universität wien

Strong Coupling Determination from Eventshape Distributions

André H. Hoang
University of Vienna

Collaboration with: D. Kolodrubetz, V. Mateu, I. Stewart

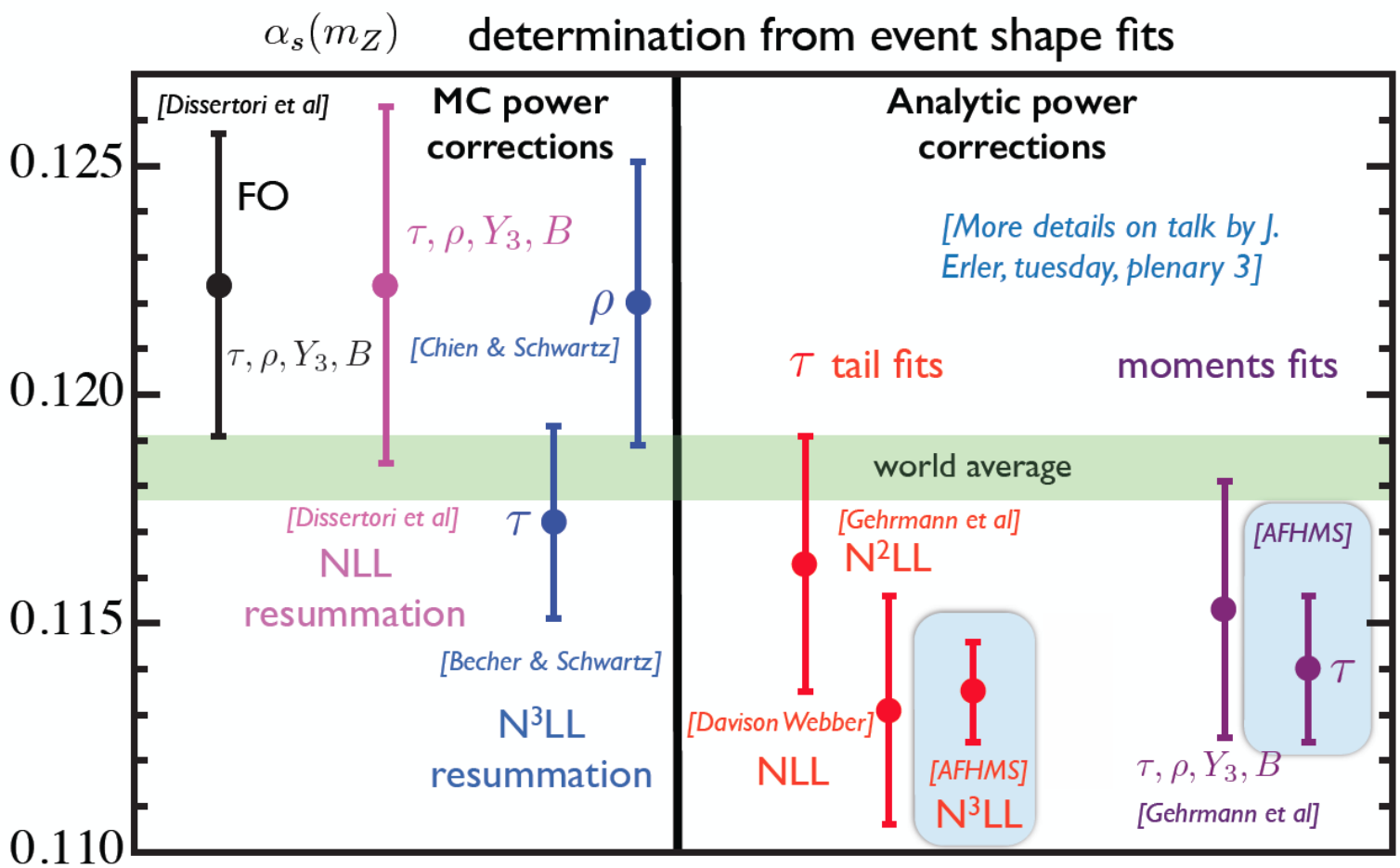
International Workshop on Future Linear Colliders
Belgrade, October 5-10, 2014

Outline

- Motivation & Introduction
- Factorization & Log resummation at N^3LL order
- Singular vs. Non-singular corrections
- Power Corrections
- Fits & Final result
- Conclusions & Outlook

Motivation

Eventshape analyses with analytic power corrections consistently get lower values for the strong coupling.



C-Parameter Definition

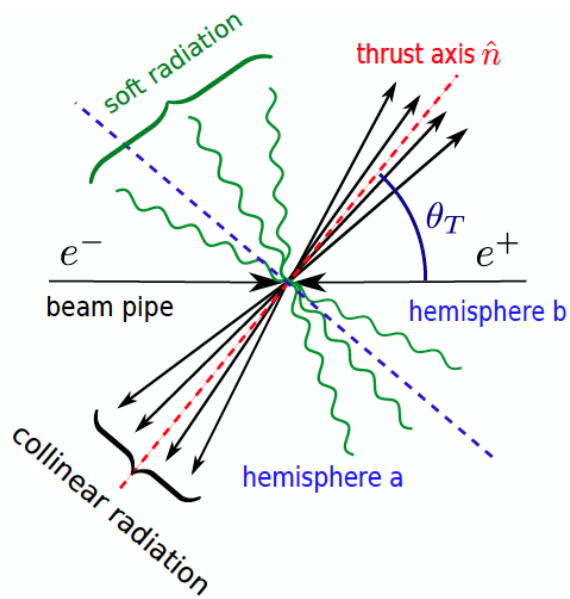
linearized momentum tensor $\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$ with eigenvalues $\lambda_{1,2,3}$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$C = 3\left(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3\right) = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

IR and collinear safe

- Double sum
- Does not require minimization



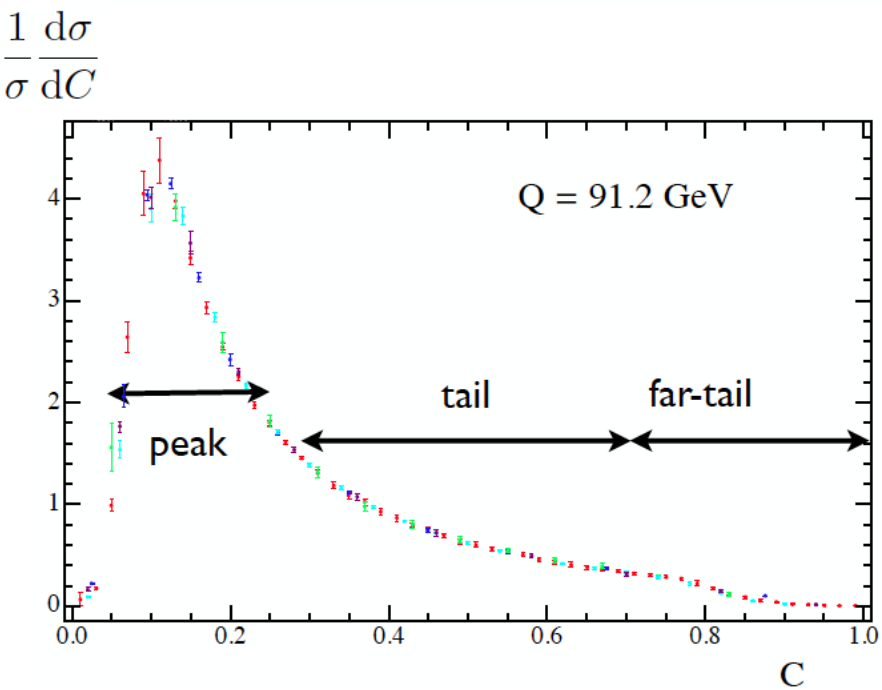
C-Parameter Definition

linearized momentum tensor $\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$ with eigenvalues $\lambda_{1,2,3}$

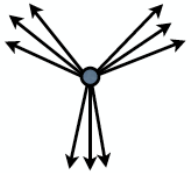
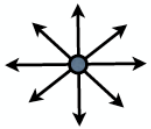
$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$C = 3\left(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3\right) = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

IR and collinear safe



Continuous transition from 2-jet to 3-jet, ... multi-jet events

dijet	$C \sim 0$	
three jets	$C \sim 0.75$	
spherical	$C \sim 1$	

Resummation of Large Logarithms

Event shapes are not inclusive quantities

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}} = -\frac{2\alpha_s}{3\pi}\frac{1}{C}\left(3 + 4\log\frac{C}{6} + \dots\right)$$

- Large logarithms at small C
- Fixed-order pert. theory not valid

One has to reorganize the expansion by considering $\alpha_s \log\frac{C}{6} \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(C_c) \equiv \int_0^{C_c} \mathrm{d}C \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}}$$

$$\begin{aligned}
\log\Sigma(C_c) = \alpha_s &(\log^2 C_c + \log C_c + 1) \\
\alpha_s^2 &(\log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\alpha_s^3 &(\log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\alpha_s^4 &(\log^5 C_c + \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\dots \quad &\vdots
\end{aligned}$$

LL | NLL | N^2LL | N^3LL | not known!

State of the art

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

Resummation of Large Logarithms

Event shapes are not inclusive quantities

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}} = -\frac{2\alpha_s}{3\pi}\frac{1}{C}\left(3 + 4\log\frac{C}{6} + \dots\right)$$

- Large logarithms at small C
- Fixed-order pert. theory not valid

One has to reorganize the expansion by considering $\alpha_s \log\frac{C}{6} \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(C_c) \equiv \int_0^{C_c} \mathrm{d}C \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}}$$

$$\begin{aligned}
\log\Sigma(C_c) = \alpha_s &(\log^2 C_c + \log C_c + 1) \\
\alpha_s^2 &(\log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\alpha_s^3 &(\log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\alpha_s^4 &(\log^5 C_c + \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
\dots \quad &\vdots
\end{aligned}$$

LL | NLL' | N^2LL' | N^3LL' | not known!

State of the art

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

Factorization Theorem (singular terms)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

perturbative

nonperturbative & perturbative

[Hoang & Stewart]

[VM, Thaler, Stewart]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \otimes F_e$$

Hadron mass effects taken into account, but no time to discuss them

Renormalization Group Evolution

large logs

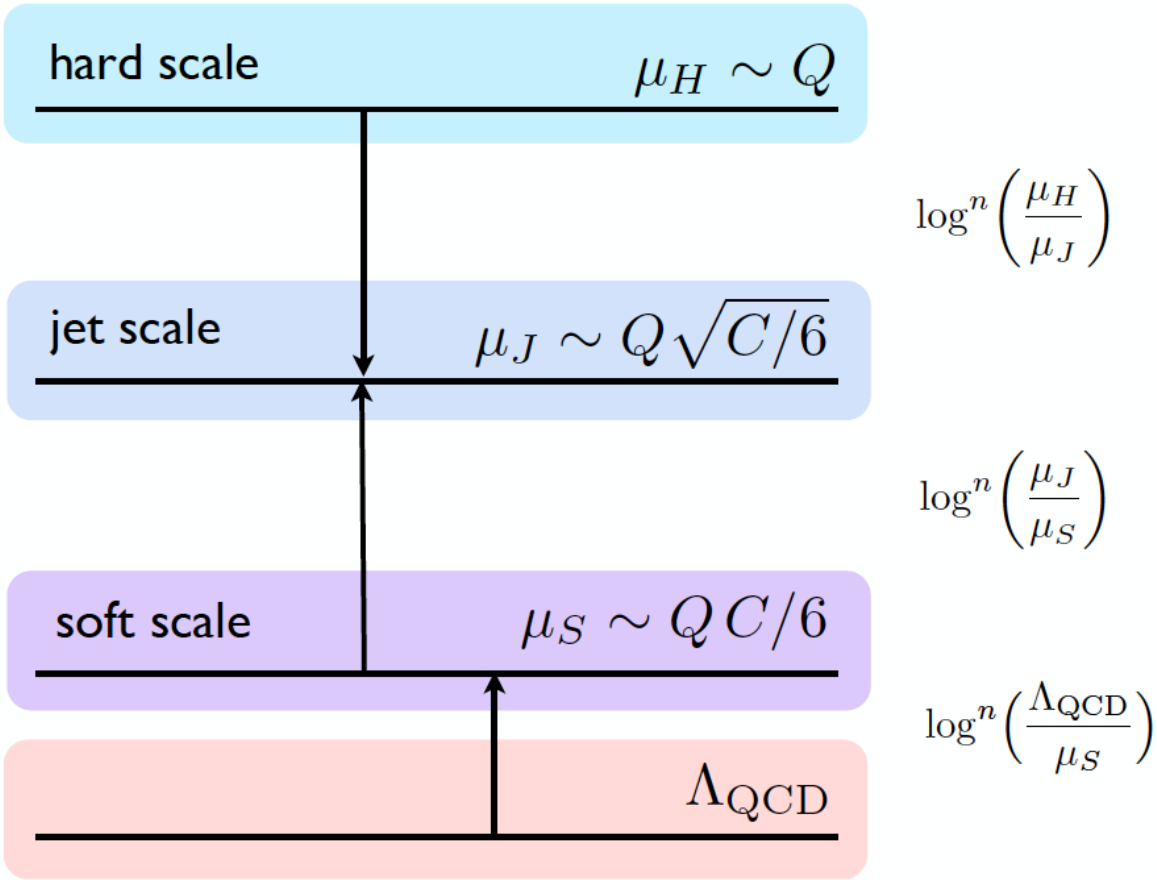
hard scale	$\mu_H \sim Q$	$\log^n\left(\frac{Q}{\mu}\right)$
jet scale	$\mu_J \sim Q\sqrt{C/6}$	$\log^n\left(\frac{Q^2 C}{6\mu^2}\right)$
soft scale	$\mu_S \sim Q\,C/6$	$\log^n\left(\frac{QC}{6\mu}\right)$
	$\Lambda_{\rm QCD}$	$\log^n\left(\frac{6\Lambda_{\rm QCD}}{QC}\right)$

The hierarchy among the scales depends on the position on the spectrum

Use profile function to describe the whole distribution

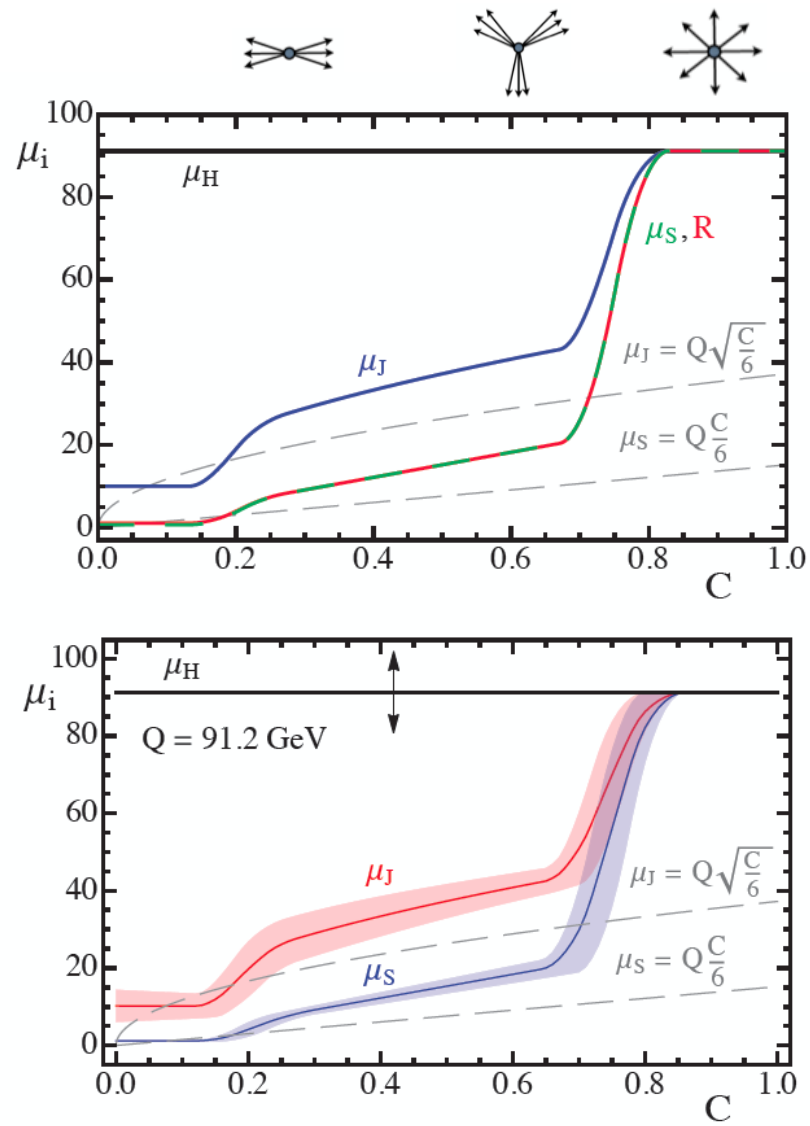
Renormalization Group Evolution

The hierarchy among the scales depends on the position on the spectrum



Renormalization Group Evolution

	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	-0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	$-1, 0, 1$
unknowns	Γ_3^{cusp}	1553.06	-1553.06 to $+4659.18$
	s_2	-43.2	-44.2 to -42.2
	j_3	0	-3000 to $+3000$
	s_3	0	-500 to $+500$
non-singular	$\epsilon_{2,\mathrm{low}}$	0	$-1, 0, 1$
	$\epsilon_{2,\mathrm{high}}$	0	$-1, 0, 1$
	$\epsilon_{3,\mathrm{low}}$	0	$-1, 0, 1$
	$\epsilon_{3,\mathrm{high}}$	0	$-1, 0, 1$



Perturbation Theory Input

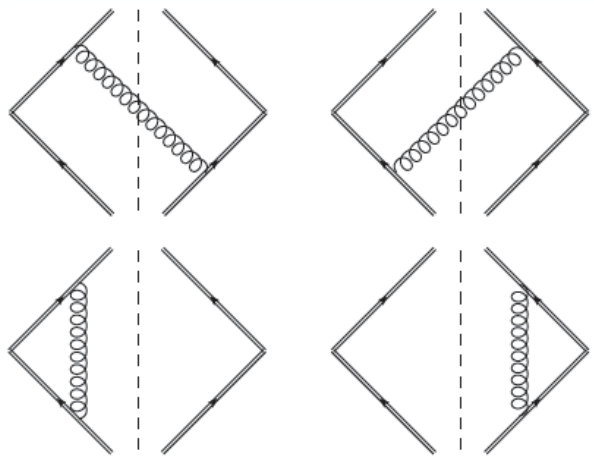
$H(Q,\mu)$	Hard function known at 3 loops	same as thrust
$J_n(s,\mu)$	Jet function known at two loops Running known at three loops	
$S_C(\ell,\mu)$	Soft function known analytically at one loop, numerically at two loops Running known at three loops	

Fixed-order predictions known at three loops → 3-loop non-log jet and soft function corr. still unknown

Mass corrections known at N^2LL and two loops → w.i.p., future update
My talk on thursday

FS QED corrections known at N^3LL → Future update planned

Soft Function Computation



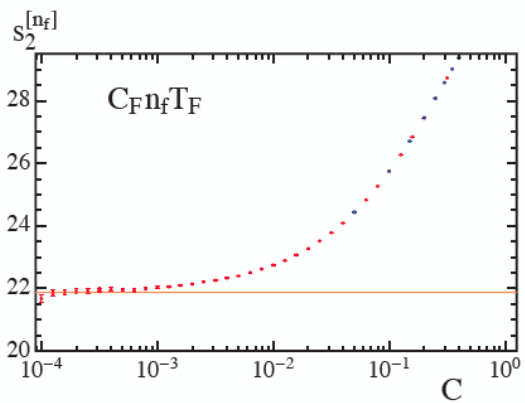
[Kolodrubetz, Hoang, VM, Stewart]

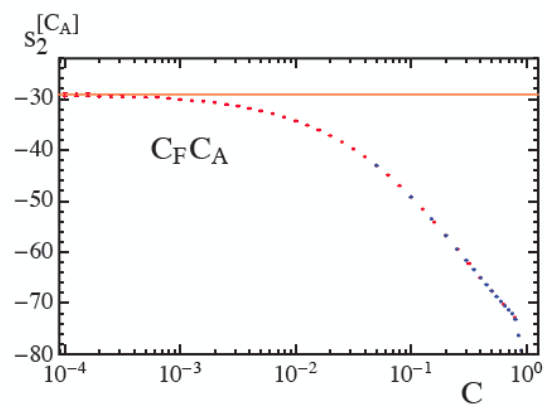
Analytic computation of soft function at 1-loop

$$S_e^{1-\text{loop}}(\ell) = \frac{2\,\alpha_s C_F e^{\epsilon\gamma_E}}{\mu\,\pi\,\Gamma(1-\epsilon)}\left(\frac{\ell}{\mu}\right)^{-1-2\epsilon} I_e(\epsilon)$$

universal formula for all event shapes

$$I_\tau(\epsilon) = \frac{1}{\epsilon} \qquad I_{\tilde{C}}(\epsilon) = \frac{1}{2}\frac{\Gamma(\epsilon)^2}{\Gamma(2\epsilon)}$$





Numerical determination at 2-loops using Event2

Kinematic Power Corrections ("Non-Singular")

$$\frac{d\hat{\sigma}_{ns}}{dC} = \frac{d\hat{\sigma}^{FO}_{full}}{dC} - \frac{d\hat{\sigma}^{FO}_{s}}{dC}$$

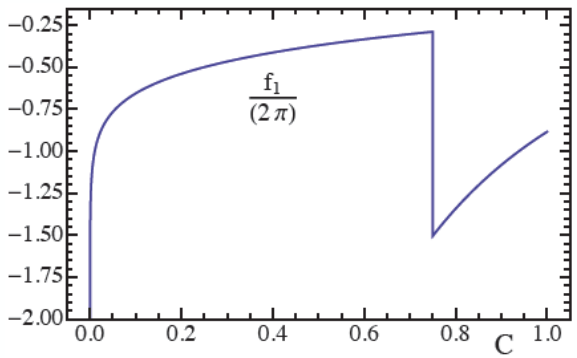
full FO

SCET with fixed scales

$$\frac{d\sigma_{ns}}{dC} = \frac{d\hat{\sigma}_{ns}}{dC} \otimes F_C$$

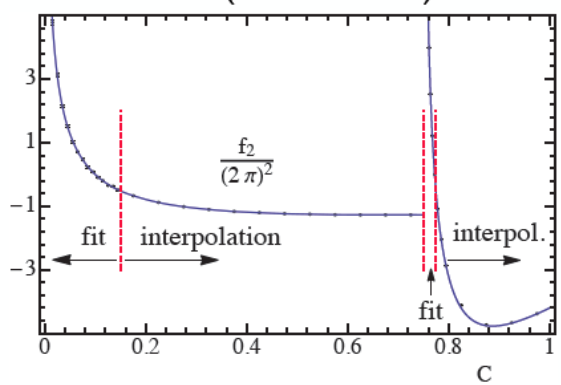
same shape function as singular terms

LO (analytically)

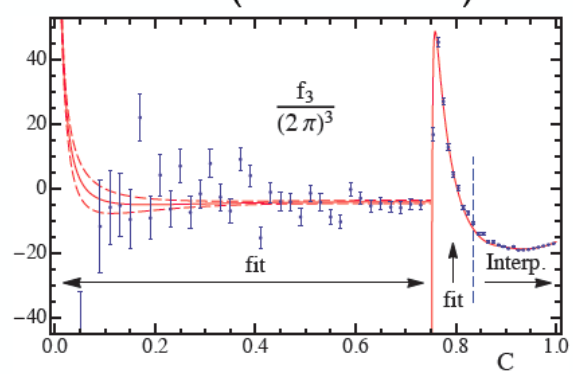


$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{ns}}{dC} = \frac{\alpha_s(Q)}{2\pi} f_1(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 f_2(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 f_3(C) + \ldots$$

NLO (from Event2)



NNLO (from EERAD3)



Non-Perturbative Power Corrections (OPE Region)

For $e \gg \frac{\Lambda_{\rm QCD}}{Q}$ Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1^{\rm e}\delta'(\ell)$$

$$\frac{{\rm d}\sigma}{{\rm d}e} = \frac{{\rm d}\hat{\sigma}}{{\rm d}e} - \frac{\Omega_1}{Q}\frac{{\rm d}}{{\rm d}e}\frac{{\rm d}\hat{\sigma}}{{\rm d}e} \simeq \frac{{\rm d}\hat{\sigma}}{{\rm d}e}\left(e - \frac{\Omega_1}{Q}\right) + \mathcal{O}\left[\left(\frac{\Lambda_{\rm QCD}}{Q\,e}\right)^2\right]$$

Universality: *[Lee & Sterman]* $\Omega_1^e = c_e\,\Omega_1^\rho$

Leading power corrections proportional to each other, calculable coefficient

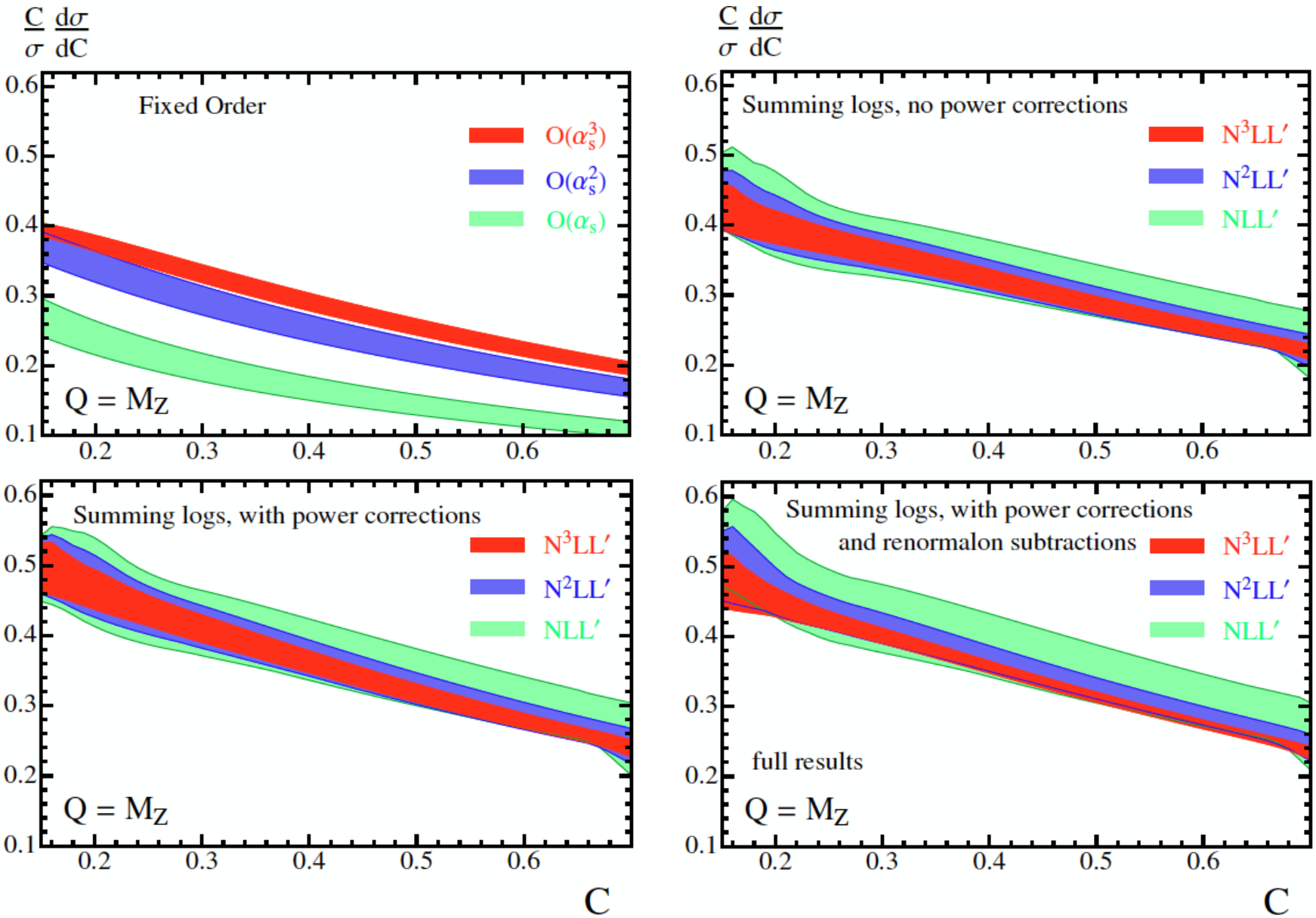
Hadron mass effect break this relation *[VM, Stewart, Thaler]*

No time to discuss this in detail

We define the gap scheme for $\Omega_1^{\rm e}$ in which it is renormalon-free

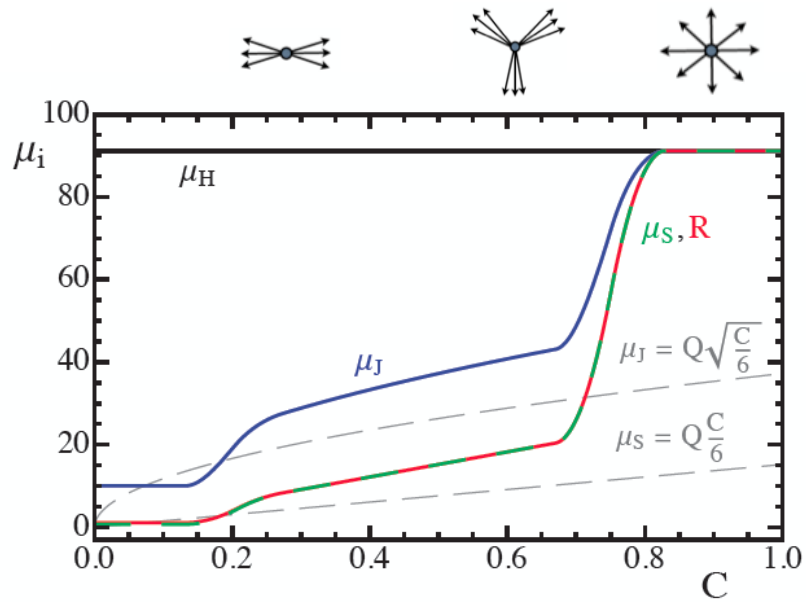
No time to discuss this in detail

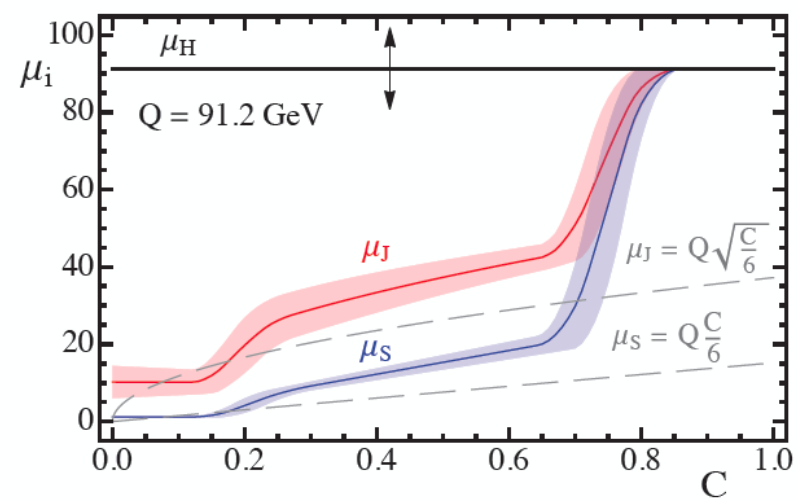
Convergence



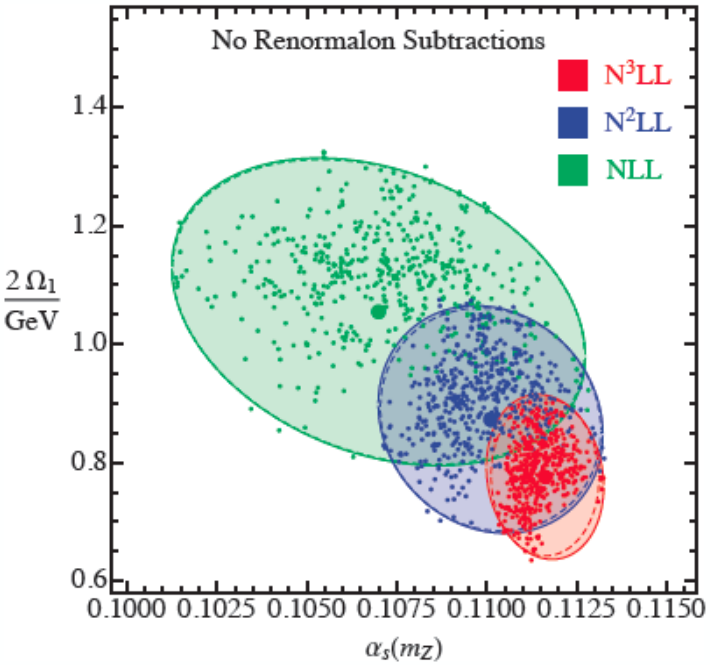
C-Parameter Tail Fits

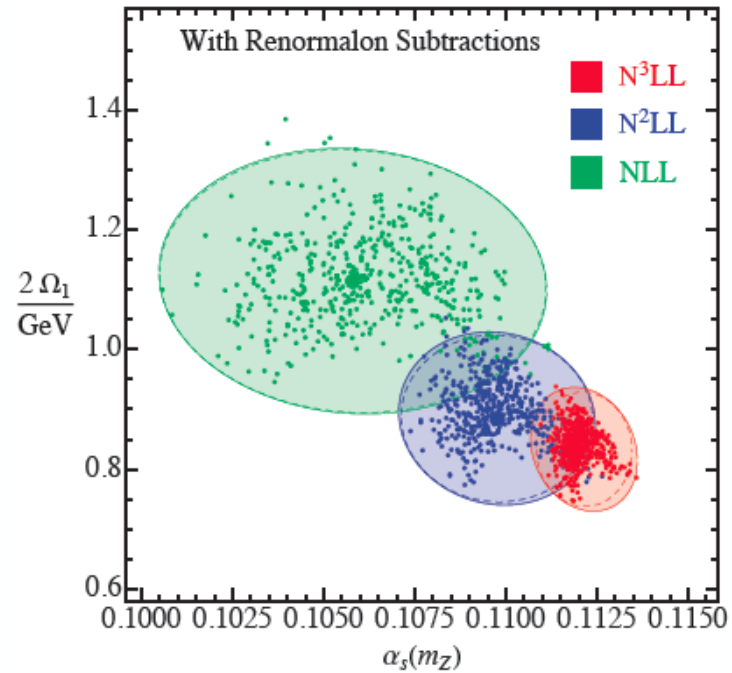
	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	−0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	−1, 0, 1
unknowns	Γ_3^{cusp}	1553.06	−1553.06 to +4659.18
	s_2	−43.2	−44.2 to −42.2
	j_3	0	−3000 to +3000
	s_3	0	−500 to +500
non-singular	$\epsilon_{2,\mathrm{low}}$	0	−1, 0, 1
	$\epsilon_{2,\mathrm{high}}$	0	−1, 0, 1
	$\epsilon_{3,\mathrm{low}}$	0	−1, 0, 1
	$\epsilon_{3,\mathrm{high}}$	0	−1, 0, 1





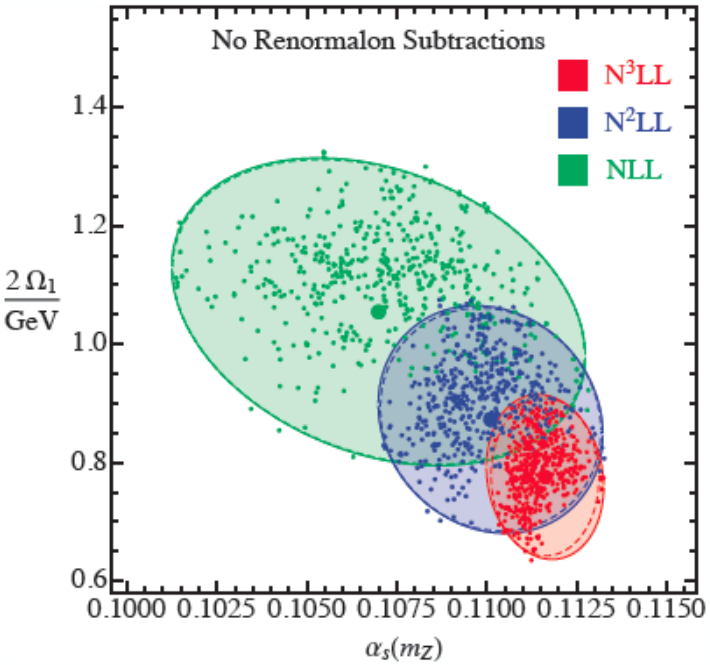
C-Parameter Tail Fits



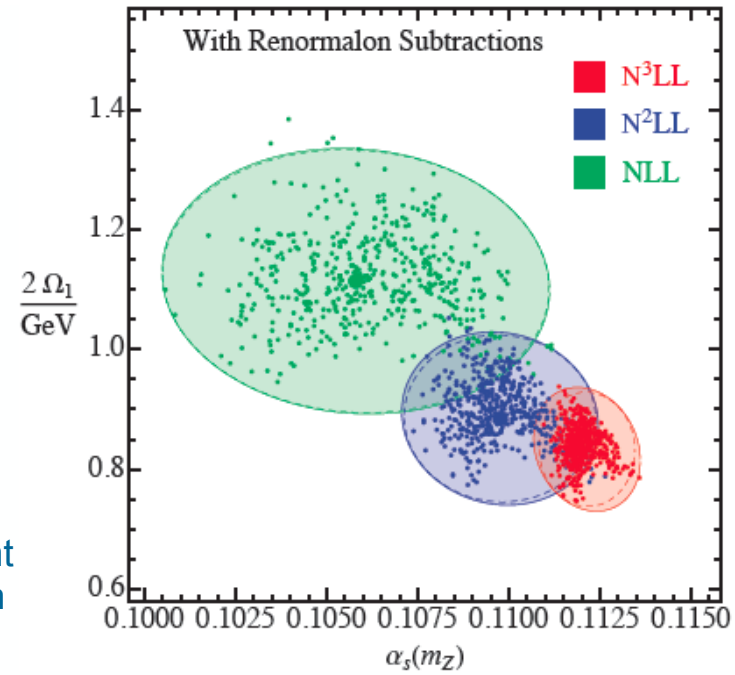


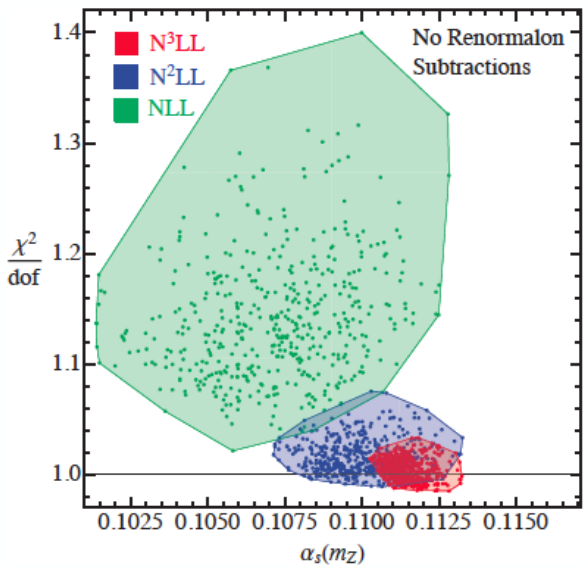
We perform global fits for energies between 35 and 206 GeV. We restrict ourselves to the tail of the distribution

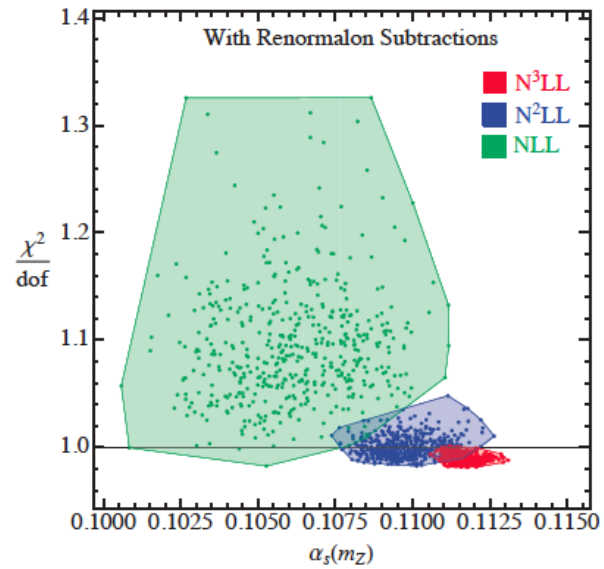
C-Parameter Tail Fits



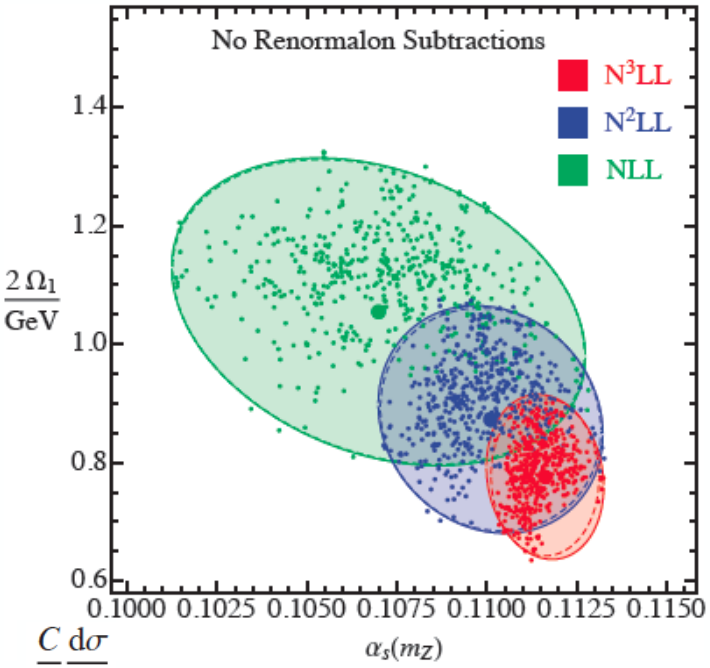
Slight improvement due to renormalon subtraction.

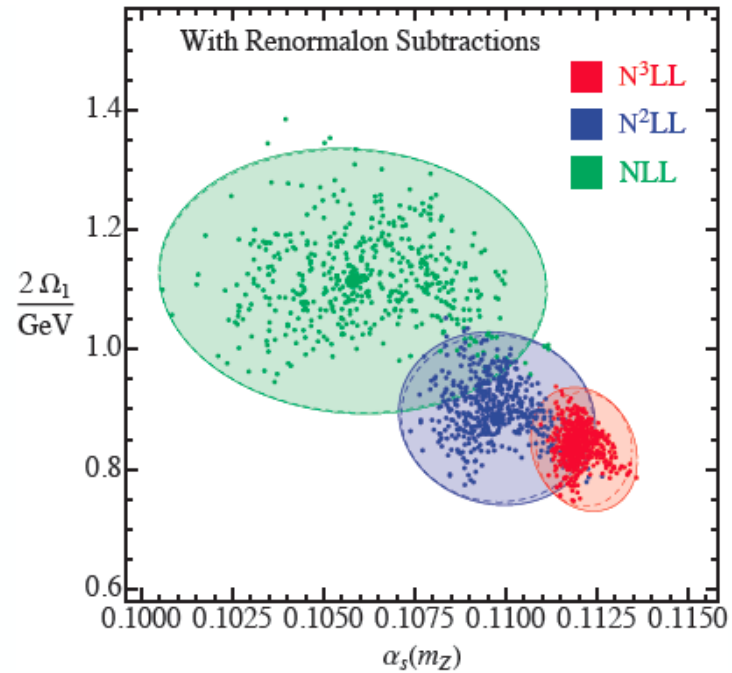


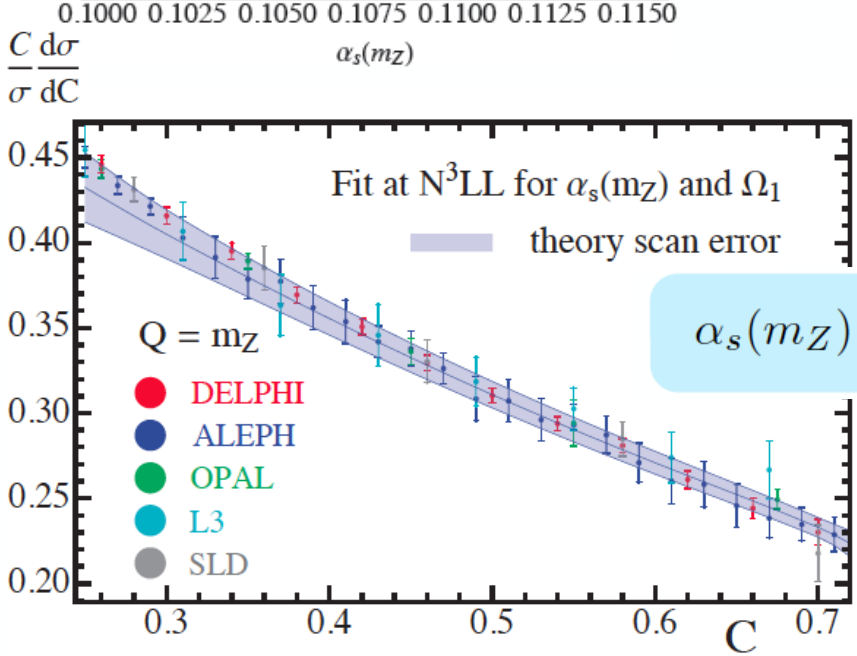




C-Parameter Tail Fits







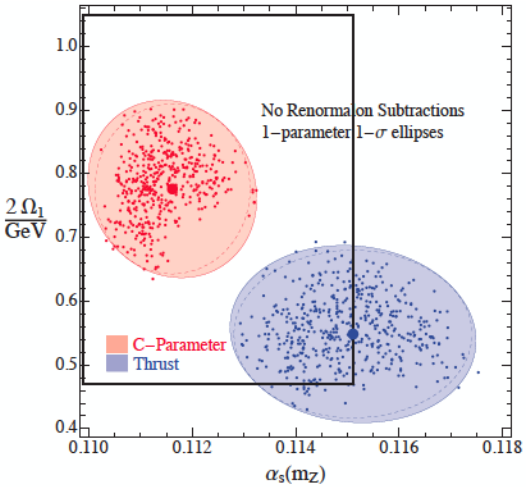
$$\alpha_s(m_Z) = 0.1121 \pm 0.0013_{\rm th} \pm 0.0006_{\rm exp} \pm 0.0002_{\rm had}$$

$$\alpha_s(m_Z) = 0.1121 \pm 0.0015$$

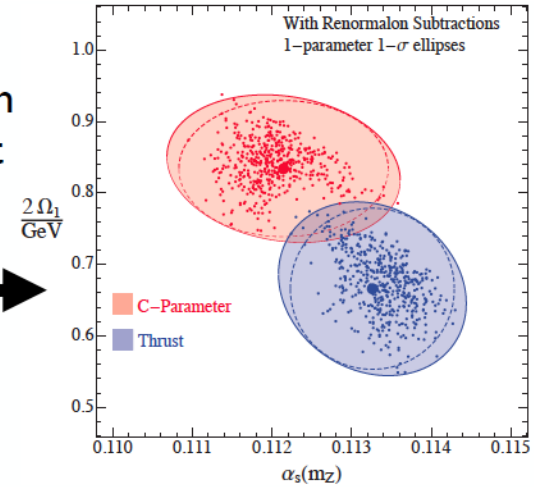
all errors combined

Universality: Thrust vs. C-Parameter

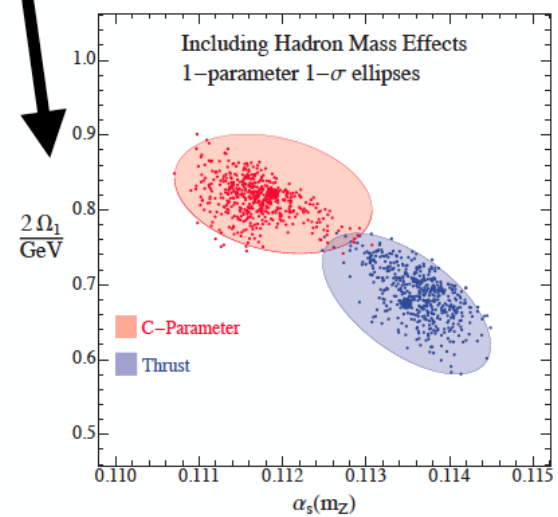
Thrust fts see
Abbate, Fickinger,
AH, Mateu, Stewart

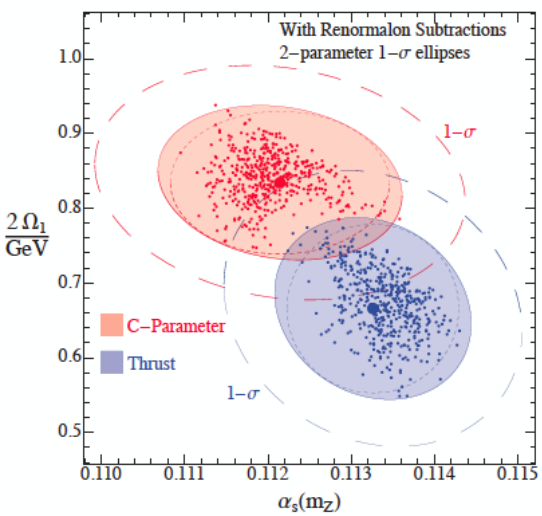


Renormalon subtraction improves α_s agreement



hadron-mass effects have small effect

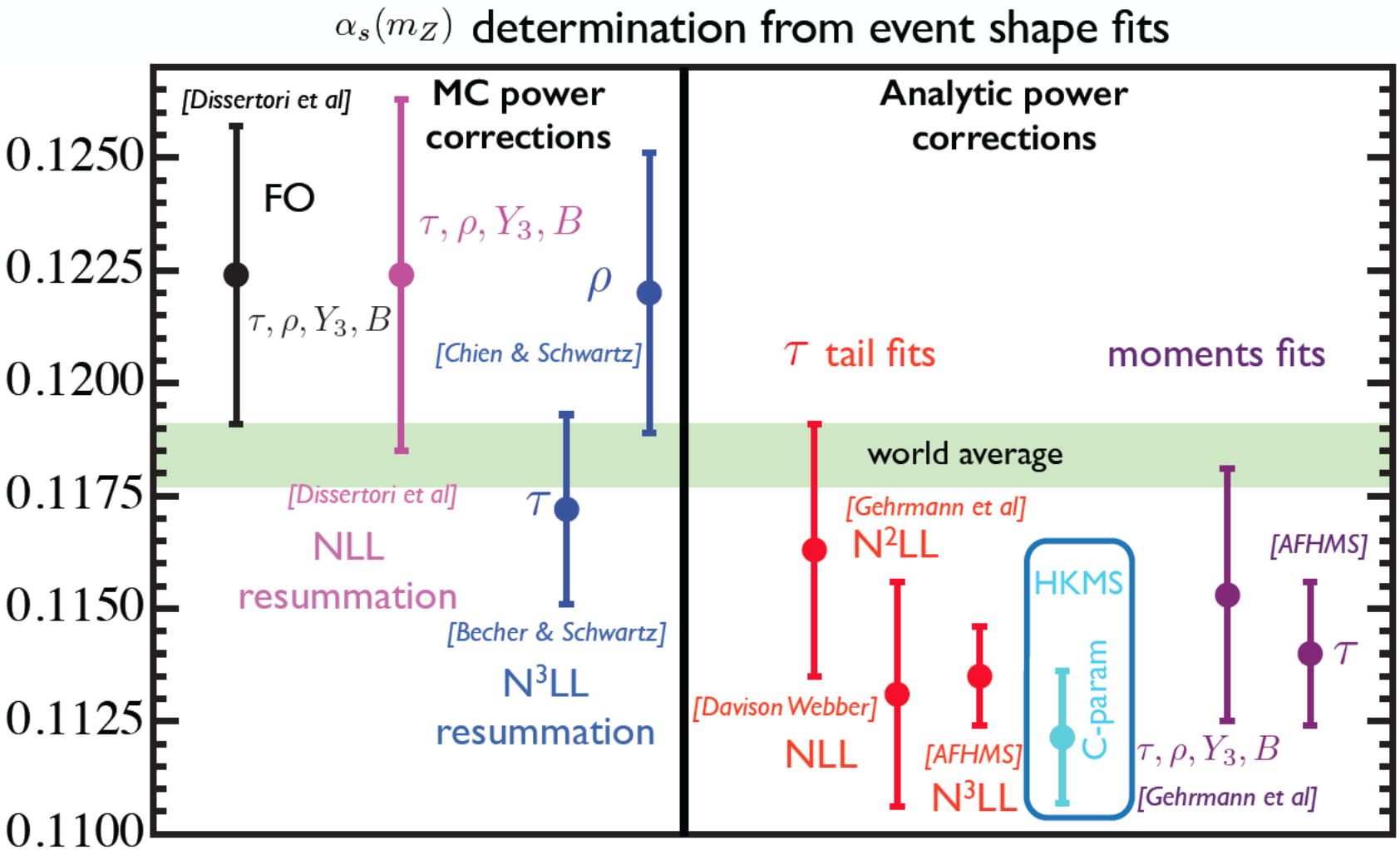




fair comparison with 2-parameter 1-σ ellipses

Result

$$\alpha_s(m_Z) = 0.1121 \pm 0.0013_{\rm th} \pm 0.0006_{\rm exp} \pm 0.0002_{\rm had}$$



Conclusions: Strong Coupling from C-Parameter

- Strong coupling again low compared to world average (-5%)
- C-parameter slightly less precise
- Consistency to the results from thrust (predicted universality confirmed from data)
- State of the art: NNNLL(') order (missing: 4-loop cusp an.dim., 3-loop jet+soft fct)

- Upcoming: bottom mass effects (see also my talk on Thursday)
- Future update: QED effects
- Upcoming: heavy jet mass

None of these effects can possibly eliminate the discrepancy to the world average.