Momentum Resolution Event-by-Event Basis

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Developped code for covariance matrix P momenta

- Last meetings i reported i was developping this code.
  - See my previous talks for more details.
- I have written marlin processor adding new LCCollection of pfos with the cov. matrix filled.
- I have repeated my calculations.
  - During that calculations some questions arise to my mind.
  - I found answer to such questions.
  - Today i talk about and share what i have learned about it (*).

(*) If you are not interested in linear algebra please jump to summary.

Understanding the new variable

Change of Basis

- We are actually doing a change of basis:
- Original base: $\mathfrak{A} = \{\tan\lambda\,, \Omega\,, \phi\,, d_0\,, z_0\,\}$
- New base: $\mathfrak{B} = \{p_x\,, p_y\,, p_z\,, \mathrm{E}\}$
- Original basis has higher rank: full description of the phenomena.
- $\mathfrak{B}$ is actually expanding just one subspace of the total space $\mathfrak{H}$ expanded by $\mathfrak{A}$.
- That looks logical: with $\mathfrak{A}$ provides position and momenta of the track. $\mathfrak{B}$ just provide momenta/energy.
- Could correlations of basis vectors $\{\tan\lambda\,, \Omega\,, \phi\,\}$ with $\{d_0\,, z_0\,\}$ have impact on correlations of $\mathfrak{B}$ basis vectors.
- Say in other words: Is it the subspace expanded by $\{d_0\,, z_0\,\}$ orthogonal to the subspace generated by $\{p_x\,, p_y\,, p_z\,, \mathrm{E}\}$?
- Or alternatively: Should i use the full covariance matrix in helicity parameters space ($\mathfrak{A}$) when traslating it to the new basis ($\mathfrak{B}$)?

- My first thought was saying: no, i dont need it.
- Then i saw other experiments use my same expressions (CDF, LHCb).
- But, is it just a valid aproximation? Is it exact? I want to know it.
- Good opportunity to learn something new.

Jacobian helix parameters to momenta space

- After some derivative exercises ...

$$\mathfrak{J} = \begin{bmatrix} \frac{\partial P_x}{\partial tan\lambda} & \frac{\partial P_y}{\partial tan\lambda} & \frac{\partial P_z}{\partial tan\lambda} & \frac{\partial E}{\partial tan\lambda} \\ \frac{\partial P_x}{\partial \Omega} & \frac{\partial P_y}{\partial \Omega} & \frac{\partial P_z}{\partial \Omega} & \frac{\partial E}{\partial \Omega} \\ \frac{\partial P_x}{\partial d_0} & \frac{\partial P_y}{\partial d_0} & \frac{\partial P_z}{\partial d_0} & \frac{\partial E}{\partial d_0} \\ \frac{\partial P_x}{\partial z_0} & \frac{\partial P_y}{\partial z_0} & \frac{\partial P_z}{\partial z_0} & \frac{\partial E}{\partial z_0} \\ \frac{\partial P_x}{\partial \phi} & \frac{\partial P_y}{\partial \phi} & \frac{\partial P_z}{\partial \phi} & \frac{\partial E}{\partial \phi} \end{bmatrix} = \frac{-1}{\Omega} \begin{bmatrix} 0 & 0 & -\Omega P_T & -\frac{P_z^2\,\Omega}{E\,tan\lambda} \\ P_x & P_y & P_z & \frac{P^2}{E} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ P_y\,\Omega & -P_x\,\Omega & 0 & 0 \end{bmatrix}$$

$\longrightarrow$ $\Sigma_i' = \mathfrak{J}^T\, \Sigma_i\, \mathfrak{J}$, covariance matrix in momenta space.

($\Sigma_i' = \mathfrak{J}\, \Sigma_i\, \mathfrak{J}^T$ if you define jacobian as the transposed of quoted above)

Is there effect on d0 z0 on the Covariance matrix?

- Original base: $\mathfrak{A} = \{\tan\lambda\, , \Omega\, , \phi\, , \mathrm{d}_0\, , \mathrm{z}_0\, \}$
- New base: $\mathfrak{B} = \{\mathrm{p_x}\, , \mathrm{p_y}\, , \mathrm{p_z}\, , \mathrm{E}\}$

$$
\begin{aligned}
\mathrm{p_x} &= \mathrm{p_T}\, cos\phi \\
\mathrm{p_y} &= \mathrm{p_T}\, sin\phi \\
\mathrm{p_z} &= \mathrm{p_T} \tan\lambda \\
E^2 &= (a\, \frac{B_z}{\Omega\, cos\,\lambda})^2 + m^2 \\
&= (\frac{\mathrm{p_T}}{cos\,\lambda})^2 + m^2
\end{aligned}
$$

$$
\mathrm{p_T} = |\frac{\kappa}{\Omega}|
$$

$$
\kappa = |a\, B_z|\ (constant)
$$

Momenta does not depend on d_0 , z_0

- $\mathrm{p_x} = \mathrm{p_x}\, (\tan\lambda\, , \Omega, \phi)$
- $\mathrm{p_y} = \mathrm{p_y}\, (\tan\lambda\, , \Omega, \phi)$
- $\mathrm{p_z} = \mathrm{p_z}\, (\tan\lambda\, , \Omega)$

Change of cov. matrix

1. $\Sigma_i' = \mathfrak{J}^T\, \Sigma_i\, \mathfrak{J}$
2. Σ_i cov. matrix in $\mathfrak{A}$.
3. Σ_i' cov. matrix in $\mathfrak{B}$.

- Should i include full matrix (rank 5) in the item 1)?
- The goal of this report is to answer this question.

Is there effect on d0 z0 on the Covariance matrix?

Momenta does not depend on d_0 , z_0

- $p_x = p_x(\tan\lambda, \Omega, \phi)$
- $p_y = p_y(\tan\lambda, \Omega, \phi)$
- $p_z = p_z(\tan\lambda, \Omega)$

Intuitively space/momentum are independent measurements, but... are they?

- Position and momenta info. comming as a result of track fitting. So, eventually they are not independent measurements: the info comes from same fits.
- The covariance in helix parameters comes from track fitting.
- The covariance in $\mathfrak{A}$ space is a symmetric 5x5 matrix with (generally) non null elements.
- That means, every variable has some correlation with others: $Cov(i, i) \neq 0 , \forall\, i, j$
- In particular, d_0 or z_0 correlation on $\tan\lambda$.
- So, as p_x depend on $\tan\lambda$, why not d_0 , z_0 effect on, for instance $\tan\lambda$, be translated to p_x when we go from $\mathfrak{A}$ to $\mathfrak{B}$?
- For me it is not obvious why those correlations should canceled.

Same Result using full matrix or not

Comparison cov. matrix using 3x3 and 5x5 helix matrix

	cov xx	cov yx	cov yy	cov zx	cov zy	cov zz	...
cov 3x3	1.57576e-05	-3.9027e-06	2.10397e-06	2.88209e-05	-7.63759e-06	5.71544e-05	...
cov 5x5	1.57576e-05	-3.9027e-06	2.10397e-06	2.88209e-05	-7.63759e-06	5.71544e-05	...

- The covariance matrix is exactly the same.
- That means d_0 , z_0 correlations canceled identically.

Cancelation Proof I

- To show this cancelation is useful to order the base vectors in the following way:
- $\mathfrak{A} = \{d_0 , z_0 , \tan\lambda , \Omega , \phi\}$
- $\mathfrak{B} = \{p_x , p_y , p_z , E\}$
- Now, the jacobian looks like (first two rows are null):

$$\mathfrak{J} = \begin{bmatrix} \frac{\partial P_x}{\partial d_0} & \frac{\partial P_y}{\partial d_0} & \frac{\partial P_z}{\partial d_0} & \frac{\partial E}{\partial d_0} \\ \frac{\partial P_x}{\partial z_0} & \frac{\partial P_y}{\partial z_0} & \frac{\partial P_z}{\partial z_0} & \frac{\partial E}{\partial z_0} \\ \frac{\partial P_x}{\partial tan\lambda} & \frac{\partial P_y}{\partial tan\lambda} & \frac{\partial P_z}{\partial tan\lambda} & \frac{\partial E}{\partial tan\lambda} \\ \frac{\partial P_x}{\partial \Omega} & \frac{\partial P_y}{\partial \Omega} & \frac{\partial P_z}{\partial \Omega} & \frac{\partial E}{\partial \Omega} \\ \frac{\partial P_x}{\partial \phi} & \frac{\partial P_y}{\partial \phi} & \frac{\partial P_z}{\partial \phi} & \frac{\partial E}{\partial \phi} \end{bmatrix} = \frac{-1}{\Omega} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\Omega P_T & -\frac{P_z^2\,\Omega}{E\ tan\lambda} \\ P_x & P_y & P_z & \frac{P^2}{E} \\ P_y\,\Omega & -P_x\,\Omega & 0 & 0 \end{bmatrix}$$

- The cov. matrix in the original basis looks like:

$$\mathfrak{A} = \begin{bmatrix} Cov[d_0, d_0] & Cov[d_0, z_0] & Cov[d_0, tan\,\lambda] & Cov[d_0, \Omega] & Cov[d_0, \phi] \\ Cov[z_0, d_0] & Cov[z_0, z_0] & Cov[z_0, tan\,\lambda] & Cov[z_0, \Omega] & Cov[z_0, \phi] \\ Cov[tan\,\lambda, d_0] & Cov[tan\,\lambda, z_0] & \color{red}{Cov[tan\,\lambda, tan\,\lambda]} & \color{red}{Cov[tan\,\lambda, \Omega]} & \color{red}{Cov[tan\,\lambda, \phi]} \\ Cov[\Omega, d_0] & Cov[\Omega, z_0] & \color{red}{Cov[\Omega, tan\,\lambda]} & \color{red}{Cov[\Omega, \Omega]} & \color{red}{Cov[\Omega, \phi]} \\ Cov[\phi, d_0] & Cov[\phi, z_0] & \color{red}{Cov[\phi, tan\,\lambda]} & \color{red}{Cov[\phi, \Omega]} & \color{red}{Cov[\phi, \phi]} \end{bmatrix}$$

Cancelation Proof II

- $\mathfrak{A} = \{\mathrm{d}_0 , \mathrm{z}_0 , \tan\lambda , \Omega , \phi \}$
- $\mathfrak{B} = \{\mathrm{p_x} , \mathrm{p_y} , \mathrm{p_z} , \mathrm{E}\}$

$\Sigma'_i = \mathfrak{J}^T \Sigma_i \mathfrak{J}$
$\Sigma'_i = (b_{ij})$
$\Sigma_i = (a_{ij})$
$\mathfrak{J} = (h_{ij})$

$$b_{ij} = \sum_{r=1}^{5} \sum_{m=1}^{5} h_{ri} a_{rm} \cdot h_{mj}$$
$$= \sum_{m=1}^{5} (\sum_{r=1}^{5} h_{ri} a_{rm}) \cdot h_{mj}$$
$$= \sum_{m=1}^{5} (0 \cdot a_{1m} + 0 \cdot a_{2m} + \sum_{r=3}^{5} h_{ri} a_{rm}) \cdot h_{mj}$$
$$= () \cdot h_{1j} + () \cdot h_{2j} + \sum_{m=3}^{5} (0 \cdot a_{1m} + 0 \cdot a_{2m} + \sum_{r=3}^{5} h_{ri} a_{rm}) \cdot h_{mj}$$
$$= \sum_{m=3}^{5} (\sum_{r=3}^{5} h_{ri} a_{rm}) \cdot h_{mj}$$

- (a_{ij}) elements related with d_0 , z_0 does not contribute to (b_{ij}) (see previous slide).
- Geometrically that means: subspaces generated by $\{\mathrm{d}_0 , \mathrm{z}_0 \}$ and $\mathfrak{B}$ are orthogonal.
- From an experimental point of view: i dont need to use the full (5x5) covariance matrix in helicity parameters (just the 3x3).

Marlin Processor

- I have written a new Marlin Processor filling the covariance matrix in P.
- Output is a new LCCollection copy of PandoraPFOs but with non null cov. matrix.
- This code should be included in new releases of `ILCSOFT`.

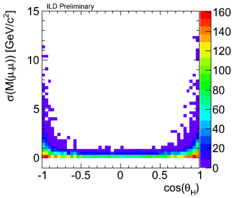
- Code will be available very soon (hopefully this evening) at kekcc here:
- /hsm/ilc/grid/JB/users/calancha/code/marlin/momentumCov
- Example and xmlfile will be provided in same directory as well.

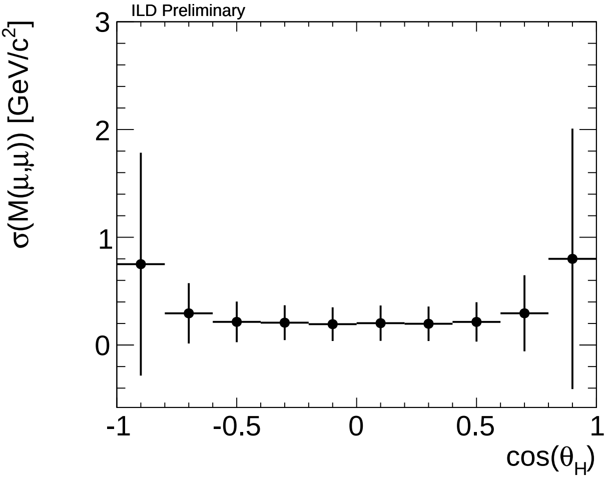
Summary

- Developed piece of code to get covariance matrix of charged particles in momenta space.
  - Understood why i just need $\{\tan\lambda\,,\,\Omega\,,\,\phi\,\}$ variables in the calculation.
- A new LCCollection is added to the event (copy of PandoraPFOs) with filled cov. matrix.
- Used it to calculate dimuon mass error event-by-event,
- but it is useful on its own (directly related with ILD tracking).

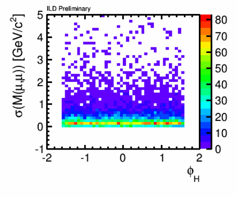
BACK UP

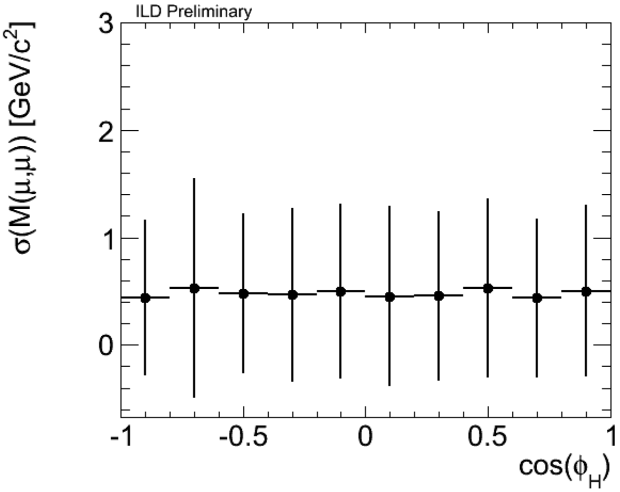
- Last meeting a show some scatter plots with the new variable.
- Plots were not clear due to not right choice of the axis.
- Its more clear to plot profile plots to see the dependence of the two variables.





- Better precision at central region (tracks have more hits).





- No dependence on azimutal angle.

Gaussian Fit [-2,2]