

Constraints on extra neutral Higgs bosons through the Higgs coupling measurement

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7th October 2014 Belgrade Serbia

Introduction

4 July 2012

Large Hadron Collider (**LHC**) experiments

Discovery of a Higgs boson h

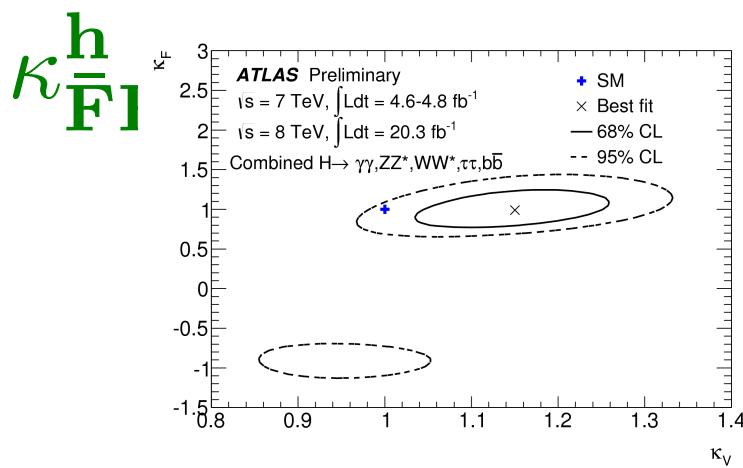
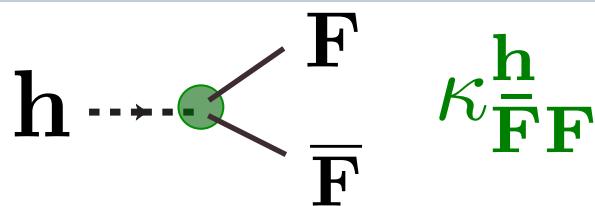
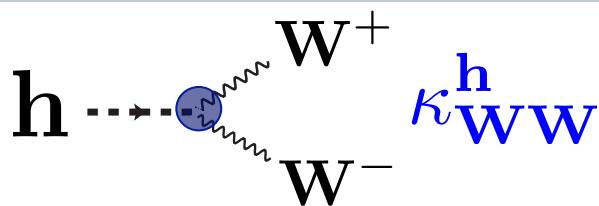
- ✓ The last particle predicted from the SM
- ✓ A key particle to solve the mystery of EWSB

Introduction

4 July 2012

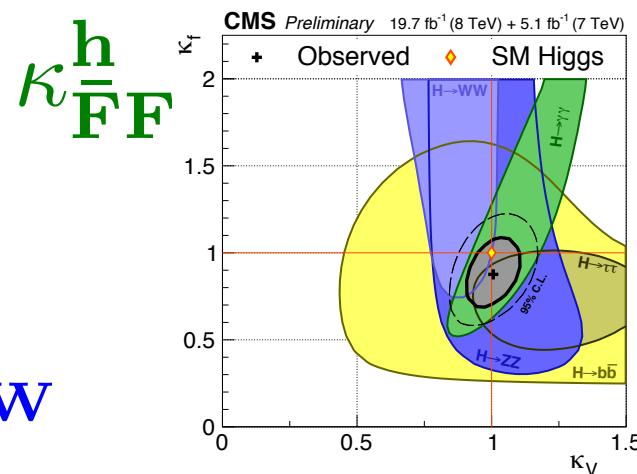
Large Hadron Collider (**LHC**) experiments

Discovery of a Higgs boson h



ATLAS-CONF-2014-009

κ_{WW}^h



CMS PAS-HIG-14-009

κ_{WW}^h

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4 July 2012

Future

Large Hadron Collider (**LHC**) experiments

Discovery of a Higgs boson h

The last particle predicted in the SM
A key particle to solve the mystery of EWSB

International Linear Collider (**ILC**) experiments and **LHC**

Precise measurements of a Higgs boson



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A key particle to solve the mystery of EWSB

International Linear Collider (**ILC**) experiments and **LHC**

Precise measurements of a Higgs boson

Can we get a new prospect of new physics through
the precise measurement of a Higgs boson?
(the main theme of my study)

Introduction

Can we get a new prospect of new physics through the precise measurement of the SM-like Higgs boson?

If the measured value of Higgs coupling strengths turns out to deviate from the SM values



Violation of perturbative unitarity

Inconsistency with electroweak precise measurements



New particles other than the 125GeV Higgs boson need to exist

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h

The SM-like Higgs boson

ϕ_2^0

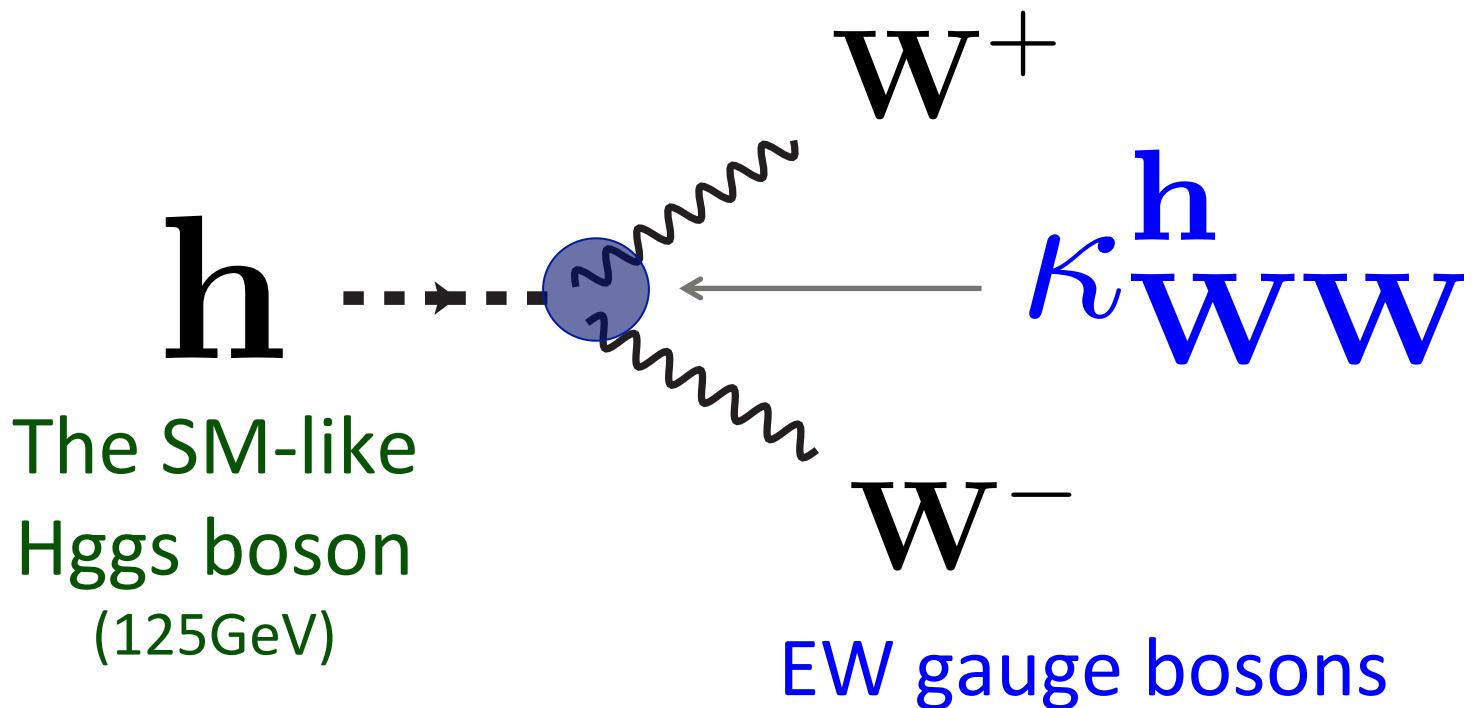
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$\phi_{N_0}^0$

Extra neutral Higgs bosons

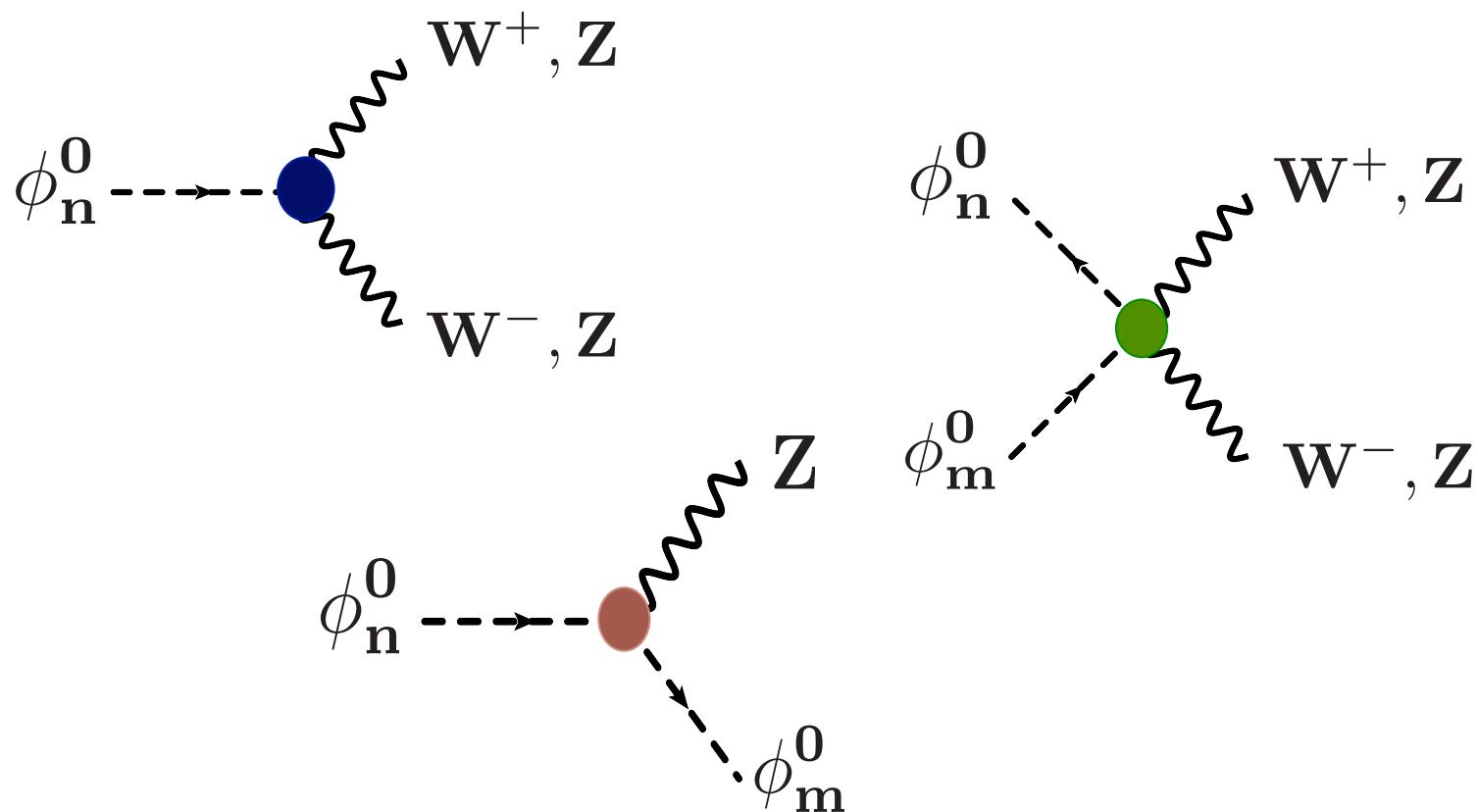
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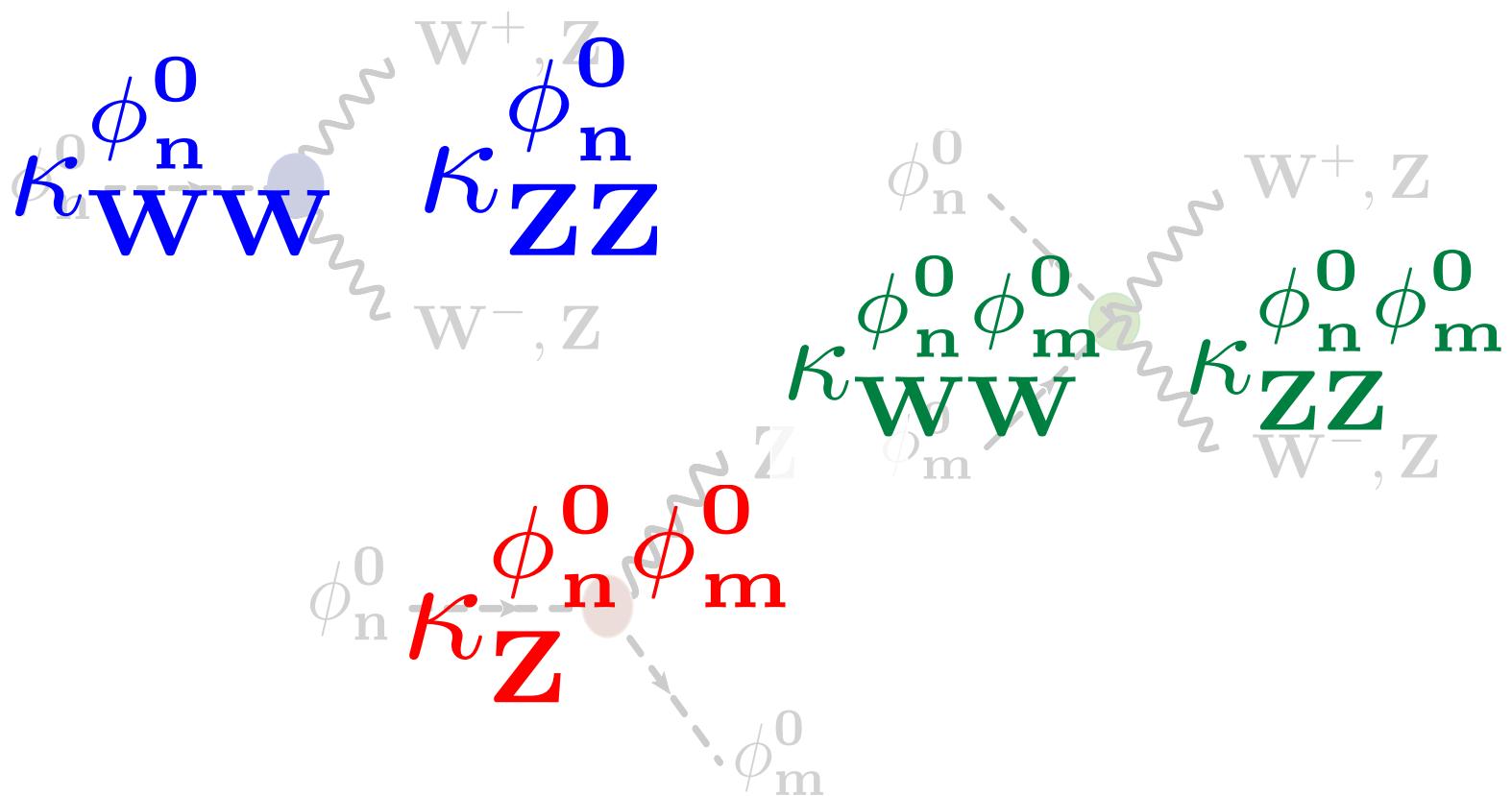
The Higgs coupling parameters

Interaction between neutral Higgs bosons and EW gauge bosons



The Higgs coupling parameters

Interaction between neutral Higgs bosons and EW gauge bosons

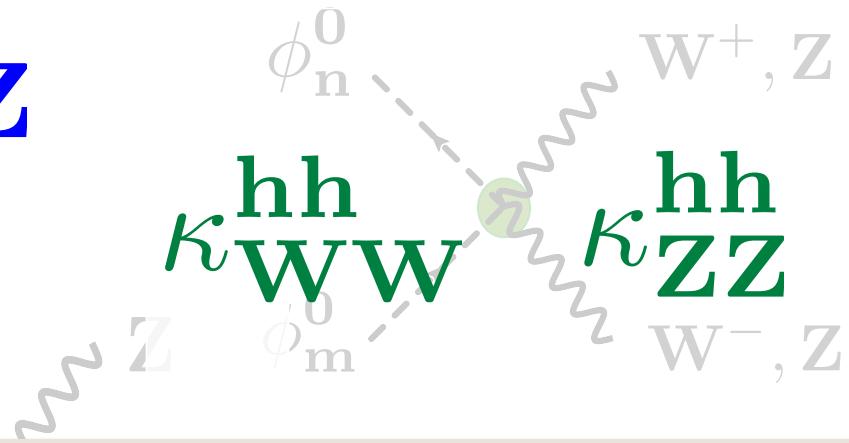


The Higgs coupling parameters

Interaction between neutral Higgs bosons and EW gauge bosons



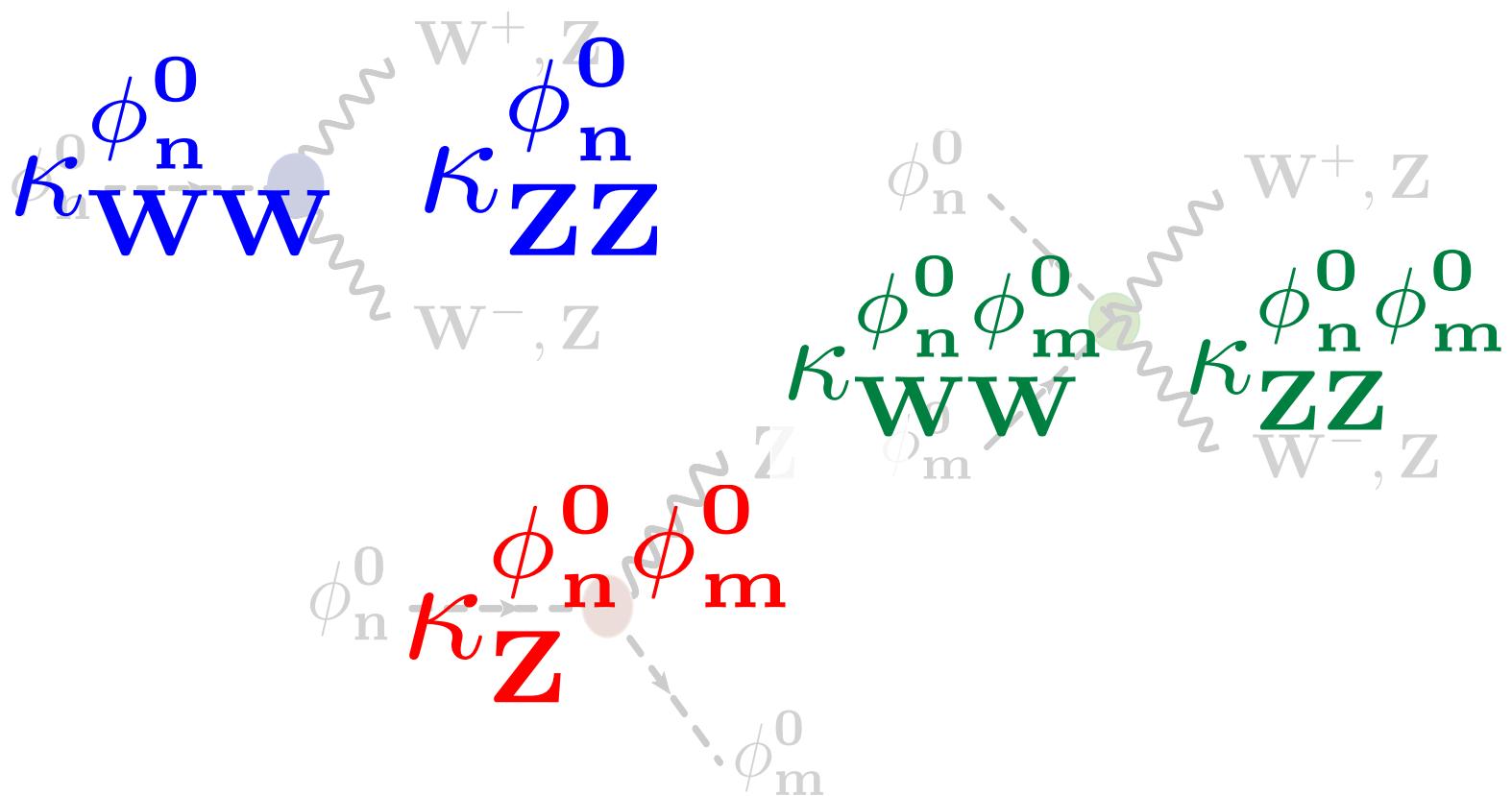
The Standard Model



$$\kappa_{WW}^h = \kappa_{ZZ}^h = \kappa_{WW}^{hh} = \kappa_{ZZ}^{hh} = 1$$

The Higgs coupling parameters

Interaction between neutral Higgs bosons and EW gauge bosons



The Higgs coupling parameters

Interaction between neutral Higgs bosons and EW gauge bosons

RN, Tsumura, Tanabashi (2014)

$$\begin{aligned}
 \mathcal{L}_\phi = & -v \sum_{n=1}^{N_0} \kappa_{\mathbf{WW}}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_+] \text{tr}[U^\dagger D^\mu U \tau_-] \\
 & - \frac{v}{4} \sum_{n=1}^{N_0} \kappa_{\mathbf{ZZ}}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \text{tr}[U^\dagger D^\mu U \tau_3] \\
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 & - \frac{1}{8} \sum_{m=1}^{N_0} \sum_{n=1}^{N_0} \kappa_{\mathbf{ZZ}}^{\phi_m^0 \phi_n^0} \phi_m^0 \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \text{tr}[U^\dagger D^\mu U \tau_3] \\
 & - \frac{i}{4} \sum_{m=1}^{N_0} \sum_{n=1}^{N_0} \kappa_{\mathbf{Z}}^{\phi_m^0 \phi_n^0} (\phi_m^0 \overset{\leftrightarrow}{\partial}_\mu \phi_n^0) \text{tr}[U^\dagger D^\mu U \tau_3]
 \end{aligned}$$

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Interaction between neutral Higgs bosons and EW gauge bosons

$$\mathcal{L}_\phi = -v \sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_+] \text{tr}[U^\dagger D^\mu U \tau_-]$$

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RN, Tsumura, Tanabashi (2014)

$$-\frac{V}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \kappa_{WW}^{\phi_m^0 \phi_n^0} \phi_m^0 \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_+] \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_-]$$

$$-\frac{v}{4} \sum_{m=1}^{N_0} \sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_m^0 \phi_n^0} \phi_m^0 \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3]$$

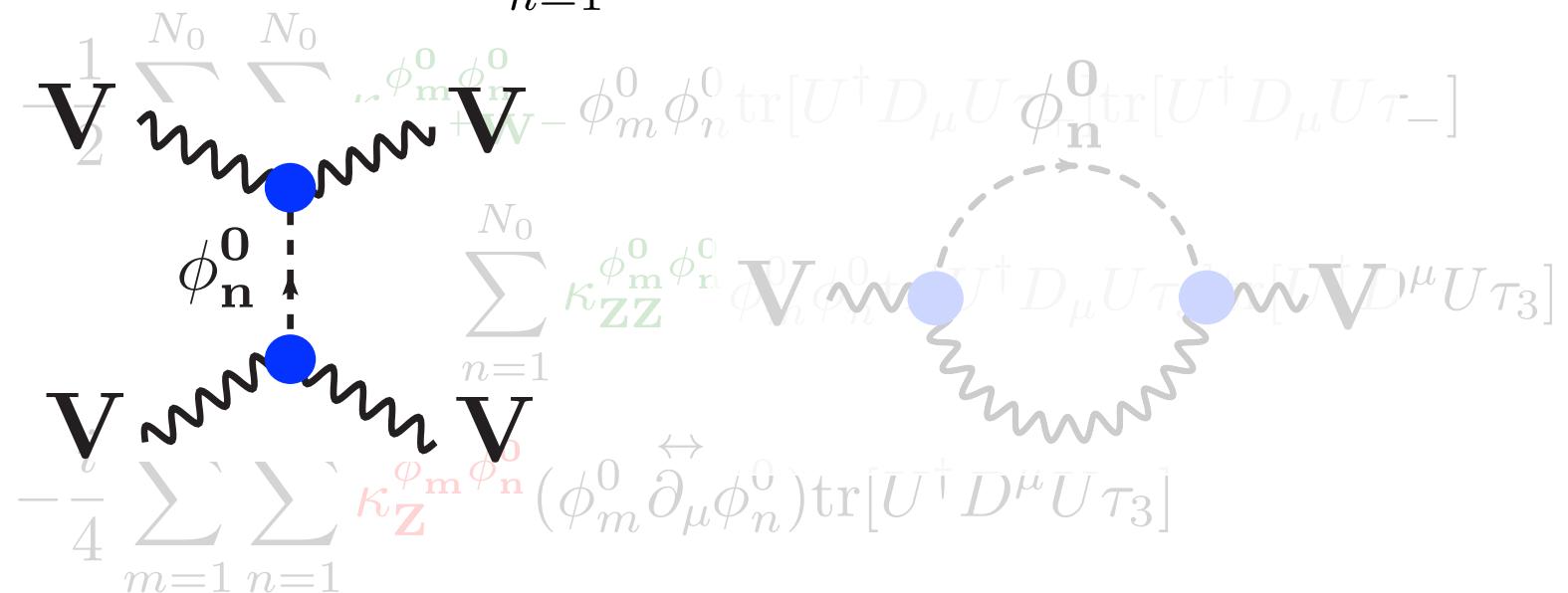
$$(\phi_m^0 \partial_\mu \phi_n^0) \text{tr}[U^\dagger D^\mu U \tau_3]$$

The Unitarity of scattering amplitude

Interaction between neutral Higgs bosons and EW gauge bosons

$$\mathcal{L}_\phi = -v \sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_+] \text{tr}[U^\dagger D^\mu U \tau_-]$$

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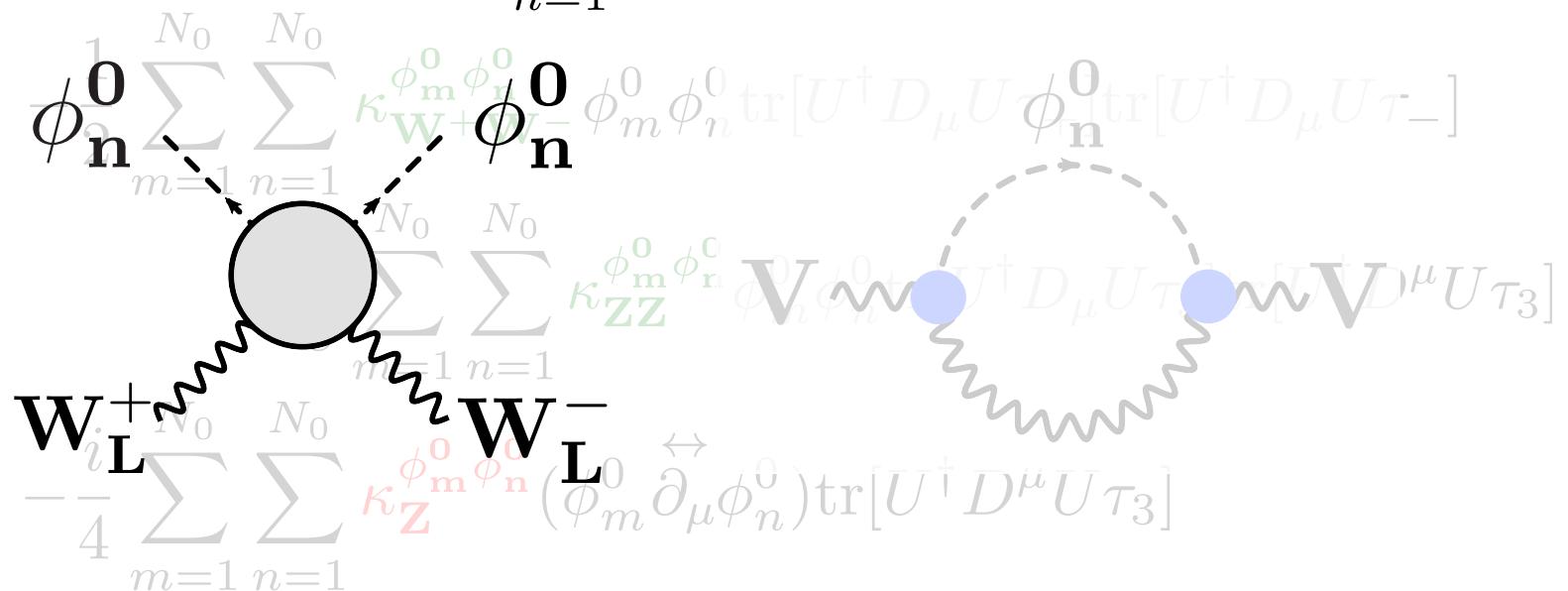


The Unitarity of scattering amplitude

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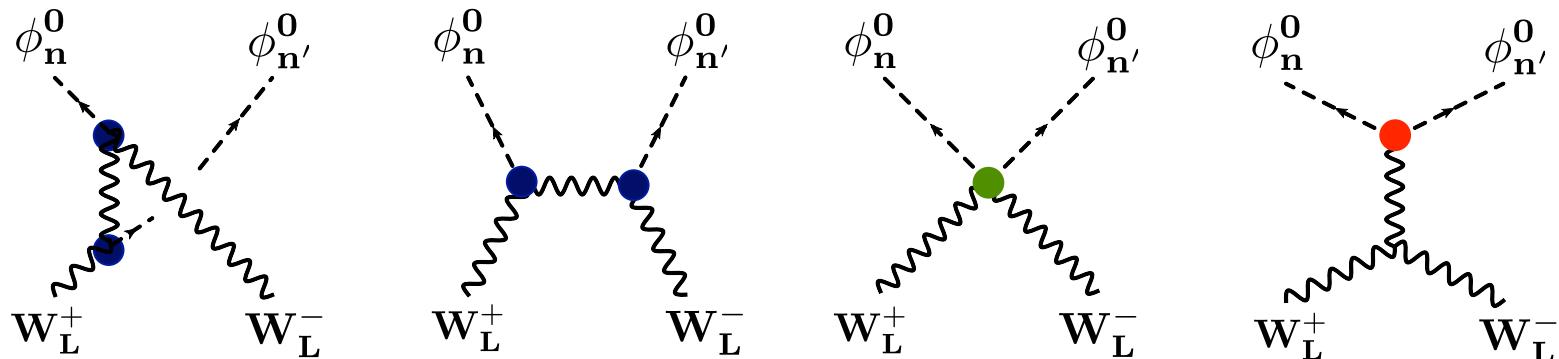
$$\mathcal{L}_\phi = -v \sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_+] \text{tr}[U^\dagger D^\mu U \tau_-]$$

$$- \frac{v}{4} \sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \text{tr}[U^\dagger D^\mu U \tau_3]$$



Scattering amplitude ($W_L^+ W_L^- \rightarrow \phi_n^0 \phi_{n'}^0$)

The high energy behavior of scattering amplitude of $W_L^+ W_L^- \rightarrow \phi_n^0 \phi_{n'}^0$.



$$\mathcal{A}_{W_L^+ W_L^- \rightarrow \phi_n^0 \phi_{n'}^0} \sim \frac{s}{v^2} \left\{ \kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_{n'}^0} - \kappa_{WW}^{\phi_n^0 \phi_{n'}^0} \right\} - \frac{i(t-u)}{v^2} \left\{ \kappa_Z^{\phi_n^0 \phi_{n'}^0} \right\}.$$

Requiring the cancellation of enhanced term in the amplitude, we obtain

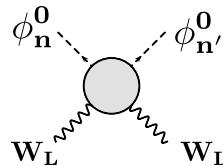
$$\kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_{n'}^0} = \kappa_{WW}^{\phi_n^0 \phi_{n'}^0}$$

$$\kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$

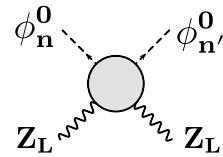
“ Unitarity conditions ”

J.F.Gunion et al (1991)

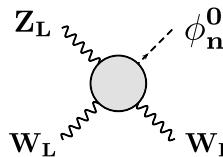
Scattering amplitudes in high energy limit



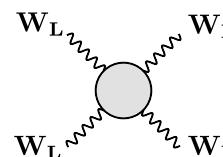
$$\mathcal{A}_{W_L^+ W_L^- \rightarrow \phi_n^0 \phi_{n'}^0} \sim \frac{\mathbf{s}}{v^2} \left\{ \kappa_{\mathbf{WW}}^{\phi_n^0} \kappa_{\mathbf{WW}}^{\phi_{n'}^0} - \kappa_{\mathbf{WW}}^{\phi_n^0 \phi_{n'}^0} \right\} - \frac{i(\mathbf{t} - \mathbf{u})}{v^2} \left\{ \kappa_{\mathbf{Z}}^{\phi_n^0 \phi_{n'}^0} \right\}.$$



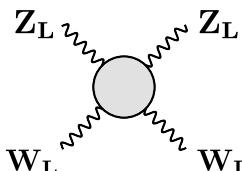
$$\mathcal{A}_{Z_L Z_L \rightarrow \phi_n^0 \phi_{n'}^0} \sim \frac{\mathbf{s}}{v_Z^2} \left\{ \rho_0 \kappa_{\mathbf{ZZ}}^{\phi_n^0} \kappa_{\mathbf{ZZ}}^{\phi_{n'}^0} - \kappa_{\mathbf{ZZ}}^{\phi_n^0 \phi_{n'}^0} + \sum_{m=1}^{N_0} \kappa_{\mathbf{Z}}^{\phi_n^0 \phi_m^0} \kappa_{\mathbf{Z}}^{\phi_{n'}^0 \phi_m^0} \right\}$$



$$\mathcal{A}_{W_L^+ W_L^- \rightarrow \phi_n^0 Z_L} \sim \frac{\mathbf{s}}{vv_Z} \left\{ \sum_{m=1}^{N_0} \kappa_{\mathbf{Z}}^{\phi_n^0 \phi_m^0} \kappa_{\mathbf{WW}}^{\phi_m^0} \right\} - \frac{i(\mathbf{t} - \mathbf{u})}{\rho_0 vv_Z} \left\{ \kappa_{\mathbf{WW}}^{\phi_n^0} - \rho_0 \kappa_{\mathbf{ZZ}}^{\phi_n^0} \right\}$$



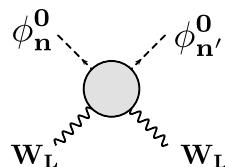
$$\mathcal{A}_{W_L^+ W_L^- \rightarrow W_L^+ W_L^-} \sim \frac{\mathbf{u}}{v^2} \left\{ -4 + \frac{3}{\rho_0} + \sum_{n=1}^{N_0} (\kappa_{\mathbf{WW}}^{\phi_n^0})^2 \right\}.$$



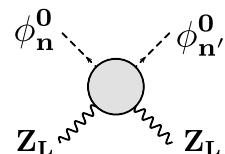
$$\mathcal{A}_{W_L^+ W_L^- \rightarrow Z_L Z_L} \sim \frac{\mathbf{s}}{v^2} \left\{ \frac{1}{\rho_0} - \rho_0 \sum_{n=1}^{N_0} \kappa_{\mathbf{WW}}^{\phi_n^0} \kappa_{\mathbf{ZZ}}^{\phi_n^0} \right\}.$$

Unitarity conditions

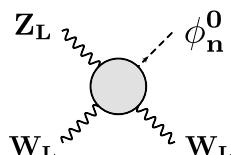
J.F.Gunion et al (1991)



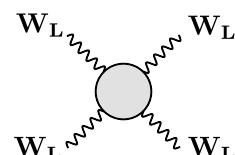
$$\kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_{n'}^0} = \kappa_{WW}^{\phi_n^0 \phi_{n'}^0}, \quad \kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$



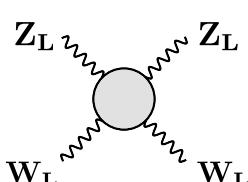
$$\kappa_{ZZ}^{\phi_n^0} \kappa_{ZZ}^{\phi_{n'}^0} = \kappa_{ZZ}^{\phi_n^0 \phi_{n'}^0}$$



$$\kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0}$$



$$\rho_0 = 1$$



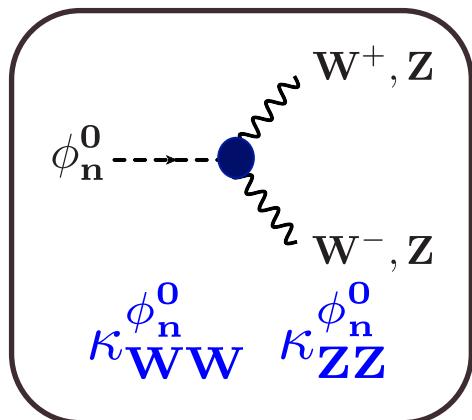
$$\sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_n^0} = 1$$

Unitarity conditions

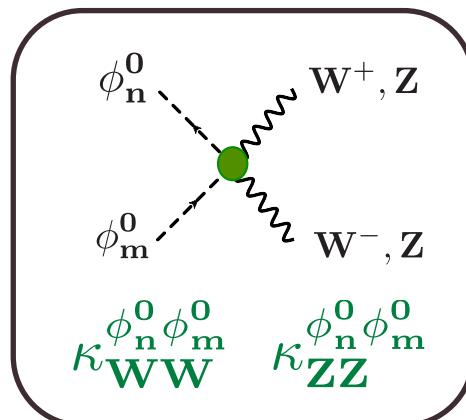
J.F.Gunion et al (1991)

Interaction between neutral Higgs bosons and EW gauge bosons

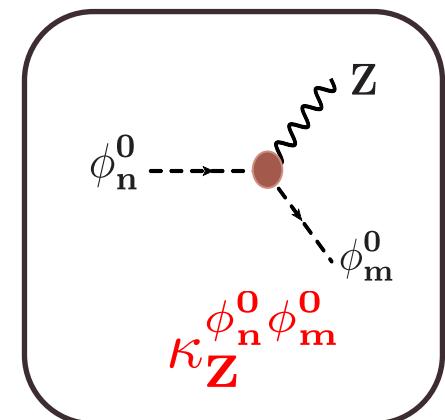
Higgs - V - V



Higgs - Higgs - V - V



Higgs - Higgs - V



- ✓ In order to make theory unitary, the following conditions (“**Unitarity conditions**”) should be satisfied among Higgs couplings and tree-level ρ parameter.

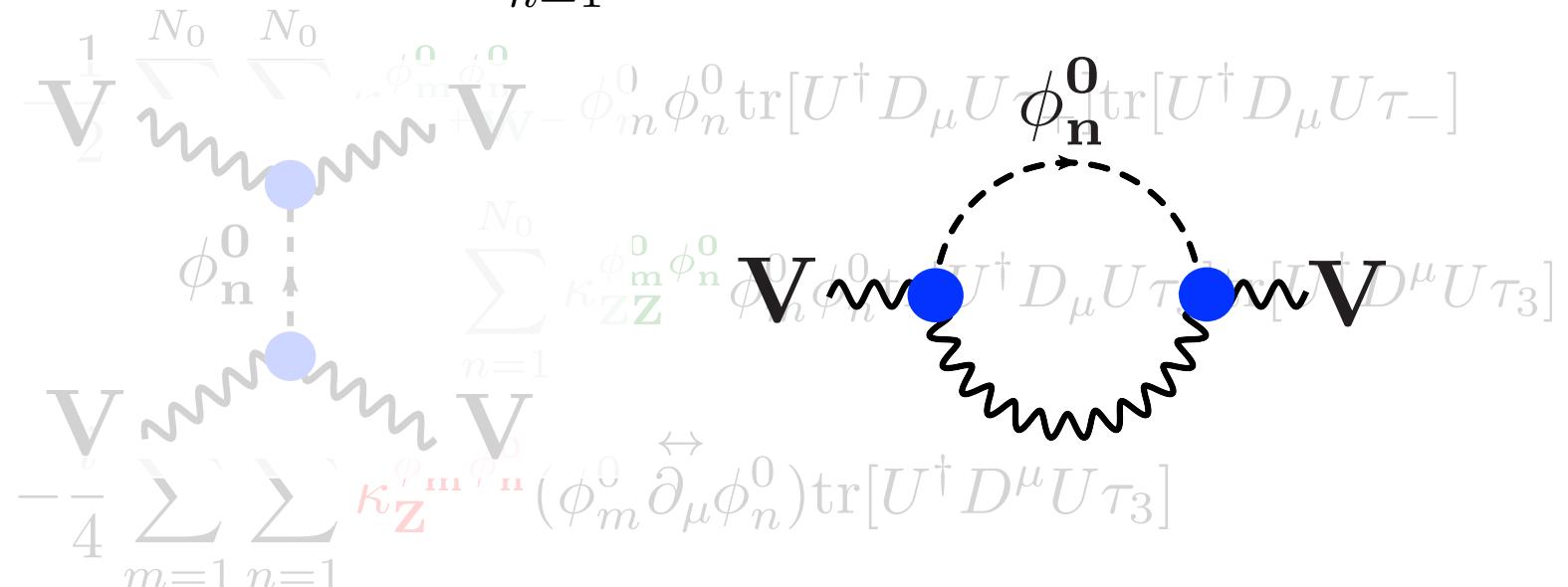
$$\rho_0 = 1 , \quad \kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0 , \quad \kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0} \equiv \kappa_V^{\phi_n^0} ,$$

$$\kappa_{WW}^{\phi_n^0 \phi_{n'}^0} = \kappa_{ZZ}^{\phi_n^0 \phi_{n'}^0} = \kappa_V^{\phi_n^0} \kappa_V^{\phi_{n'}^0} \quad \text{and} \quad \sum_{n=1}^{N_0} \kappa_V^{\phi_n^0} \kappa_V^{\phi_n^0} = 1$$

Interaction between neutral Higgs bosons and EW gauge bosons

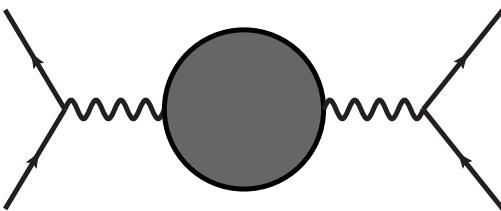
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$$-\frac{v}{4} \sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \text{tr}[U^\dagger D^\mu U \tau_3]$$



The electroweak oblique corrections

The electroweak precision measurements of the $f\bar{f} \rightarrow f'\bar{f}'$ scattering amplitudes



In order to compare our models with the electroweak precision measurements,
We evaluate the oblique correction parameters (called “**S,T and U parameters**”)

M.E.Peskin and T.Takeuchi (1990)

$$S \equiv 16\pi \left\{ \Pi'_{33}(0) - \Pi'_{3Q}(0) \right\} - 16\pi \left\{ \Pi'_{33}(0) - \Pi'_{3Q}(0) \right\} |_{SM},$$

$$T \equiv \frac{4}{\alpha v^2} \left\{ \Pi_{11}(0) - \Pi_{33}(0) \right\} - \frac{4}{\alpha v^2} \left\{ \Pi_{11}(0) - \Pi_{33}(0) \right\} |_{SM},$$

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The Higgs coupling parameters

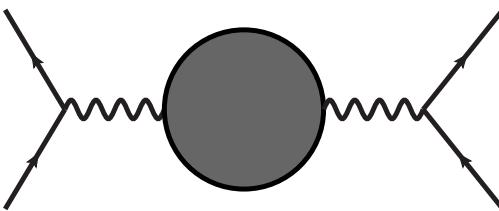
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RN, Tsumura, Tanabashi (2014)

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 & - \frac{1}{8} \sum_{m=1}^{N_0} \sum_{n=1}^{N_0} \kappa_{\mathbf{ZZ}}^{\phi_m^0 \phi_n^0} \phi_m^0 \phi_n^0 \text{tr}[U^\dagger D_\mu U \tau_3] \text{tr}[U^\dagger D^\mu U \tau_3] \\
 & - \frac{i}{4} \sum_{m=1}^{N_0} \sum_{n=1}^{N_0} \kappa_{\mathbf{Z}}^{\phi_m^0 \phi_n^0} (\phi_m^0 \overset{\leftrightarrow}{\partial}_\mu \phi_n^0) \text{tr}[U^\dagger D^\mu U \tau_3]
 \end{aligned}$$

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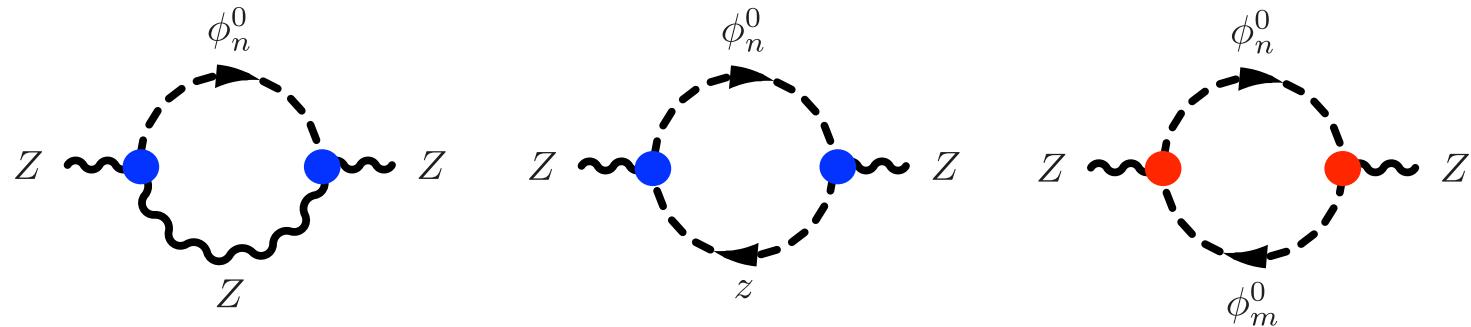
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The electroweak oblique corrections

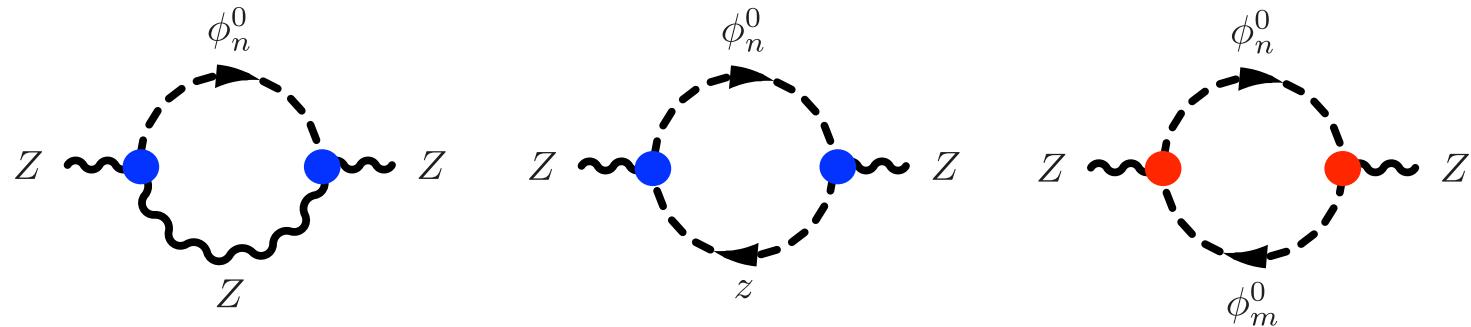
For example, **UV divergent terms of S parameters** at one loop are obtained by



$$S = \frac{1}{12} \left[1 - \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{Z}\mathbf{Z}}^{\phi_n^0} \right)^2 - \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_{\mathbf{Z}}^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2} + S_{\text{finite}},$$

The electroweak oblique corrections

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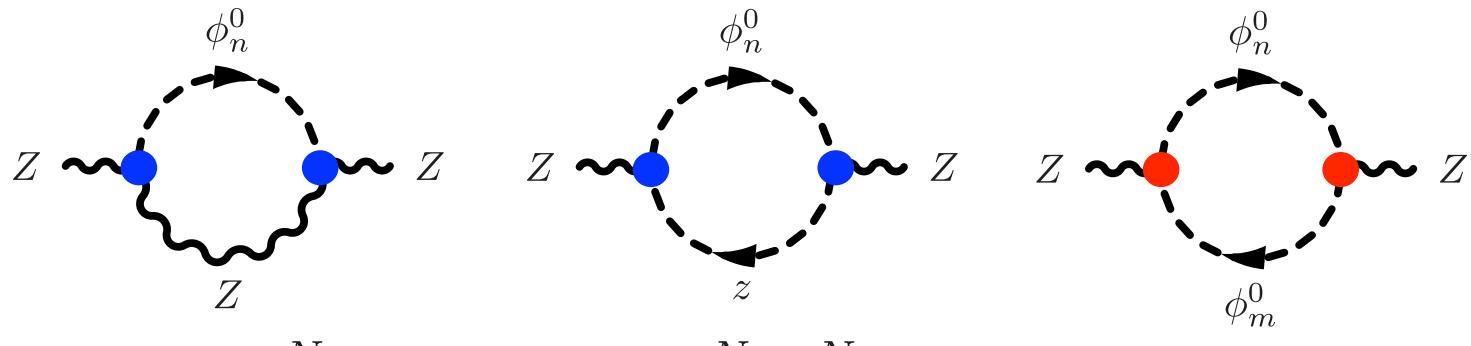


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The electroweak oblique corrections

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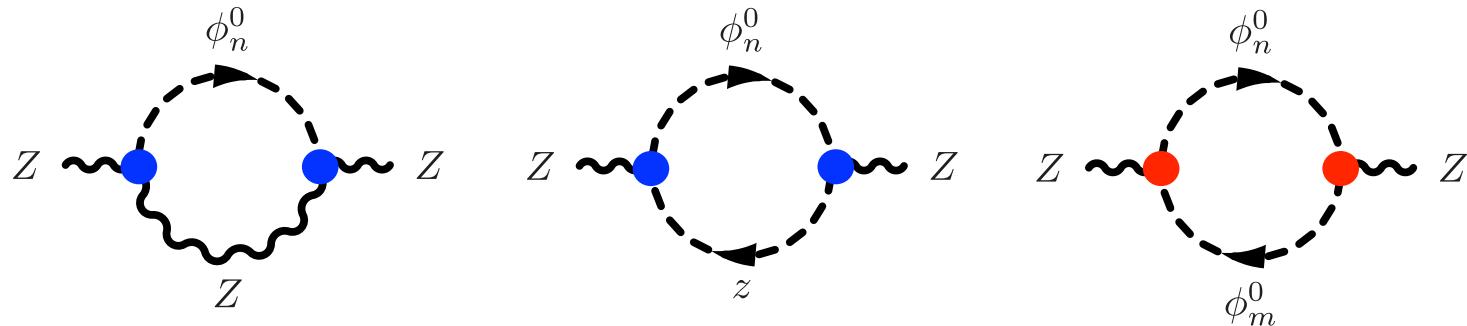


$$1 - \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{Z}\mathbf{Z}}^{\phi_n^0} \right)^2 - \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_{\mathbf{Z}}^{\phi_n^0 \phi_m^0} \right)^2 = 0$$

Let us recall “unitarity conditions”, $\sum_{n=1}^{N_0} \left(\kappa_{\mathbf{Z}\mathbf{Z}}^{\phi_n^0} \right)^2 = 1$ and $\kappa_{\mathbf{Z}}^{\phi_n^0 \phi_m^0} = 0$

The electroweak oblique corrections

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The finiteness of S parameters is automatically guaranteed
once unitarity conditions are satisfied among Higgs couplings

RN, Tsumura, Tanabashi (2014)

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} - 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} + 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

$$\begin{aligned} T_{log} = & \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{M_{\phi_n^0}^2}{v^2} \right. \\ & \left. - \frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2}, \end{aligned}$$

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} - 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} + 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{M_{\phi_n^0}^2}{v^2} \right. \\ \left. - \frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

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Let us recall “unitarity conditions” again,

$$\kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0 \quad \kappa_{WW}^{\phi_n^0 \phi_{n'}^0} = \kappa_{ZZ}^{\phi_n^0 \phi_{n'}^0} \quad \kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0} \equiv \kappa_V^{\phi_n^0} \quad \sum_{n=1}^{N_0} \kappa_V^{\phi_n^0} \kappa_V^{\phi_n^0} = 1$$

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} - 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} + 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{M_{\phi_n^0}^2}{v^2} \right. \\ \left. - \frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

The finiteness of S,T and U parameters is automatically guaranteed
once unitarity conditions are satisfied among Higgs couplings

RN, Tsumura, Tanabashi (2014)

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

We can test any model including arbitrary number of neutral Higgs bosons.

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

※ We don't consider extensions including charged Higgs bosons and fermions for simplicity.

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{\Lambda^2}{v^2} \right]$$

✓ Perturbative unitarity bound

$$-\frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

✓ Electroweak precision tests

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

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RN, Tsumura, Tanabashi (2014)

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

We can test any model including arbitrary

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

※ We don't consider extensions including charged Higgs bosons and fermions for simplicity.

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{\Lambda^2}{v^2} \right]$$

✓ Perturbative unitarity bound

$$-\frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

✓ Electroweak precision tests

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

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Unitarity constraints

Let us consider the S-wave transition matrix among $\mathbf{W}_L \mathbf{W}_L$ and $\mathbf{Z}_L \mathbf{Z}_L$ states,

$$\mathcal{T} = \begin{pmatrix} t_0^{W_L^+ W_L^- \rightarrow W_L^+ W_L^-} & \frac{1}{\sqrt{2}} t_0^{W_L^+ W_L^- \rightarrow Z_L Z_L} \\ \frac{1}{\sqrt{2}} t_0^{W_L^+ W_L^- \rightarrow Z_L Z_L} & \frac{1}{2} t_0^{Z_L Z_L \rightarrow Z_L Z_L} \end{pmatrix}$$

The S-wave amplitude : $t_0 = \frac{1}{32\pi} \int_{-1}^1 \mathcal{A}(s, \cos \theta) d\cos \theta.$

Unitarity constraints

Let us consider the S-wave transition matrix among $\mathbf{W}_L \mathbf{W}_L$ and $\mathbf{Z}_L \mathbf{Z}_L$ states.

Assuming that Higgs couplings satisfy “unitarity condition”,

$$\kappa_{\mathbf{Z}}^{\phi_n^0 \phi_{n'}^0} = 0 \quad \kappa_{\mathbf{WW}}^{\phi_n^0 \phi_{n'}^0} = \kappa_{\mathbf{ZZ}}^{\phi_n^0 \phi_{n'}^0} \quad \kappa_{\mathbf{WW}}^{\phi_n^0} = \kappa_{\mathbf{ZZ}}^{\phi_n^0} \equiv \kappa_{\mathbf{V}}^{\phi_n^0} \quad \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{V}}^{\phi_n^0} \right)^2 = 1$$

, for sufficiently high energy scale $s \gg M_{\phi_n^0}^2$, we obtain

$$\mathcal{T} = -\frac{1}{8\pi v^2} \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{V}}^{\phi_n^0} \right)^2 M_{\phi_n^0}^2 \begin{pmatrix} 1 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{3}{4} \end{pmatrix}$$

Perturbative unitarity requires the maximum eigenvalue of \mathcal{T} should be satisfy

$$|t_0^{\max}| = \frac{1}{8\pi v^2} \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{V}}^{\phi_n^0} \right)^2 M_{\phi_n^0}^2 \times \frac{5}{4} \leq \frac{1}{2}$$

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, for sufficiently high energy scale $s \gg M_{\phi_n^0}^2$, we obtain

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Perturbative unitarity requires

$$\frac{1}{8\pi v^2} \left[\left(\kappa_{\mathbf{V}}^{\mathbf{h}} \right)^2 M_h^2 + \left\{ 1 - \left(\kappa_{\mathbf{V}}^{\mathbf{h}} \right)^2 \right\} \mathbf{M}_{\mathbf{H}}^2 \right] \times \frac{5}{4} \leq \frac{1}{2}$$

The electroweak oblique corrections

UV divergent terms of T and U parameters at one loop are obtained by

We can test any model including arbitrary number of neutral Higgs bosons.

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} + 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

※ We don't consider extensions including charged Higgs bosons and fermions for simplicity.

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{\Lambda^2}{v^2} \right]$$

Perturbative unitarity bound

$$-\frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

✓ Electroweak precision tests

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

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RN, Tsumura, Tanabashi (2014)

The electroweak precision tests

Assuming that Higgs couplings satisfy “unitarity condition”,

$$\kappa_{\mathbf{Z}}^{\phi_n^0 \phi_{n'}^0} = 0 \quad \kappa_{\mathbf{WW}}^{\phi_n^0 \phi_{n'}^0} = \kappa_{\mathbf{ZZ}}^{\phi_n^0 \phi_{n'}^0} \quad \kappa_{\mathbf{WW}}^{\phi_n^0} = \kappa_{\mathbf{ZZ}}^{\phi_n^0} \equiv \kappa_{\mathbf{V}}^{\phi_n^0} \quad \sum_{n=1}^{N_0} \left(\kappa_{\mathbf{V}}^{\phi_n^0} \right)^2 = 1$$

The expressions of finite corrections to the S and T parameters are obtained by

(for sufficiently heavy ϕ_n^0 ($n \geq 2$)) .

$$S \geq S_H \simeq \frac{1}{12} \left[1 - \left(\kappa_{\mathbf{V}}^{\mathbf{h}} \right)^2 \right] \left[\ln \frac{\mathbf{M}_{\mathbf{H}}^2}{M_h^2} + 0.86 \right] > 0$$

$$T \leq T_H \simeq -\frac{3(M_Z^2 - M_W^2)}{16\pi^2 v^2 \alpha} \left[1 - \left(\kappa_{\mathbf{V}}^{\phi_n^0} \right)^2 \right] \left[\ln \frac{\mathbf{M}_{\mathbf{H}}^2}{M_h^2} - 1.05 \right] < 0$$

The electroweak precision tests

The Gifitter Group (2014)

$$S = 0.06 \pm 0.09 \quad T = 0.10 \pm 0.07$$

The electroweak oblique corrections

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We can test any model including arbitrary number of neutral Higgs bosons.

$$T_{quad} = \sum_{n=1}^{N_0} \left[\kappa_{WW}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{WW}^{\phi_n^0} \right)^2 - \kappa_{ZZ}^{\phi_n^0 \phi_n^0} 2 \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 + \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \frac{\Lambda^2}{(4\pi)^2 \alpha v^2},$$

※ We don't consider extensions including charged Higgs bosons and fermions for simplicity.

$$T_{log} = \sum_{n=1}^{N_0} \left[\left\{ -\kappa_{WW}^{\phi_n^0 \phi_n^0} + \left(\kappa_{WW}^{\phi_n^0} \right)^2 + \kappa_{ZZ}^{\phi_n^0 \phi_n^0} - \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right\} \frac{\Lambda^2}{v^2} \right]$$

✓ Perturbative unitarity bound

$$-\frac{3}{4} g^2 \left\{ \left(\kappa_{WW}^{\phi_n^0} \right)^2 - 1 \right\} + \frac{3}{4} g_Z^2 \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - 1 \right\} \right] \frac{1}{(4\pi)^2 \alpha} \ln \frac{\Lambda^2}{\mu^2},$$

✓ Electroweak precision tests

$$U_{log} = \frac{1}{12\pi} \left[\sum_{n=1}^{N_0} \left\{ \left(\kappa_{ZZ}^{\phi_n^0} \right)^2 - \left(\kappa_{WW}^{\phi_n^0} \right)^2 \right\} + \frac{1}{2} \sum_{n=1}^{N_0} \sum_{m=1}^{N_0} \left(\kappa_Z^{\phi_n^0 \phi_m^0} \right)^2 \right] \ln \frac{\Lambda^2}{\mu^2},$$

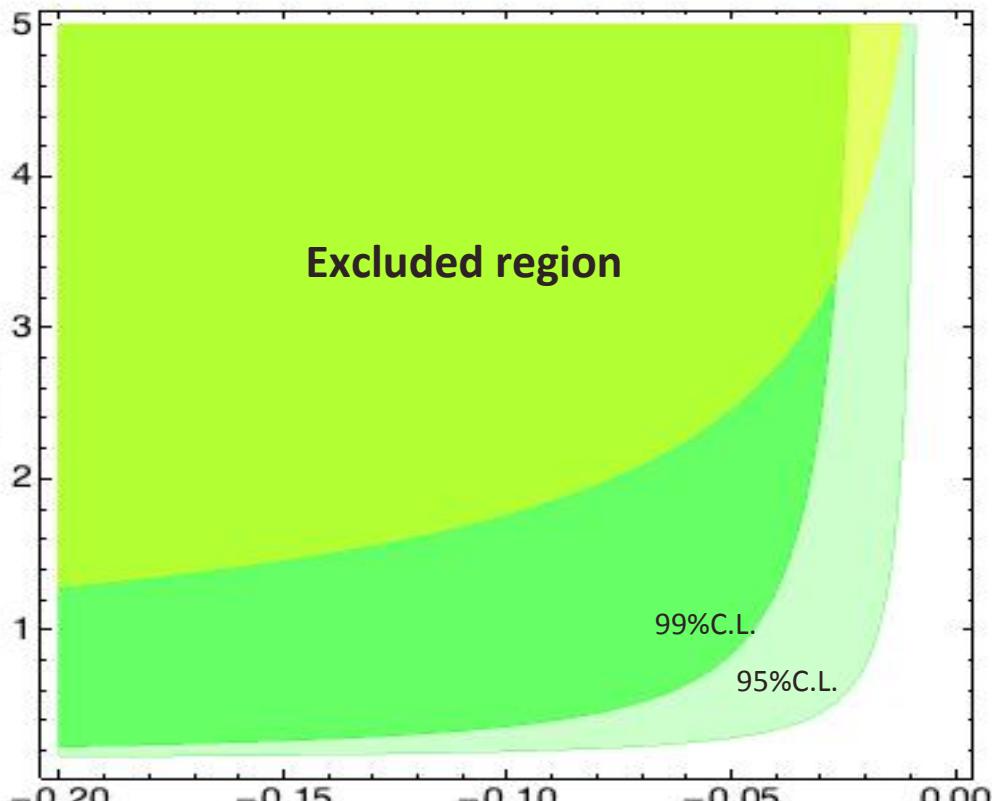
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RN, Tsumura, Tanabashi (2014)

Constraints on “heavy” Higgs bosons

M_H

The mass of
the second lightest
extra neutral
Higgs boson
[TeV]



RN, Tsumura, Tanabashi (2014)

$$(\Delta \kappa_W^h \equiv \kappa_W^h - 1)$$

The deviation
of hVV
coupling
from the SM
prediction

$$\Delta \kappa_V^h$$

Perturbative unitarity

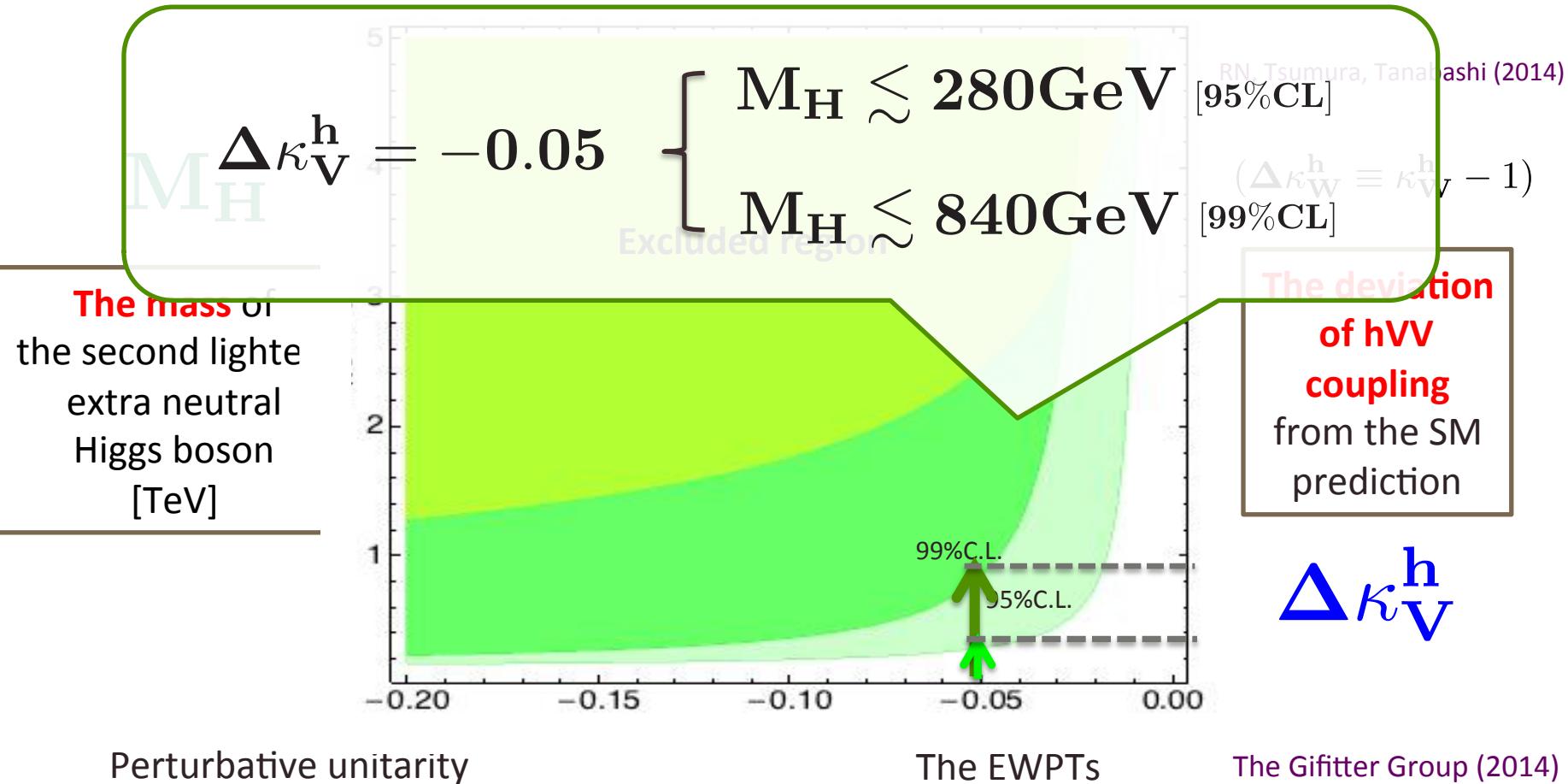
$$\frac{5}{32\pi v^2} \left[(\kappa_V^h)^2 M_h^2 + \left\{ 1 - (\kappa_V^h)^2 \right\} M_H^2 \right] \leq \frac{1}{2}$$

The EWPTs

The Gifitter Group (2014)

$$S = 0.06 \pm 0.09 \quad T = 0.10 \pm 0.07$$

Constraints on “heavy” Higgs bosons



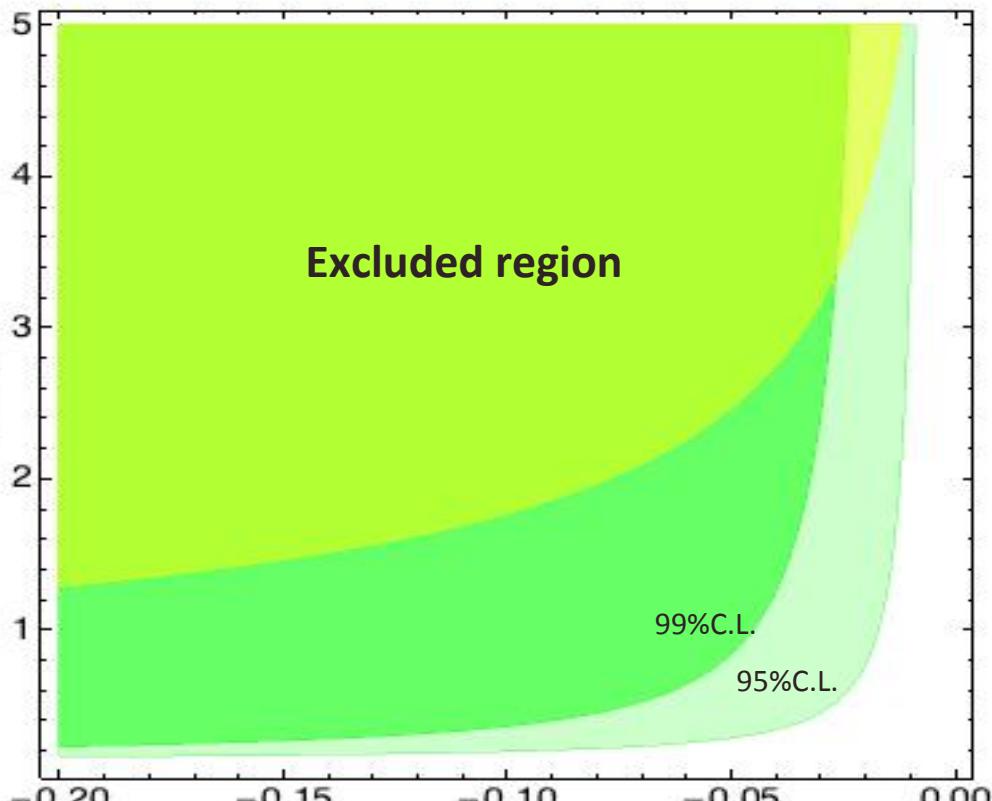
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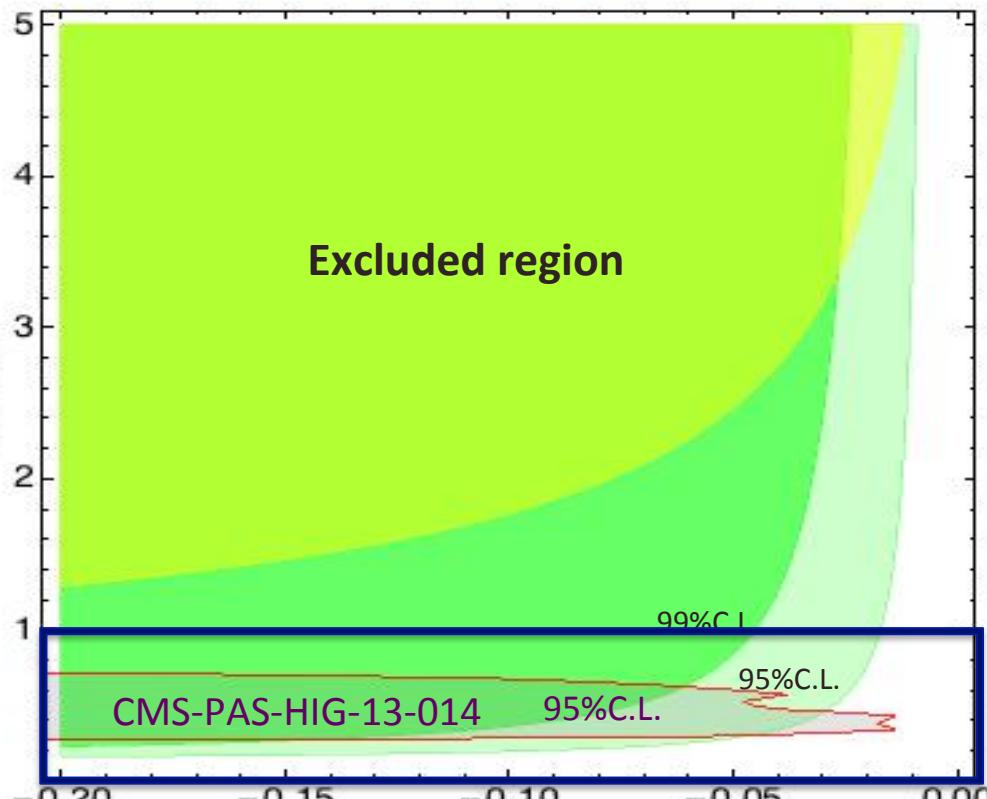
$$\Delta \kappa_V^h$$

The Gifitter Group (2014)

Constraints on “heavy” Higgs bosons

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The mass of
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RN, Tsumura, Tanabashi (2014)

$$(\Delta \kappa_W^h \equiv \kappa_W^h - 1)$$

The deviation
of hVV
coupling
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$$\Delta \kappa_V^h$$

Perturbative unitarity

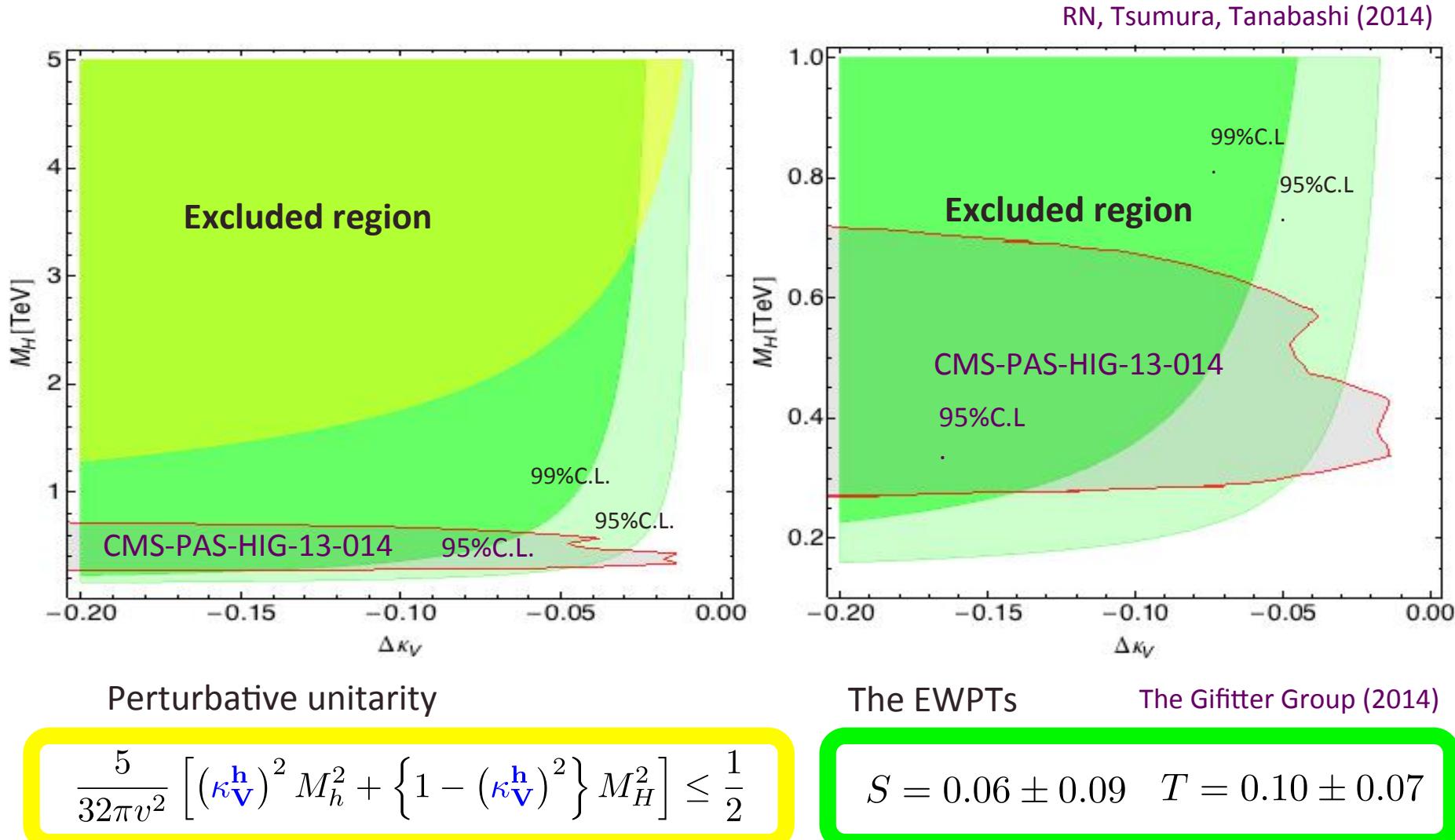
$$\frac{5}{32\pi v^2} \left[(\kappa_V^h)^2 M_h^2 + \left\{ 1 - (\kappa_V^h)^2 \right\} M_H^2 \right] \leq \frac{1}{2}$$

The EWPTs

The Gifitter Group (2014)

$$S = 0.06 \pm 0.09 \quad T = 0.10 \pm 0.07$$

Constraints on “heavy” Higgs bosons



Summary

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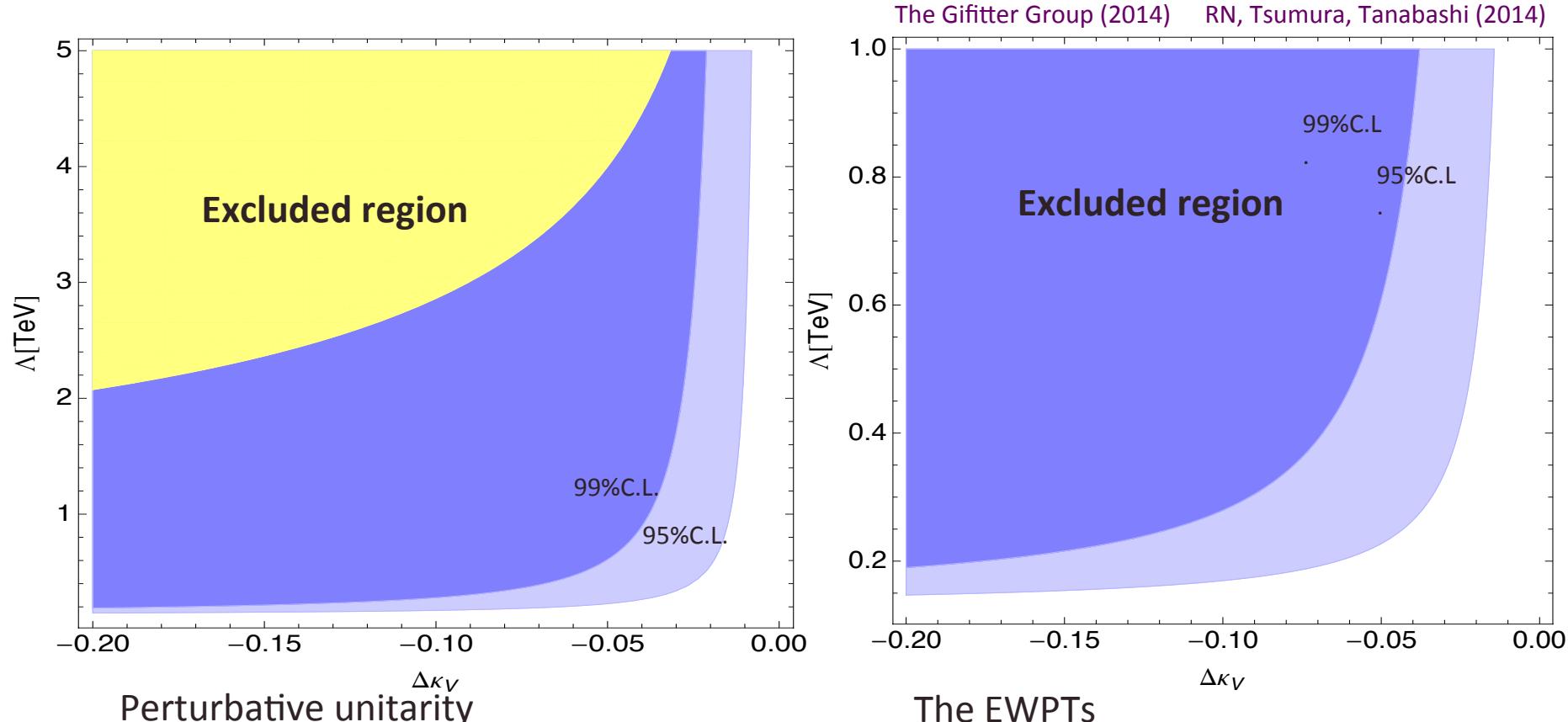
- ✓ We introduce **arbitrary number of neutral Higgs bosons in the electroweak symmetry breaking sector.**
- ✓ We (re)derive conditions among Higgs coupling which need to be satisfied to maintain the unitarity of the high energy scattering amplitude of weak gauge bosons (“**Unitarity conditions**”).
- ✓ We find that the finiteness of S, T and U parameters at one loop are **automatically guaranteed** once unitarity conditions are satisfied.
- ✓ Using unitarity conditions, we can test extended EWSB model including only neutral Higgs bosons by the unitarity limit and EWPTs limit. That gives us strong constraint on the mass of extra neutral Higgs boson as function of the deviation of the SM-like Higgs coupling.

$$\frac{5}{32\pi v^2} \left[(\kappa_V^H)^2 M_H^2 + \left\{ 1 - (\kappa_V^H)^2 \right\} M_H^2 \right] \leq \frac{1}{2}$$

$$S = 0.06 \pm 0.09 \quad T = 0.10 \pm 0.07$$

Thank you for your attentions.

Constraints on the SM cutoff



$$\frac{1}{16\pi v^2} \left[1 - (\kappa_{\mathbf{V}}^{\mathbf{h}})^2 \Lambda^2 \right] \leq \frac{1}{2}$$

$$M_H^{\text{upper}} \simeq 0.63 \Lambda^{\text{upper}}$$

$$S \simeq \frac{1}{12} \left[1 - (\kappa_{\mathbf{V}}^{\mathbf{h}})^2 \right] \ln \frac{\Lambda^2}{M_h^2} \quad T \simeq -\frac{3(M_Z^2 - M_W^2)}{16\pi^2 v^2 \alpha} \left[1 - (\kappa_{\mathbf{V}}^{\mathbf{h}})^2 \right] \ln \frac{\Lambda^2}{M_h^2}$$

$$M_H^{\text{upper}} \simeq 1.69 \Lambda^{\text{upper}}$$