_2$
$\tan\beta$	8	9.5
μ	358.5	
$\mu_{eff} = \lambda x$		484
A_λ		4200
A_κ		-120
A_{u_3}	1928.	2500.
A_{d_3}	2500.	2000.
A_{e_3}	1500.	2000
M_{l_3}	300	300.
M_{e_3}	300	300.
M_{Q_3}	1500	1050.
$M_{Q_{1,2}}$	1500.	1500.
M_{u_3}	1500	1000.
$M_{u_{1,2}}$	1500	1500.
Md_3	1500.	800
$Md_{1,2}$	1500.	1500

	MSSM	NMSSM
m_{S_1}	124.60	124.60
m_{S_2}	4470	335.2
m_{S_3}		4471
m_{P_1}	4470	250.8
m_{P_2}		4471
$m_{H^\pm}$	4472	4472

- In the NMSSM, S_2 and P_2 are singlet-like at 99.99%.

Backup: data fit to MSSM and model distinction

χ^2-fit with NMSSM $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$, $m_{\tilde{\chi}_1^\pm}$, $\sigma_{L,R}(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ and $\sigma_{L,R}(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ to MSSM parameters:

M_1 [GeV]	M_2 [GeV]	μ [GeV]	$\tan\beta$
362.7 $\pm$0.4	108.3$\pm$0.1	519.6$\pm$8.7	unconstrained $\gtrsim 8$

Fit result excludes that the "data" are consistent with the MSSM (χ^2/d.o.f. $= 220.8/11$).

Moreover, observing the NMSSM $m_{\tilde{\chi}_3^0} = 364.7 \pm 1.8$ GeV. Away from fit $m_{\tilde{\chi}_3^0} \in [520, 532]$ GeV !!

$\tilde{\chi}_3^0$	$\tilde{B}$	$\tilde{W}$	$\tilde{H}_a$	$\tilde{H}_b$	$\tilde{S}$
NMSSM	31.0%	0.0%	0.0%	0.3%	68.8%
MSSM fit	0.1%	0.6%	38.0%	61.3%	

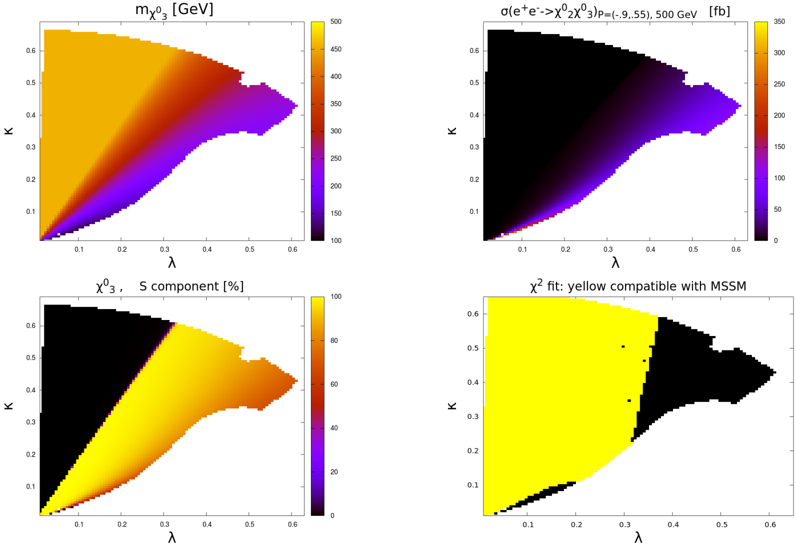
One can also look at gaugino properties through precision observables.

Backup: Heavy singlino, case 1, $\mu < M_1 < M_2$

[GeV]	NMSSM
M_1	450
M_2	1600
M_3	1600
$\tan\beta$	27
$\mu_{eff} = \lambda x$	120
A_λ	3000
A_κ	-30
A_{u_3}	3300.
A_{d_3}	2000.
A_{e_3}	2000
M_l	300.
M_e	300.
M_Q	1500.
M_u	1500.
M_d	1500.

Backup: Heavy singlino, case 1, $\mu < M_1 < M_2$

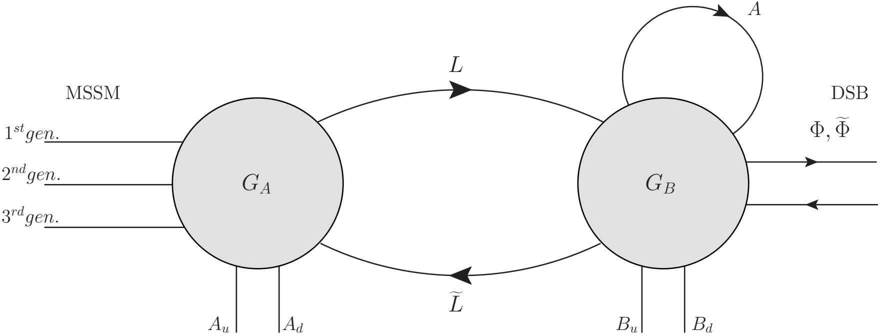
Detecting $\tilde{\chi}_3^0$ at LHC/TeV-LC can point to the NMSSM; while in the MSSM $m_{\tilde{\chi}_3^0}$=454 GeV, in the NMSSM:



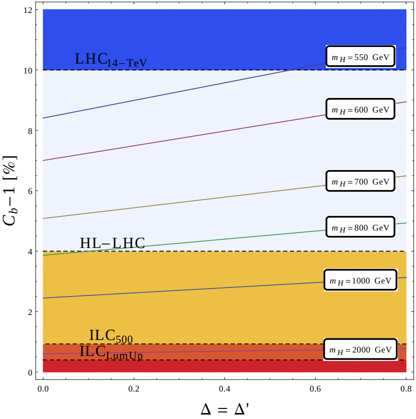
Backup: Heavy singlino, case 2, $M_2 < M_1 < \mu$

[GeV]	NMSSM
M_1	240
M_2	105
M_3	600
$\tan\beta$	9.2
$\mu_{eff} = \lambda x$	505
A_λ	3700
A_κ	-40
A_{u_3}	3700.
A_{d_3}	2500.
A_{e_3}	1500
$M_{l_{1,2}}$	300.
M_{l_3}	500.
$M_{e_{1,2}}$	300.
M_{e_3}	500.
$M_{Q_{1,2}}$	1500
M_{Q_3}	1800.
M_{u_3}	1500.
M_{d_3}	1500.

Backup: possible UV completion for quiver model



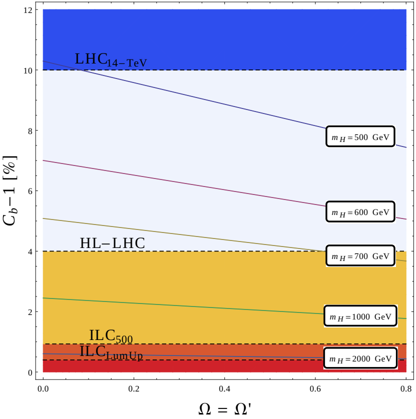
Backup: coupling enhancement in the vector Higgs case, LHC vs ILC



The expected precisions on the Higgs couplings and total width at LHC and ILC are obtained from a constrained 7-parameter fit assuming no non-SM production or decay modes.

The fit assumes universality:
$c_u \equiv c_t = c_c,\ c_d \equiv c_b = c_s,\ c_l \equiv c_\tau = c_\mu$

$$c_b^{\text{vector}} \simeq \left(1 - \frac{m_h^2}{m_H^2}\right)^{-1} \left(1 + \frac{[g_2^2(1 + \Delta_2) + \frac{3}{5} g_1^2 (1 + \Delta_1)]v^2}{4 m_H^2}\right)$$



The expected precisions on the Higgs couplings and total width at LHC and ILC are obtained from a constrained 7-parameter fit assuming no non-SM production or decay modes.

The fit assumes universality:
$c_u \equiv c_t = c_c, \; c_d \equiv c_b = c_s, \; c_l \equiv c_\tau = c_\mu$

$$c_b^{\text{chiral}} \simeq \left(1 - \frac{m_h^2}{m_H^2}\right)^{-1} \left(1 + \frac{[g_2^2(1 - \Omega_2) + \frac{3}{5} g_1^2 (1 - \Omega_1)] v^2}{4 m_H^2}\right)$$