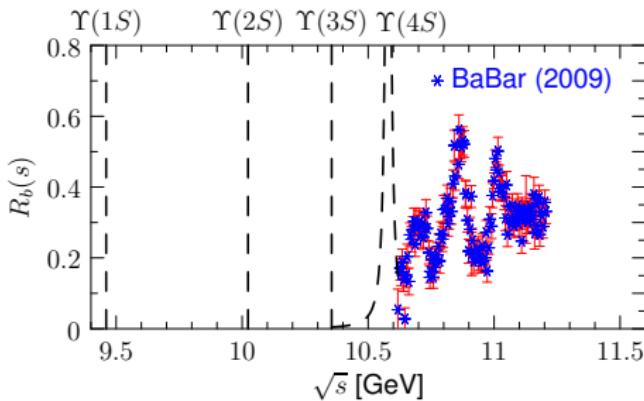
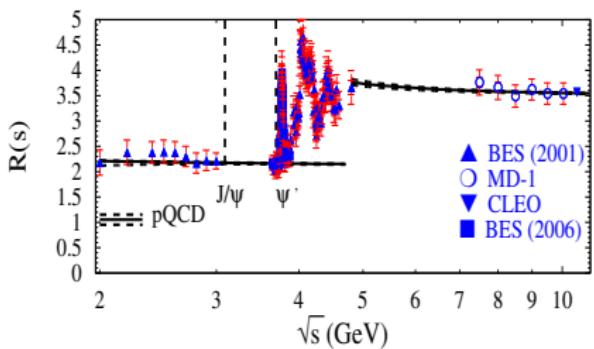


Bottom and charm masses

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Charm and bottom production at e^+e^- colliders



$$R_Q(s) = \frac{\sigma(e^+e^- \rightarrow Q\bar{Q} X)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Sum rules

- Determine m_Q from moments:

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \frac{R_Q(s)}{s^{n+1}}$$
$$\mathcal{M}_n^{\text{th}} = \mathcal{M}_n^{\text{exp}}$$

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- large n :
 - + better data
 - + more sensitivity to m_Q
 - larger non-perturbative effects

$n \approx 1$ for charm

$n \ll 20$ for bottom

Relativistic sum rules

$n \approx 1$

- No data for continuum → use theory prediction for

$$\sqrt{s} > 4.8 \text{ GeV (charm)}$$

$$\sqrt{s} > 11.24 \text{ GeV (bottom)}$$

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$$\sqrt{s} > 4.8 \text{ GeV (charm)}$$

$$\sqrt{s} > 11.24 \text{ GeV (bottom)}$$

- Compute moments from dispersion relation

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \frac{R_Q(s)}{s^{n+1}} = \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_Q(q^2) \Big|_{q^2=0}$$

$$\Pi_Q(q^2) = \overbrace{\text{---}}^{\bar{q}} + \text{---} + \text{---} + \text{---} + \dots$$

- Analytic results for $n \leq 3$

[Chetyrkin, Kühn, Sturm 06; Boughezal, Czakon, Schutzmeier 06; AM, Maierhöfer, Marquard, Smirnov 09]

Approximate results up to $n = 10$

[Hoang, Mateu, Zebarjad 08; Kiyo, AM, Maierhöfer, Marquard 09; Greynat, Peris 10]

Relativistic sum rules

Results

n	$m_c(3 \text{ GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

[Chetyrkin, Kühn, AM, Maierhöfer, Marquard, Steinhauser 09]

Non-relativistic sum rules

$n \approx 10$

$$\mathcal{M}_n = \sum_{\text{bound states}} (\text{residue}) + \int_{4m_b^2}^{\infty} ds \frac{R_b(s)}{s^{n+1}}$$

- Smearing range $E_{\text{kin}} \sim m_b/n$

$$\Rightarrow v = \sqrt{\frac{E_{\text{kin}}}{m_b}} \sim \frac{1}{\sqrt{n}} \ll 1, \quad v \sim \alpha_s$$

- Bottom quarks are **non-relativistic**
- Bound states: **binding potential is not a perturbation**

\Rightarrow Effective theory description

Non-relativistic sum rules

Potential non-relativistic QCD

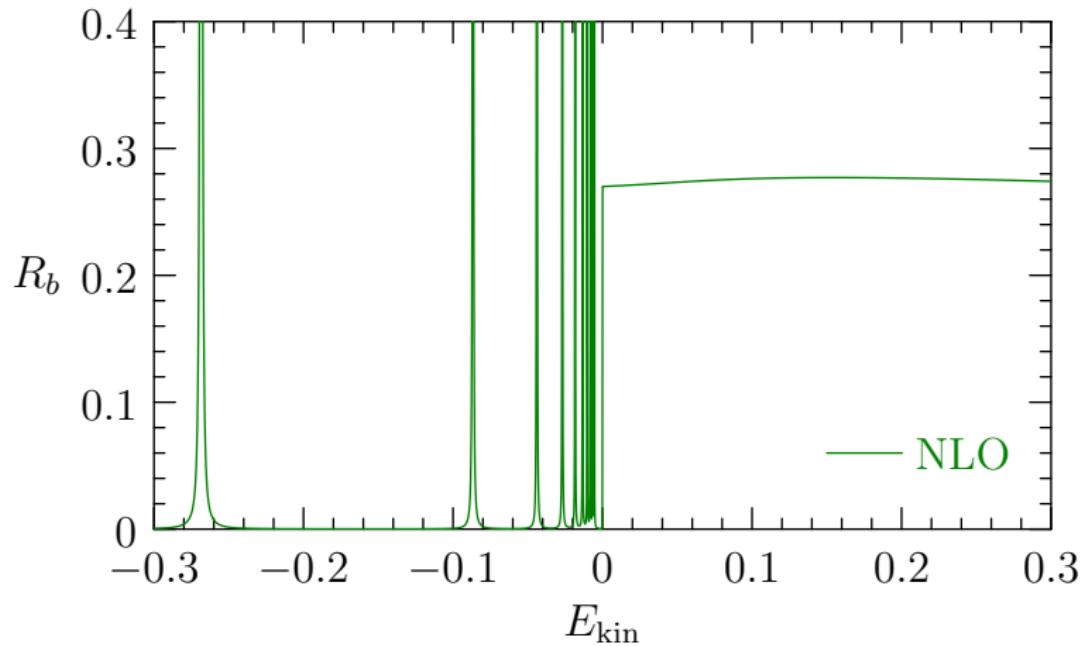
[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left(i\partial_0 + g_s A_0(t, \mathbf{0}) + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} \right) \psi \\ & + \int d^3 \mathbf{r} [\psi^\dagger \psi] (x + \mathbf{r}) \left(-\frac{C_F \alpha_s}{r} + \delta V(r) \right) [\chi^\dagger \chi](x) \\ & - g_s \psi^\dagger \mathbf{x} \cdot \mathbf{E}(t, \mathbf{0}) \psi + \mathcal{L}_{\text{anti-quark}}\end{aligned}$$

- Describes non-relativistic (anti-)quarks interacting via colour Coulomb potential
- Higher orders:
 - ▶ corrections to kinetic energy
 - ▶ corrections to potential
 - ▶ ultrasoft gluons ($k \sim m_b v^2$)

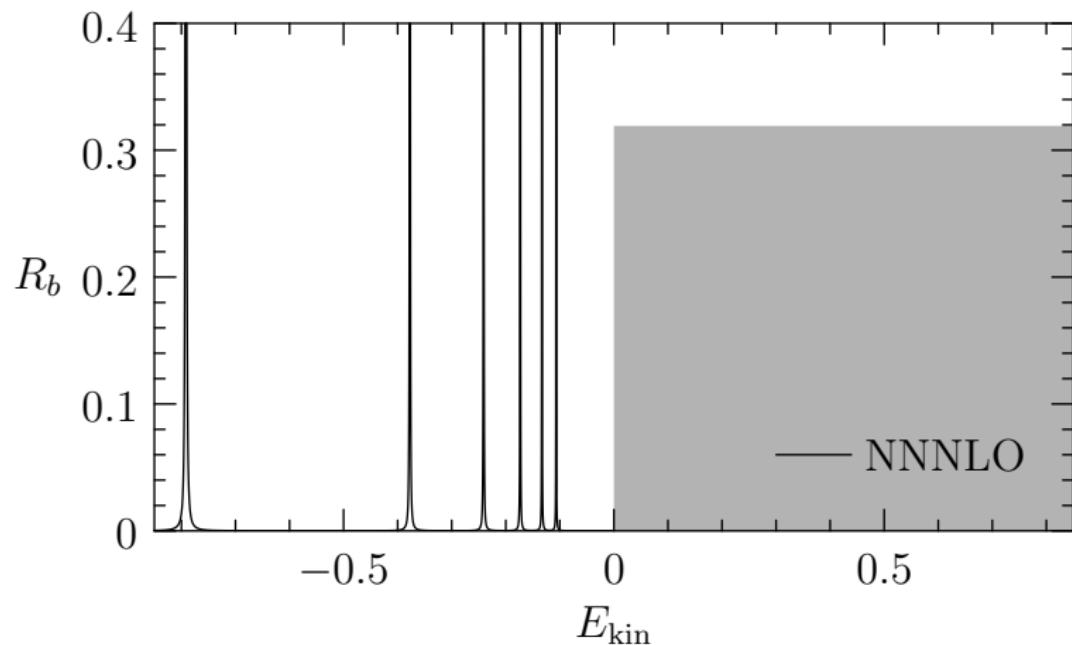
Non-relativistic sum rules

The cross section in PNRQCD



Non-relativistic sum rules

The cross section in PNRQCD



[Anzai, Beneke, Kiyo, Kniehl, Marquard, Penin, Piclum, Schuller, Smirnov, Seidel, Steinhauser, Sumino 02-14]

Non-relativistic sum rules

Technical details

- Use $R_b = 0.3 \pm 0.2$ for experimental continuum

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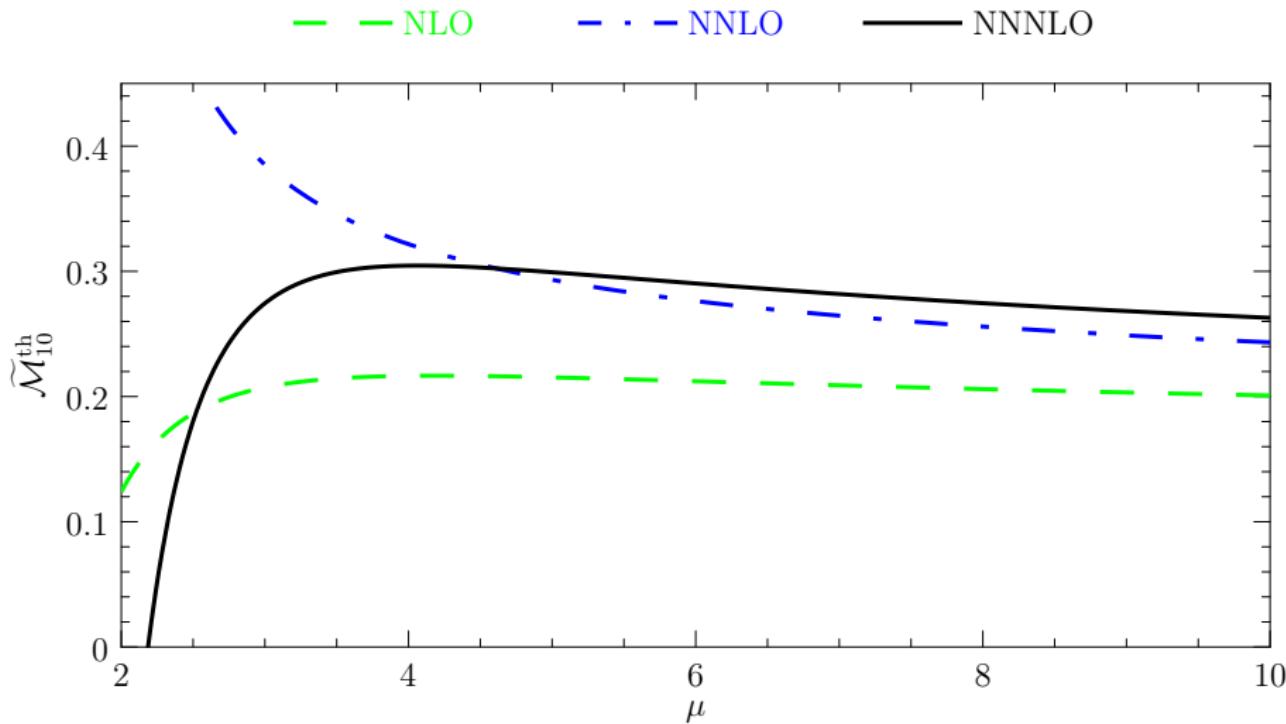
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- Overall renormalisation scale $\mu = m_b^{\text{PS}}$, vary $3 \text{ GeV} \leq \mu \leq 10 \text{ GeV}$
(independent variation of $\overline{\text{MS}}$ mass scale)

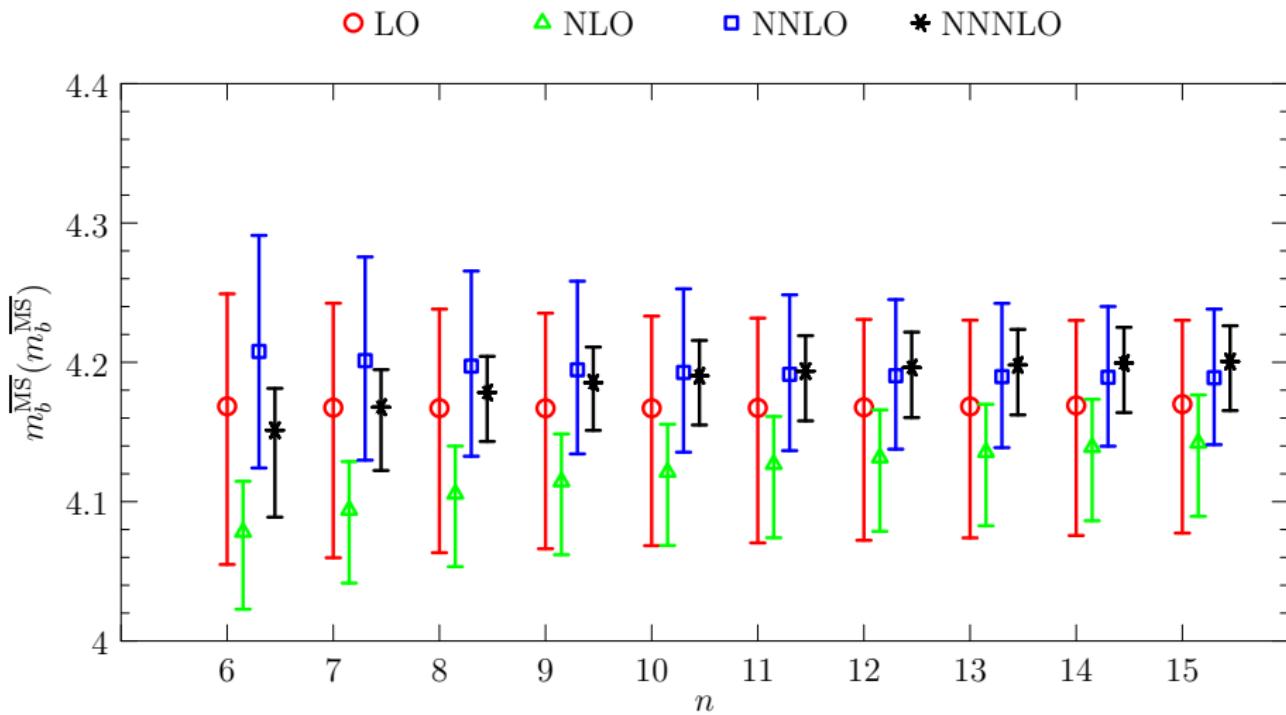
Non-relativistic sum rules

Scale dependence

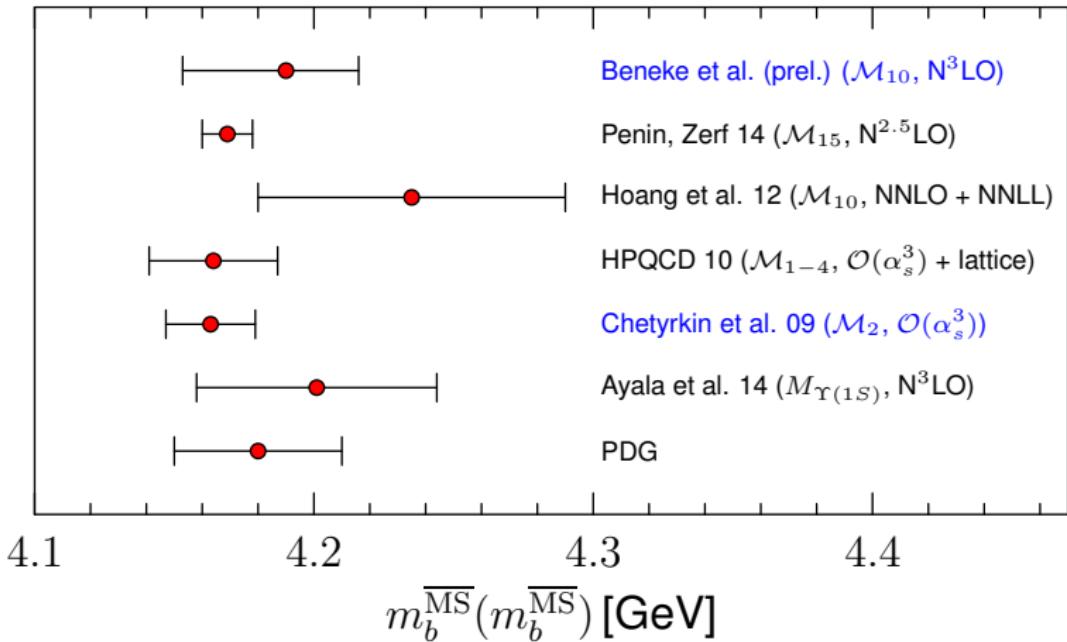


Non-relativistic sum rules

Preliminary results



The bottom quark mass



Conclusion

- NNNLO sum rules allow determinations of m_c , m_b with uncertainties $\sim 1\%$
- No cross section data for bottom continuum $\sqrt{s} \geq 11.24 \text{ GeV}$
- Non-relativistic sum rules limited by theory
- Good agreement between different methods (DIS, B decays, finite-energy sum rules, lattice, ...)