

Using the Hadronic Recoil Cross Section Measurement in Higgs Coupling Fits

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Mark Thomson's analysis of $\sigma(ZH)$ with $Z \rightarrow q\bar{q}$ uses two measurements to obtain the cross section:

$$\sigma(ZH) = \sigma(ZH) \cdot BR(visible) + \sigma(ZH) \cdot BR(invisible)$$

$$\sigma(ZH) \cdot BR(visible)$$

Model Independent?

- ★ Combining visible + invisible analysis: wanted M.I.
 - i.e. efficiency independent of Higgs decay mode

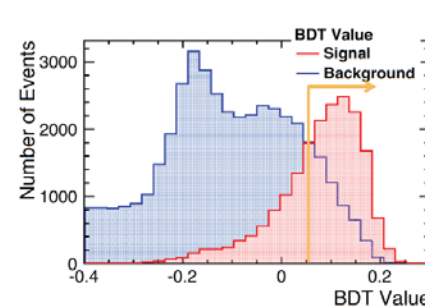
Decay mode	$\epsilon_{\mathcal{L} > 0.65}^{vis}$	$\epsilon_{\mathcal{L} > 0.60}^{vis}$	$\epsilon^{vis} + \epsilon^{invis}$
H \rightarrow invis.	<0.1 %	22.0 %	22.0 %
H $\rightarrow q\bar{q}/gg$	22.2 %	<0.1 %	22.2 %
H $\rightarrow WW^*$	21.6 %	0.1 %	21.7 %
H $\rightarrow ZZ^*$	20.2 %	1.0 %	21.2 %
H $\rightarrow \tau^+\tau^-$	24.7 %	0.3 %	24.9 %
H $\rightarrow \gamma\gamma$	25.8 %	<0.1 %	25.8 %
H $\rightarrow Z\gamma$	18.5 %	0.3 %	18.8 %

Very similar efficiencies

$$\sigma(ZH) \cdot BR(invisible)$$

BDT Selection

- ★ Preliminary results (7 variable BDT selection)



Signal		
Channel	Efficiency	
Z H \rightarrow qq invis.	20.7 %	
Backgrounds		
Channel	Efficiency	Events
qqlv	<0.1 %	900
qqll	<0.1 %	4
qqvv	1.5 %	2414

- ★ Assuming no invisible decays (1 sigma stat. error):

$$\Rightarrow \Delta\sigma_{invis} = \pm 0.57 \%$$

(CLIC beam spectrum, 500 fb⁻¹ @ 350 GeV, no polarisation)

In order to use this cross section measurement in our Higgs analyses we have to quantify the penalty associated with the fact that $\sigma(ZH) \cdot BR(visible)$ is "almost model independent". By how much must we blow up $\Delta\sigma(ZH) \cdot BR(visible)$ to account for the fact that the efficiencies differ by as much as 7%?



Model Independent?



- ★ **Combining visible + invisible analysis: wanted M.I.**
 - **i.e. efficiency independent of Higgs decay mode**

Decay mode	$\epsilon_{\mathcal{L}>0.65}^{\text{vis}}$	$\epsilon_{\mathcal{L}>0.60}^{\text{vis}}$	$\epsilon^{\text{vis}} + \epsilon^{\text{invis}}$
H \rightarrow invis.	<0.1 %	22.0 %	22.0 %
H \rightarrow q \bar{q} /gg	22.2 %	<0.1 %	22.2 %
H \rightarrow WW*	21.6 %	0.1 %	21.7 %
H \rightarrow ZZ*	20.2 %	1.0 %	21.2 %
H \rightarrow $\tau^+\tau^-$	24.7 %	0.3 %	24.9 %
H \rightarrow $\gamma\gamma$	25.8 %	<0.1 %	25.8 %
H \rightarrow Z γ	18.5 %	0.3 %	18.8 %
H \rightarrow WW* \rightarrow q \bar{q} q \bar{q}	21.3 %	<0.1 %	21.3 %
H \rightarrow WW* \rightarrow q \bar{q} lv	21.9 %	<0.1 %	21.9 %
H \rightarrow WW* \rightarrow q \bar{q} $\tau\nu$	22.1 %	<0.1 %	22.1 %
H \rightarrow WW* \rightarrow lvlv	24.8 %	0.1 %	25.0 %
H \rightarrow WW* \rightarrow lv $\tau\nu$	20.5 %	0.8 %	22.1 %
H \rightarrow WW* \rightarrow $\tau\nu\tau\nu$	16.4 %	2.5 %	18.9 %

Very similar efficiencies

Look at wide range of WW topologies

We have used an approach where we use all of our $\sigma \cdot BR$ measurements for visible Higgs decays to obtain an estimate of the average signal efficiency for $\sigma(ZH) \cdot BR(visible)$. It is then straightforward to propagate the $\sigma \cdot BR$ errors, as well as the systematic errors on the individual decay mode efficiencies for the $\sigma(ZH) \cdot BR(visible)$ selection, to the error on $\sigma(ZH) \cdot BR(visible)$.

Let

$$\Psi \equiv \sigma(ZH) \cdot BR(visible)$$

Ω = Number of signal + background events in $\sigma(ZH) \cdot BR(visible)$ analysis

B = Predicted number of background events in $\sigma(ZH) \cdot BR(visible)$ analysis

Ξ = Average efficiency for signal events to pass $\sigma(ZH) \cdot BR(visible)$ analysis

L = luminosity

$$\Psi = \frac{\Omega - B}{L \Xi} = \frac{1}{\Xi} \sum_i \psi_i \xi_i = \sum_i \psi_i \quad \text{where}$$

$$\psi_i = \sigma(ZH) \cdot BR_i$$

ξ_i = efficiency for events from Higgs decay i to pass $\sigma(ZH) \cdot BR(visible)$ analysis

$$\Xi = \frac{\sum_i \psi_i \xi_i}{\sum_i \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L \eta_i}$$

ω_i = Number of signal + background events in $\sigma(ZH) \cdot BR_i$ analysis

β_i = Predicted number of background events in $\sigma(ZH) \cdot BR_i$ analysis

η_i = efficiency for Higgs decay i to pass $\sigma \cdot BR_i$ analysis

K_i = number of signal + background events common to had Z recoil
and $\sigma \cdot BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis

ε_i = number of signal + background events unique to $\sigma \cdot BR_i$ analysis

$$\Omega = E + \sum_i K_i \quad S \equiv \Omega - B \quad T \equiv \frac{\sqrt{S+B}}{S}$$

$$\omega_i = K_i + \varepsilon_i \quad s_i \equiv \omega_i - \beta_i \quad \tau_i \equiv \frac{\sqrt{s_i + \beta_i}}{s_i}$$

$$\lambda_i \equiv \frac{K_i}{\omega_i} \quad N \equiv L \sigma_{ZH} \quad r_i \equiv BR_i \quad \delta_i \equiv \xi_i - \Xi$$

$$\left(\frac{\Delta \Psi}{\Psi} \right)^2 = T^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 (\delta_i^2 - 2\lambda_i \eta_i \delta_i) + \Delta \xi_i^2 \right] \right\}$$

This is our result for the error on $\sigma(ZH) \cdot BR(\text{visible})$ given the approach outlined on page 4

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = T^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 [\tau_i^2 \delta_i^2 + \Delta\xi_i^2] \right\}$$

Assume $\sqrt{s} = 350$ GeV and $L=500 \text{ fb}^{-1}$

$$N = L \sigma_{ZH} = 45383 \quad r_i = BR_i = (1 - BR_{BSM}) BR_i(SM) \quad \tau_i(SM) = \frac{\Delta\sigma \bullet BR_i(SM)}{\sigma \bullet BR_i(SM)} = \frac{\sqrt{s_i + \beta_i}}{s_i}$$

From Mark Thomson's presentation at the ILD Meeting Oshu City Sep 8, 2014:

$$T = \frac{\sqrt{S+B}}{S} = 0.014 \quad \Omega=S+B = 17738$$

$\xi_i(SM)$ are taken from the table on page 21 of Mark's presentation.

We assume that Mark's vis+invis efficiency values on page 21 cover all possible BSM decays since they cover all SM decays from completely invisible to fully hadronic multi-jet decays. Assuming no knowledge of the properties of the BSM decays we can then set

$$\xi_{BSM} = 0.5 * [\xi_{vis+invis}(\text{max}) + \xi_{vis+invis}(\text{min})] = 0.5 * [0.258 + 0.188] = 0.22$$

$$\Delta\xi_{BSM} = 0.5 * [\xi_{vis+invis}(\text{max}) - \xi_{vis+invis}(\text{min})] = .035$$

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = T^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2 \right] \right\}$$

We next obtain the error $\tau_{BSM} = \frac{\Delta\sigma \bullet BR_{BSM}}{\sigma \bullet BR_{BSM}}$ from Michael Peskin's Higgs coupling fit program. We do not use the $\sum_i BR_i = 1$ constraint, and to begin with we only use the leptonic recoil σ_{ZH} measurement. This provides a model independent measurement of g_{BSM} . For $\sqrt{s} = 350$ GeV, $L=500 \text{ fb}^{-1}$ Michael's program gives $\frac{\Delta g_{BSM}}{g_{BSM}} = 0.032$ which we multiply by two to get $\tau_{BSM} = \frac{\Delta\sigma \bullet BR_{BSM}}{\sigma \bullet BR_{BSM}} = 0.064$. **We assume that $r_{BSM}(\text{true}) = 0$** and therefore set the measured $r_{BSM} = \tau_{BSM} = 0.064$. This gives a model independent $\frac{\Delta\Psi}{\Psi} = 0.014 * 1.27 = 0.018$.

We then add this new model independent hadronic recoil σ_{ZH} measurement as input to Michael's program to obtain a new error $\tau_{BSM} = 0.041$. Setting $r_{BSM} = \tau_{BSM} = 0.041$ we then obtain a new model independent hadronic recoil σ_{ZH} error of $\frac{\Delta\Psi}{\Psi} = 0.014 * 1.12 = 0.016$.

Iterating again we arrive at $r_{BSM} = \tau_{BSM} = 0.039$ and $\frac{\Delta\Psi}{\Psi} = 0.014 * 1.11 = 0.016$. Further iterations don't give any improvement. Our best model independent hadronic recoil cross section error is $\Delta\sigma_{ZH} = 0.016$.

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = T^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2 \right] \right\}$$

We have shown that $\frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2 \right] = 0.11$ for $\sqrt{s} = 350$ GeV, $L=500 \text{ fb}^{-1}$.

How does this scale with luminosity?

$$\frac{N^2}{\Omega} \propto L \quad \tau_i^2 \propto L^{-1} \quad r_i^2 \text{ is independent of lumi except } r_{BSM}^2 = \tau_{BSM}^2 \propto L^{-1}.$$

If we assume $\Delta\xi_i = 0$ except $\Delta\xi_{BSM} = 0.035$ then

$$\frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2 \right] = 0.11 \text{ independent of the luminosity at } \sqrt{s} = 350 \text{ GeV.}$$

Caveats for hadronic recoil systematic error calculation :

These results assume that the true $r_{BSM} = BR(H \rightarrow BSM) = 0$.

As the true r_{BSM} grows we need to keep the product $r_{BSM} \Delta \xi_{BSM}$ constant to maintain the same systematic error. For example

true r_{BSM} required $\Delta \xi_{BSM}$

.05 0.027

.10 0.014

.15 0.0091

.20 0.0068

These $\Delta \xi_{BSM}$ requirements may seem stringent for the larger values of true r_{BSM} . However as r_{BSM} grows we will have more BSM decays to analyze and the required improvement in Monte Carlo modelling of the BSM decays should follow.

1st Five Years of ILC Running

Model Independent Higgs Couplings $\Delta g_i/g_i$

\sqrt{s} L	Scenario B	Scenario D-500	
	250 GeV 360 fb ⁻¹	350 GeV 470 fb ⁻¹	
σ_{ZH} meas.	l^+l^- only	l^+l^- only	$l^+l^- + q\bar{q}$
$\gamma\gamma$	14.9 %	11.0	11.0 %
gg	5.2 %	3.3	3.1 %
WW	4.0 %	1.7	1.0 %
ZZ	1.1 %	1.5	0.72 %
$b\bar{b}$	4.4 %	2.4	2.0 %
$\tau^+\tau^-$	4.7 %	3.0	2.8 %
$c\bar{c}$	5.6 %	4.1	3.9 %
$\Gamma_T(h)$	9.6 %	7.1	4.9 %

Further improvement in the Higgs coupling measurements can be obtained using the constraint $\sum_i BR_i = 1$ as first noted by Michael Peskin.

This constraint is model independent so long as the error in $BR(H \rightarrow BSM)$ is included in the fit. **What error in $BR(H \rightarrow BSM)$ is required to produce an improvement in Higgs coupling measurements ?**

1st Five Years of ILC Running

Model Independent Higgs Couplings $\Delta g_i/g_i$

\sqrt{s} L	Scenario D-500					
	350 GeV 470 fb ⁻¹					
σ_{ZH} meas.	$l^+l^- + q\bar{q}$	$l^+l^- + q\bar{q}$	$l^+l^- + q\bar{q}$	$l^+l^- + q\bar{q}$	$l^+l^- + q\bar{q}$	$l^+l^- + q\bar{q}$
$BR(H \rightarrow BSM)$ (95% CL)	no meas.	<7.2%	<3.6%	<1.8%	<0.9%	<0.09%
$\gamma\gamma$	11.0 %	10.9 %	10.9 %	10.9 %	10.9 %	10.9 %
gg	3.1 %	3.0 %	2.9 %	2.9 %	2.9 %	2.9 %
WW	1.0 %	0.94 %	0.82 %	0.71 %	0.67 %	0.65 %
ZZ	0.72 %	0.67 %	0.60 %	0.53 %	0.51 %	0.50 %
$b\bar{b}$	2.0 %	1.8 %	1.6 %	1.5 %	1.4 %	1.4 %
$\tau^+\tau^-$	2.8 %	2.7 %	2.6 %	2.5 %	2.5 %	2.4 %
$c\bar{c}$	3.9 %	3.8 %	3.7 %	3.7 %	3.7 %	3.7 %
$\Gamma_T(h)$	4.9 %	4.4 %	3.6 %	2.8 %	2.5 %	2.3 %

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Summary

- The systematic error for the model dependence of Mark Thomson's hadronic recoil Higgstrahlung cross section measurement has been shown to be 11% of the statistical error assuming no knowledge of the properties of any BSM Higgs decays. This result is tailored for the context where $BR(H \rightarrow BSM)$ is small.
- If $BR(H \rightarrow BSM)$ is not small then analysis of BSM decays will improve the error on the efficiency for such events to pass the hadronic recoil analysis. It may be possible to maintain the 11 % systematic error using the improved efficiency error. Of course we have a different Higgs physics program if $BR(H \rightarrow BSM)$ is not small.
- A good understanding of $\sigma \cdot BR(H \rightarrow BSM)$ is required to squeeze the last little bit of model independent Higgs coupling precision out of the data.

Backup Slides

Let

$$\Psi \equiv \sigma(ZH) \cdot BR(visible)$$

Ω = Number of signal + background events in $\sigma(ZH) \cdot BR(visible)$ analysis

B = Predicted number of background events in $\sigma(ZH) \cdot BR(visible)$ analysis

Ξ = Average efficiency for signal events to pass $\sigma(ZH) \cdot BR(visible)$ analysis

L = luminosity

$$\Psi = \frac{\Omega - B}{L \Xi} = \frac{1}{\Xi} \sum_i \psi_i \xi_i = \sum_i \psi_i \quad \text{where}$$

$$\psi_i = \sigma(ZH) \cdot BR_i$$

ξ_i = efficiency for events from Higgs decay i to pass $\sigma(ZH) \cdot BR(visible)$ analysis

$$\Xi = \frac{\sum_i \psi_i \xi_i}{\sum_i \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L \eta_i}$$

ω_i = Number of signal + background events in $\sigma(ZH) \cdot BR_i$ analysis

β_i = Predicted number of background events in $\sigma(ZH) \cdot BR_i$ analysis

η_i = efficiency for Higgs decay i to pass $\sigma \cdot BR_i$ analysis

K_i = number of signal + background events common to had Z recoil
and $\sigma \cdot BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis

ε_i = number of signal + background events unique to $\sigma \cdot BR_i$ analysis

$$\Omega = E + \sum_i K_i \quad S \equiv \Omega - B \quad T \equiv \frac{\sqrt{S+B}}{S}$$

$$\omega_i = K_i + \varepsilon_i \quad s_i \equiv \omega_i - \beta_i \quad \tau_i \equiv \frac{\sqrt{s_i + \beta_i}}{s_i}$$

$$\lambda_i \equiv \frac{K_i}{\omega_i} \quad N \equiv L \sigma_{ZH} \quad r_i \equiv BR_i \quad \delta_i \equiv \xi_i - \Xi$$

$$(\Delta\Psi)^2 = \left(\frac{\partial\Psi}{\partial\Omega}\right)^2 V_{\Omega\Omega} + \left(\frac{\partial\Psi}{\partial\Xi}\right)^2 V_{\Xi\Xi} + 2\frac{\partial\Psi}{\partial\Omega}\frac{\partial\Psi}{\partial\Xi} V_{\Omega\Xi}$$

$$\frac{\partial\Psi}{\partial\Omega} = \frac{1}{L\Xi} = \frac{\Psi}{\Omega}\left(1 - \frac{B}{\Omega}\right)^{-1} \qquad \frac{\partial\Psi}{\partial\Xi} = -\frac{\Omega - B}{L\Xi^2} = -\frac{\Psi}{\Xi}$$

$$V_{\Omega\Omega} = E + \sum_i K_i = \Omega$$

$$V_{\Xi\Xi} = \frac{1}{L^2\Psi^2} \sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2} (\varepsilon_i + K_i)$$

$$V_{\Omega\Xi} = \frac{1}{L\Psi} \sum_i \frac{\xi_i - \Xi}{\eta_i} K_i$$

$$\begin{aligned}
\left(\frac{\Delta\Psi}{\Psi}\right)^2 &= \frac{1}{\Omega^2} \left(1 - \frac{B}{\Omega}\right)^{-2} V_{\Omega\Omega} + \frac{1}{\Xi^2} V_{\Xi\Xi} - \frac{2}{\Omega\Xi} \left(1 - \frac{B}{\Omega}\right)^{-1} V_{\Omega\Xi} \\
&= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} + \frac{1}{L^2 \Xi^2 \Psi^2} \sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2} (\varepsilon_i + K_i) - \frac{2}{L \Omega \Xi \Psi} \left(1 - \frac{B}{\Omega}\right)^{-1} \sum_i \frac{\xi_i - \Xi}{\eta_i} K_i \\
&= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} + \frac{1}{L^2 \Xi^2 \Psi^2} \sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2} (L\eta_i\psi_i + \beta_i) - \frac{2}{L \Omega \Xi \Psi} \left(1 - \frac{B}{\Omega}\right)^{-1} \sum_i \frac{\xi_i - \Xi}{\eta_i} \lambda_i (L\eta_i\psi_i + \beta_i) \\
&= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} \left[1 + \frac{L}{\Omega} \sum_i \frac{(\xi_i - \Xi)^2}{\eta_i} \psi_i \left(1 + \frac{\beta_i}{s_i}\right) - \frac{2L}{\Omega} \sum_i (\xi_i - \Xi) \psi_i \lambda_i \left(1 + \frac{\beta_i}{s_i}\right) \right] \\
&= \frac{S+B}{S^2} \left\{ 1 + \frac{L}{\Omega} \sum_i (\xi_i - \Xi) \psi_i \left(\frac{s_i + \beta_i}{s_i^2} \right) [(\xi_i - \Xi)L\psi_i - 2\lambda_i s_i] \right\} \\
&= T^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \tau_i^2 [\delta_i^2 - 2\lambda_i \eta_i \delta_i] \right\}
\end{aligned}$$

What if we don't do a hadronic Z recoil measurement and instead only use $\sigma(ZH) \cdot BR_i$ to calculate $\sigma(ZH) \cdot BR(\text{visible}) = \sum_i \sigma(ZH) \cdot BR_i$?

$$\Psi' = \sum_i \psi_i \quad \psi_i = \frac{\omega_i - \beta_i}{L \xi_i}$$

$$(\Delta\Psi')^2 = \sum_i \left(\frac{\partial\Psi'}{\partial\omega_i} \right)^2 \omega_i, \quad \frac{\partial\Psi'}{\partial\omega_i} = \frac{1}{L\eta'_i}$$

$$(\Delta\Psi')^2 = \frac{1}{L^2} \sum_i = \frac{1}{L^2} \sum_i \frac{s_i + \beta_i}{\xi_i^2}$$

$$\begin{aligned} \left(\frac{\Delta\Psi'}{\Psi'} \right)^2 &= \left(\sum_i \frac{\omega_i - \beta_i}{L \xi_i} \right)^{-2} \frac{1}{L^2} \sum_i \frac{s_i + \beta_i}{\xi_i^2} \\ &= \frac{S+B}{S^2} \frac{L}{\Omega} \Xi^2 \sum_i \frac{\psi_i}{\xi_i} \left(1 + \frac{\beta_i}{s_i} \right) \end{aligned}$$

Compare this now with our formula for $\left(\frac{\Delta\Psi}{\Psi} \right)^2$ for $\lambda_i = 1$:

$$\begin{aligned} \left(\frac{\Delta\Psi}{\Psi} \right)^2 &= \frac{S+B}{S^2} \left\{ 1 + \frac{1}{\Omega} \sum_i \omega_i \left[\left(1 - \frac{\Xi}{\xi_i} \right)^2 - 2 \left(1 - \frac{\Xi}{\xi_i} \right) \right] \right\} \\ &= \frac{S+B}{S^2} \left\{ 1 + \frac{1}{\Omega} \sum_i \omega_i \left[1 - \frac{2\Xi}{\xi_i} + \left(\frac{\Xi}{\xi_i} \right)^2 - 2 + 2 \frac{\Xi}{\xi_i} \right] \right\} = \left(\frac{\Delta\Psi'}{\Psi'} \right)^2 \end{aligned}$$

