Matrix Element Method for ILC Physics Analysis

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svn co https://svnsrv.desy.de/basic/physsim/Physsim/trunk

http://ilcphys.kek.jp/meeting/physics/archives/2009-05-19/GGGuide.pdf

Apr. 16 @ ILD Soft&Ana Meeting

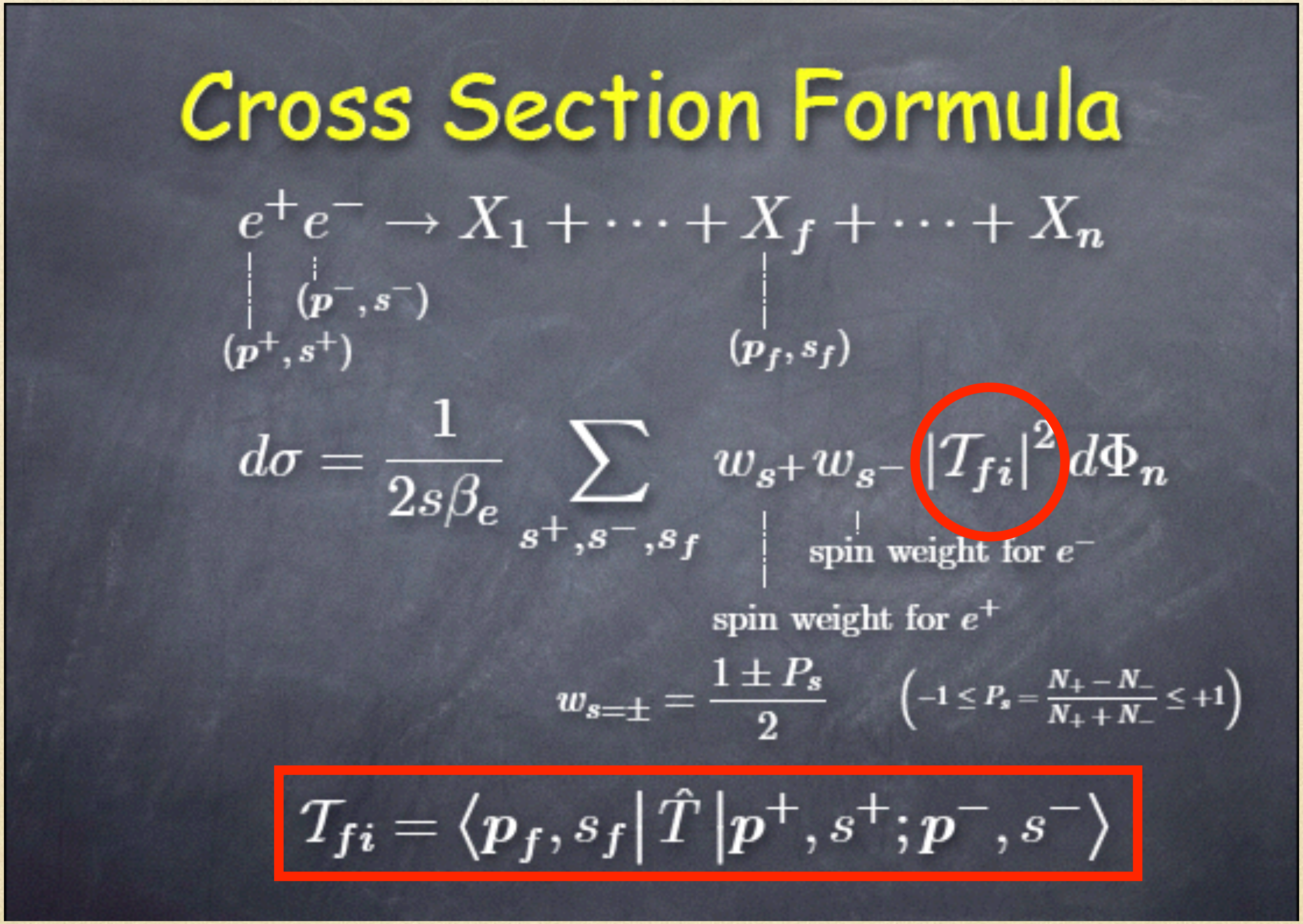
MEM: maximal use of event kinematics

(approach the true likelihood of each event)

- one of the Multivariate methods
- first used for precision top mass measurement at D0
- recently used for the H—>ZZ*—>4l analysis in Higgs discovery by CMS
- not being widely used comparing to BDT, MLP, but to me MEM is even more interesting

What is Matrix Element (Amplitude)

(squared ~ differential cross section)



Cross Section Formula

$$e^+e^- \to X_1 + \cdots + X_f + \cdots + X_n$$

(p^+, s^+) (p^-, s^-) (p_f, s_f)

$$d\sigma = \frac{1}{2s\beta_e} \sum_{s^+, s^-, s_f} w_{s^+} w_{s^-} |\mathcal{T}_{fi}|^2 \, d\Phi_n$$

spin weight for e^-

spin weight for e^+

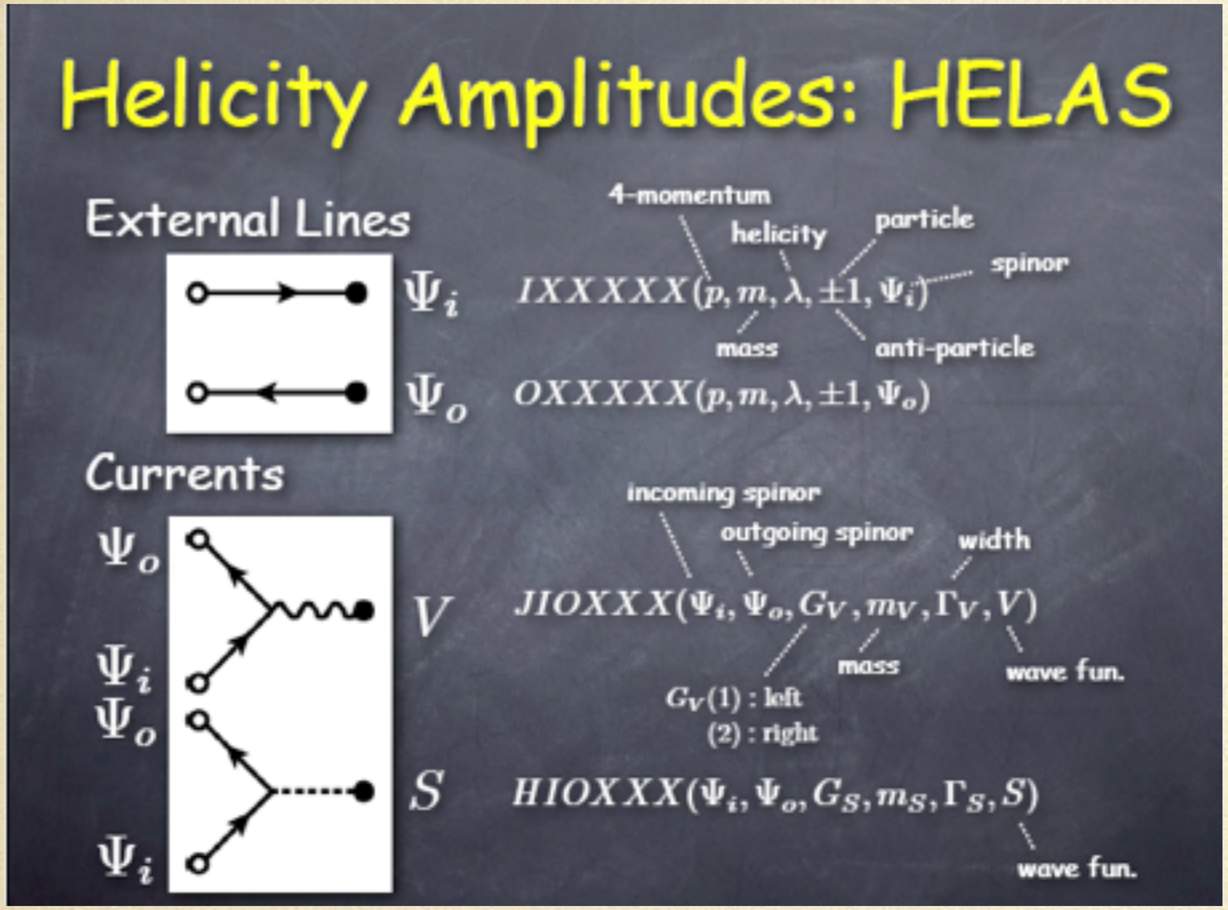
$$w_{s=\pm} = \frac{1 \pm P_s}{2} \qquad \left(-1 \le P_s = \frac{N_+ - N_-}{N_+ + N_-} \le +1 \right)$$

$$\boxed{\mathcal{T}_{fi} = \langle p_f, s_f | \hat{T} | p^+, s^+; p^-, s^- \rangle}$$

(technically, |ME|^2 is the weight of each phase space point)

tools to calculate ME (thank K.Fujii for C++ version!)

original fortran version by H. Murayama, etc.



Helicity Amplitudes: HELAS

External Lines

$IXXXXX(p, m, \lambda, \pm 1, \Psi_i)$

4-momentum
helicity
particle
spinor
mass
anti-particle

$OXXXXX(p, m, \lambda, \pm 1, \Psi_o)$

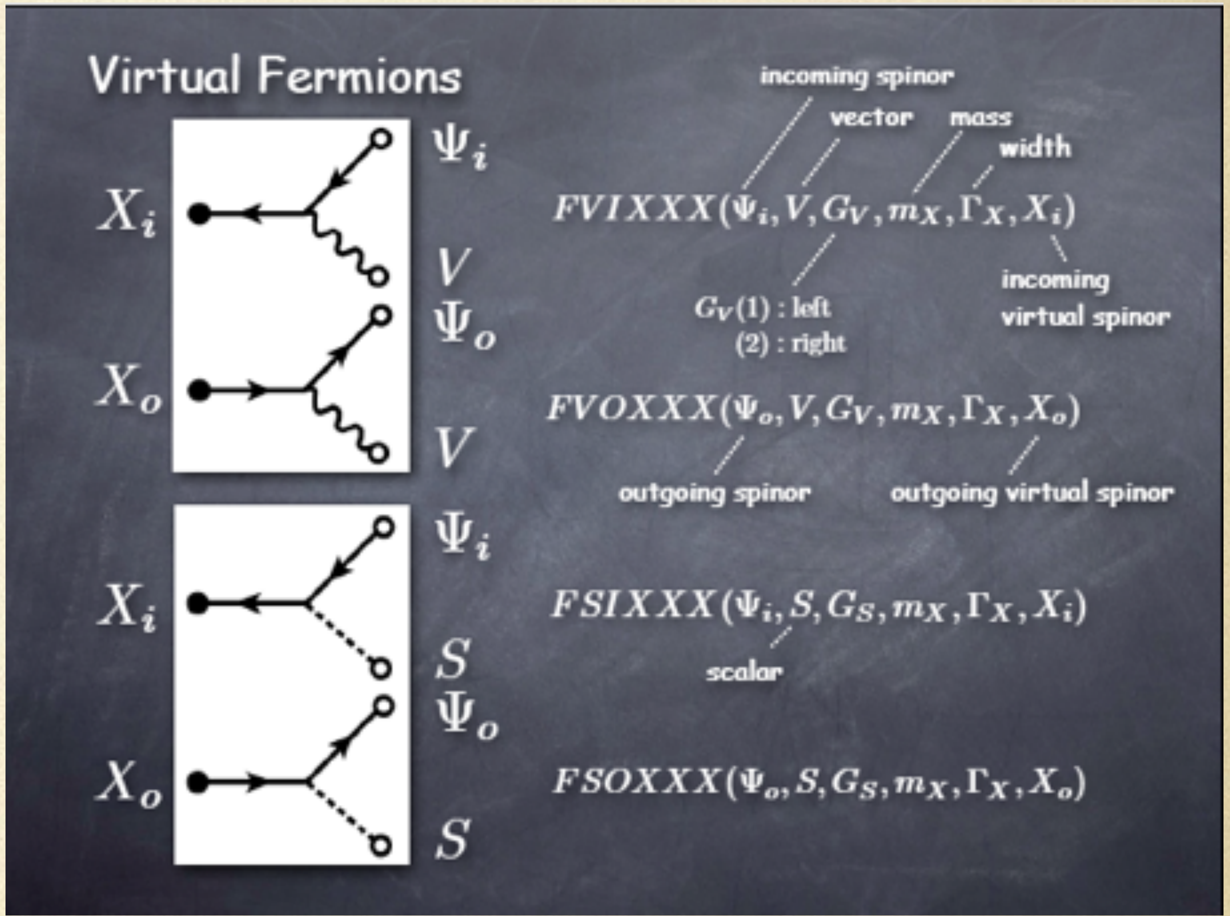
Currents

$JIOXXX(\Psi_i, \Psi_o, G_V, m_V, \Gamma_V, V)$

incoming spinor
outgoing spinor
width
$G_V(1)$: left
(2) : right
mass
wave fun.

$HIOXXX(\Psi_i, \Psi_o, G_S, m_S, \Gamma_S, S)$

wave fun.



Virtual Fermions

$FVIXXX(\Psi_i, V, G_V, m_X, \Gamma_X, X_i)$

incoming spinor
vector
mass
width
$G_V(1)$: left
(2) : right
incoming virtual spinor

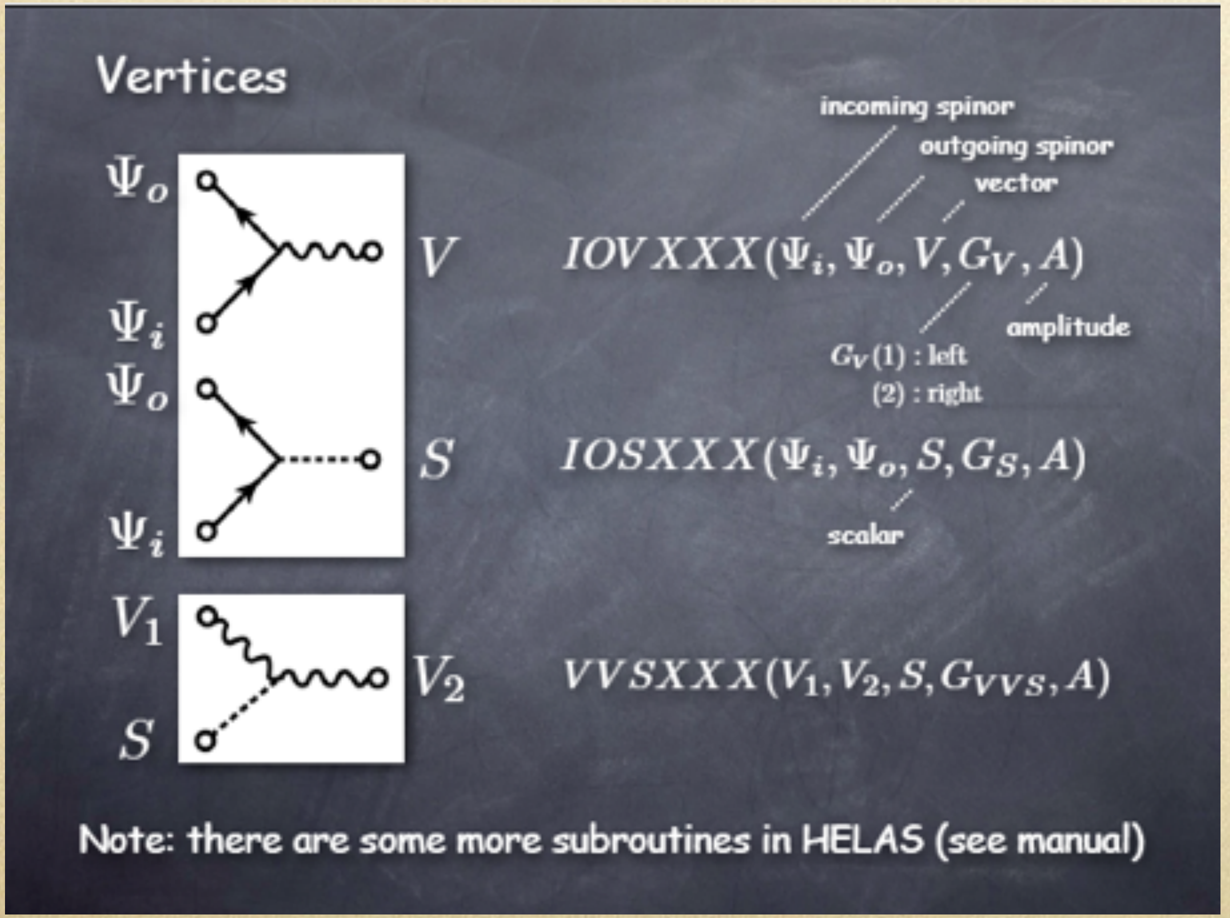
$FVOXXX(\Psi_o, V, G_V, m_X, \Gamma_X, X_o)$

outgoing spinor
outgoing virtual spinor

$FSIXXX(\Psi_i, S, G_S, m_X, \Gamma_X, X_i)$

scalar

$FSOXXX(\Psi_o, S, G_S, m_X, \Gamma_X, X_o)$



Vertices

$IOVXXX(\Psi_i, \Psi_o, V, G_V, A)$

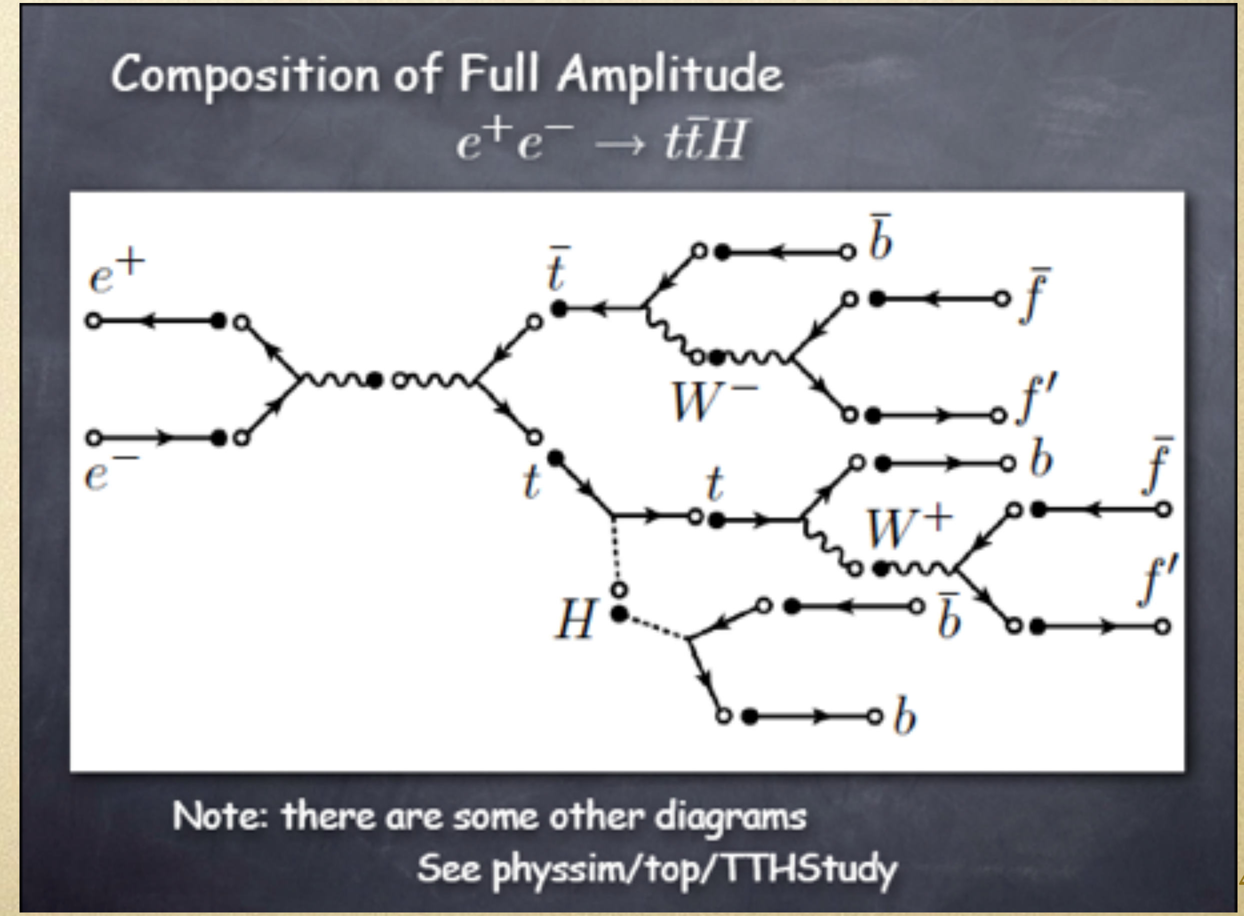
incoming spinor
outgoing spinor
vector
$G_V(1)$: left
(2) : right
amplitude

$IOSXXX(\Psi_i, \Psi_o, S, G_S, A)$

scalar

$VVSXXX(V_1, V_2, S, G_{VVS}, A)$

Note: there are some more subroutines in HELAS (see manual)



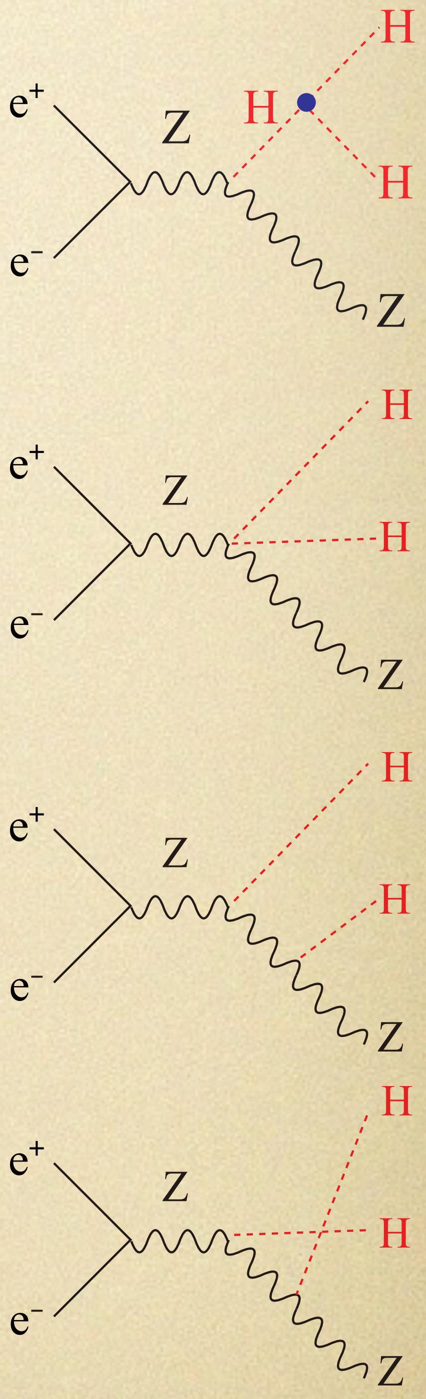
Composition of Full Amplitude

$e^+e^- \to t\bar{t}H$

Note: there are some other diagrams
See physsim/top/TTHStudy

example core code to calculate matrix element

```
//---------------------------
// Double Higgs Production Amplitude
//————————————
HELFermion em(fK[0], kM_e, fHelInitial[0], +1, kIsIncoming);
HELFermion ep(fK[1], kM_e, fHelInitial[1], -1, kIsOutgoing);
HELScalar  h1(fP[0]);
HELScalar  h2(fP[1]);
HELFermion f (fP[2], fM[2], fHelFinal [2], +1, kIsOutgoing);
HELFermion fb(fP[3], fM[3], fHelFinal [3], -1, kIsIncoming);

Double_t v     = 2.*kM_w/kGw;
Double_t ghhh  = -TMath::Power(fMass,2)/v*3.;
Double_t gzzh  = kGz*kM_z;
Double_t gzzhh = kGz*kGz/2.;

HELVector zf(fb, f, glz, grz, kM_z, gamz);
HELVector zs(em, ep, glze, grze, kM_z, gamz);

HELScalar hh(h1, h2, ghhh, fMass, 0.);
HELVertex amp1(zs, zf, hh, gzzh);         // HHH self-coupling

HELVertex amp2(zs, zf, h1, h2, gzzhh);    // ZZHH 4-point

HELVector vz1(zf, h1, gzzh, kM_z, gamz);
HELVertex amp3(zs, vz1, h2, gzzh);        // double H-strahlung

HELVector vz2(zf, h2, gzzh, kM_z, gamz);
HELVertex amp4(zs, vz2, h1, gzzh);        // double H-strahlung

Complex_t amp  = amp1 + amp2 + amp3 + amp4;
```



develop ME tools within Marlin

- idea: provide some libraries to calculate ME for specified channels where only four momentum of final state particles need be fed.
- core libraries implemented: HELLib (C++ HELAS, helicity amplitude subroutines for feynman diagrams).
- LCME (linear collider matrix element libs), so far major Higgs production implemented: LCMEZH, LCMENNH, LCMEEEH, LCMEZHH, LCMENNHH, LCMEEEZ.
- verified by using MC truth information.
- v1.0 released, available now on svn.
- typical way to use it: check out; compile; include libPhyssim.so in your $MARLIN_DLL; using namespace lcme; follow example marlin processor (included in the package).

svn co https://svnsrv.desy.de/basic/physsim/Physsim/trunk

example code in your marlin processor

```
  // initialize LCMEZHH with Higgs mass of 125 GeV and beam
polarisations P(e-,e+) = (0.,0.)
  _zhh = new LCMEZHH("LCMEZHH","ZHH",125.,0.,0.);
  // set mode of Z decay
  _zhh->SetZDecayMode(5);
```

```
  // ----------------------------------
  // calculate the matrix element
  // ----------------------------------
  // put four-momenta of final states to an array
  TLorentzVector vLortzMC[4] = {lortzLep1MC, lortzLep2MC, lortzH1MC, lortzH2MC};
  // pass kinematics to ME object
  _zhh->SetMomentumFinal(vLortzMC);
  // matrix element can be given for each combination of initial and final helicities
  Int_t vHelLL[2] = {-1,-1};
  Int_t vHelLR[2] = {-1,1};
  Int_t vHelRL[2] = {1,-1};
  Int_t vHelRR[2] = {1,1};
  Double_t dSigmaLL = _zhh->GetMatrixElement2(vHelLL);
  Double_t dSigmaLR = _zhh->GetMatrixElement2(vHelLR);
  Double_t dSigmaRL = _zhh->GetMatrixElement2(vHelRL);
  Double_t dSigmaRR = _zhh->GetMatrixElement2(vHelRR);
  // if no combination of helicities specified, final combinations are summed
  // and initial combinations are weighted by beam polarisations
  Double_t dSigma    = _zhh->GetMatrixElement2();
  // that's all need to do to get matrix elment for each event
  // ----------------------------------
```

Detector Effect

- unfortunately, the four momentum we measured have resolution —> we need detector transfer function (jet-energy resolution, momentum resolution, etc.) and integrate all possible truth four momentum.

- and even worse, some four momentum can not be measured (missing neutrinos) —> integrate all possible truth four momentum.

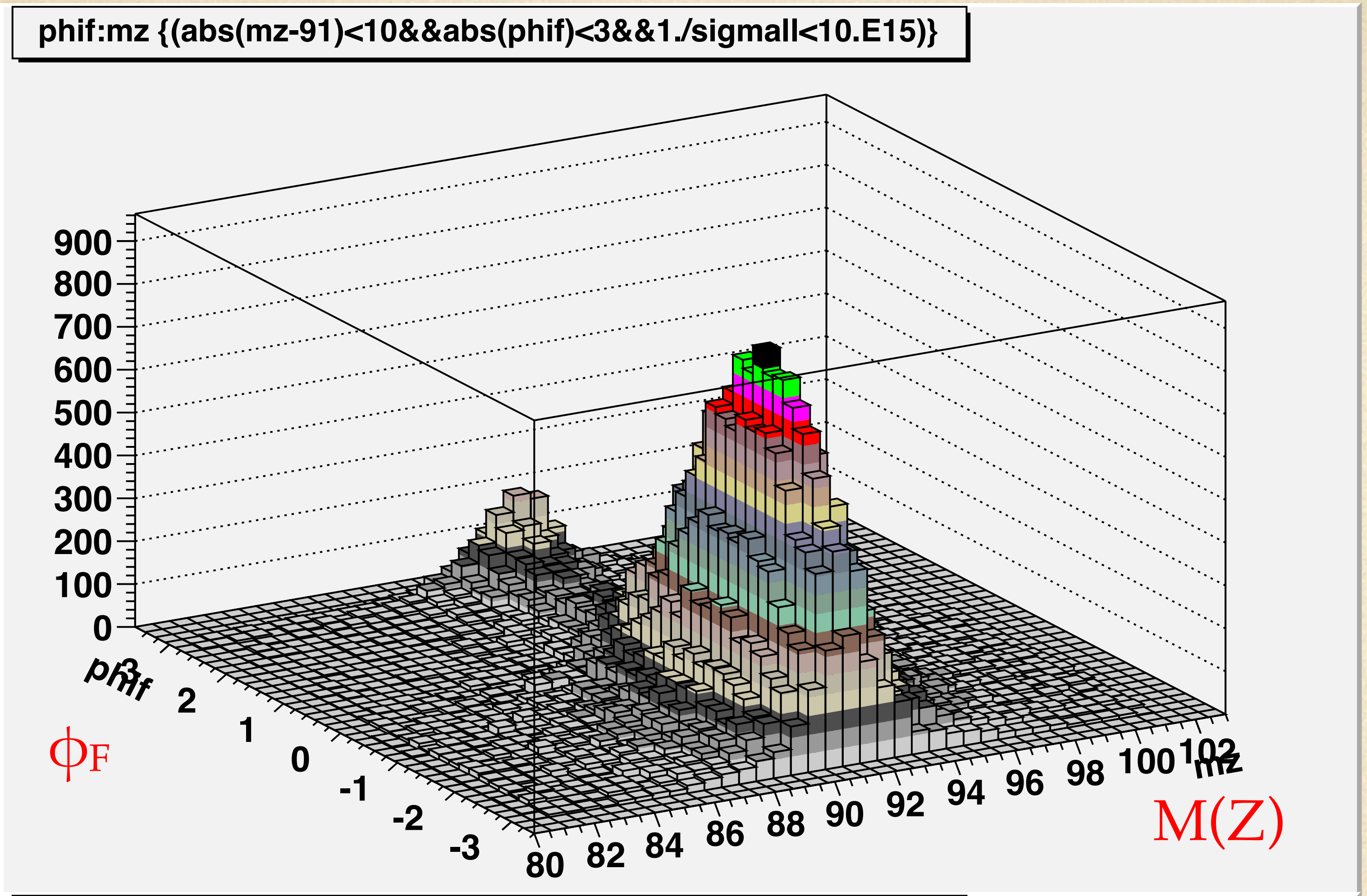
$$L(\mathbf{p}_i^{\mathrm{vis}}|\mathbf{a}) = \frac{1}{\sigma_{\mathbf{a}}}[\underline{\prod_{j\in\mathrm{inv.}}\int\frac{d^3p_j}{(2\pi)^3 2E_j}}][\underline{\prod_{k\in\mathrm{vis.}}\int\frac{d^3p_k}{(2\pi)^3 2E_k}W_i(\mathbf{p}_i^{\mathrm{vis}}|p_k,\mathbf{a})]}|M(p_j,p_k;\mathbf{a})|^2$$

verification

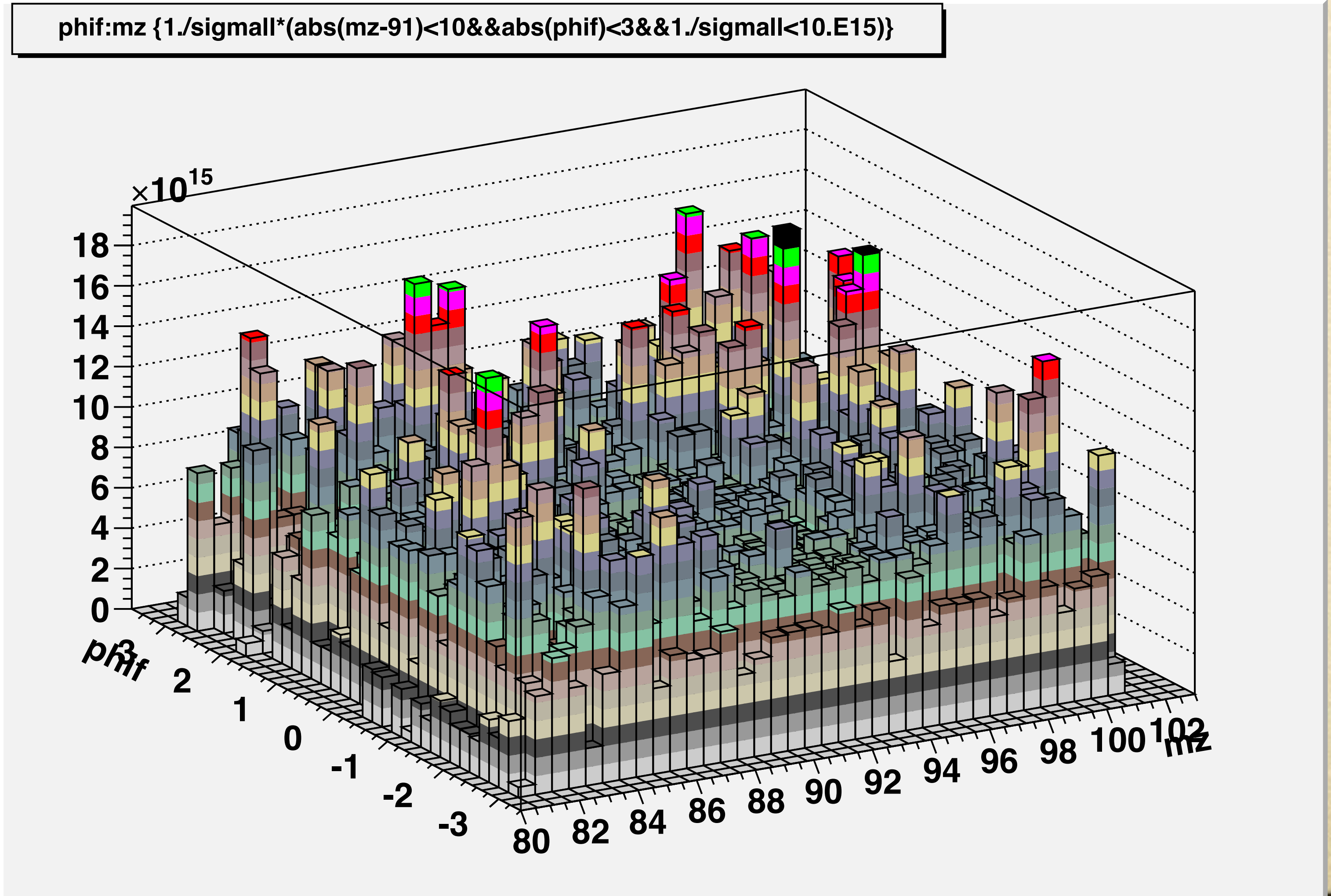
$$L(\mathbf{p}_i^{\mathrm{vis}}|\mathbf{a}) = \frac{1}{\sigma_{\mathbf{a}}}[\prod_{j\in\mathrm{inv.}}\int\frac{d^3p_j}{(2\pi)^3 2E_j}][\prod_{k\in\mathrm{vis.}}\int\frac{d^3p_k}{(2\pi)^3 2E_k}W_i(\mathbf{p}_i^{\mathrm{vis}}|p_k,\mathbf{a})]|M(p_j,p_k;\mathbf{a})|^2$$

- stdhep events are generated without any ISR and BS, helicity combinations of both initial and final states can be controlled.
- detector transfer function ($W(p_i | p_k, a)$) become a delta function, and no invisible variables. (test with MC information).
- ME calculated for each event in this way should be exactly as same as the ME used in event generation (as event weights).
- to verify the calculated ME, if each event is weighted by (1./ | ME | ^{2}), all variables should be uniformly distributed.

verification: ZHH



original events

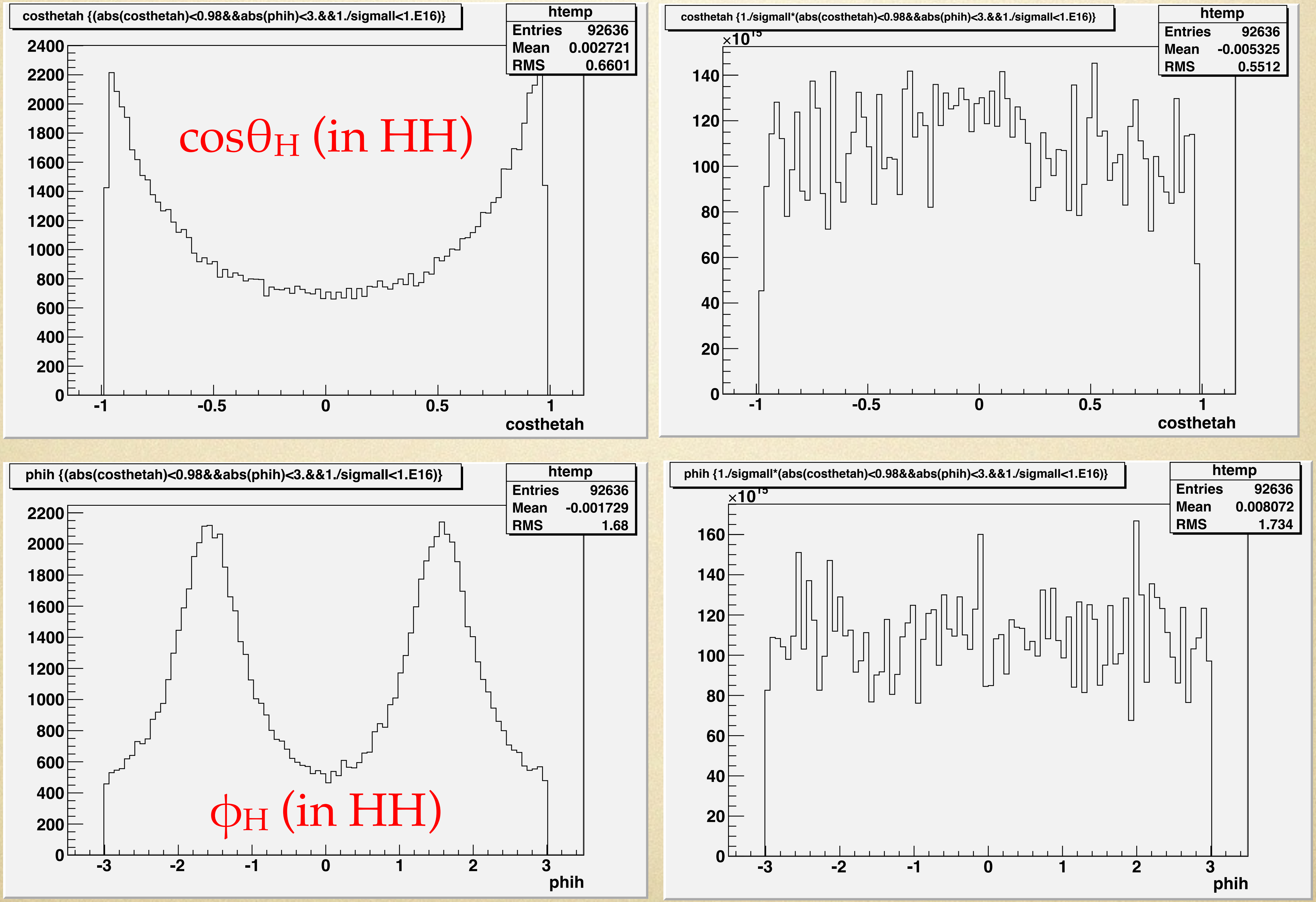


weighted by $\frac{1}{|\mathrm{ME}|^2}$

verification:
$\nu\nu HH$

original events

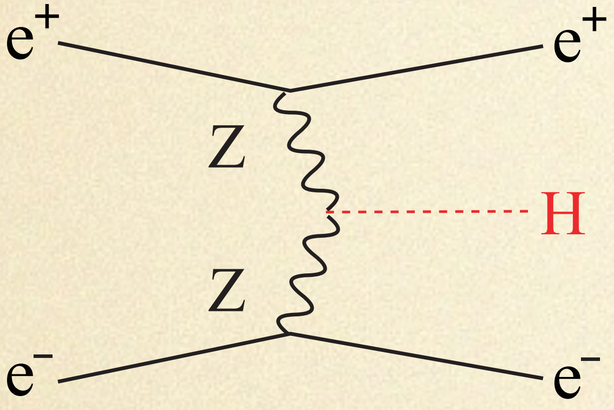
weighted by $\frac{1}{|ME|^2}$

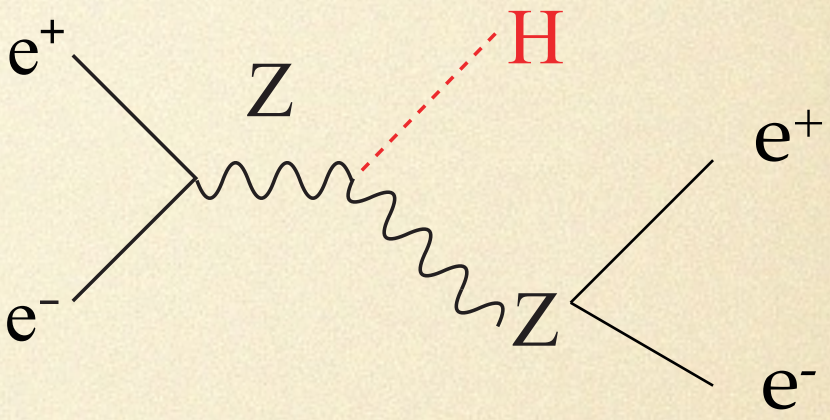


$\cos\theta_H$ (in HH)

ϕ_H (in HH)

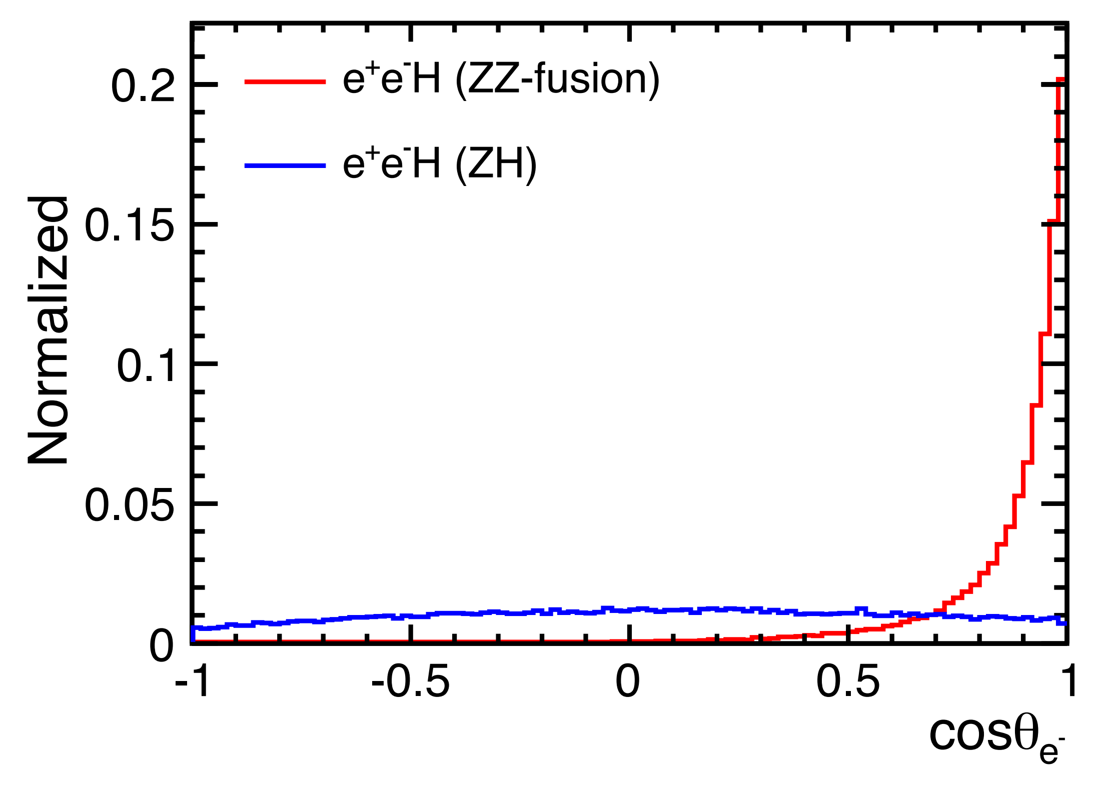
first application: e+e- —> e+e-H

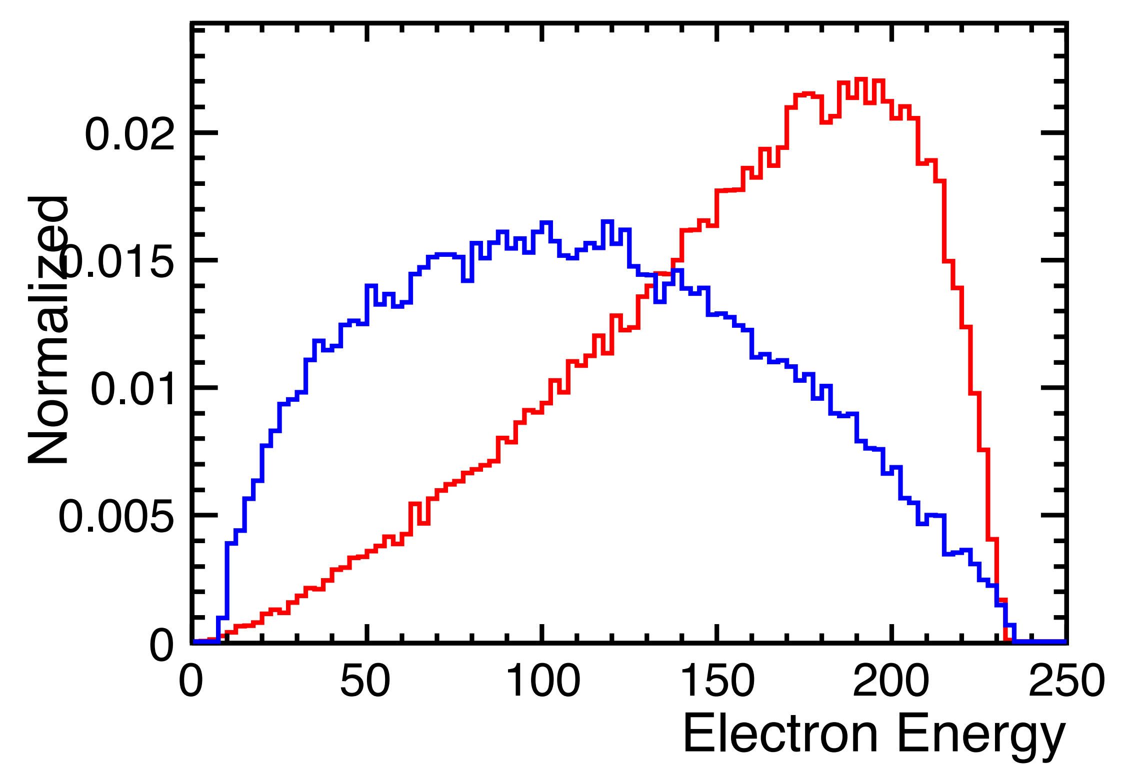


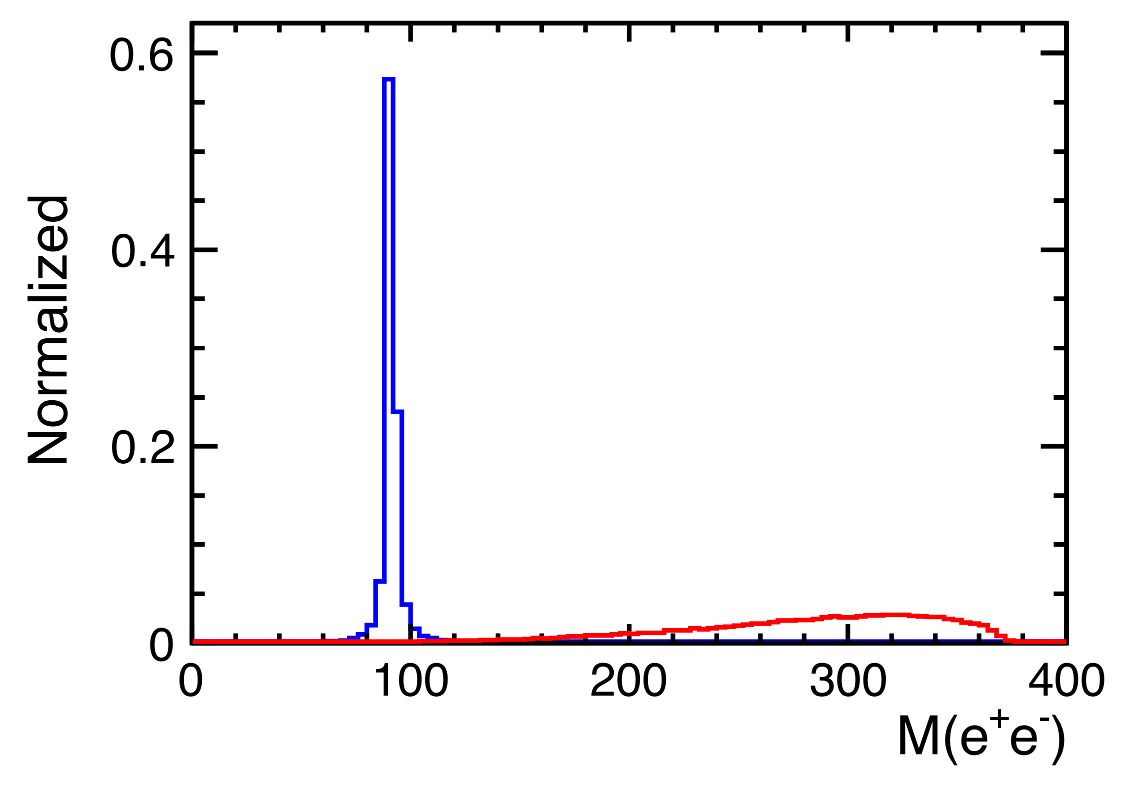


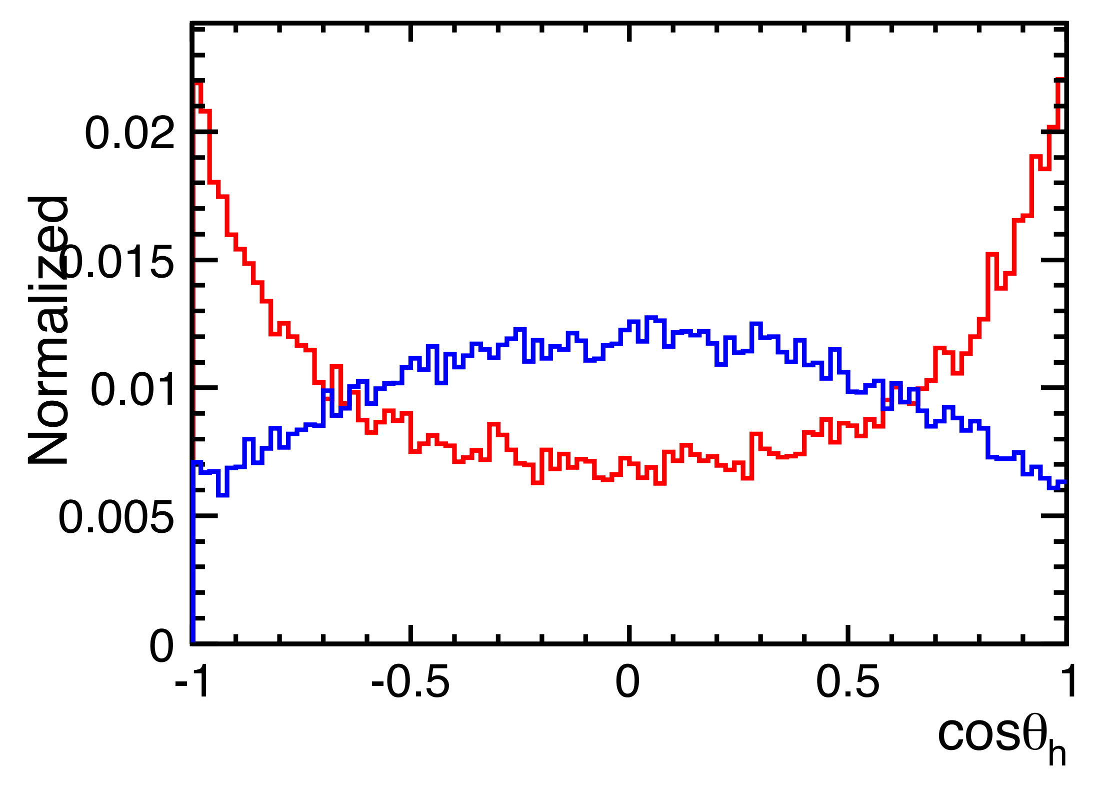
- to discover the Higgs production channel via ZZ-fusion.
- provide another HZZ coupling measurement, important at high energies (σ_S=0.7/7.5/22.8 fb @ 250/500/1000 GeV).
- crucial to discriminate events via ZH production.
- relatively straightforward to apply ME method, since kinematics can be fully reconstructed! and LCMEEEH and LCMEZH are ready.
- a new analysis @ ILC is being setup, samples are generated with DBD softwares, with restrictions in generators.

some distributions



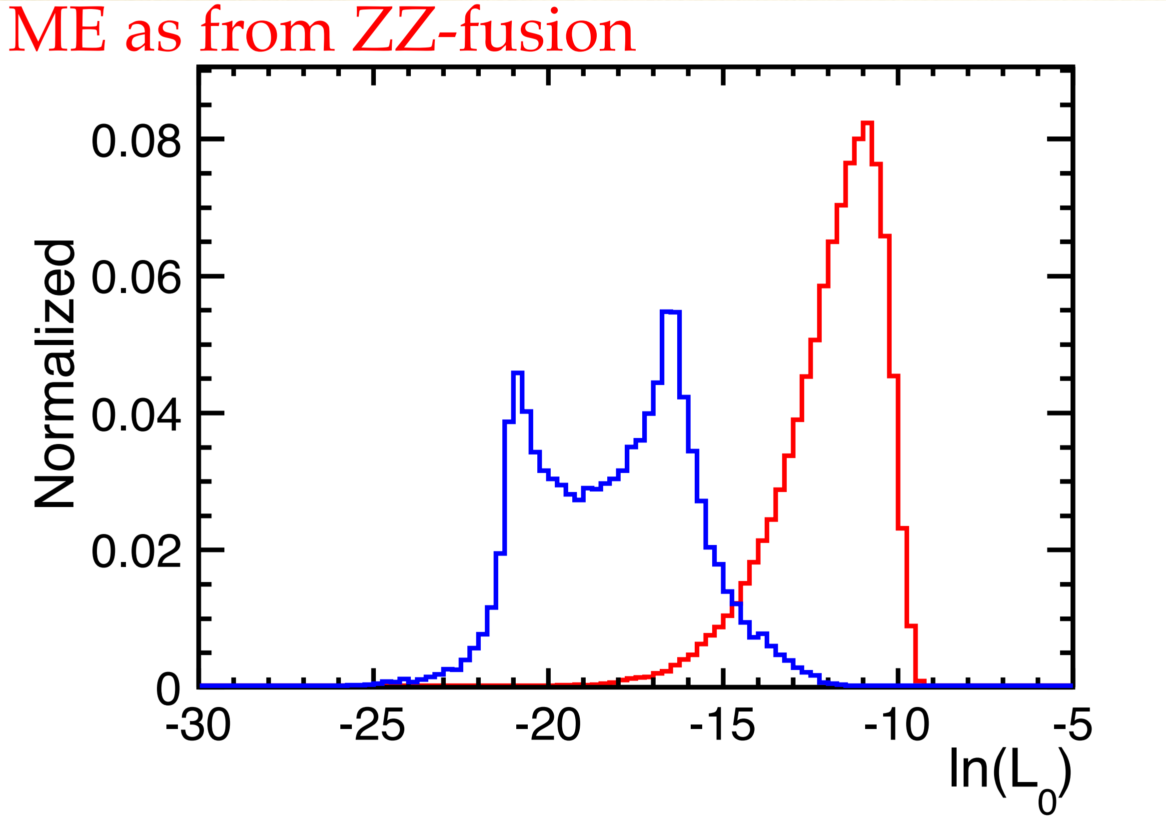




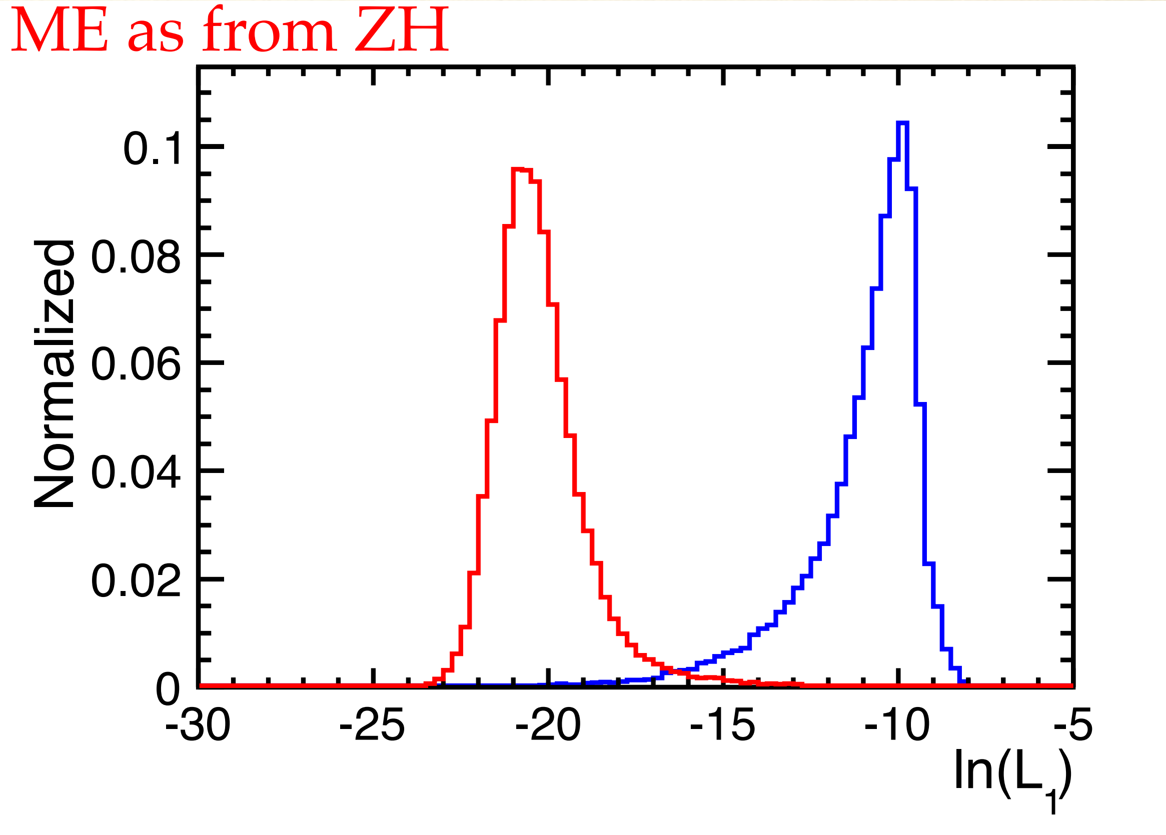


matrix elements (generator level)

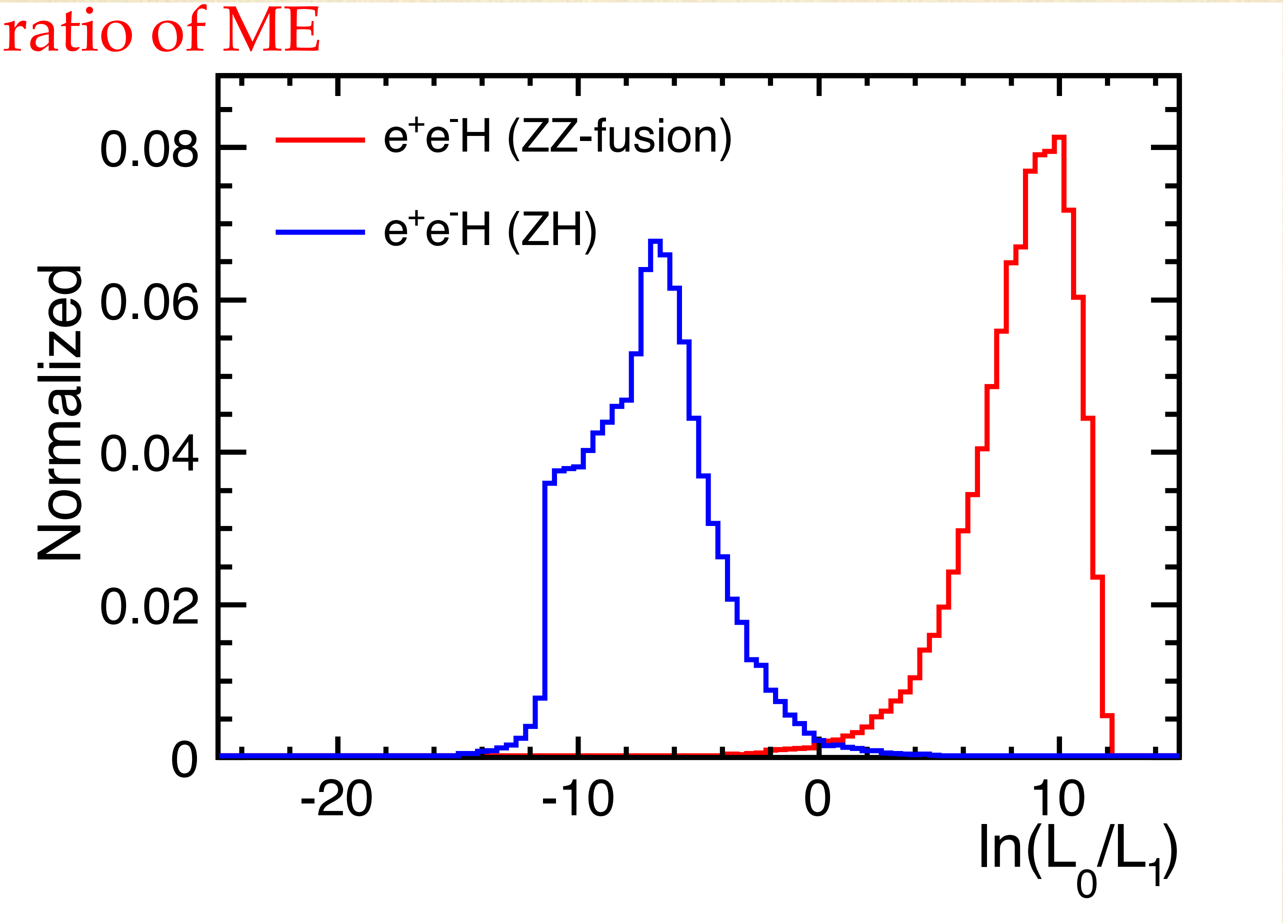
ME as from ZZ-fusion



ME as from ZH



ratio of ME



L0: ME from ZZ-fusion

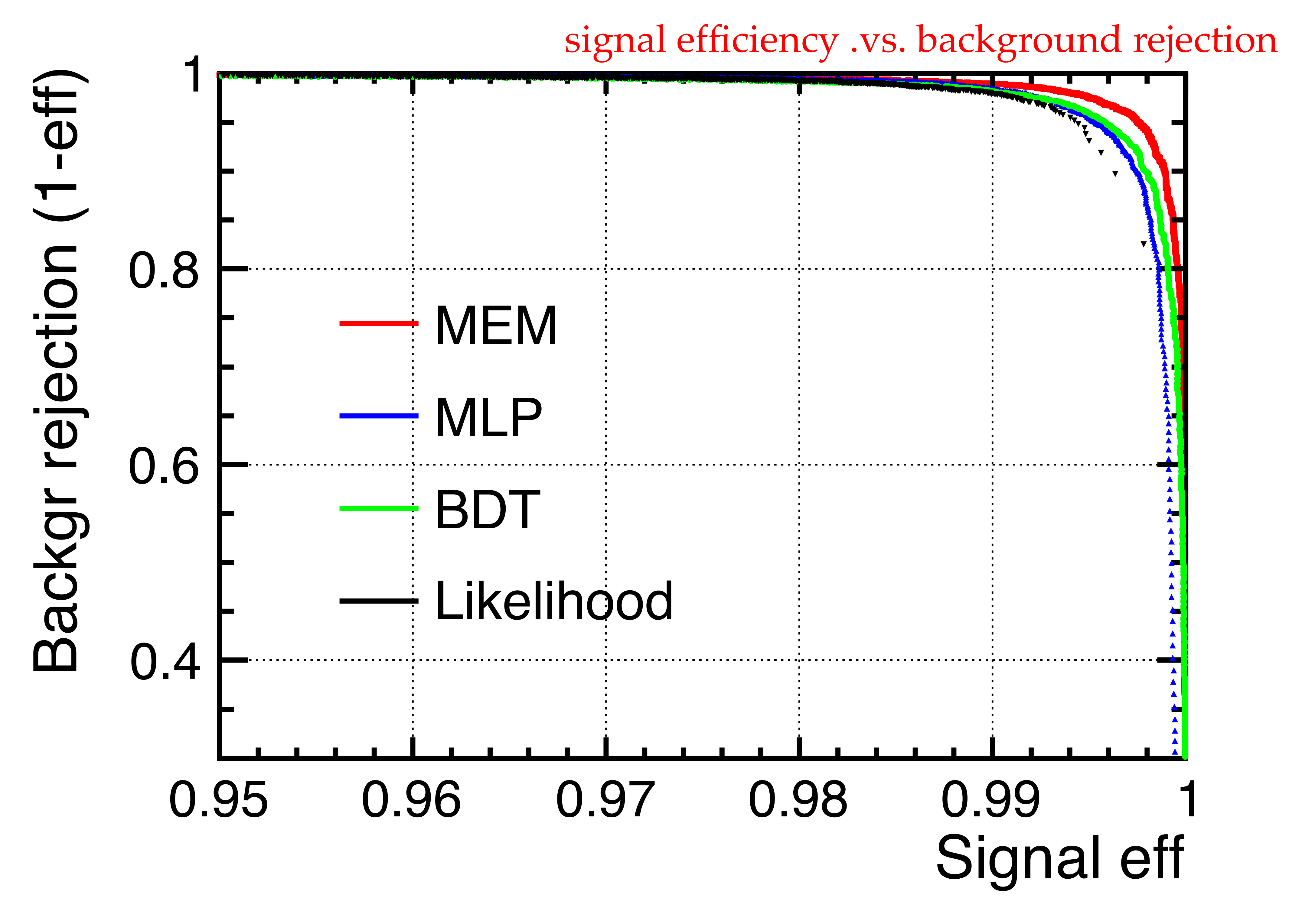
L1: ME from ZH

(Neyman-Pearson lemma)

one question: would MEM be better than other MVA methods such as MLP, BDT, Likelihood?

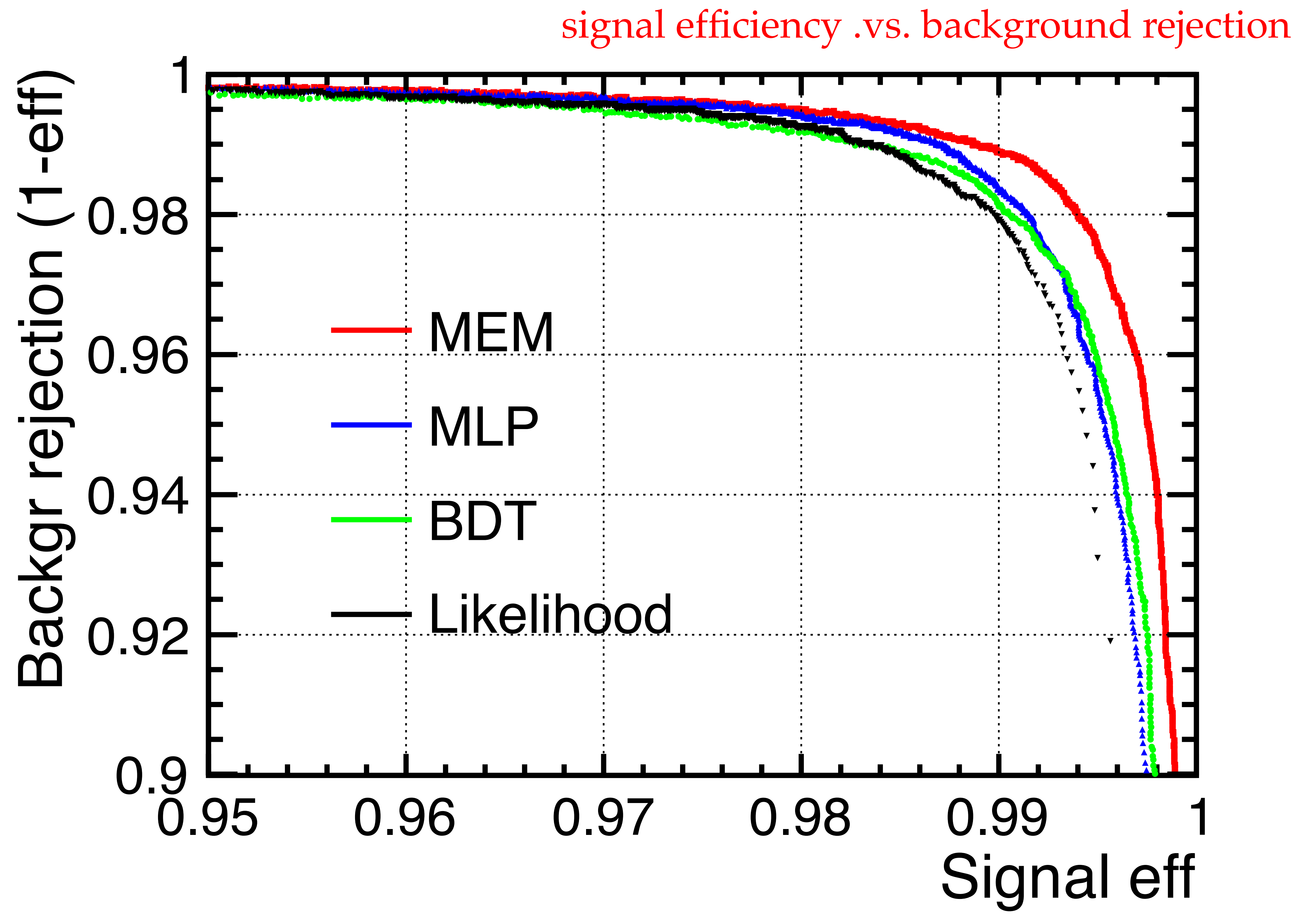
- principle experiment: we can do this test based on generator information.
- degree of freedom: ECM rest-frame (ecm,$\cos\theta_H$,ϕ_H); Z rest-frame (mZ,$\cos\theta_F$,ϕ_F); or (energy electron, $\cos\theta_e$,ϕ_e);
- input all possible kinematics, but, not to confuse usual MVA training too much, remove some variables which have almost same distributions, such as some phi distributions.

physicist .vs. statistician



all methods look impressive, excellent background rejection!

physicist .vs. statistician



it's not actually a surprise since true likelihood is only known by physicist; but it's still impressive that usual MVA work so perfectly without any knowledge of physics!

$e^+ + e^- \to e^+e^-H \to e^+e^-b\bar{b}$

full simulation @ 500GeV
samples with DBD software

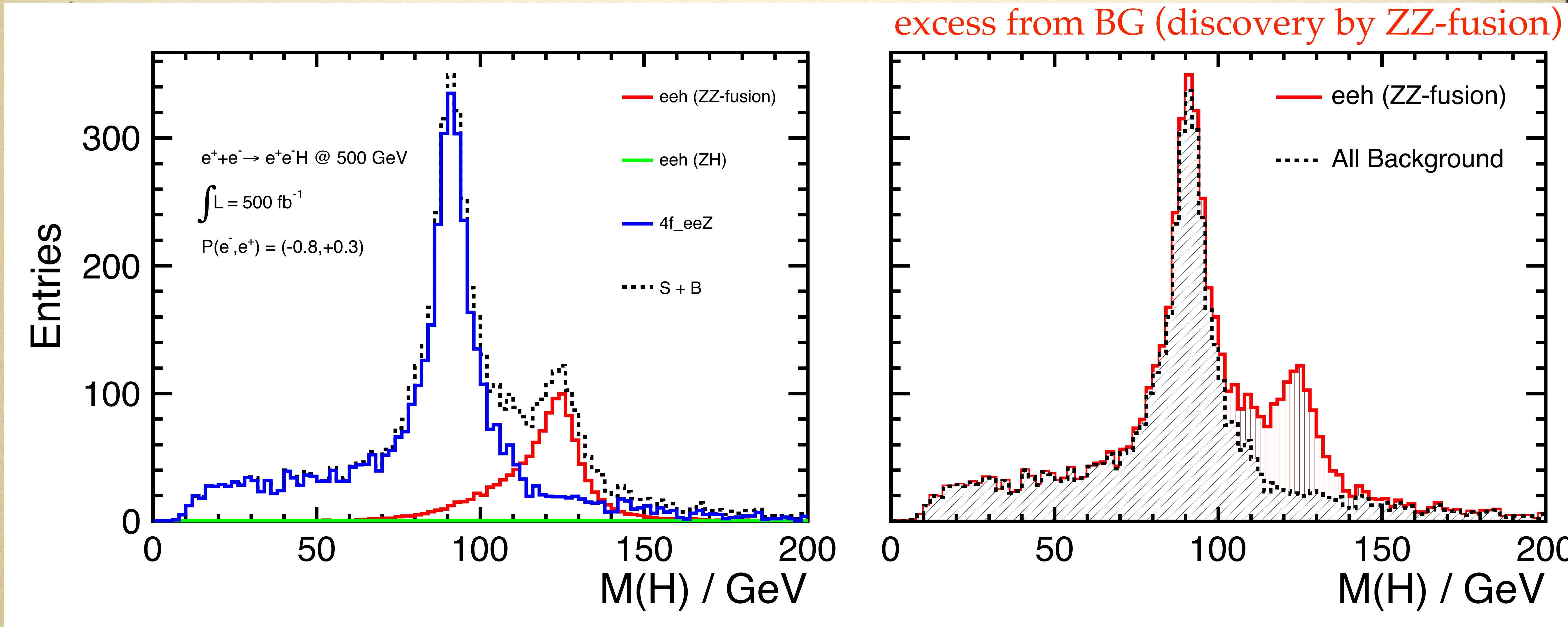
pre-selection:

- select two isolated electron w/ BS and FSR recovered (MVA)
- fastjet (kt) algorithm to remove the very forward overlaid particles
- two jets clustering and flavor tagging

final-selection:

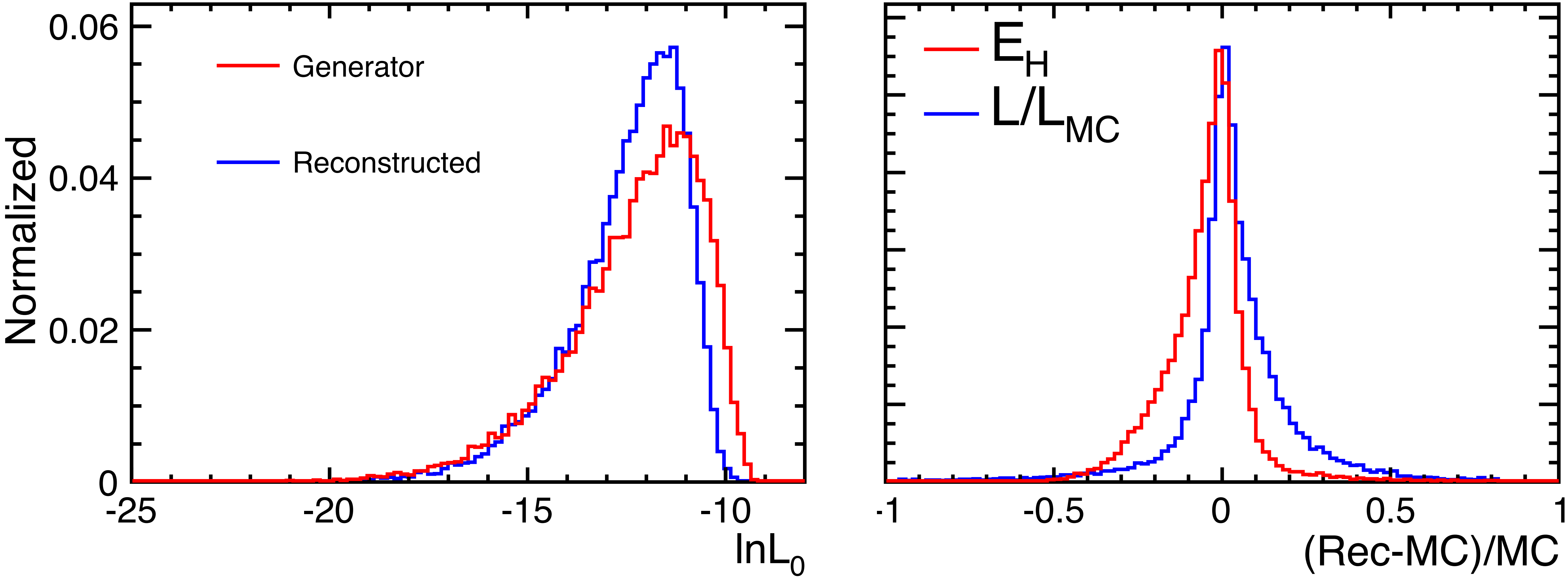
- more strict cuts on electrons: MVA>0.8, $P_1+P_2>200$, cosθ12<-0.14 (cut1)
- two-jets: Npfos > 8 (cut2)
- PtH > 10, MissPt < 60 (cut3)
- flavor-tagging: btag1 + 2*btag2 > 1.0 (cut4)
- Higgs mass: (105, 155), recoil mass > 110 (cut5)

Higgs Mass



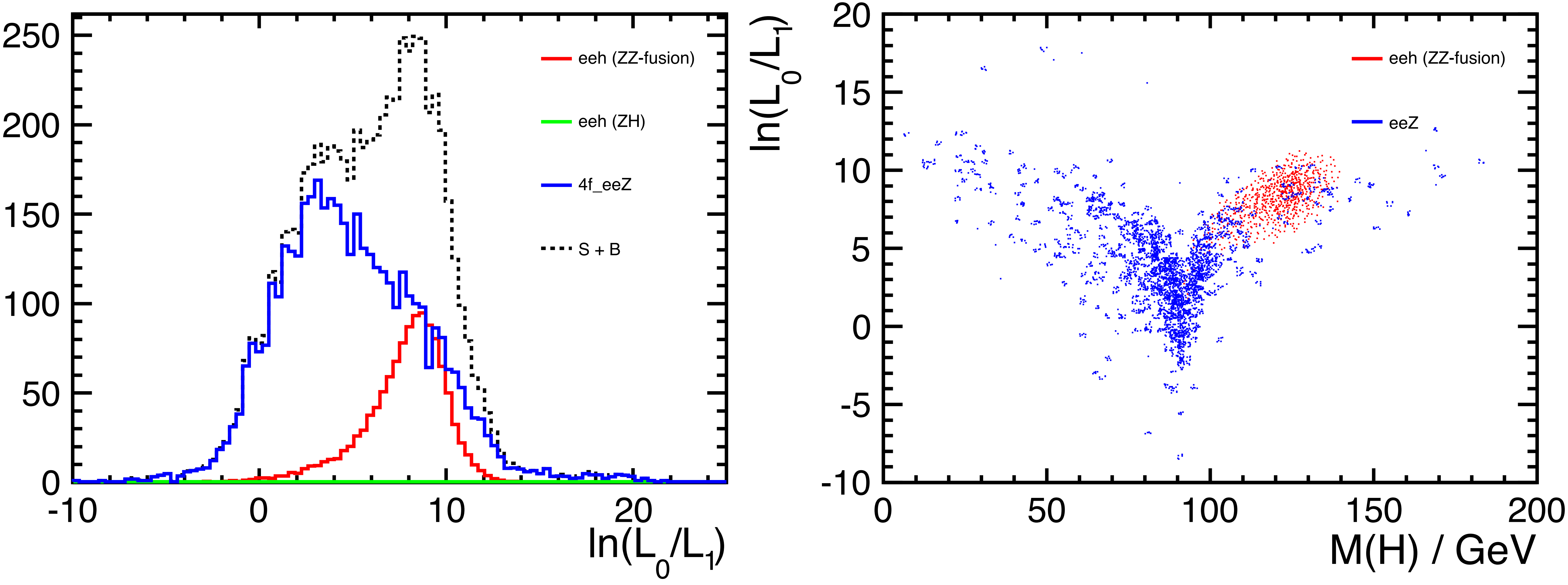
statistics of BG after all cuts is not enough, plots are scaled from those before flavor-tagging cuts.

Matrix Element comparison with MC truth



not so bad, thanks to our state-of-art detector!

Matrix Element with BG eeZ



H decay needs be implemented

signal and backgrounds (reduction table)

$$e^+ + e^- \to e^+e^-H \to e^+e^-b\bar{b}$$

Polarization: (e-,e+)=(-0.8,+0.3) $E_{\rm cm} = 500{\rm GeV}, M_H = 125{\rm GeV}$ $\int {\rm L} = 500\ {\rm fb}^{-1}$

preliminary!

	Expected	pre-selction	cut1	cut2	cut3	cut4	cut5
eeh (fusion)	3.74×10	2685	2313	1930	1886	1179	935 (918)
eeh (ZH)	1.72×10	1541	30.7	8.8	7.6	3.2	2.5
eeZ	4.88×10	301261	115353	28073	26601	4261	300
BG	2.78×10	3.72×10	1.38×10	2.97×10	2.72×10	4471	359
significance	0.41	2.5	3.6	7.2	7.4	15.3	25.5

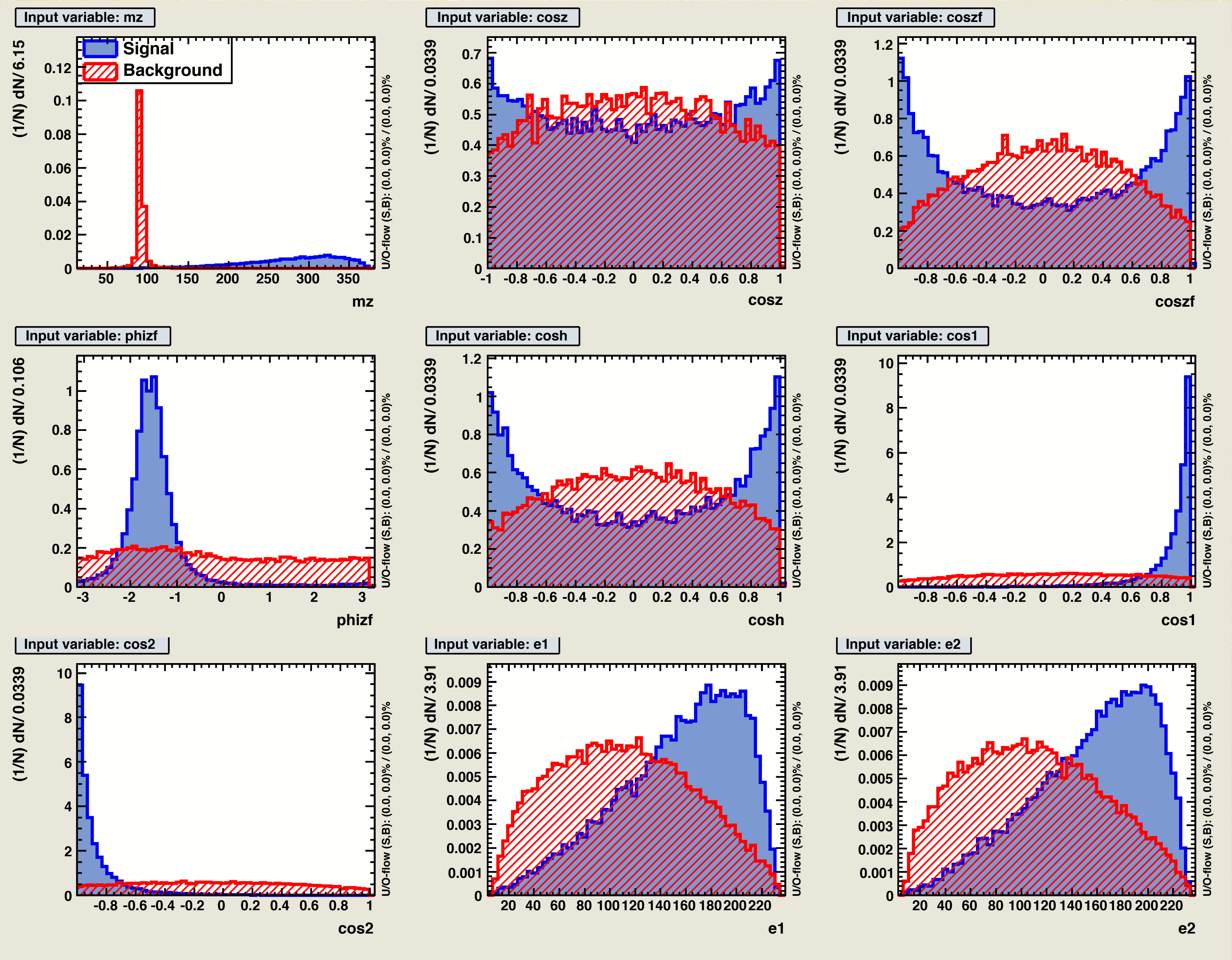
$$\frac{\delta(\sigma_{eeH} \cdot {\rm Br}(H \to b\bar{b}))}{\sigma \cdot {\rm Br}} = 3.9\%$$

summary and next step

- so far ME tools look like working well, but still more details need be checked by various analyses; welcome to test it.
- challenging technics next is how to integrate ME efficiently in analysis with detector transfer function; welcome to join the effort.
- essentially MEM can be applied to all other analyses; ultimate goal is to apply for Higgs self-coupling analysis; recently we found promising application for recoil mass study.

back up

input to MVA



some experience (adjusting input variables):

a.) remove unnecessary input variables (no difference or highly correlated)

b.) BDT is very robust, usually can handle correlations well

c.) Likelihood is very sensitive to very sharp distributions

d.) MLP is sensitive to correlated variables, moderate to sharp distribution.