



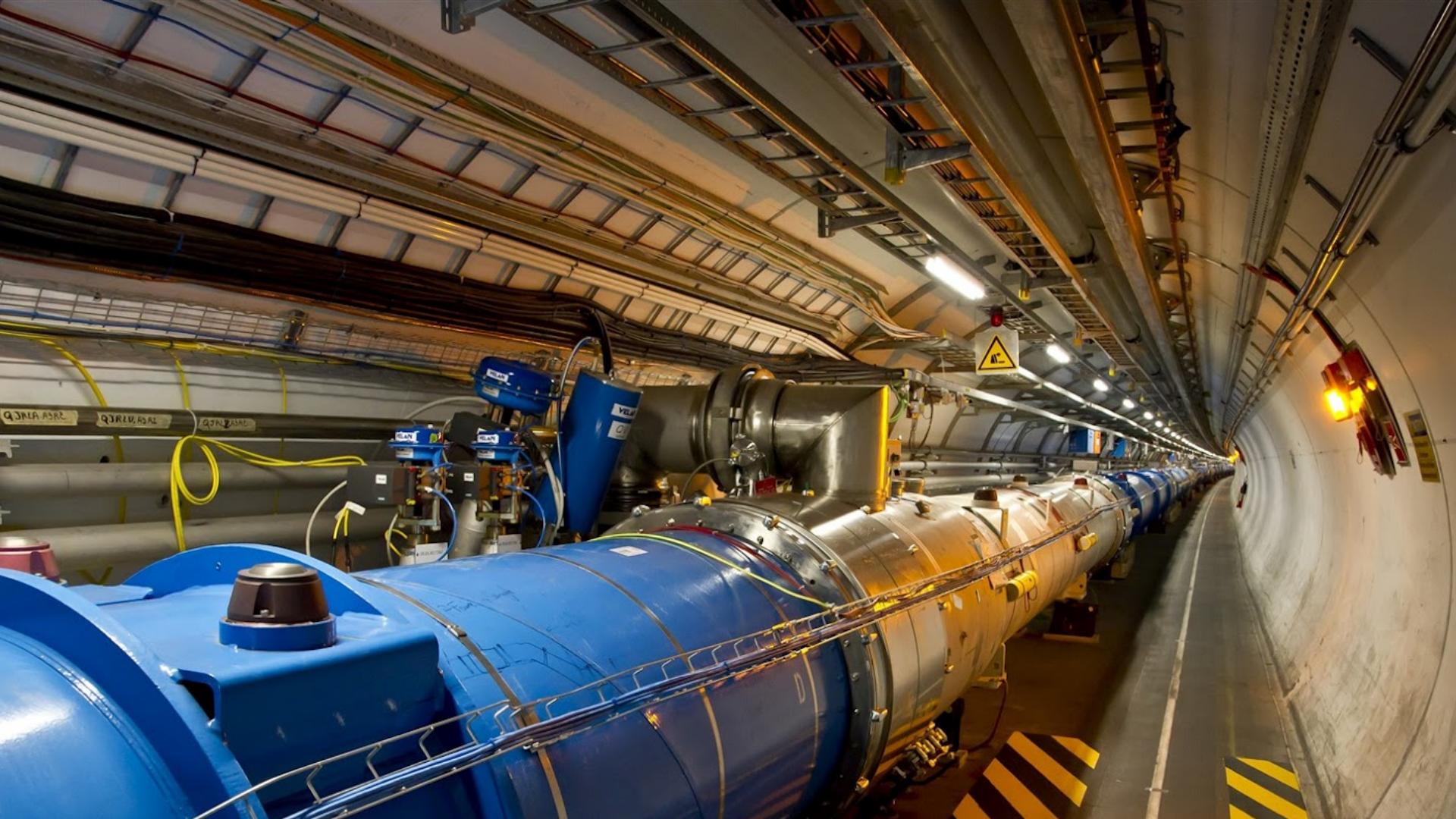
Study of systematic errors on scalar boson mass

LCWS14 Belgrade 2014/10/07

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On behalf of ATLAS and CMS

Overview

- Introduction to the LHC, ATLAS, and CMS
- Details on systematic uncertainties at the LHC
 - Electron and muon momentum scales
 - Photon momentum scales
 - Theoretical inputs
- Future prospects (from Snowmass 2013)
 - Including comparisons of LHC and linear colliders

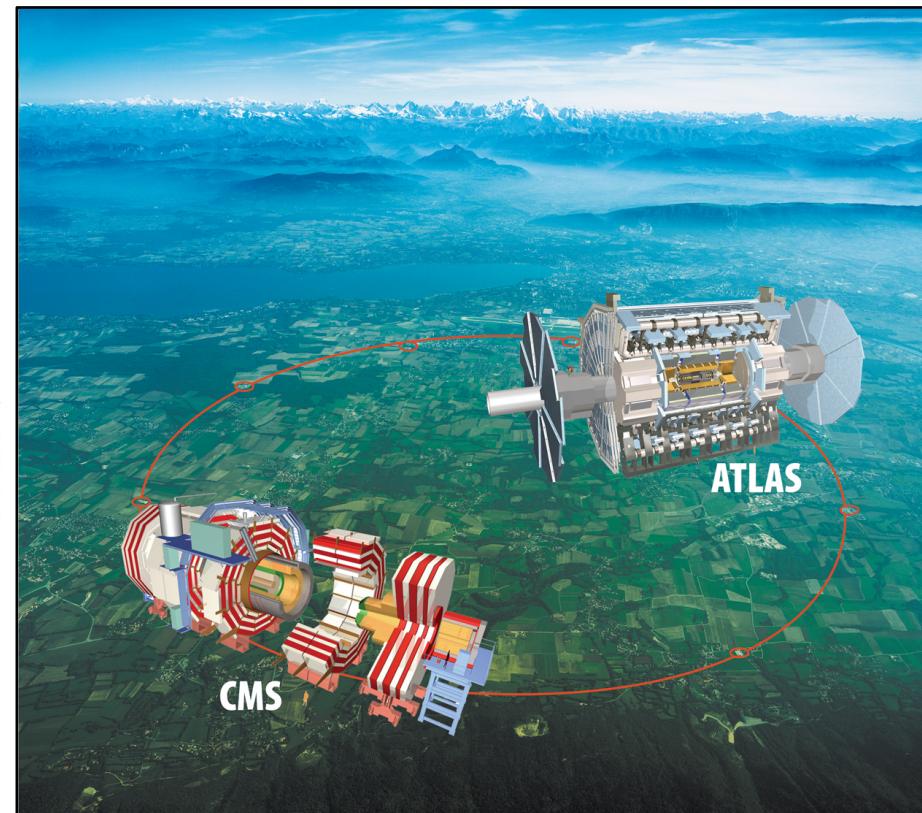
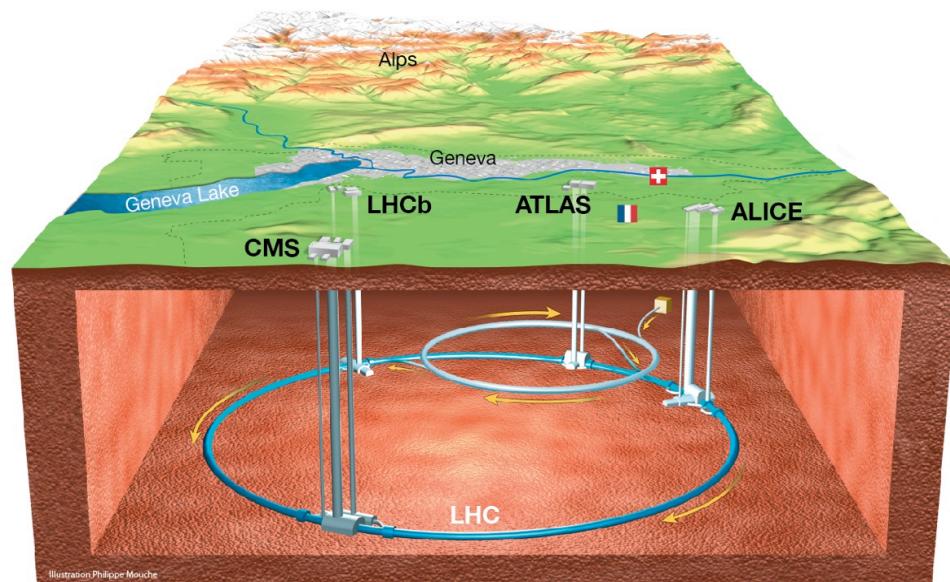


Physics at the world's highest energy collider

THE LHC, ATLAS AND CMS

The LHC

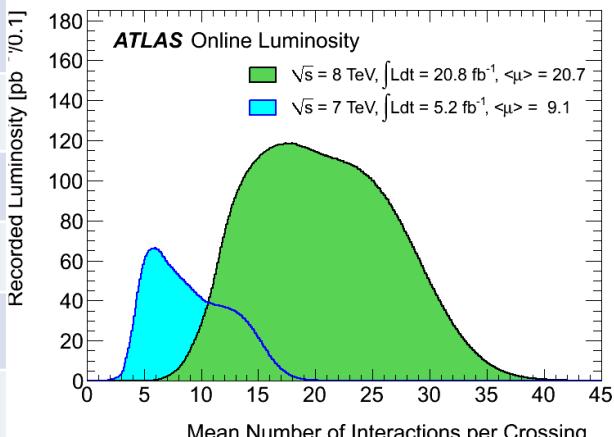
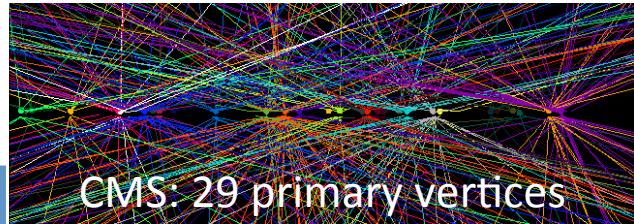
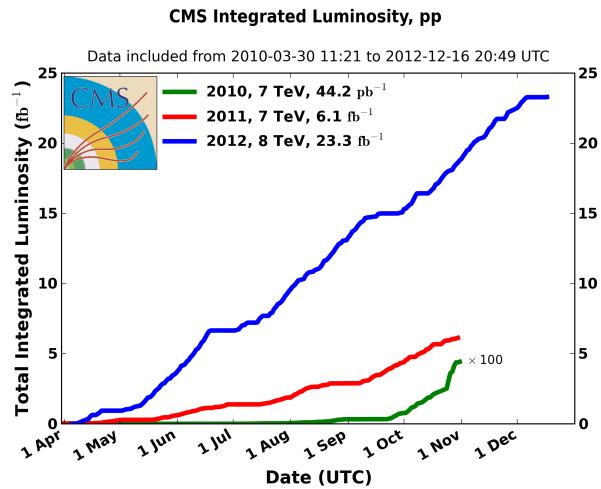
- The LHC is a 27 km long circular pp collider
 - Also capable of PbPb and Pbp collisions
- Located at CERN, Geneva, on the Franco-Swiss border



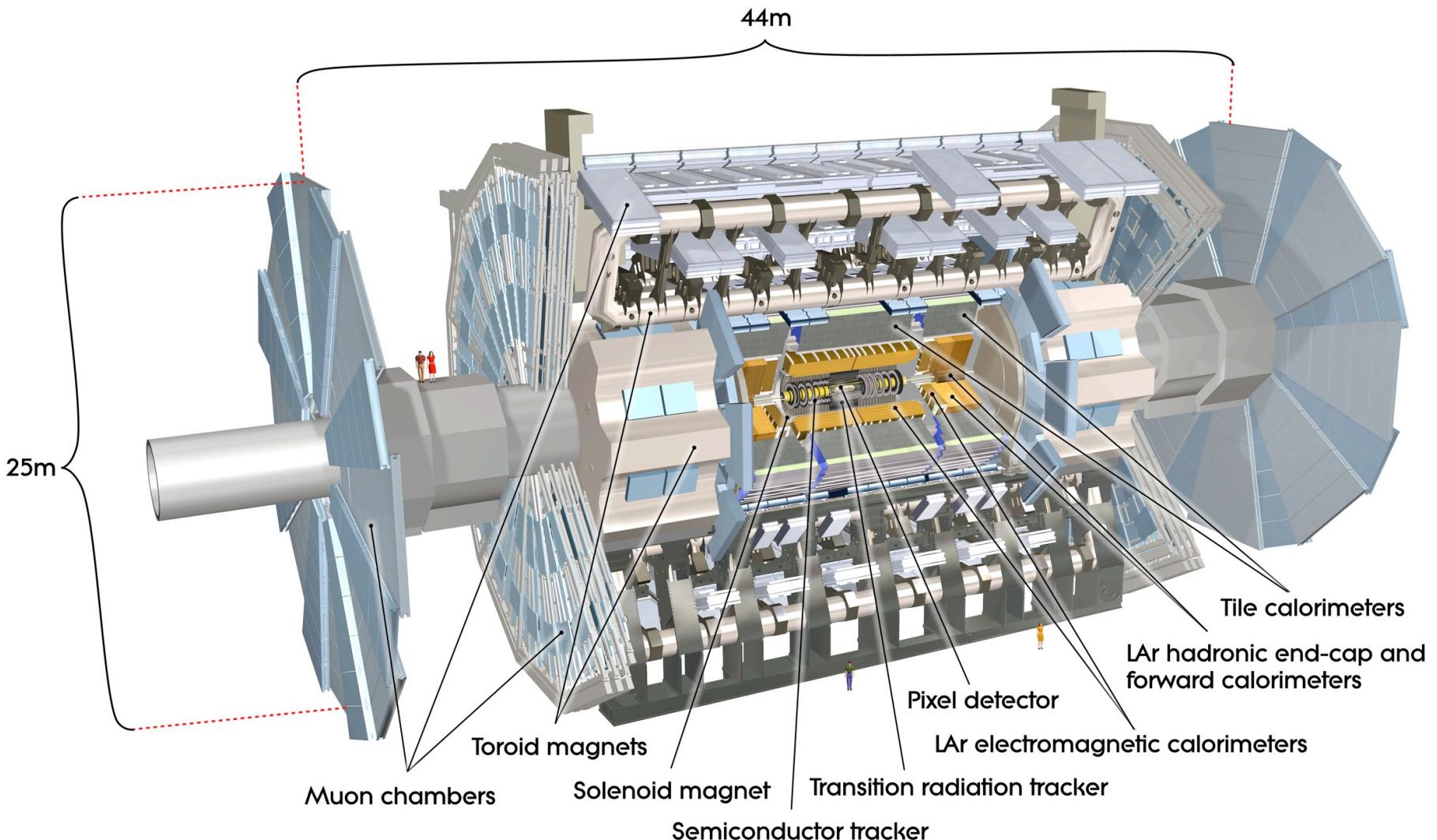
Beam conditions at the LHC

- The beam conditions of the LHC are shown in the table below
- The high luminosity at the LHC comes at the cost of multiple interactions per bunch crossing
 - Known as pileup, $\langle\mu\rangle$

Parameter	Design	2011	2012	2015
Beam energy (TeV)	7	3.5	4	6.5
Particles per bunch (10^{11})	1.15	1.5	1.6-1.7	1.15
Number of bunches	2808	1380	1374	2590
Bunch spacing (ns)	25	50	25	25
Interactions per crossing, $\langle\mu\rangle$	~ 20	$\sim 6-11$	~ 40	~ 50
Peak luminosity ($10^{34} \text{ cm}^{-2} \text{ s}^{-1}$)	1.0	0.36	0.77	1.7



The ATLAS detector



The CMS detector

CMS DETECTOR

Total weight : 14,000 tonnes
Overall diameter : 15.0 m
Overall length : 28.7 m
Magnetic field : 3.8 T

STEEL RETURN YOKE

12,500 tonnes

SILICON TRACKERS

Pixel ($100 \times 150 \mu\text{m}$) $\sim 16\text{m}^2 \sim 66\text{M}$ channels
Microstrips ($80 \times 180 \mu\text{m}$) $\sim 200\text{m}^2 \sim 9.6\text{M}$ channels

SUPERCONDUCTING SOLENOID

Niobium titanium coil carrying $\sim 18,000\text{A}$

MUON CHAMBERS

Barrel: 250 Drift Tube, 480 Resistive Plate Chambers
Endcaps: 468 Cathode Strip, 432 Resistive Plate Chambers

PRESHOWER

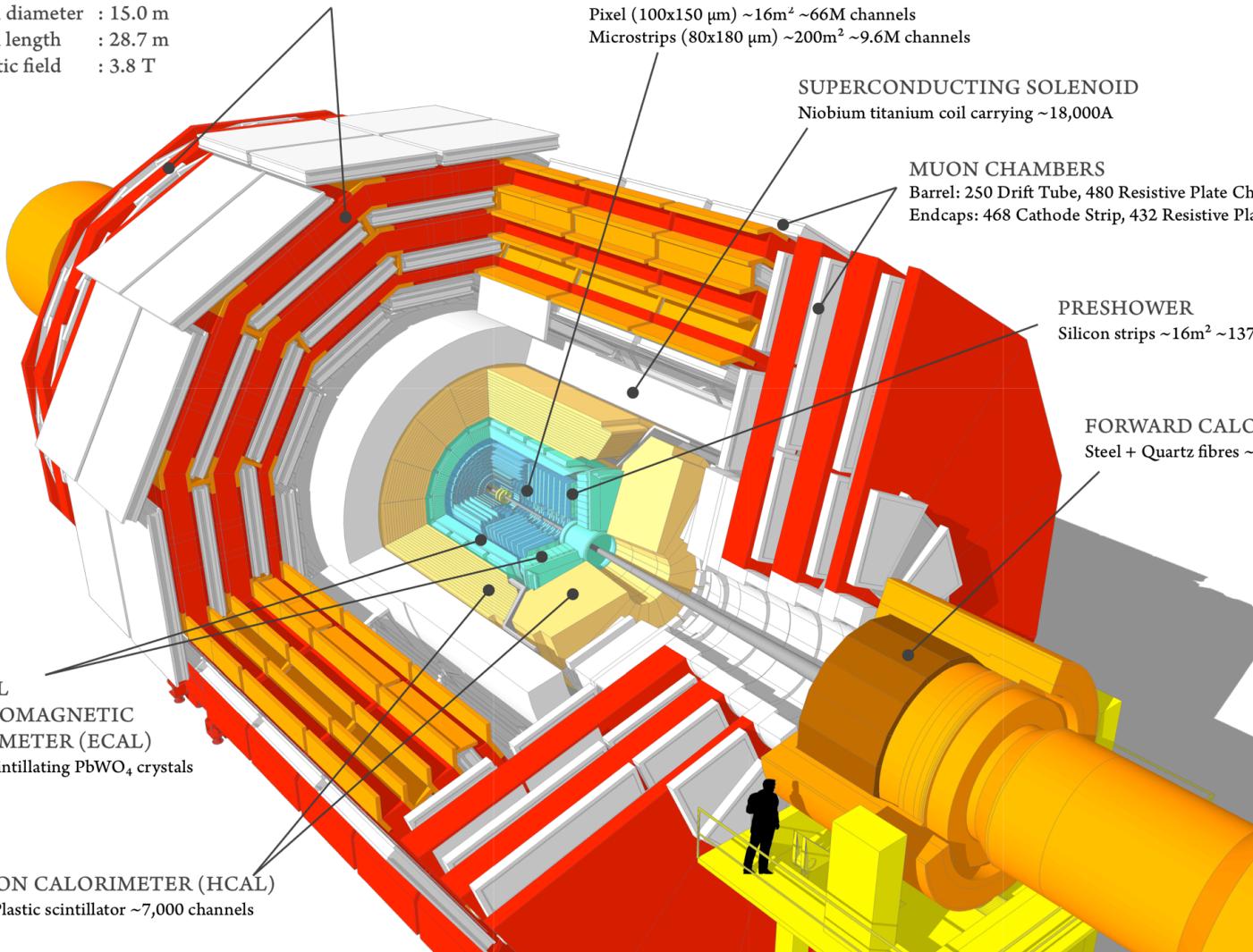
Silicon strips $\sim 16\text{m}^2 \sim 137,000$ channels

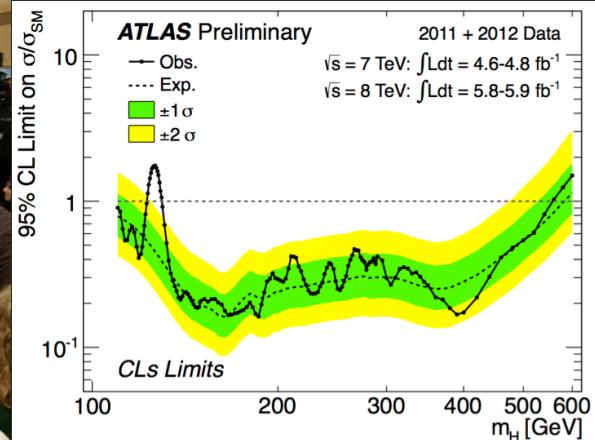
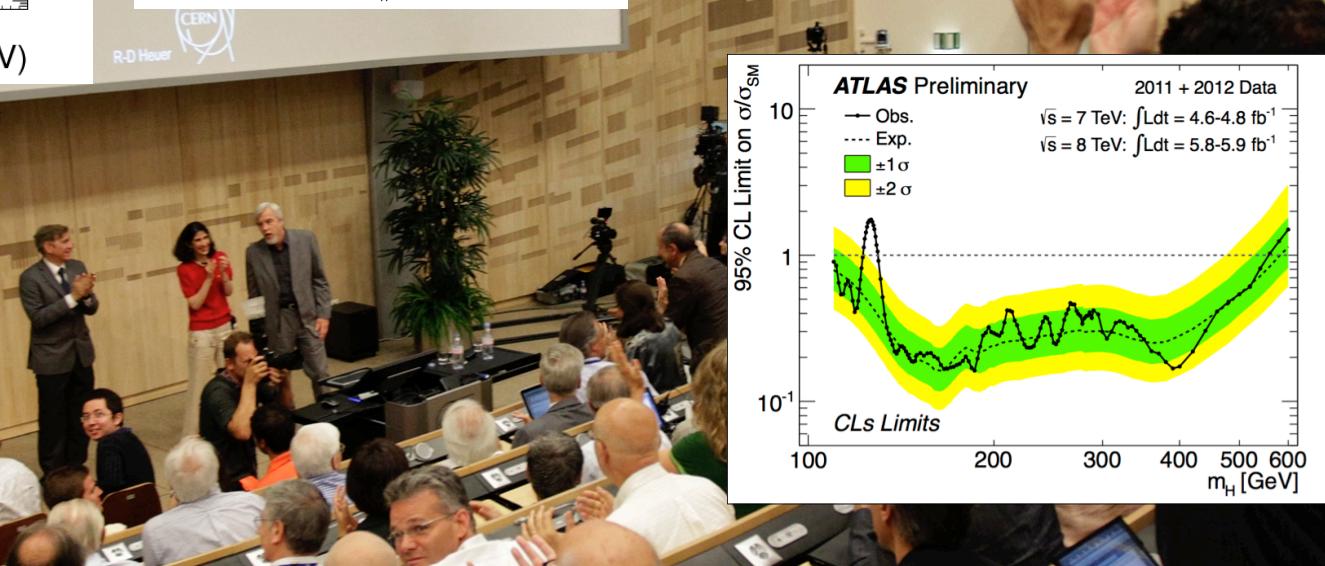
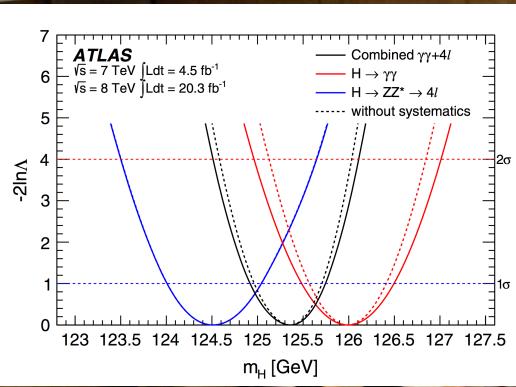
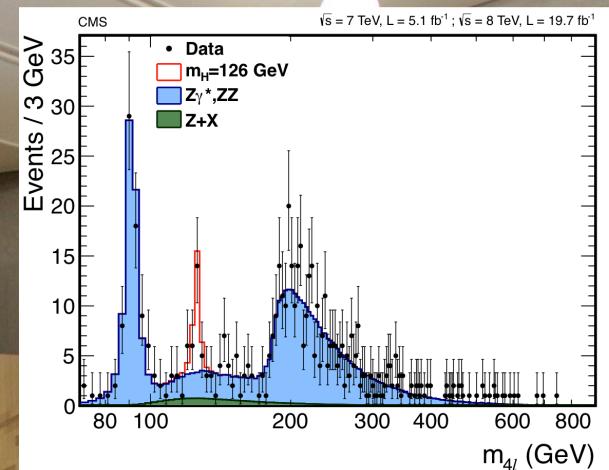
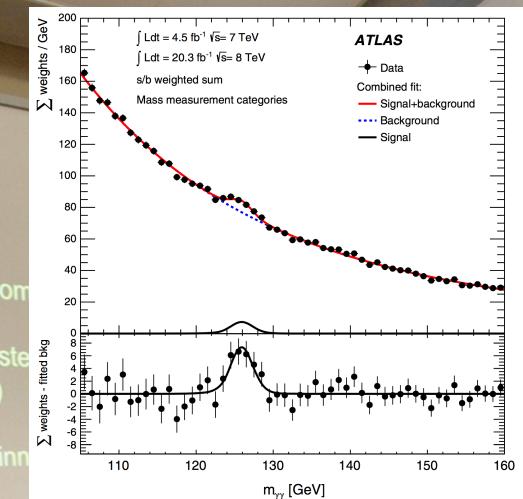
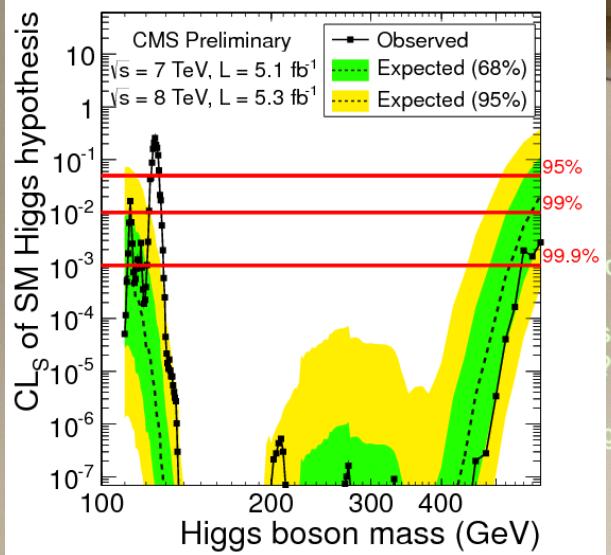
FORWARD CALORIMETER

Steel + Quartz fibres $\sim 2,000$ Channels

CRYSTAL ELECTROMAGNETIC CALORIMETER (ECAL)
 $\sim 76,000$ scintillating PbWO₄ crystals

HADRON CALORIMETER (HCAL)
Brass + Plastic scintillator $\sim 7,000$ channels



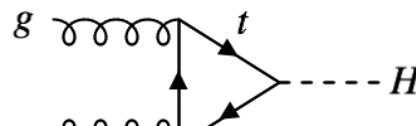


Measuring the mass and width

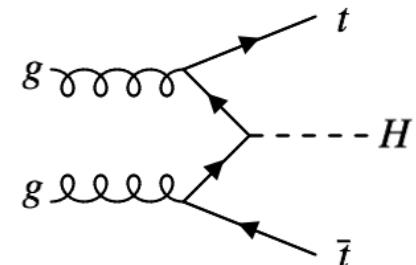
SCALAR BOSON AT THE LHC

Scalar boson search at the LHC

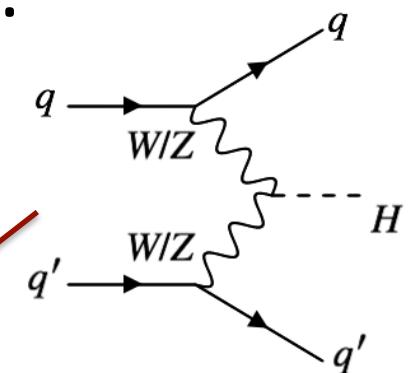
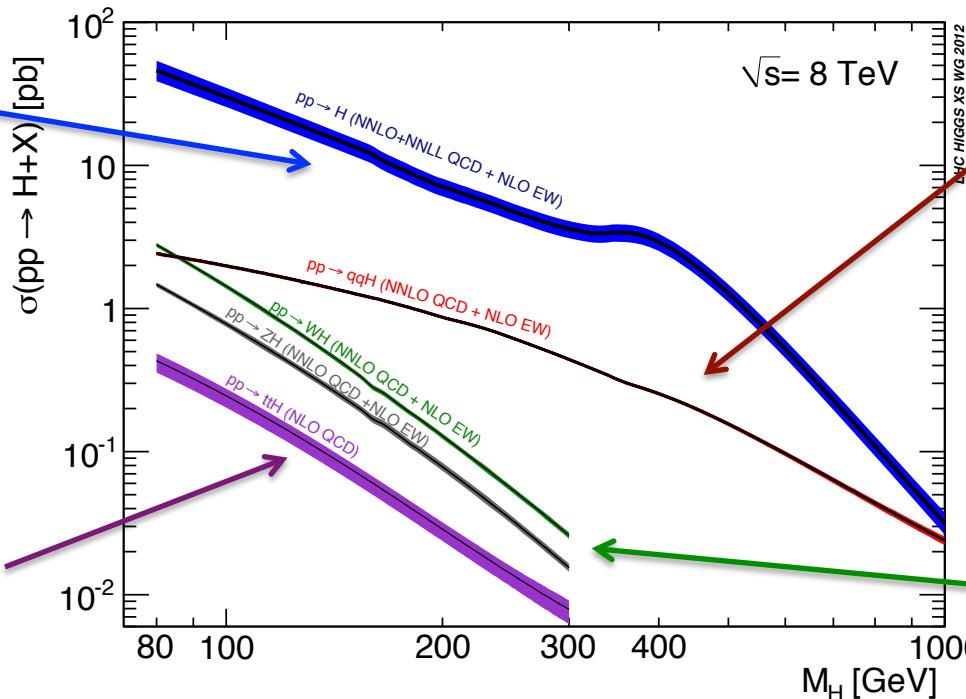
- At the LHC things events are “messy”
- There are four main production modes:



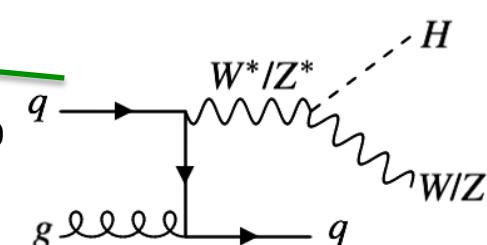
gluon gluon fusion



tt associated production



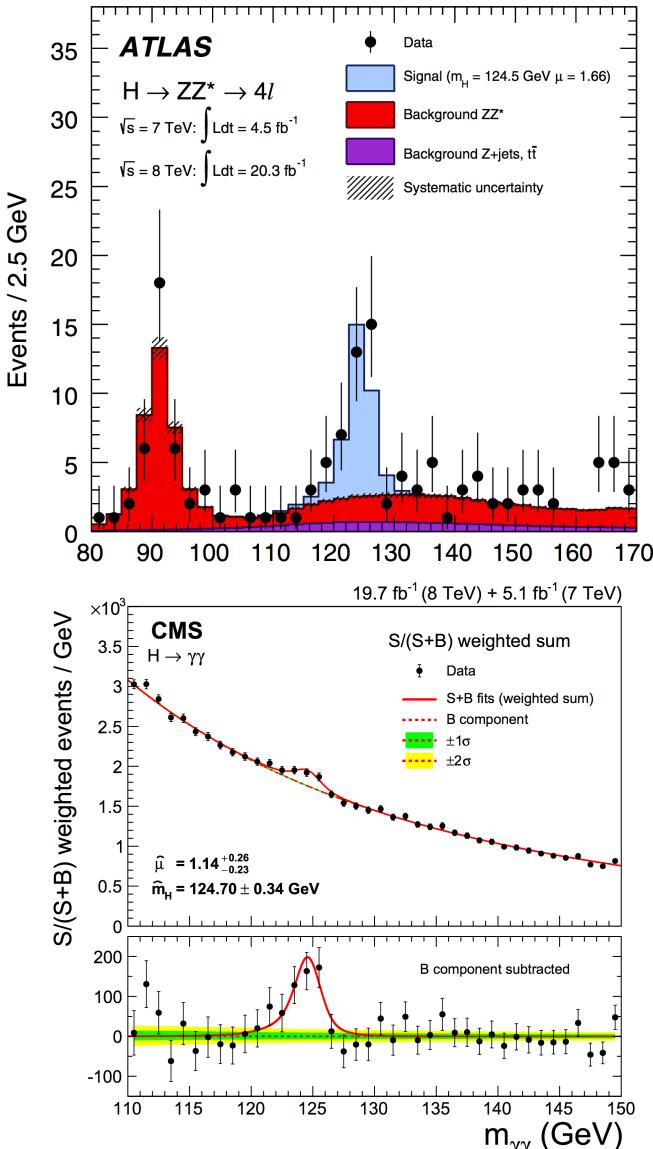
Vector boson fusion



WZ associated production

Scalar boson search at the LHC

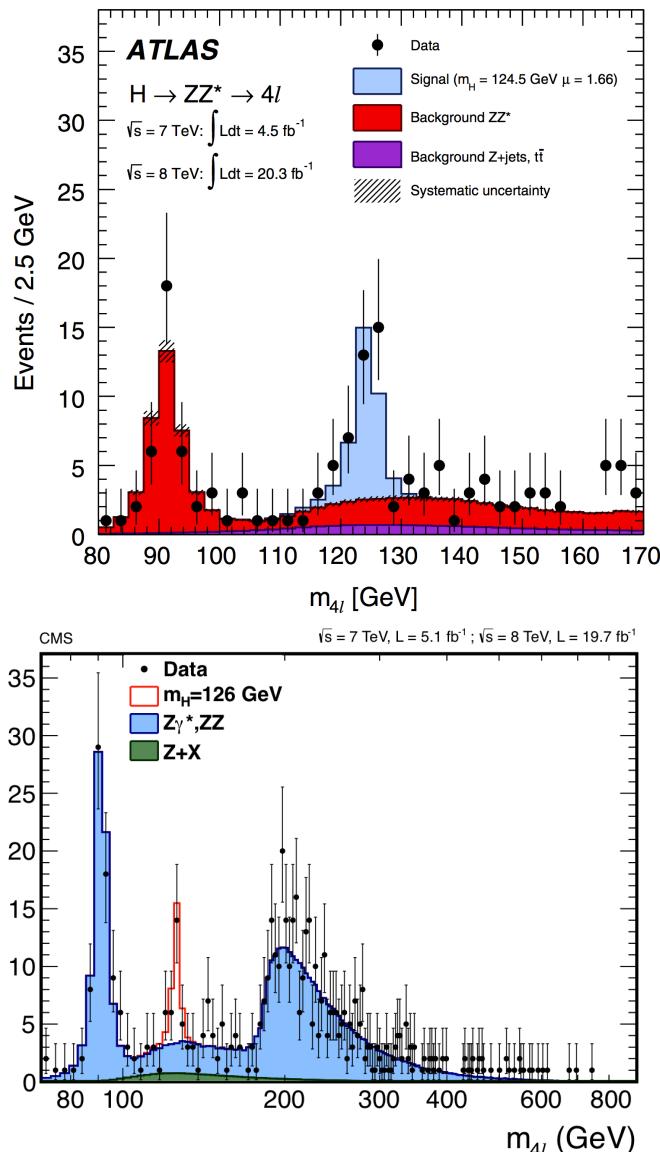
- The mass measurements at the LHC are dominated by two fine resolution final states:
 - $H \rightarrow ZZ^* \rightarrow llll$
 - $H \rightarrow \gamma\gamma$
- These measurements have motivated dedicated studies of lepton, photon energy scales
- Despite having higher branching fractions, $H \rightarrow bb$, $H \rightarrow \tau\tau$, $H \rightarrow WW^*$ all have poor mass resolution
- $H \rightarrow Z\gamma$, $Z \rightarrow \mu\mu$ are too rare for observation using LHC Run 1 data
- Scalar boson lineshape is not accessible using only on-shell measurements



$H \rightarrow ZZ^*$ at the LHC

- $H \rightarrow ZZ^* \rightarrow llll$ is the “golden mode”
 - Very fine mass resolution
 - Very low backgrounds
 - Low signal statistics
 - Use $Z \rightarrow ll$ (and/or $Z \rightarrow llll$) as a standard candle

	m_H [GeV]	statistical	systematic
ATLAS	124.5	0.52	0.06
CMS	125.6	0.4	0.2

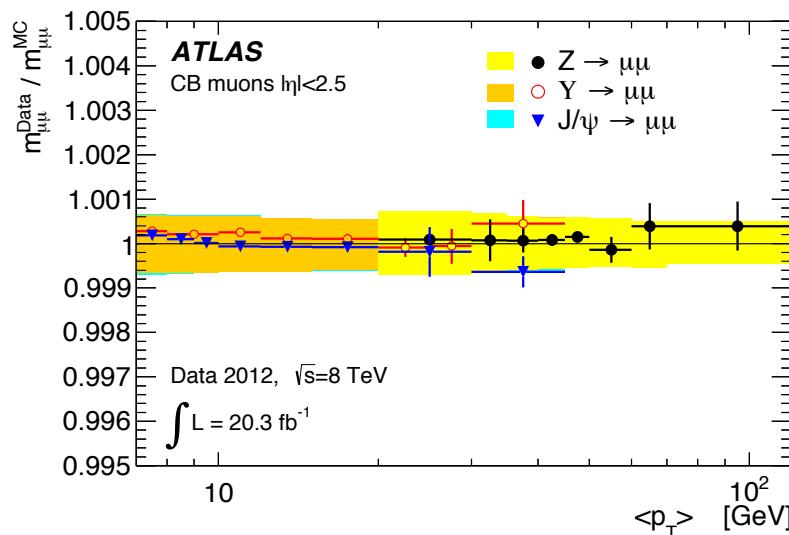
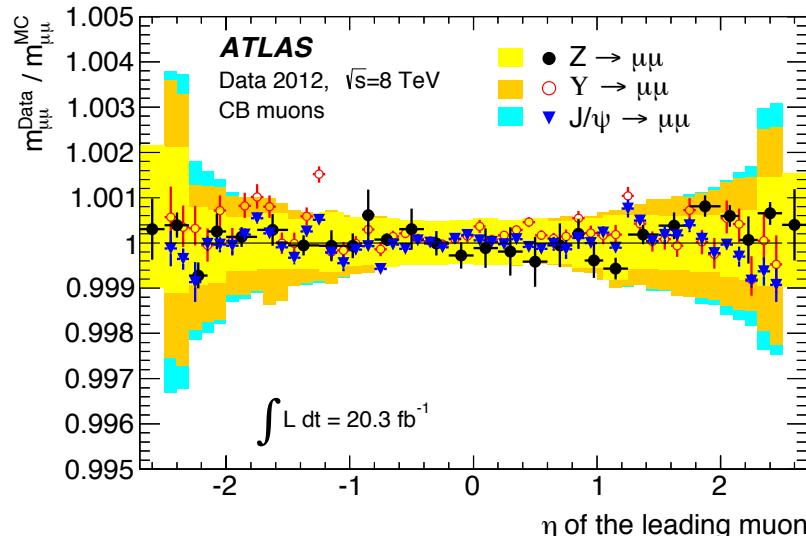


Lepton systematic uncertainties

- Lepton reconstruction must be well understood in terms of momentum scale and resolution
- To understand the momentum scale narrow resonances are used:
 - $J/\psi \rightarrow ee / \mu\mu$
 - $Y(nS) \rightarrow ee / \mu\mu$
 - $Z \rightarrow ee / \mu\mu$
 - These decays allow the validation over a wide range of transverse momentum, p_T , and pseudorapidity, η

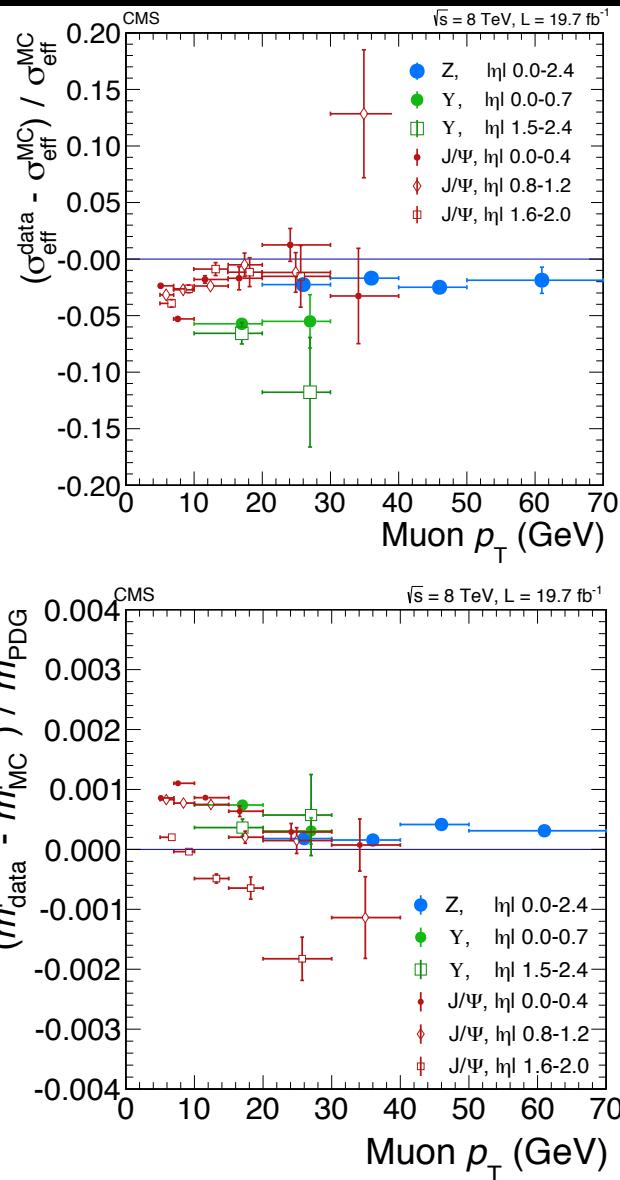
Muon calibration at ATLAS

- Momentum scales of muons in simulation are corrected to match data at the $Z \rightarrow \mu\mu$ and $J/\psi \rightarrow \mu\mu$ peaks, binned in η and p_T
- Muons are split into categories depending on the detector subsystems used:
 - Inner detector (ID)
 - Momentum scale uncertainties 0.02% ($|\eta| = 0$) – 0.2% ($|\eta| > 2$), < 0.1% overall
 - Muon systems (MS)
 - Momentum scale uncertainties < 0.2% everywhere
 - Combined ID and MS (CB)
 - Momentum scale uncertainties 0.04% (barrel) – 0.2% ($|\eta| > 2$)



Muon calibration at CMS

- Muons are calibrated by:
 1. Momentum scale and resolution measurements are made using a reference model of the Z lineshape convolved with a Gaussian
 2. Biases are determined according to shifts in the Z peak position
- Events are fitted to Z , J/ψ , $\Upsilon(nS)$ peaks, averaging over η and p_T
- Systematic uncertainties:
 - Peak position: 0.1% ($ee\mu\mu, \mu\mu\mu\mu$)
 - Peak width: 5%



Muon reconstruction uncertainties

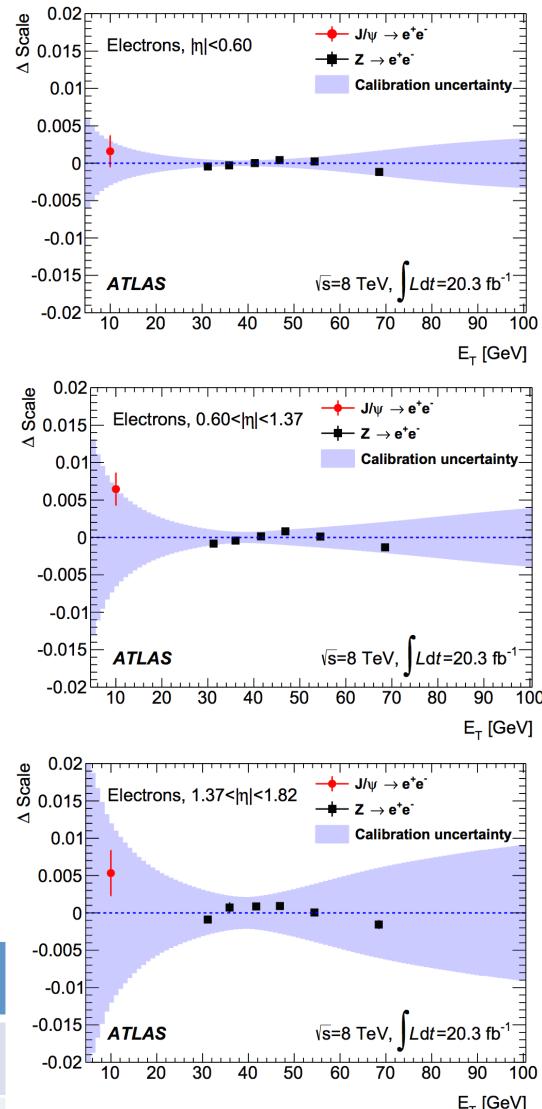
- Typical momentum scale uncertainties are of the order $\sim 0.1\%$ per muon
- Momentum scale uncertainties are determined using the whole of the LHC Run 1 datasets
 - Statistics limited, $\sim 10M$ events per resonance

	ATLAS	CMS
Momentum scale uncertainty \rightarrow mass scale	0.04%-0.2%	0.1% in 4μ final state
$n(Z \rightarrow \mu\mu)$ events	9M (corrections to MC)	14M (Corrections to MC, validation of MC)
$n(J/\psi \rightarrow \mu\mu)$ events	17M (corrections to MC)	27M (validation of MC)
$n(\Upsilon \rightarrow \mu\mu)$ events	5M (validation of MC)	15M (validation of MC)

Electron calibration at ATLAS

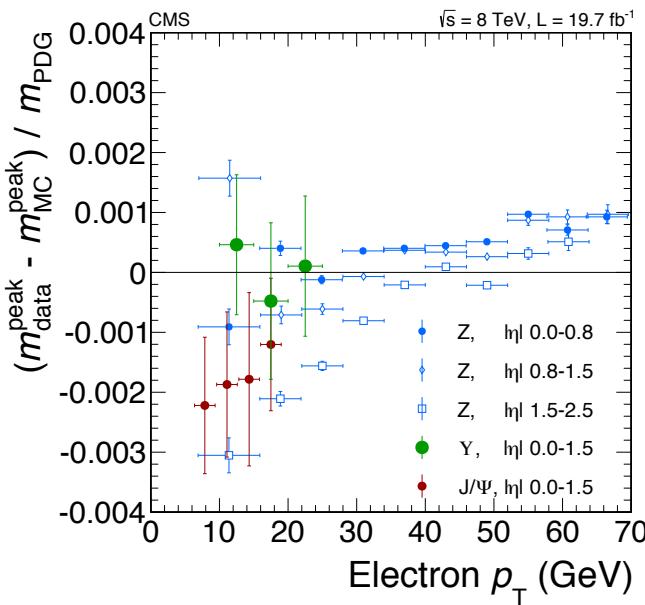
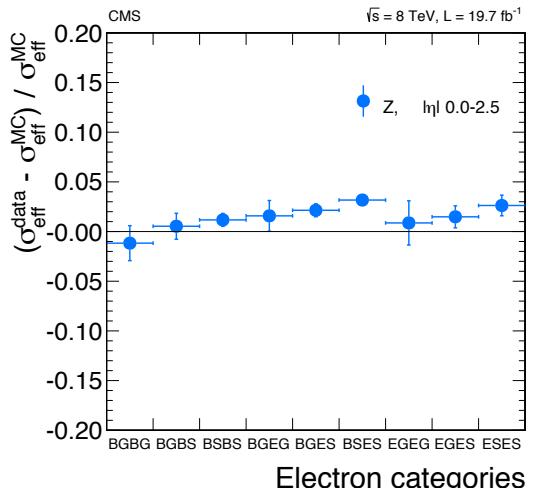
- Electron candidates are calibrated first using a multivariate discriminant using simulated (MC) events
 - Material budget is taken into account by measuring contributions to first and second Ecal layers
- Uniformity corrections are applied to take variations wrt ϕ , time and pileup in the detector into account
- Longitudinal scale variations taken into account in data
- MC and data samples compared to obtain MC based calibration
- $Z \rightarrow ee$ peak used to correct energy scale in data
- $Z \rightarrow ee$ peak used to smear E_T in MC
- Calibrated electrons validated against $J/\psi \rightarrow ll$, $Z \rightarrow ll$ samples

	$ \eta < 1.37$	$1.37 < \eta < 1.82$	$ \eta > 1.82$
$E_T \sim 11 \text{ GeV}$	0.04%	0.2%	0.05%
$E_T \sim 40 \text{ GeV}$	0.4-1%	1.1%	0.4%



Electron calibration at CMS

- Electrons are calibrated in 3 steps:
 1. Momentum scale corrections from Z mass difference between data and simulation
 - Time dependence is implicit to account for transparency loss in crystals
 2. The p_T dependence is taken into account using linearity corrections
 - $J/\psi \rightarrow ee$ and $Y(nS) \rightarrow ee$ are used to validate the $p_T < 20$ GeV region
 3. Energies of single electrons smeared with a Gaussian factor $\sim G(1, \Delta\sigma)$
- Events are then categorised according to p_T and η and fitted to Z , J/ψ , $Y(nS)$ peaks
- Systematic uncertainties:
 - Peak position: 0.1% ($ee\mu\mu$), 0.3% ($eeee$)
 - Peak width: 1.2-4%



Electron reconstruction uncertainties

- Typical energy scale uncertainties are of the order <0.5% per electron
- Energy scale uncertainties are determined using the whole of the LHC Run 1 datasets
 - Statistics limited

	ATLAS	CMS
Momentum scale uncertainty → mass scale	$E_T \sim 40 \text{ GeV}: 0.03\text{-}0.05\%$ $E_T \sim 10 \text{ GeV}: 0.4\text{-}2\%$	0.3% in 4e final state 0.1% in 2e2 μ final state
$n(Z \rightarrow ee)$ events	6.6M (corrections to MC)	10M (Corrections to MC, validation of MC)
$n(J/\psi \rightarrow ee)$ events	0.3M (corrections to MC)	5k (validation of MC)
$n(Z \rightarrow eey)$ events	0.2M (validation of MC)	
$n(Y \rightarrow ee)$ events		25k (validation of MC)

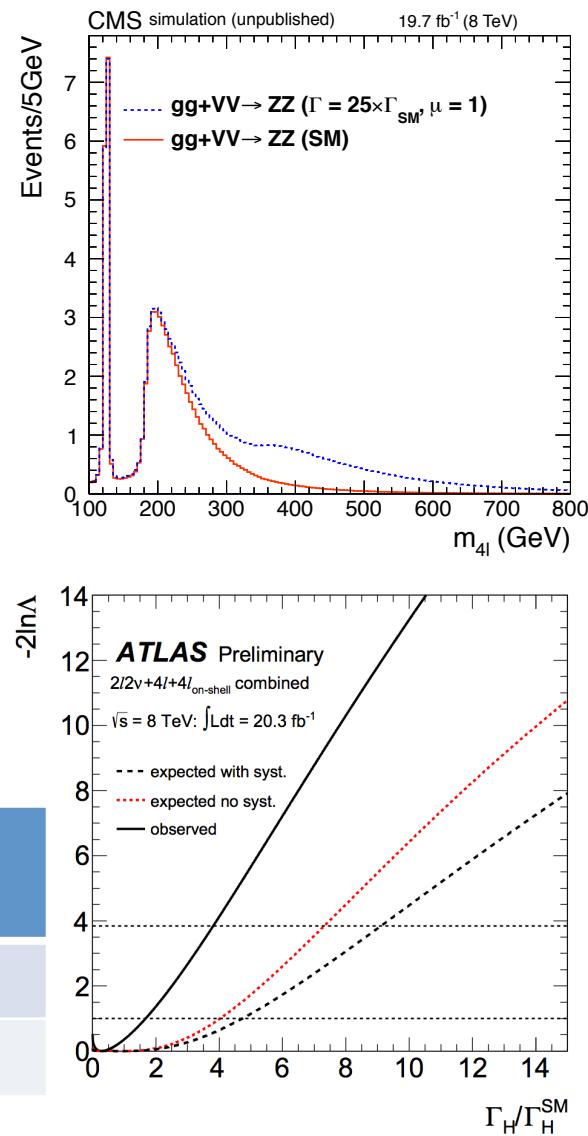
$H \rightarrow ZZ^*$ per event uncertainties

- For $H \rightarrow ZZ^*$ there are so few signal events that the mass variations between events are significant
- By taking per-event mass uncertainties the mass resolution can be improved
- Per event uncertainties help constrain the scalar boson width in the $H \rightarrow ZZ^*$ channel
- Both ATLAS and CMS use per event uncertainties
- eg At CMS the per event mass uncertainty estimation, \mathcal{D}_m is used in a likelihood function $P(\mathcal{D}_m | m_H)$
 - To separate statistical and systematic uncertainties the nuisance parameters are fixed to their best-fit values and the likelihood performed again
 - This gives $m_H = 125.6 \pm 0.4 \text{ (stat)} \pm 0.2 \text{ (syst)}$

Width estimation in ZZ*

- In a Standard Model scenario it is possible to measure the width by comparing on-shell and off-shell $gg \rightarrow ZZ^{(*)}$ mass spectra
- Dominant uncertainties come from theoretical calculations (interference of $gg \rightarrow H \rightarrow ZZ^*$ and SM processes)
 - Of the order of 20-30% for signal, 10-20% for backgrounds
- Limits on the width are:

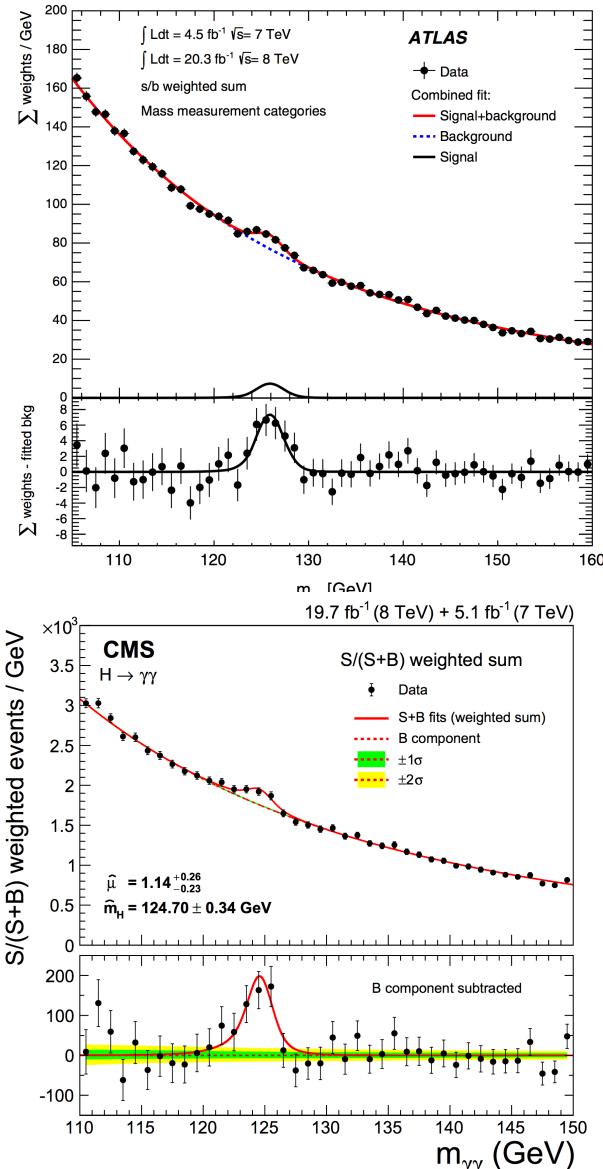
Experiment	Observed limit	Expected limit	$\Gamma_H^{\text{SM}} (m_H^{\text{SM}})$
ATLAS	24 MeV	35 MeV	4.15 MeV (125.5 GeV)
CMS	22 MeV	33 MeV	4.15 MeV (125.6 GeV)



$H \rightarrow \gamma\gamma$ at the LHC

- $H \rightarrow \gamma\gamma$ has the following features:
 - High statistics in the low mass (< 140 GeV) regime
 - Simple final state
 - Large backgrounds
 - Energy scales measured using detector response to $X \rightarrow e^+e^-$
 - Also $Z \rightarrow ll\gamma$ at ATLAS
 - Further sensitivity gains through categorisation and weighting

	m_H [GeV]	statistical	systematic
ATLAS	125.98	0.42	0.28
CMS	124.70	0.31	0.15



Photon reconstruction at ATLAS

- Mass uncertainties in the $H \rightarrow \gamma\gamma$ final state are dominated by photon energy scale uncertainties ($\sim 0.25\%$)
- Calibration very similar to electrons
- Uncertainties balance sheet:

Source of uncertainty	Uncertainty (unconverted photons)	Uncertainty (converted photons)
$Z \rightarrow ee$ calibration	0.02-0.11%	0.02-0.11%
LAr cell non-linearity	0.09-0.39%	0.06-0.29%
Layer calibration	0.11-0.13%	0.05-0.10%
ID material	0.06-0.10%	0.05-0.06%
Other material	0.07-0.35%	0.04-0.20%
Conversion reconstruction	0.02-0.05%	0.02-0.06%
Lateral shower shape	0.04-0.07%	0.09-0.19%
Total	0.23-0.59%	0.21-0.47%

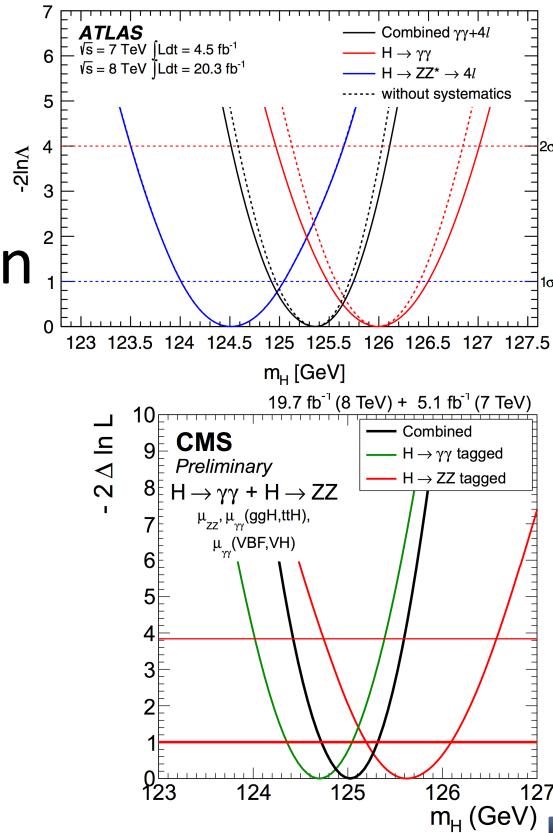
Photon reconstruction at CMS

- Mass uncertainties in the $H \rightarrow \gamma\gamma$ final state are dominated by three main factors
- Calibration very similar to electrons
- Uncertainties balance sheet is less detailed than that of ATLAS:

Source of uncertainty	Uncertainty (GeV)
Electron-photon differences	0.10
Linearity of the energy scale	0.10
Energy scale calibration and resolution	0.05
Other contributions	0.04
Total	0.15

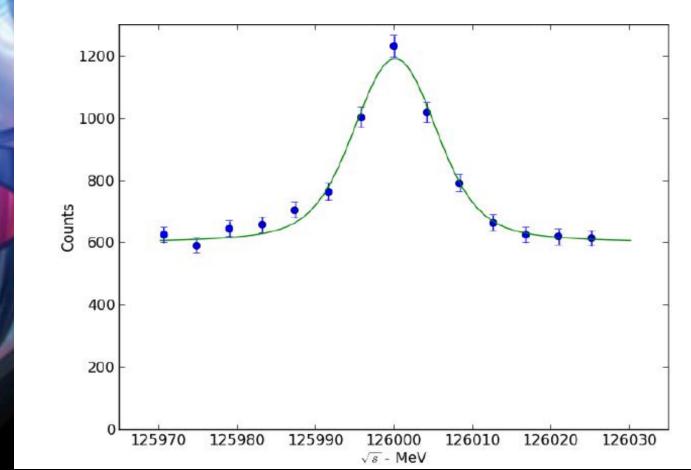
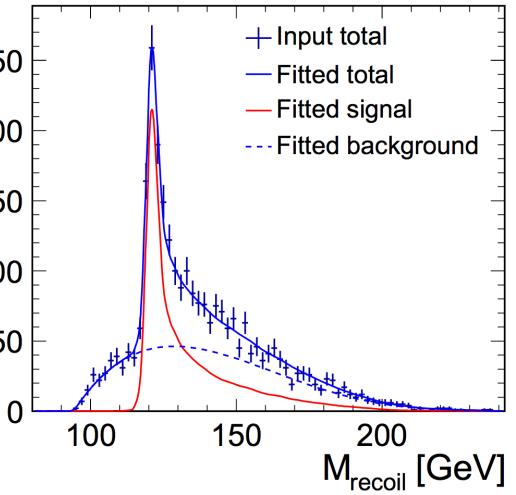
Mass combinations

- Combining the mass measurements gives a good cross check
 - Masses are consistent within 2.1σ between modes and across both experiments
- Statistical uncertainties are larger than systematic uncertainties
- Correlations between e and γ scale factors taken into account



m_H (GeV)	ATLAS	CMS
$H \rightarrow ZZ^* \rightarrow llll$	124.51 ± 0.52 (stat) ± 0.06 (syst)	125.6 ± 0.4 (stat) ± 0.2 (syst)
$H \rightarrow \gamma\gamma$	125.98 ± 0.42 (stat) ± 0.28 (syst)	124.7 ± 0.31 (stat) ± 0.15 (syst)
Combined	125.36 ± 0.37 (stat) ± 0.18 (syst)	125.03 ± 0.27 (stat) ± 0.15 (syst)

Events



The next stages

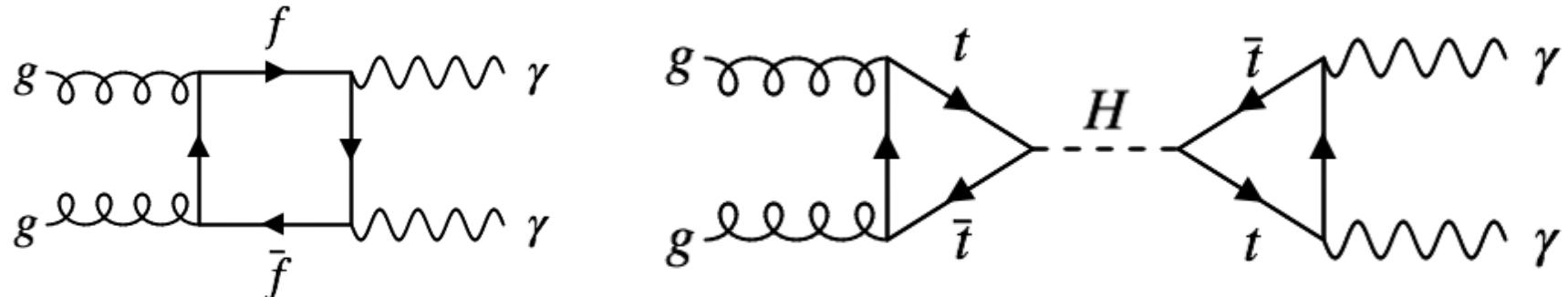
FUTURE PROSPECTS

Future prospects for the LHC

- Projections were presented at Snowmass 2013
 - <http://www.snowmass2013.org/>
- Benchmark projections given at 300 fb^{-1} and 3000 fb^{-1}
- Mass measurements at the LHC will be dominated by systematic uncertainties (from lepton and photon energy/momentum scales)
- These uncertainties will decrease as statistics increase
- Optimistic projections of mass resolution (at 14 TeV):
 - 300 fb^{-1} : 70 MeV
 - 3000 fb^{-1} : 25 MeV
 - (These are taken from Snowmass 2013, not official ATLAS/CMS projections)

$gg \rightarrow \gamma\gamma$ calculations

- Mass peak slightly shifted due to higher order interference effects
- Many box diagrams contribute to $gg \rightarrow \gamma\gamma$, eg:



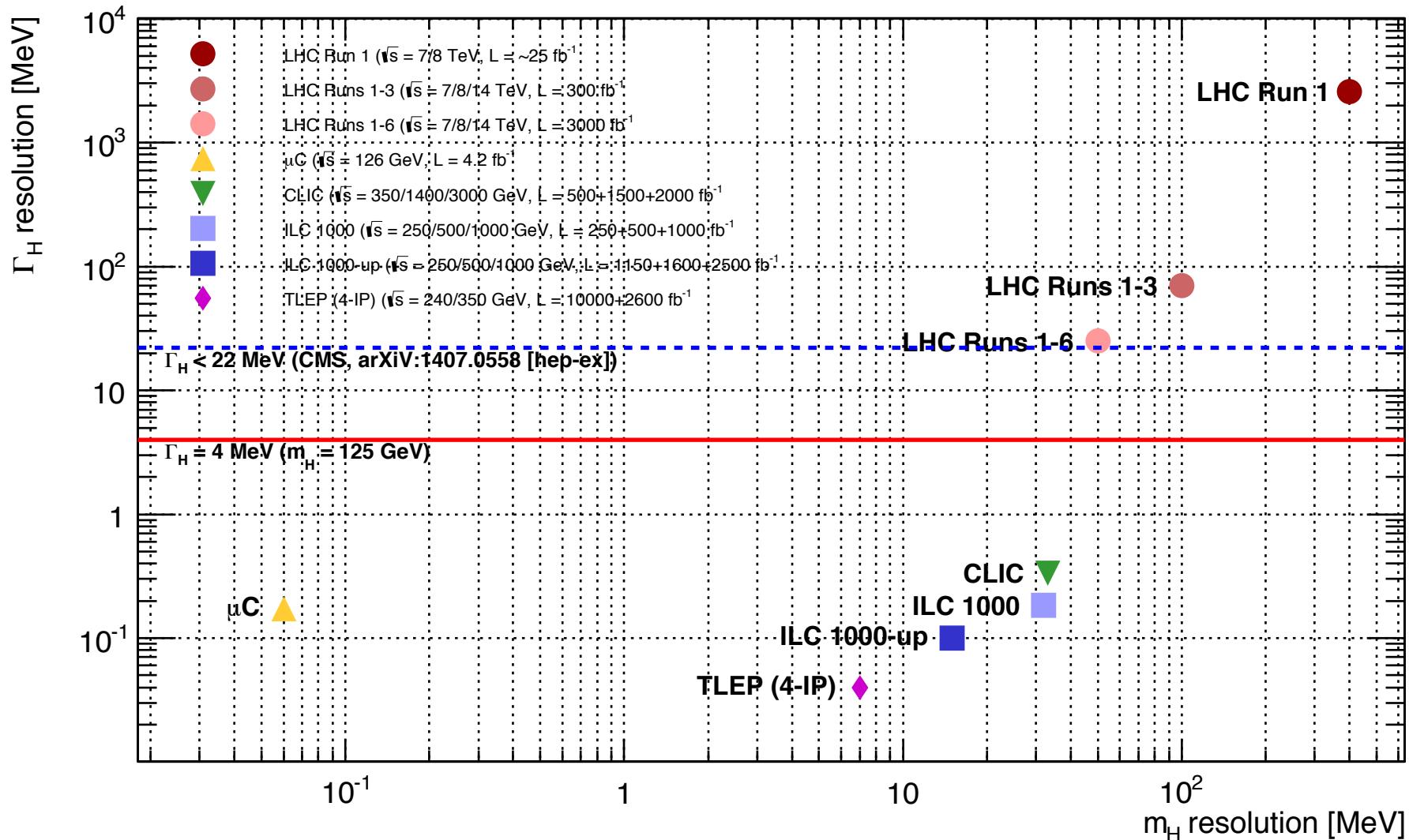
- Interference leads to a shift in the mass peak
- Expect ~ 70 MeV shift (depending on the width)
- This will become a problem as we become more precise

Future prospects for lepton colliders

- e^+e^- : See a return to the LEP-like Higgstrahlung
 - Resolution limited by Z momentum resolution
- $\mu^+\mu^-$: See a return to the LEP-like lineshape measurement

Facility	ILC500	ILC1000	ILC1000-up	CLIC	TLEP (4 IP)	μC
\sqrt{s} (GeV)	250/ 500	250/ 500/ 1000	250/ 500/ 1000	350/ 1400/ 3000	240/ 350	126
L (fb^{-1})	250+ 500	250+ 500+ 1,000	1,150+ 1,600+ 2,500	500+ 1,500+ 2000	10,000+ 2,600	4.2
m_H (MeV)	32	32	15	33	7	0.06
Γ_H	5.0%	4.6%	2.5%	8.4%	1.0%	4.3%

Future prospects for colliders

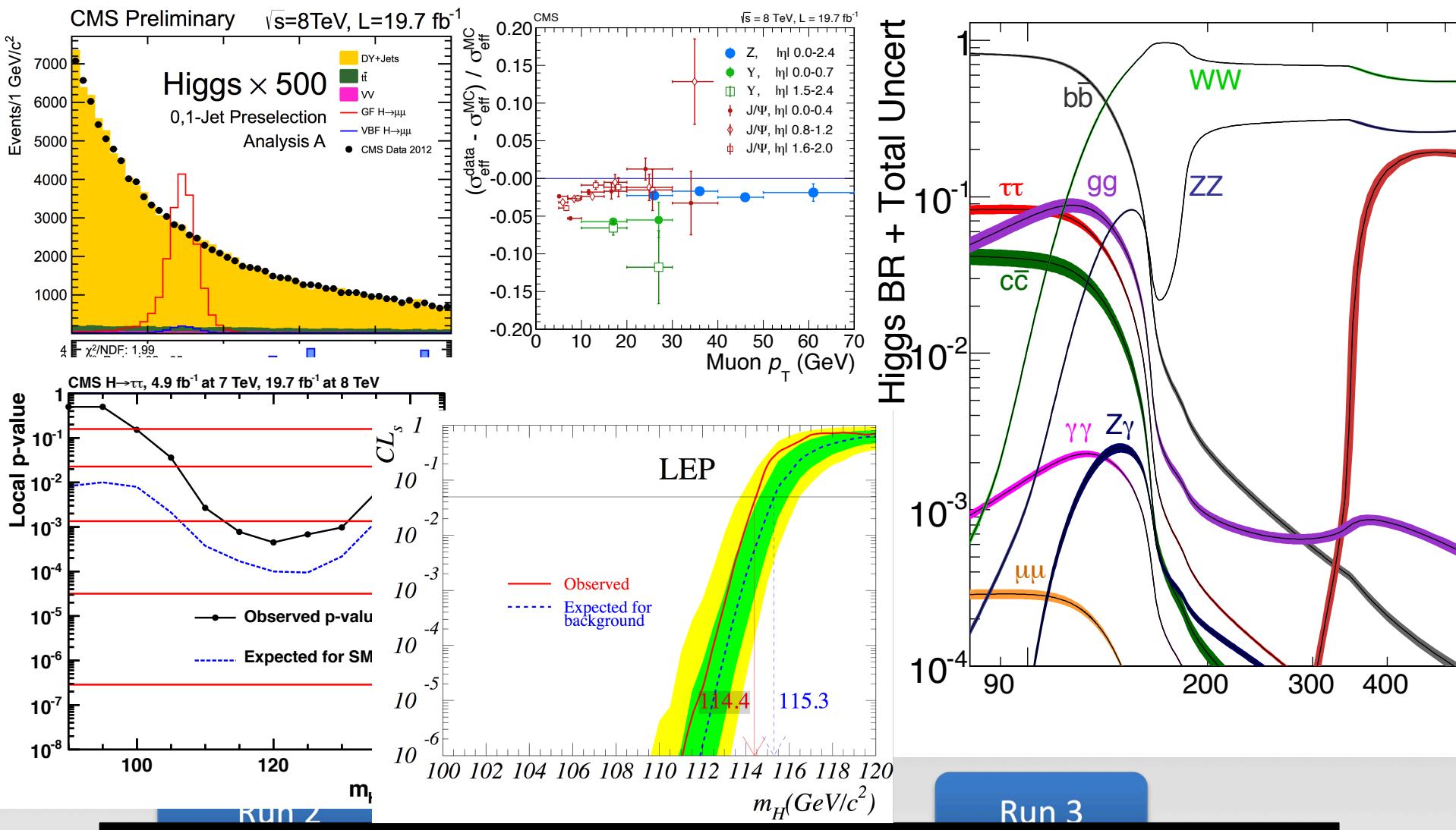




CONCLUSIONS

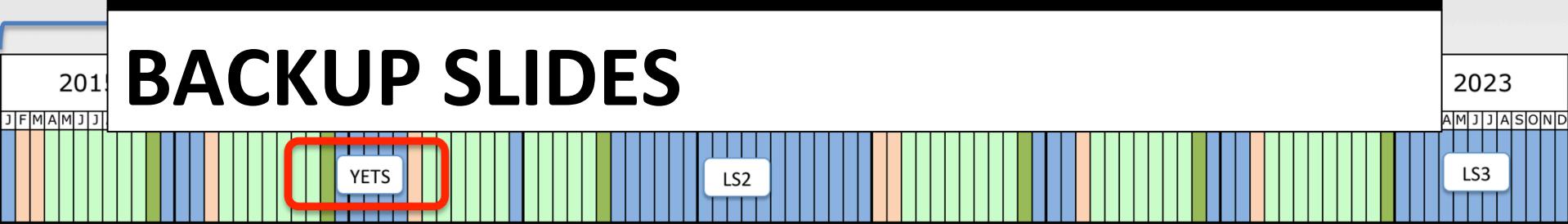
Conclusions

- ATLAS and CMS have discovered the scalar boson with <1% mass precision
 - Very challenging conditions in the detector
 - Studies push mass resolution to the limits
 - Systematic uncertainties dominated by lepton and photon scale factors
 - Expect improvement to ~ 25 MeV in the future at the LHC (3000 fb^{-1} , estimates taken from Snowmass 2013)
 - With $\sim 10M$ events per resonance, $\sim 0.5\%$ scale factor uncertainty
- Further improvement requires a linear collider
 - Thank you for your attention



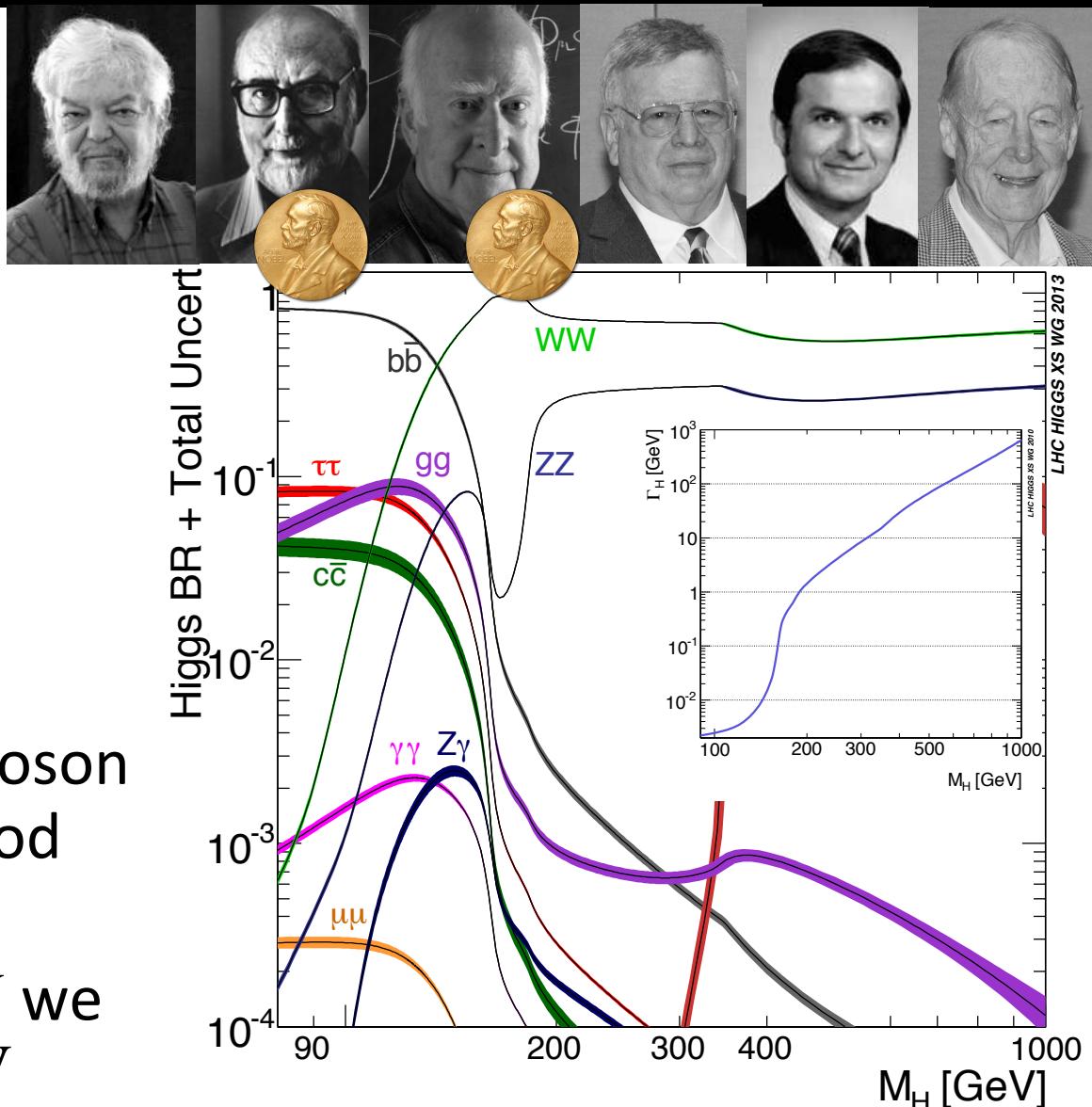
Need more information?

BACKUP SLIDES



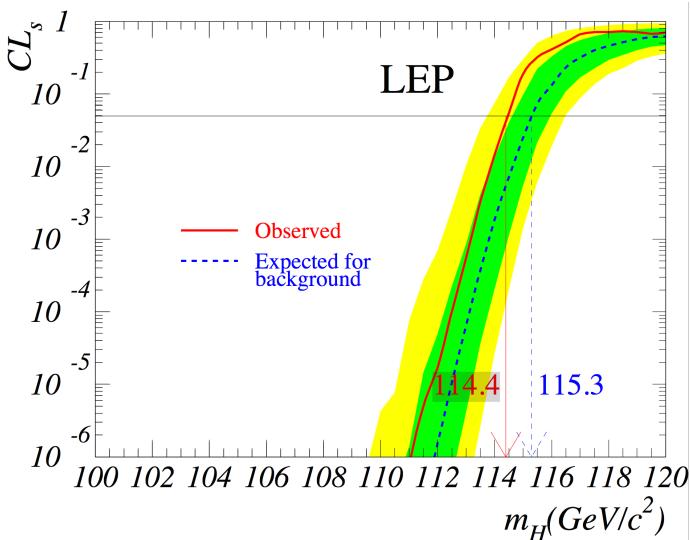
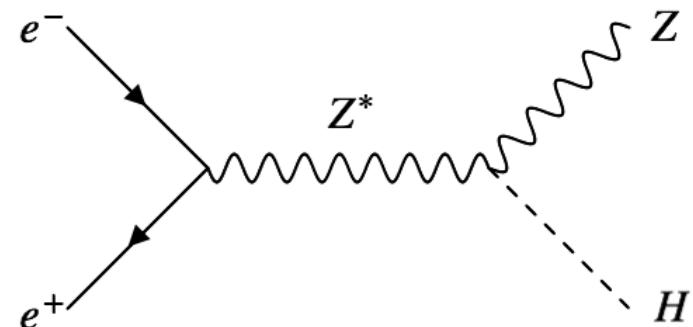
Why care about the scalar boson mass?

- The scalar boson was postulated in 1964 by Brout & Englert, Higgs, and Guralnik & Hagen & Kibble
- Its couplings depend strongly on its mass
- The width constrains invisible decays
- Measuring the scalar boson mass and width is a good probe of new physics
- At a mass of ~ 125 GeV we can expect $\Gamma_H \sim 4$ MeV



Higgs search at LEP, Tevatron

- The LEP accelerator was a circular e^+e^- collider located at CERN
- LEP was dismantled to make space for the LHC
- One search strategy was to look for Higgstrahlung ($e^+e^- \rightarrow Z^* \rightarrow ZH$)
- Can be independent of the final decay mode
- At the end of data taking (2000) the four main experiments had excluded the scalar boson up to a mass of 114.4 GeV
- Mass resolution after discovery would be dominated by Z boson four momentum resolution
- Tevatron ($p\bar{p} \sqrt{s} = 1.8$ TeV) looked for $H \rightarrow b\bar{b}$, seeing 3σ hints at ~ 125 GeV



Scalar boson search at the LHC

- We don't know the centre of mass frame
- Final states must be explicitly reconstructed

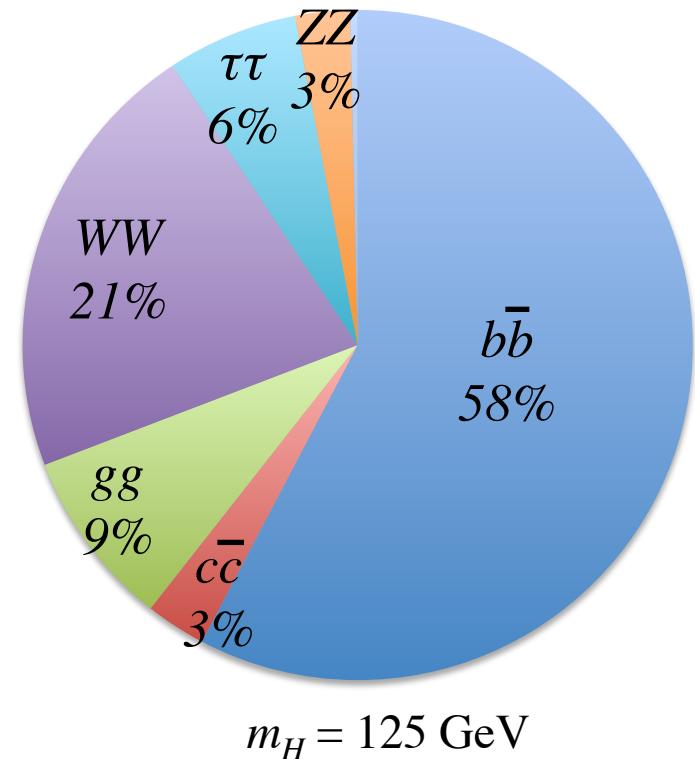
$- H \rightarrow q\bar{q}, H \rightarrow gg$
 $- H \rightarrow invisible$
 $- H \rightarrow WW^*, H \rightarrow \tau\tau$

$- H \rightarrow ZZ^*$
 $- H \rightarrow \gamma\gamma,$
 $- H \rightarrow Z\gamma, H \rightarrow \mu\mu$

Poor mass resolution

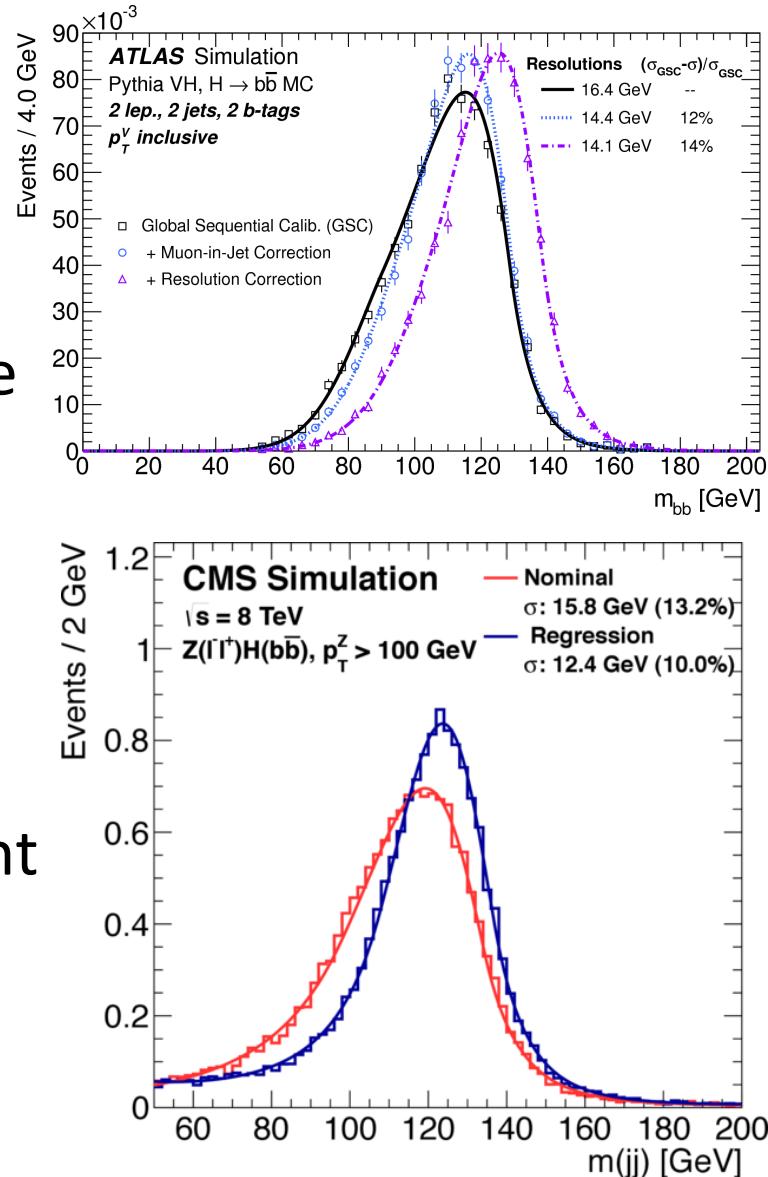
Good mass resolution

Very rare decays



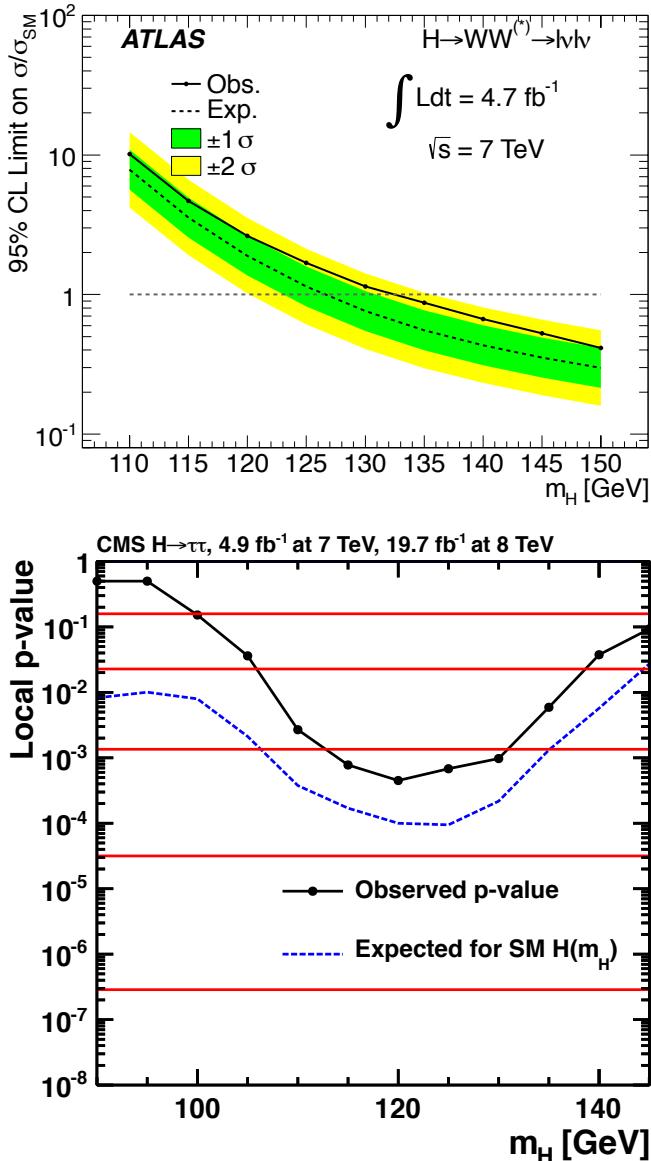
QCD decays, $H \rightarrow invisible$ at the LHC

- Out of $H \rightarrow gg$ and $H \rightarrow qq$, only $H \rightarrow bb$ is observable
 - $\sim 70\%$ of all final states
 - QCD backgrounds are far too large
 - $H \rightarrow bb$ only observable in associate production with W/Z
 - Very large uncertainties due to jet energy scale, jet energy resolution
 - Poor mass resolution
- $H \rightarrow invisible$ is very challenging:
 - High QCD, large pileup environment
 - Large uncertainties on missing transverse energy
 - Very poor mass resolution



$H \rightarrow WW^*$, $H \rightarrow \tau\tau$ at the LHC

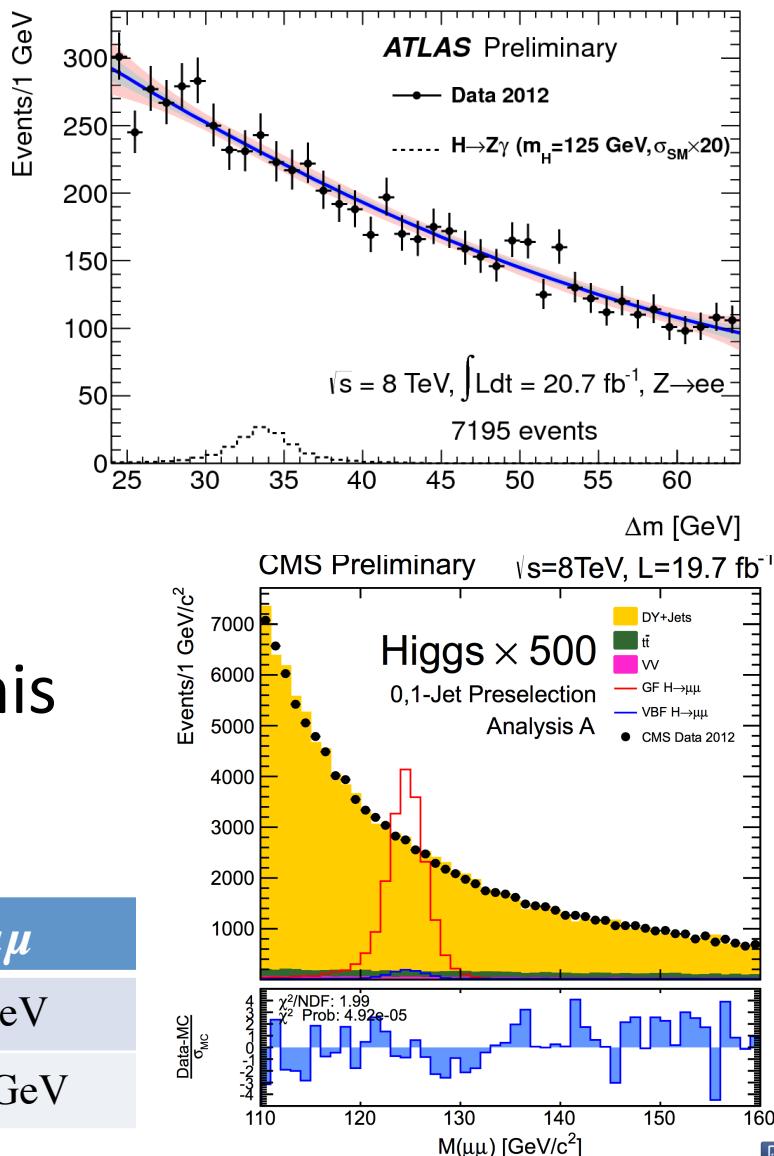
- Relatively high branching fractions
 - $\sim 27\%$ of all final states
- $e^+ \mu^-$ modes are quite clean
 - At least two neutrinos mean poor mass resolution
 - A better measure of signal strength than mass
- $Z \rightarrow \tau\tau$ is a background for $H \rightarrow \tau\tau$
- In two slides we find 97% of scalar boson decays not useful for precision mass measurements...



$H \rightarrow Z\gamma, H \rightarrow \mu\mu$ at the LHC

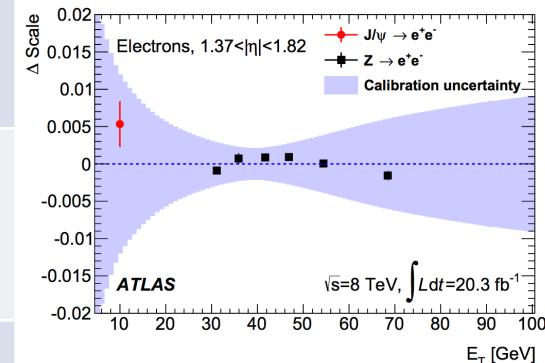
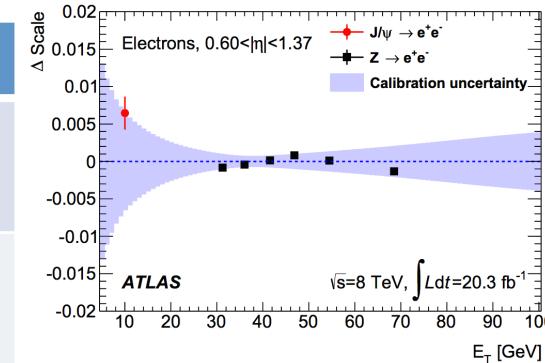
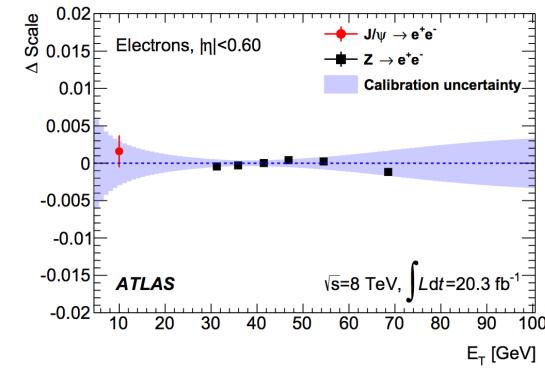
- These rare decays can in principle give fine mass resolution
 - Very high backgrounds and low signal statistics
 - Not visible in LHC Run 1 or LHC Run 2
 - Worth keeping an eye out for this in the 3000 fb^{-1} dataset

Mass resolution	$H \rightarrow Z\gamma$	$H \rightarrow \mu\mu$
ATLAS	5.0%($e e \gamma$) / 2.8%($\mu \mu \gamma$)	$\sim 5\text{-}6 \text{ GeV}$
CMS	1-3%	1.6-5.9 GeV



Electron and photon calibration at ATLAS

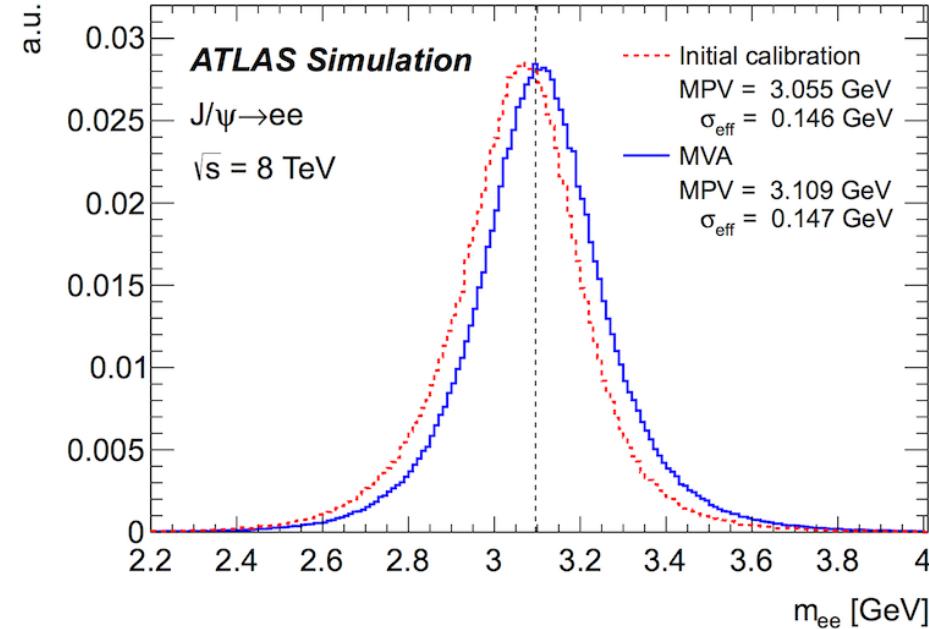
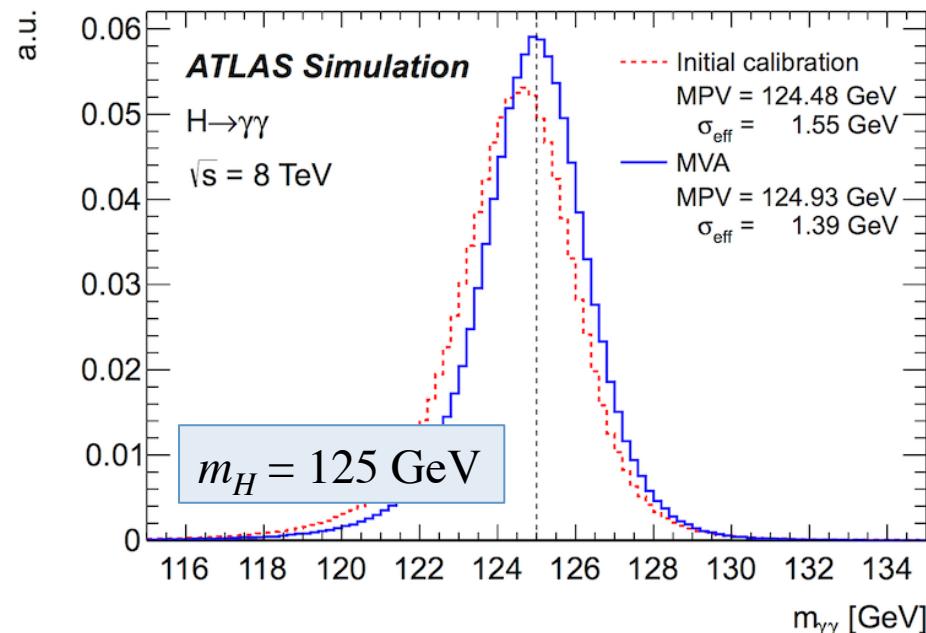
- Electrons and photons at ATLAS are reconstructed primarily in the LAr ecal
- Uncertainties are larger for photons



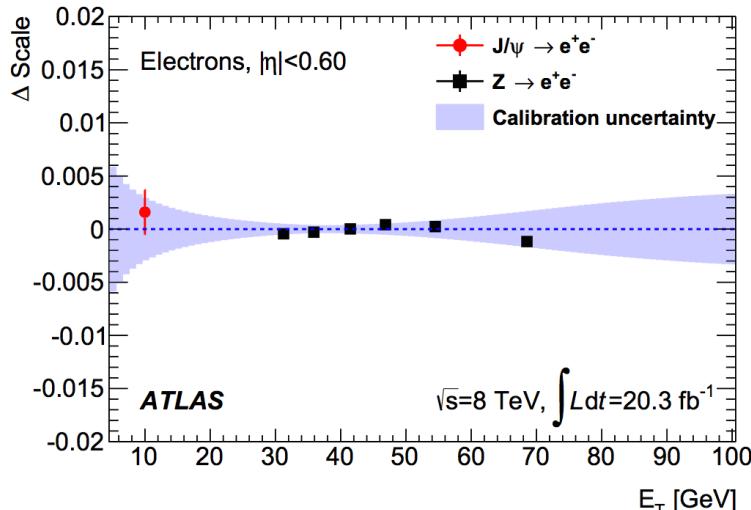
Source	Uncertainty
Non-linearities at cell level	0.1% in barrel $\leq 1\%$ for $1.5 < \eta < 1.7$
Relative calibration between calo layers	0.10-0.15%
Material before calorimeter	0.1-0.3% (unconverted photons) 0.05-0.15% (converted photons)
Misidentification of converted vs unconverted photons	0.02-0.04%
Lateral shower shape model	0.05-0.3%

Simulation improvement at ATLAS

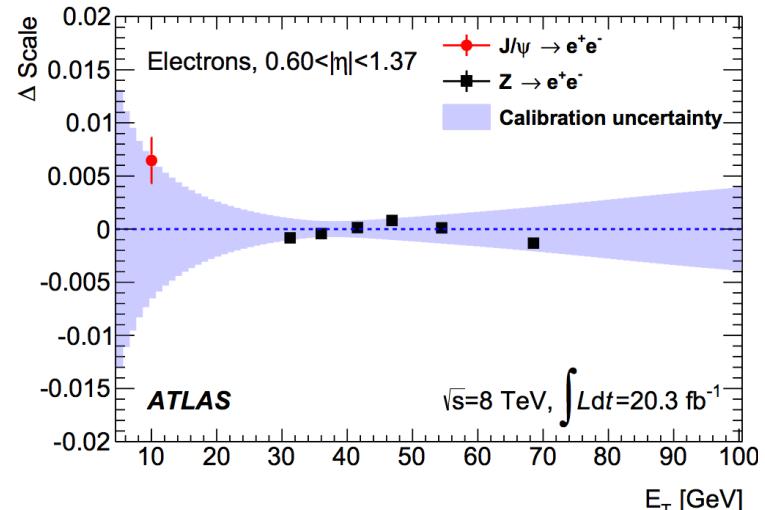
- Electron and photon reconstruction is improved at ATLAS using dedicated data/simulation studies
- Validation for electrons performed using $J/\psi \rightarrow ee$
- Validation for photons performed using $Z \rightarrow ll\gamma$



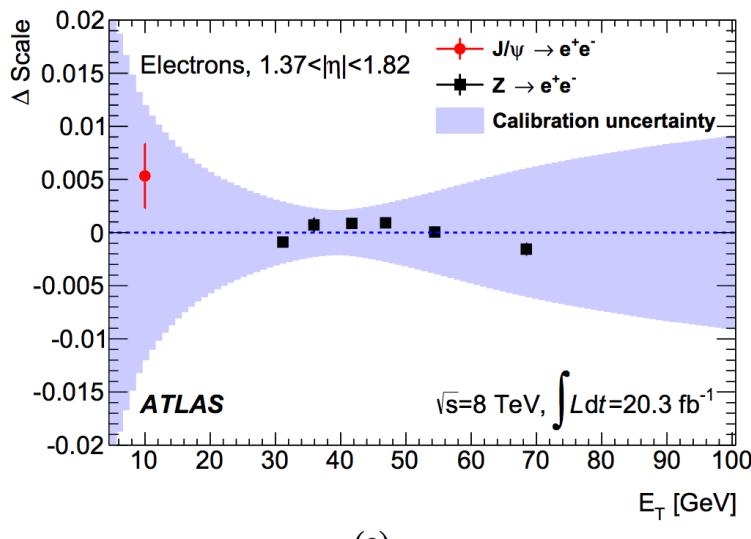
Electron scale factors at ATLAS



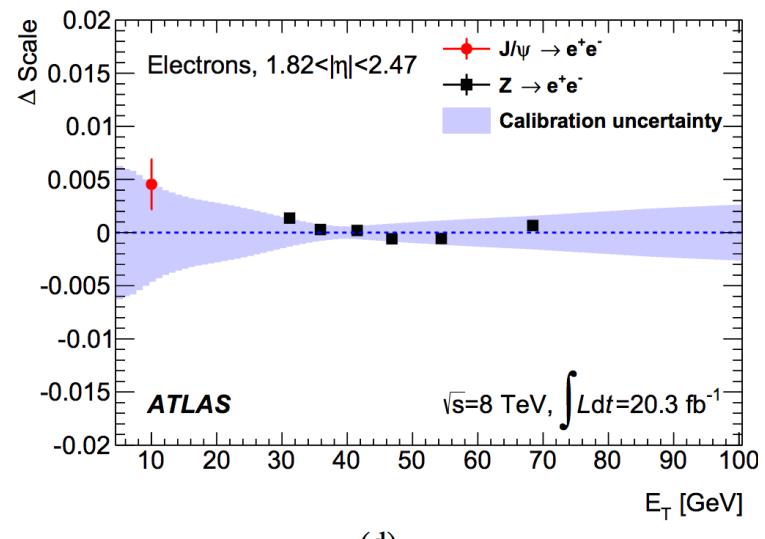
(a)



(b)

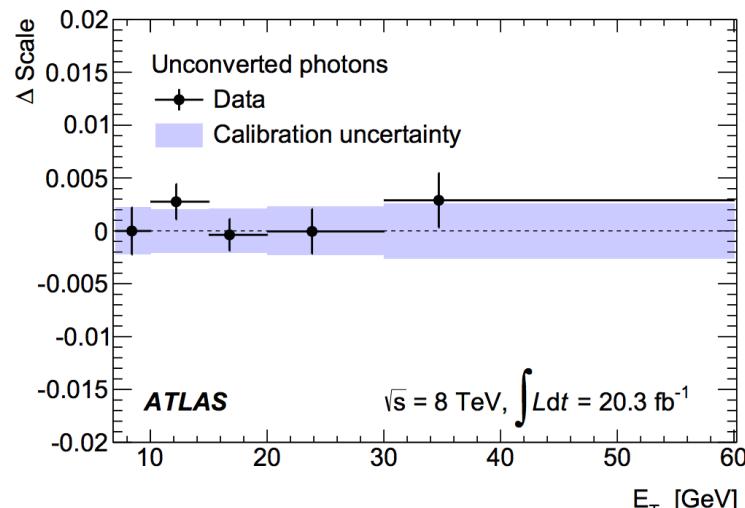


(c)

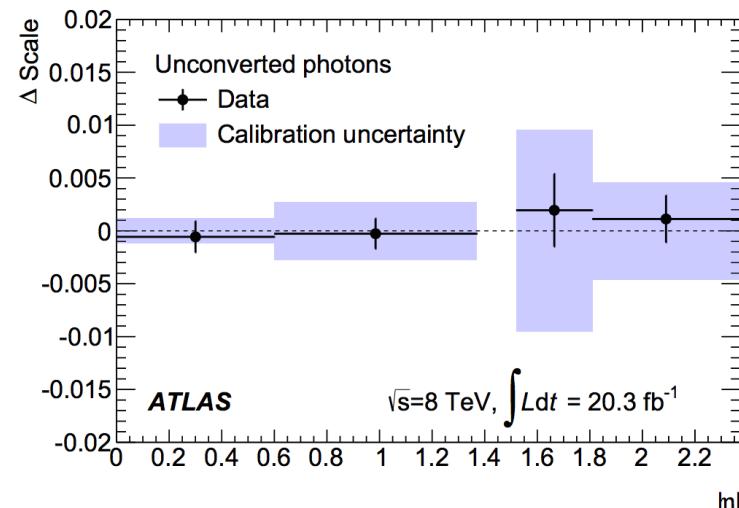


(d)

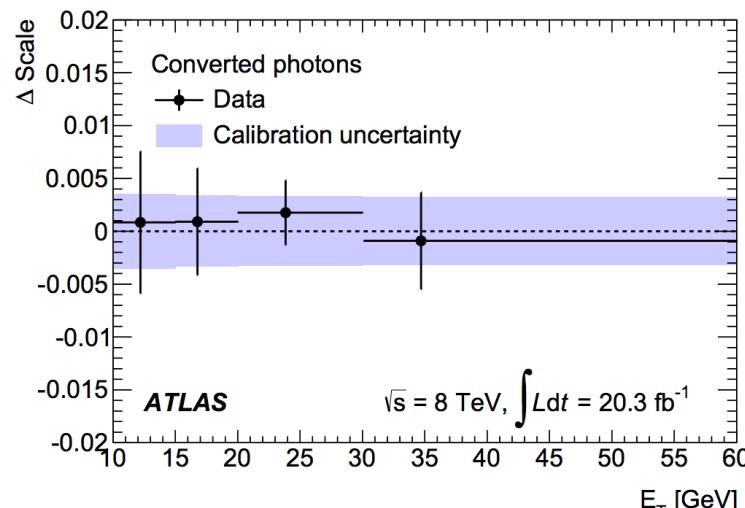
Unconverted photon scale factors at ATLAS



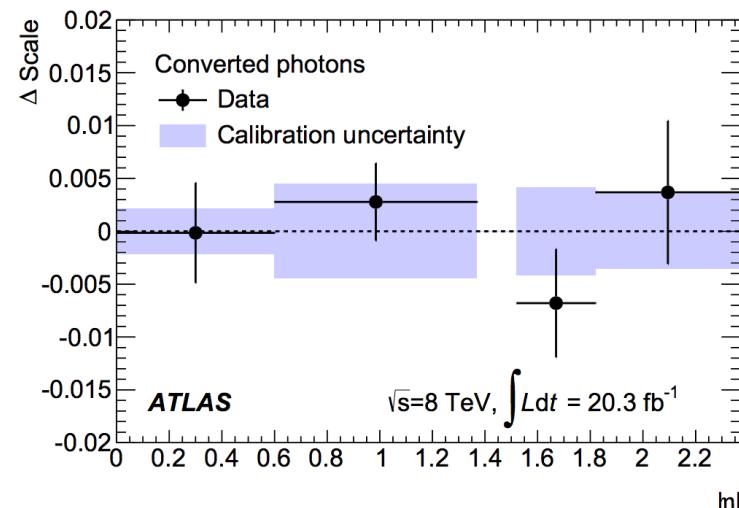
(a)



(b)

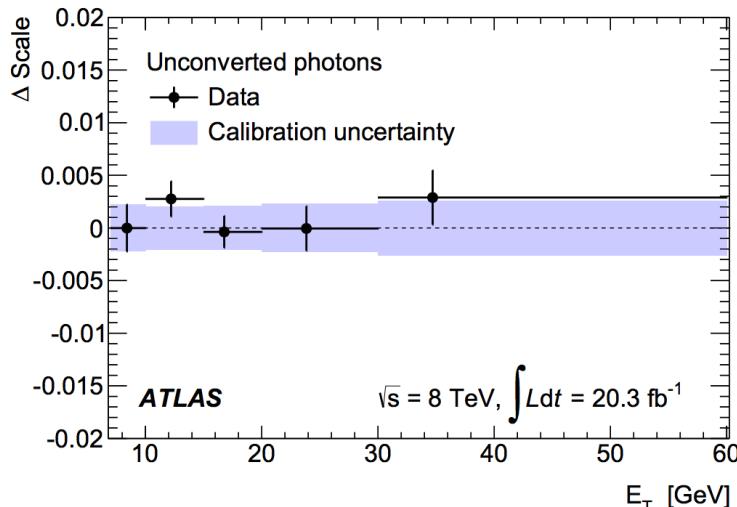


(c)

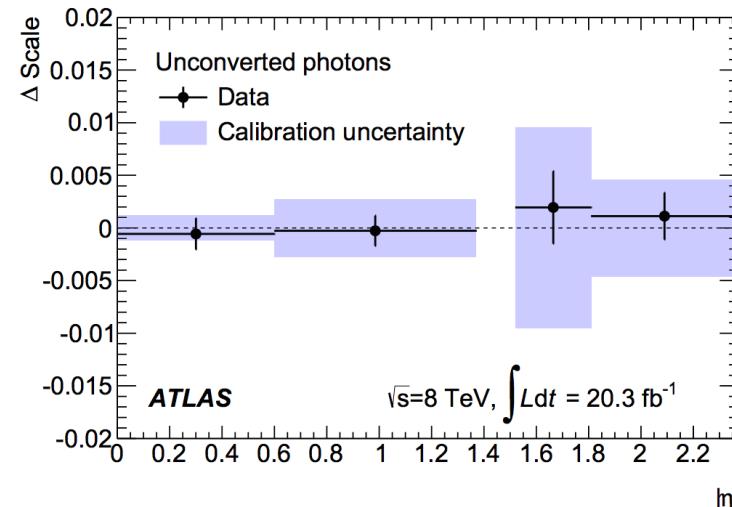


(d)

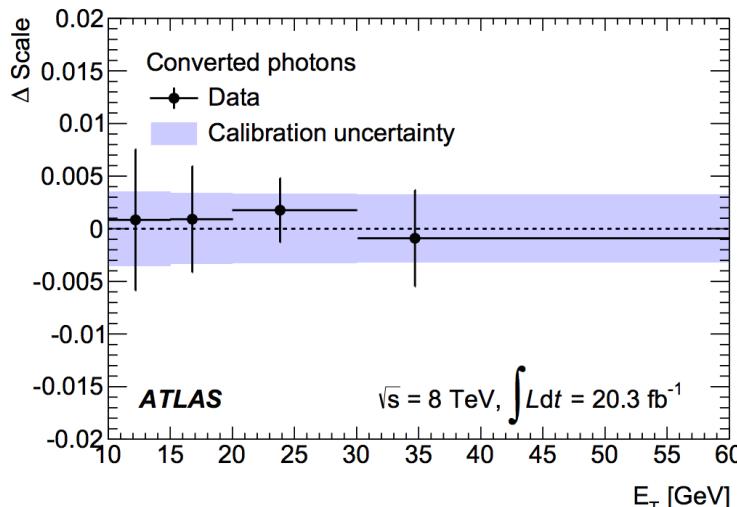
Converted photon scale factors at ATLAS



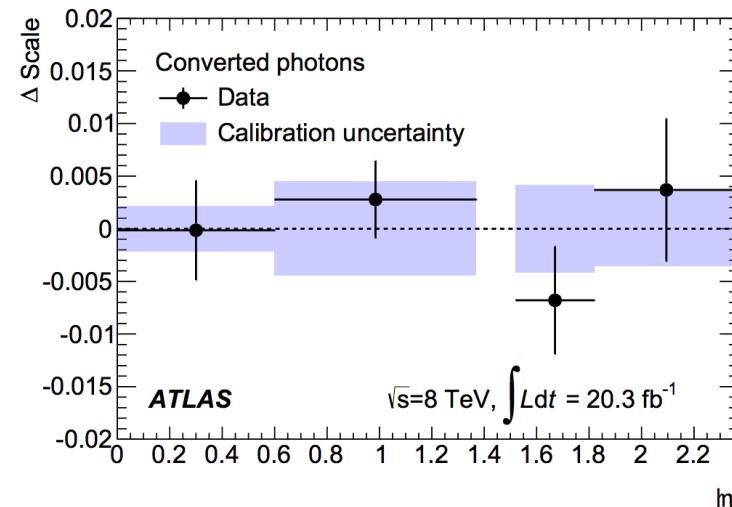
(a)



(b)



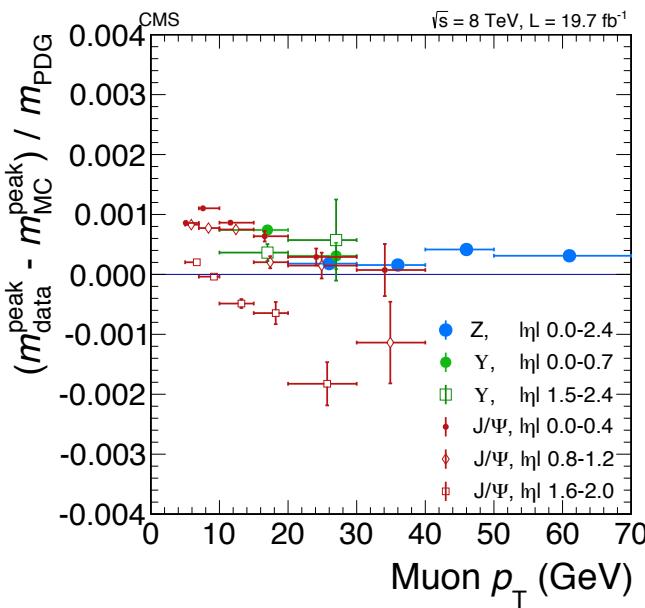
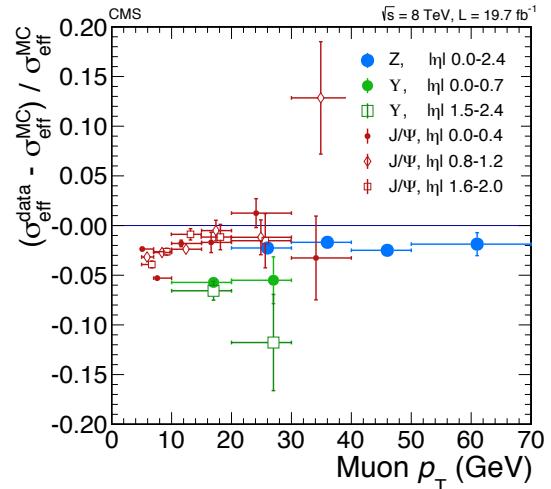
(c)



(d)

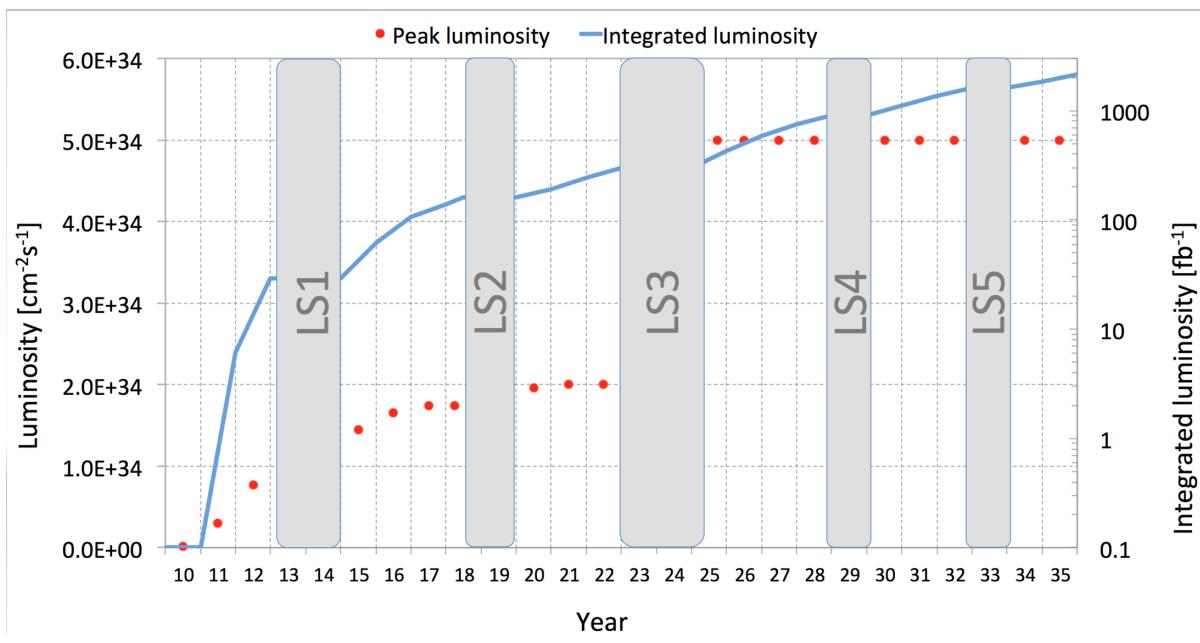
Muon calibration at ATLAS

- Muons are calibrated by:
 1. Momentum scale and resolution measurements are made using a reference model of the Z lineshape convolved with a Gaussian
 2. Biases are determined according to shifts in the Z peak position
- Events are fitted to Z , J/ψ , $\Upsilon(nS)$ peaks, averaging over η and p_T
- Systematic uncertainties:
 - Peak position: 0.1% ($ee\mu\mu, \mu\mu\mu\mu$)
 - Peak width: 5%



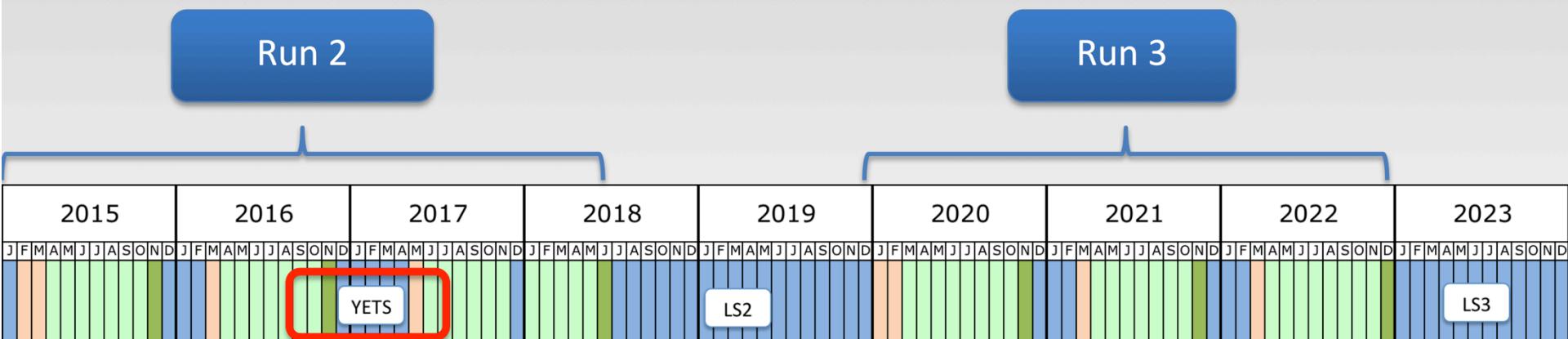
LHC timeline

- The energy and luminosity of the LHC will evolve:
 - End of Run 3, expect $\sim 300 \text{ fb}^{-1}$
 - End of Run 6, expect $\sim 3000 \text{ fb}^{-1}$
- Usual caveats apply!



Run 2

Run 3



References / sources

- Slide 1:
 - <http://cds.cern.ch/record/1474902/files/>
 - <http://www.isgtw.org/feature/how-grid-computing-helped-cern-hunt-higgs>
- Slide 3:
 - <http://home.web.cern.ch/topics/large-hadron-collider>
- Slide 4:
 - <https://cds.cern.ch/record/1708847>
 - <https://cds.cern.ch/record/1190487>
- Slide 5:
 - [1] <https://twiki.cern.ch/twiki/bin/view/CMSPublic/LumiPublicResults>
 - [2] <http://cms.web.cern.ch/news/new-world-record-first-pp-collisions-8-tev>
 - [3] <https://twiki.cern.ch/twiki/bin/view/AtlasPublic/LuminosityPublicResults>
- Slide 6:
 - <http://cds.cern.ch/record/1095924>
- Slide 7:
 - <http://cms.web.cern.ch/news/cms-detector-design>
- Slide 9:
 - <https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections>

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 - <http://arxiv.org/abs/1407.0558>
- Slide 11:
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 - <http://arxiv.org/abs/1312.5353>
- Slide 13:
 - <http://arxiv.org/abs/1406.3827>
- Slide 14:
 - <https://twiki.cern.ch/twiki/bin/view/CMSPublic/Hig13002PubTWiki>
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- Slide 25:
 - <https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/HIGG-2013-12/>
 - <https://twiki.cern.ch/twiki/bin/view/CMSPublic/Hig14009TWiki>
- Slide 26:
 - <http://www.slac.stanford.edu/econf/C1307292/docs/submittedArxivFiles/1307.5288.pdf>
 - <http://www.slac.stanford.edu/econf/C1307292/docs/submittedArxivFiles/1308.0494.pdf>
 - http://www.interactions.org/cms/?pid=2100&image_no=DE0061