Production of Jets with Massive Quarks at the ILC and the MC Mass

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Outline

- Motivation
- Monte-Carlo top quark mass
- Variable flavor number scheme (VFNS) for final state jets with massive quarks
- Secondary massive quarks
- Primary massive quarks
- Preliminary results
- Conclusions

Motivation

Top quark mass measurements at the ILC:

Rather well known/studied:

- ✓ Total cross section @ threshold: $\mathrm{NNLL}_{\mathrm{NRQCD+resum}}$, $\mathrm{NNNLO}_{\mathrm{NRQCD}}$
- ✓ Total cross section @ 500-1000 GeV: $\mathrm{NNNLO}_{\mathrm{pQCD}}$
- ✓ Top reconstruction methods á la LHC: m_t(MC)

This talk

Might be very useful also in the context of LHC !

Not so well known/studied:

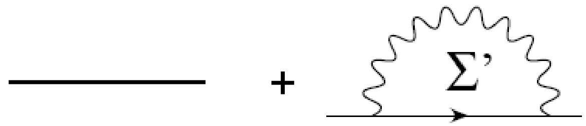
- ✓ Boosted top: top jet invariant mass
- ✓ Differential distributions @ threshold

Off-shell @ electroweak effects: e.g. width, couplings, alpha_s vs. mass

(more discriminating power)

More work needed here.

Heavy Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \Sigma^{\text{fin}}(m^0, m^0, \mu)$$

$\overline{\text{MS}}$ scheme:

$$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

→ $\overline{m}(\mu)$ is pure UV-object without IR-sensitivity

→ Useful scheme for $\mu > m$

→ Used a lot in beyond TeV physics

- Very energetic processes (E>>m)
- Total cross sections
- Off-shell massive quarks
- Away from thresholds/endpoints

Pole scheme:

$$m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

→ m^{pole} = perturbative single particle pole of perturbative S-matrix

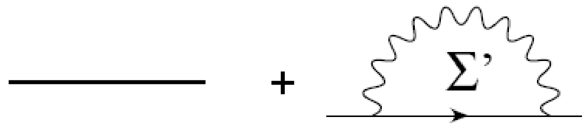
→ Absorbes all self energy corrections into the mass parameter

Should not be used if uncertainties are below 1 GeV !

→ Separation: self energy corrections ↔ inter quark/gluon interactions for all momenta

→ Has perturbative instabilities due to sensitivity to momenta < 1 GeV (Λ_{QCD})

Heavy Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \Sigma^{\text{fin}}(m^0, m^0, \mu)$$

$\overline{\text{MS}}$ scheme: $$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

Pole scheme: $$m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

MSR scheme: $$m^{\text{MSR}}(R) = m^{\text{pole}} - \Sigma^{\text{fin}}(R, R, \mu)$$

Jain, AH, Scimemi, Stewart (2008)

→ Interpolates between MS and pole mass scheme

→ Absorbes self energy corrections into the mass parameter ONLY above scale R

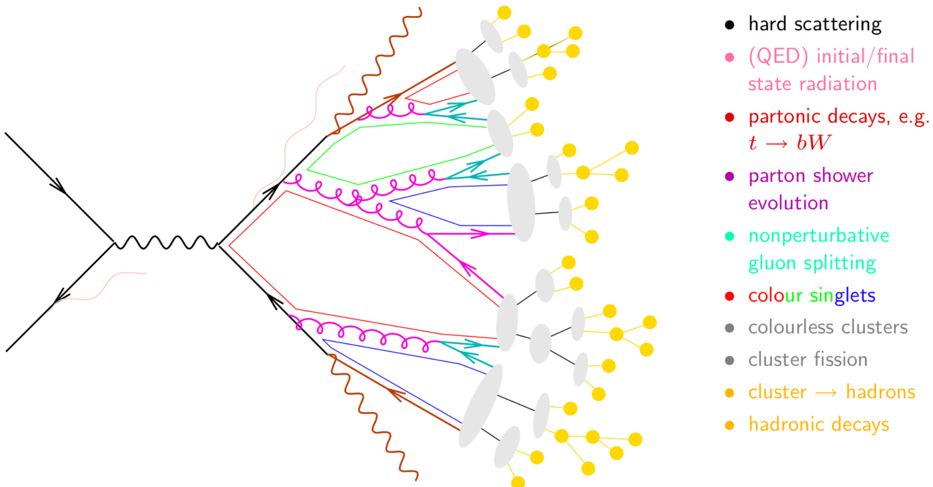
$$m_t^{\text{MSR}}(R = 0) = m^{\text{pole}}$$

$$m_t^{\text{MSR}}(R = \overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

→ Separation: self energy corrections ↔ inter quark/gluon interactions only for scales above R

→ Improved stability in perturbation theory for all classes of observables.

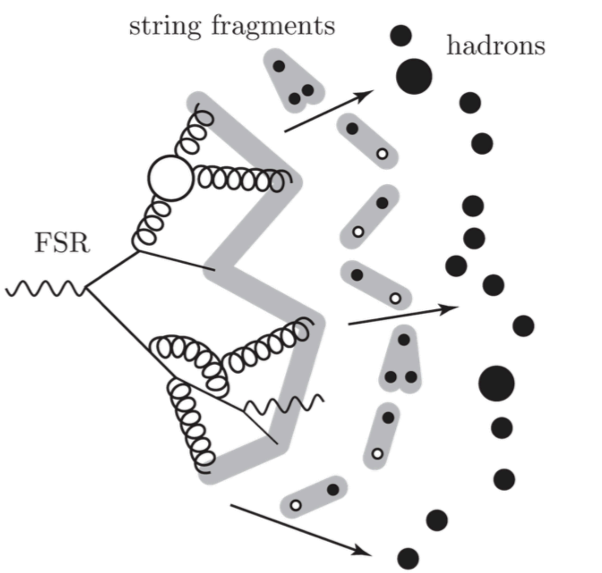
Heavy Quark Mass in the MC



Monte-Carlo QCD Calculator:

- Computes all inter-quark/gluon and radiation processes
- Computes hadronization of partons
- Electroweak radiation effects

- Does NOT calculate self-energy processes

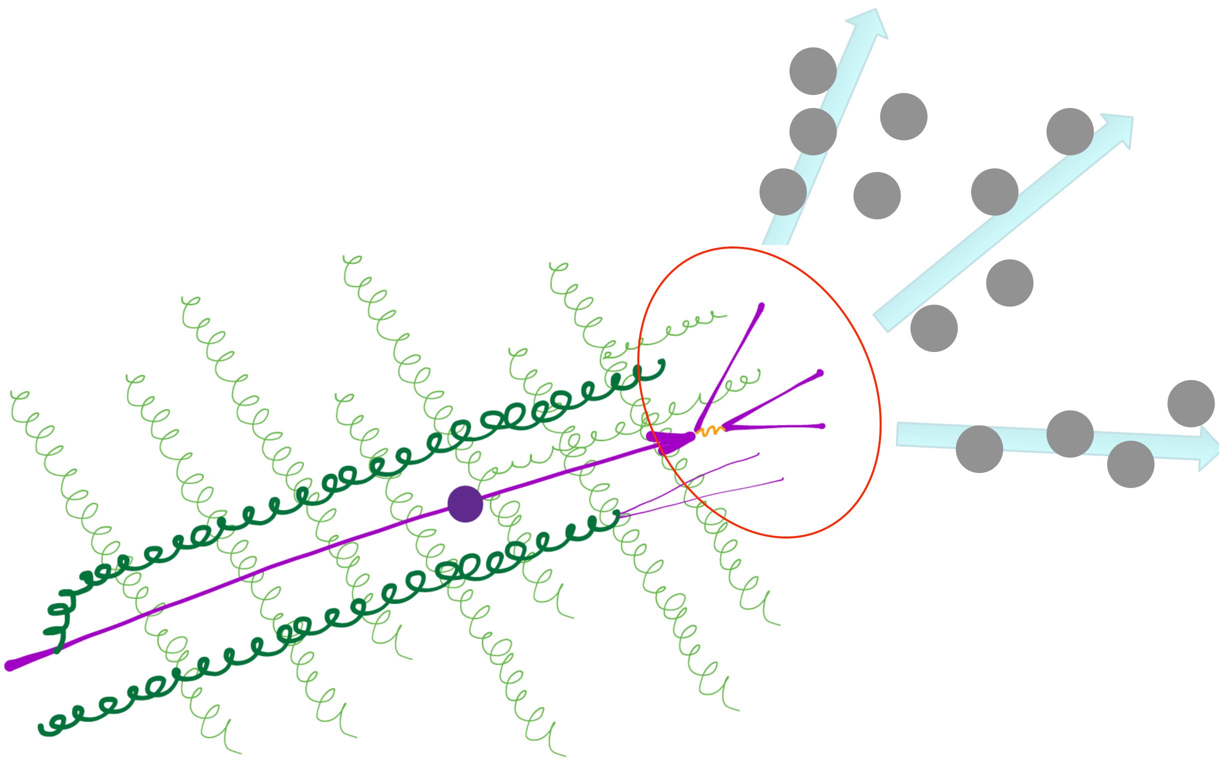


Inter-quark/gluon radiation/
Parton shower
cut-off at $\Lambda_s = 1\ \mathrm{GeV}$

Hadronization model below.

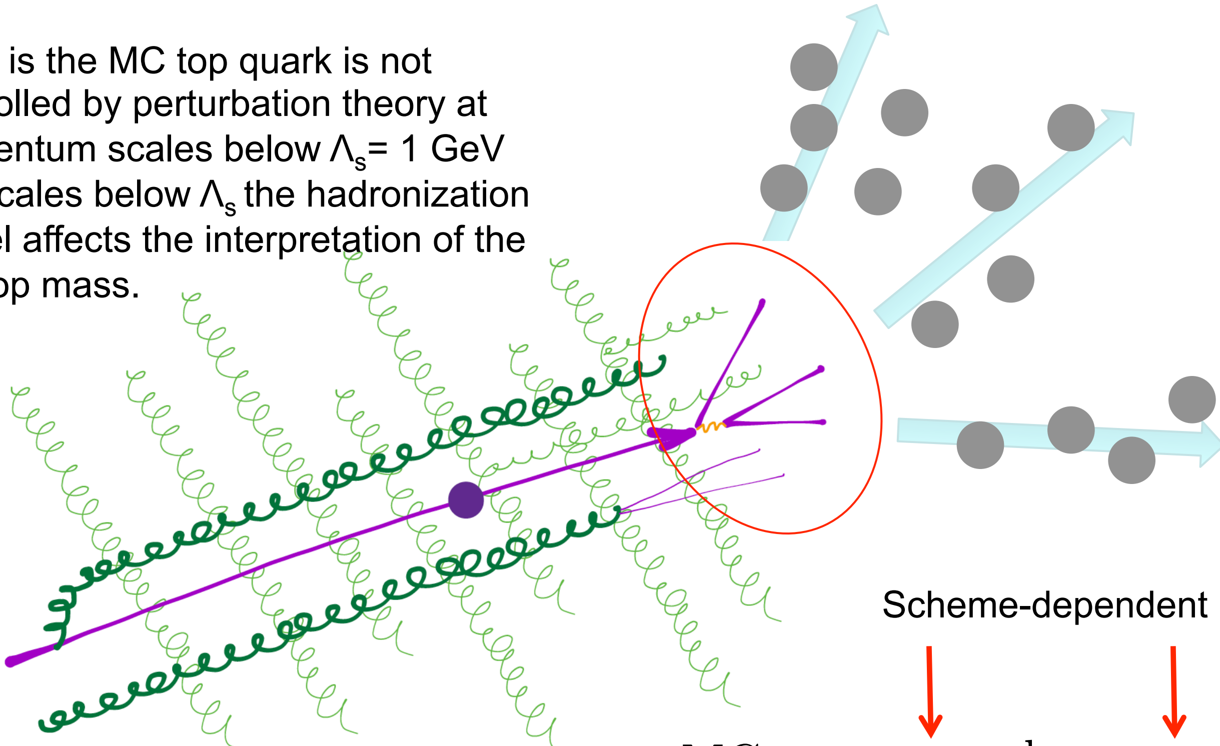
Shower, shower cut, model details affect the value of top mass.

Heavy Quark Mass in the MC



Heavy Quark Mass in the MC

- What is the MC top quark is not controlled by perturbation theory at momentum scales below Λ_s= 1 GeV
- For scales below Λ_s the hadronization model affects the interpretation of the MC top mass.



Scheme-dependent

nonperturbative
~ O(1 GeV)

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{quark}} + \Delta$$
$$= m_t^{\mathrm{MSR}}(R) + \Delta^{\mathrm{MSR}}(R)$$

Suitable scales: $R = 1 - 3\,\mathrm{GeV} \sim \Lambda_s$

MC mass has features similar to the mass of a Top-meson.

We use knowledge from B-meson physics.

AH, Stewart: arXive:0808.0222

Theory Tool to Measure the MC mass

The relation between MC mass and field theoretical mass can be made more precise by "measuring" the MC mass using a hadron level QCD prediction of a mass-dependent observable.

→ Inclusive jet invariant mass distribution:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level for comparison with MC at the hadron level

- Implementation of massive quarks into a general unified framework: valid for all quark masses and energies
- **VFNS for final state jets (with massive quarks)***

* In collaboration with: P. Pietrulewicz, V. Mateu, I. Jemos, S. Gritschacher

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD ..)
More to come …

* Also incorporates work on boosted tops: Fleming, AHH, Mantry, Stewart 2007

Theory Tools to Measure the MC mass

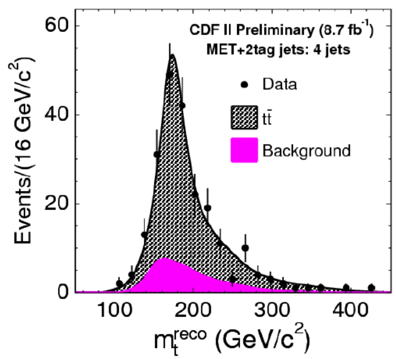
Observable: Thust in e+e-

$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q}$$

$$\stackrel{\tau\to 0}{\approx} \frac{M_1^2 + M_2^2}{Q^2}$$

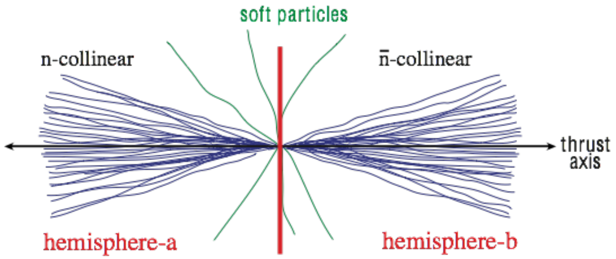
Relation to peak region of invariant mass distribution !

Other (similar) observables possible: maybe better than thrust



Theoretical methods are applicable for:

- Boosted tops: top decay products within a single jets
  → inclusive treatment viable
  → non-perturb. effects from massless quark production



All methods can also be applied directly to experimental data: $E_{cm} \geq 500$ GeV.

Comparison to the MC is a different method, since it does not depend on experimental uncertainties.

Overall structure of predictions

$$\left(\frac{d\sigma}{d\tau}\right) = \int d\ell \left[\boxed{\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}} \left(\tau - \frac{\ell}{Q}\right)} + \boxed{\left(\frac{d\sigma}{d\tau}\right)^{\text{nonsing}}_{\text{part}} \left(\tau - \frac{\ell}{Q}\right)} \right] \boxed{S^{\text{mod}}(\ell)}$$

- Most singular terms
- factorization formula
- log summations
- Mass-dependence

- Kinematically suppressed
- Taken from fixed-order pQCD calculations
- Mass-dependence

Perturbative

$$\delta(\tau),\ \left(\frac{\ln\tau}{\tau}\right)_+$$

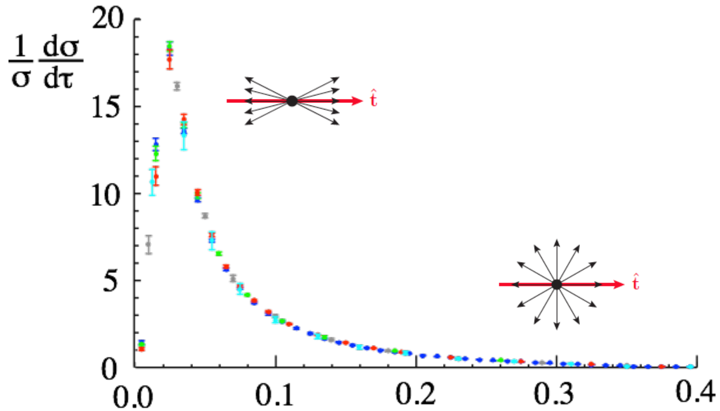
- Universal: independent on quark mass (taken from massless quark jets)

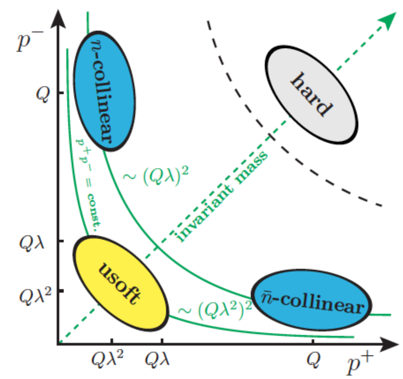
Nonperturbative

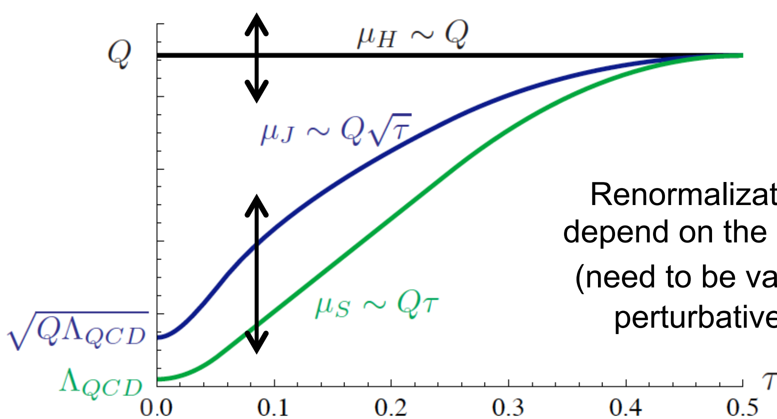
Factorization for Massless Quarks (singular)

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$

Korshemski, Sterman
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman





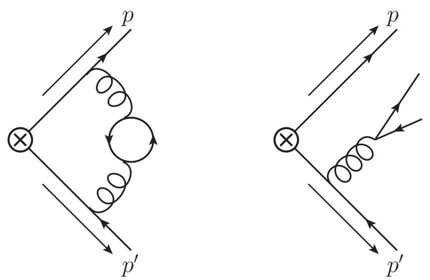


Abbate, AH, Fickinger, Mateu, Stewart

Renormalization scales that depend on the kinematic region (need to be varied to estimate perturbative uncertainty) → See my talk on Tuesday

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s)\int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

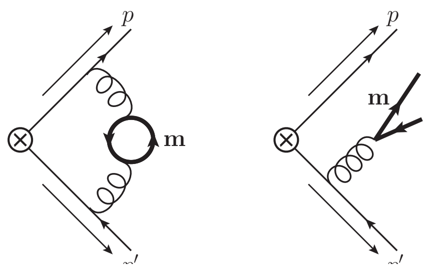
Accounting for full mass dependence



→ fully massless

- Full N^{3}LL$'$ (u.t. 4-loop cusp)+ 3-loop non-singular
- Gap scheme for soft function

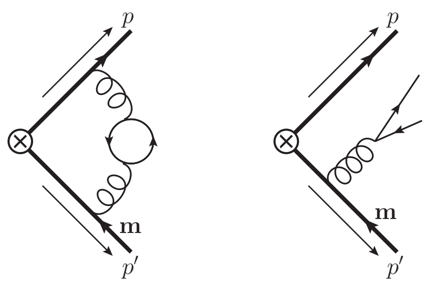
SCET authors: Becher, Schwartz,
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman
Fixed-order authors: Ge]hrmann etal, Weinzierl



→ secondary massive

- Full N^{2}LL$'$/N^{3}LL
- Four different physical situations

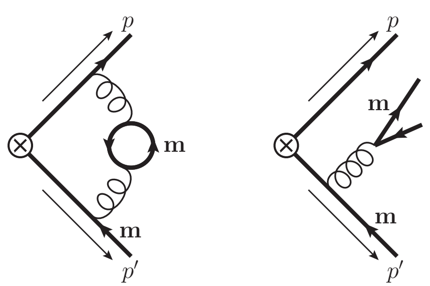
Pietrulewicz, AH, Gritschacher, Jemos 2013+2014



→ primary massive

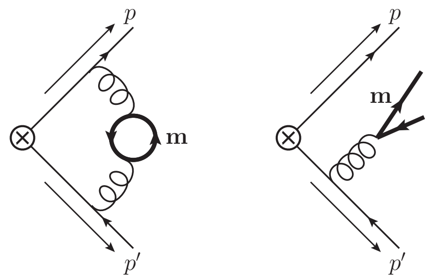
- Full N^{2}LL$'$/N^{3}LL finished
- Three different physical situations
- Massive quark loops in log resummation

Being written up



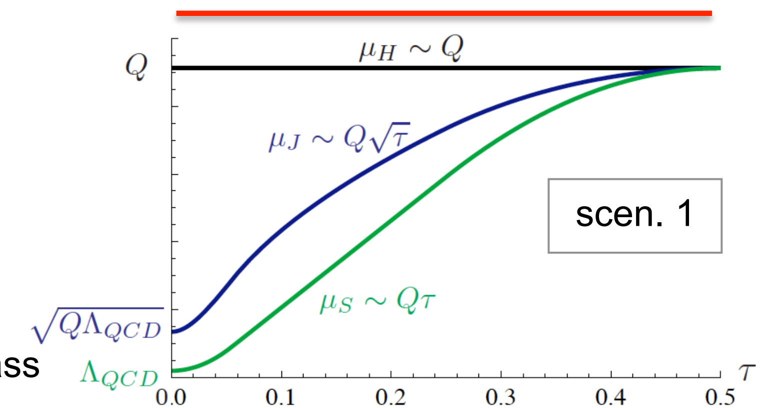
→ primary massive
secondary massive

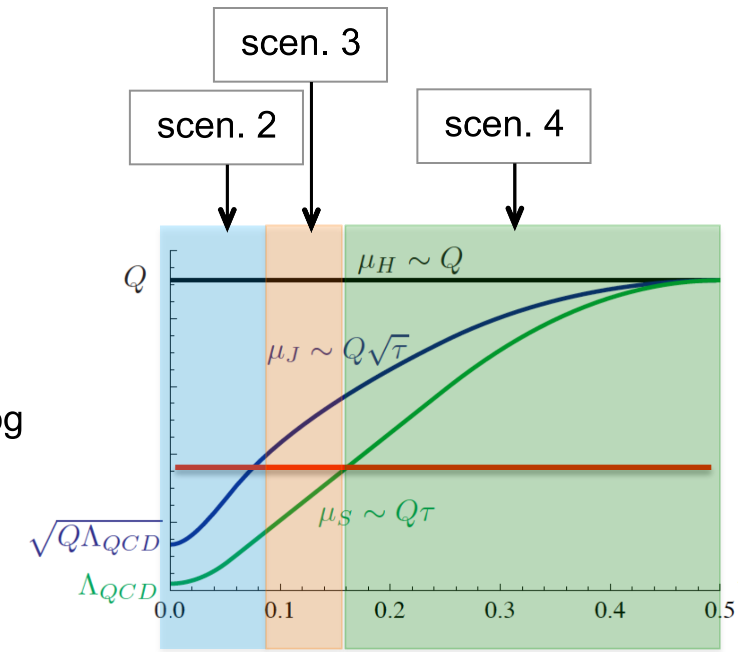
VFN Scheme: Secondary Massive Quarks



New developments:

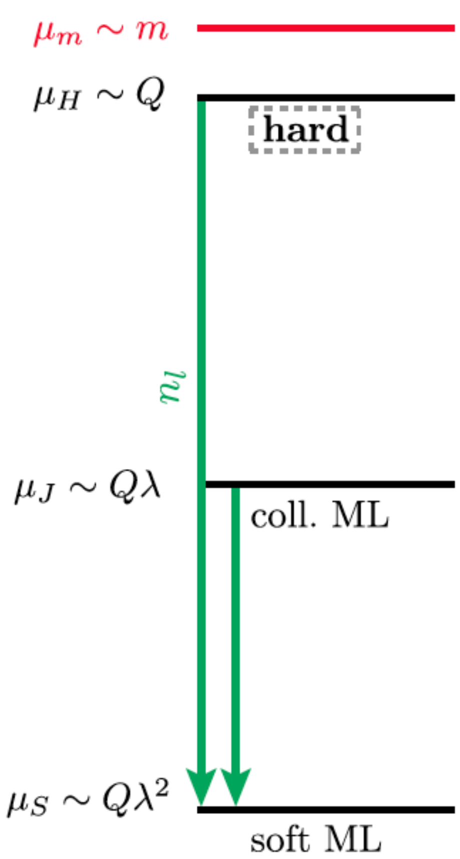
- Provided results for factors with complete mass dependence at O(as^2) [NNNLL/NNLL']
- Flavor threshold correction factors at O(as^2)
- Reconcile problem of $SCET_2$-type rapidity divergences
- Establish consistency conditions of flavor threshold matching factors (e.g. universality between thrust and DIS@ large x
- Simple implementation rules related to modified renormalization conditions
- Method treating massive quark loops within log resummation
- Removal of O(Λ_{QCD}) renormalon effects concerning mass and soft effects
- All possible kinematic regions covered (decoupling limit↔ massless limit)





VFN Scheme: Secondary Massive Quarks

Example: scenario 1 for m> Q > M_{jet} > E_{soft}

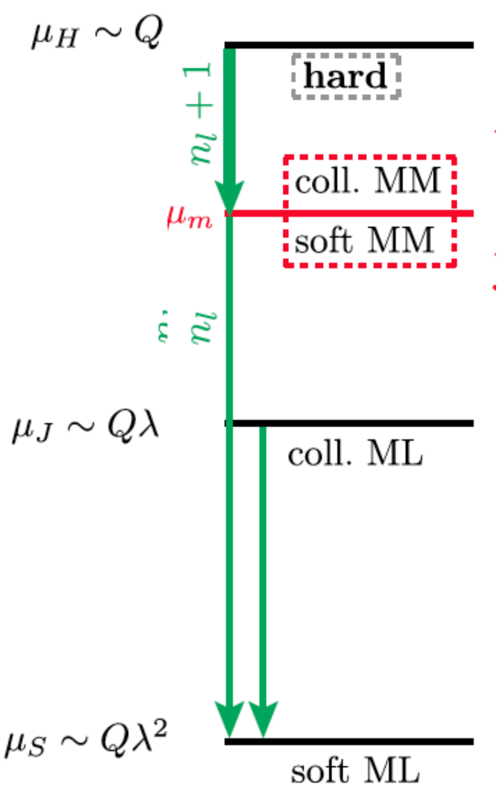


$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = Q|C^{(n_l)}(Q,m,\mu_H)|^2 |U_C^{(n_l)}(Q,\mu_H,\mu_S)|^2$$
$$\times \int ds \int ds' J^{(n_l)}(s',\mu_J) U_J^{(n_l)}(s-s',\mu_S,\mu_J)\; S^{(n_l)}\left(Q\tau - \frac{s}{Q},\mu_S\right)$$

- Same form as for massless case
- Massive quark corrections in $C^{(nl)}$ in the n_l-flavor scheme for $\alpha_s^{(n_l)}$
- Decoupling of massive quark for m≫Q
- $U^{(nl)}$ evolution factors as in the massless case for n_l quark flavors.

VFN Scheme: Secondary Massive Quarks

Example: scenario 2 for Q > m > M_{jet} > E_{soft}



$$
\begin{aligned}
\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = {} & Q|C^{(n_l+1)}(Q,m,\mu_H)|^2|U_C^{(n_l+1)}(Q,\mu_H,\mu_m)|^2 \\
& \times |\mathcal{M}_C(Q,m,\mu_m)|^2|U_C^{(n_l)}(Q,\mu_m,\mu_S)|^2 \\
& \times \int ds \int ds'\, J^{(n_l)}(s',\mu_J)\, U_J^{(n_l)}(s-s',\mu_S,\mu_J)\, S^{(n_l)}\Big(Q\tau - \frac{s}{Q},\mu_S\Big)
\end{aligned}
$$

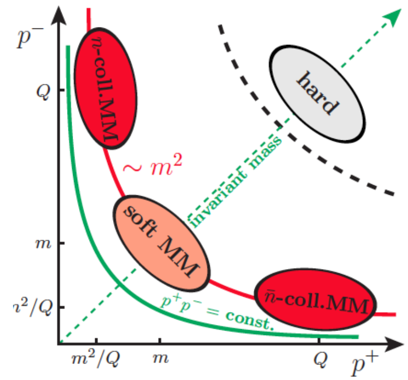
- Massive quark corrections in $C^{(nl+1)}$ in the (n_l+1)-flavor scheme for $\alpha_s^{(n_l+1)}$
- No mass singularities for $m \ll Q$ in $C^{(nl+1)}$
- $U^{(nl+1)}$ evolution factors as in the massless case for (n_l+1) quark flavors.
- Massive quark threshold correction M_C for hard coefficient evolution at μ_m~m
- Separation of massive quark loops corrections

Approach similar to massive quark threshold for PDF evolution using the SCET formalism

Rapidity Logarithms

- Secondary mass effects start at O(α_s^2)
- Counting for rapidity logs: α_s Log ~ 1
- At O(α_s^2):
  - Modified counting needed
  - Need terms at O(α_s^3 Log) and O(α_s^4 Log^2)
- Extract O(α_s^3 Log) $M_H^{(3)}$ term from DIS

Use results from Alblinger, Blümlein, etal. 2014



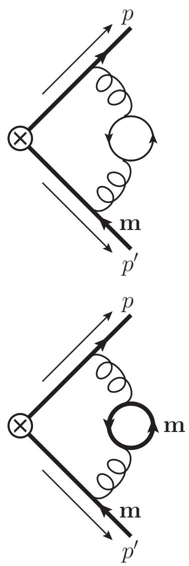
O(α_s)

O(α_s^2)

$$
\begin{aligned}
\mathcal{M}_H(Q,m,\mu_m) = 1 &+ \frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\frac{4}{3}L_m^2+\frac{40}{9}L_m+\frac{112}{27}\right\} \\
&+ \Bigg[\frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\left\{\frac{4}{9}L_m^3+\frac{38}{9}L_m^2+\left(\frac{242}{27}+\frac{2\pi^2}{3}\right)L_m-\frac{52}{9}\zeta(3)+\frac{875}{54}+\frac{5\pi^2}{9}\right\} \\
&\quad + \frac{\left(\alpha_s^{(n_l+1)}\right)^3 C_F T_F}{(4\pi)^3}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\mathcal{M}_H^{(3)}+\sum_{n=0}^{3} a_n\, L_m^n\right\} \\
&+ \frac{\left(\alpha_s^{(n_l+1)}\right)^4 C_F^2 T_F^2}{(4\pi)^4}\ln^2\left(-\frac{m^2}{Q^2}\right)\left\{\frac{8}{9}L_m^4+\frac{160}{27}L_m^3+\frac{416}{27}L_m^2+\frac{4480}{243}L_m+\frac{6272}{729}\right\}\Bigg]
\end{aligned}
$$

$$L_M = \ln\left(\frac{m^2}{\mu_m^2}\right)$$

VFN Scheme: Primary Massive Quarks

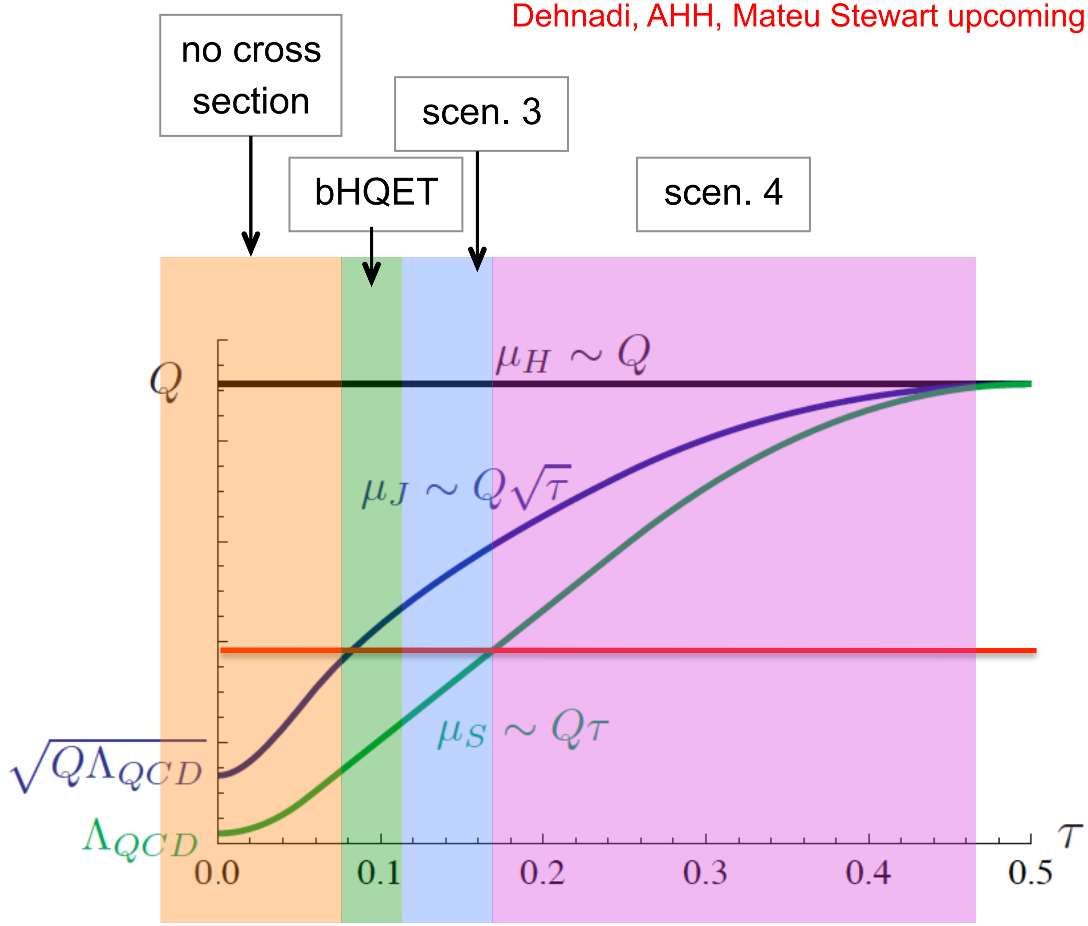


→ bHQET-type theory when the jet scale approaches the quark mass

Fleming, AHH, Mantry, Stewart 2007

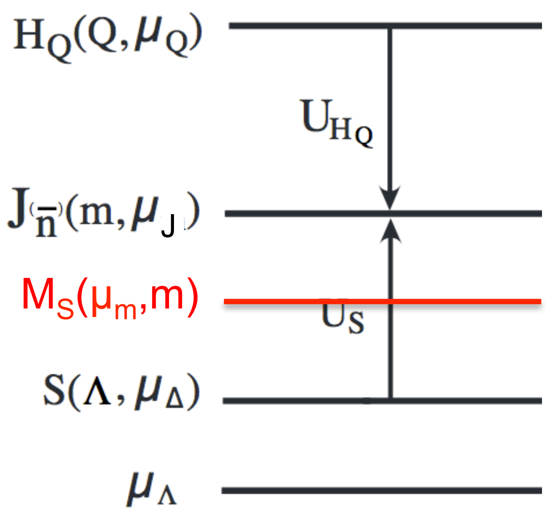
→ two SCET-type theories

Dehnadi, AHH, Mateu Stewart upcoming



VFN Scheme: Primary Massive Quarks

Example: SCET scenario 3: Q >> J > m > S



- Massive jet function (differs from massless)
- Soft matching coefficient (differs from massless)
- Universal hard coefficient
- Universal soft function
- RG-evolution analogous to massless case

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-III}} = Q\,H_Q^{(n_f)}(Q,\mu_Q)\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s\int \mathrm{d}k\,\mathrm{d}k'\,\mathrm{d}k''\,J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\,U_S^{(n_f)}(k,\mu_J,\mu_m)\,\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)\,U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\,S_{\mathrm{part}}^{(n_l)}\left(Q\tau-Q\tau_{\min}-\frac{s}{Q}-k'',\mu_S\right)$$

$$n_f = n_\ell + 1$$

VFN Scheme: Primary Massive Quarks

- Mass dependence in all FO components of all factorization theorems
- Most relevant quark mass dependence contains in the jet functions (SCET & bHQET)
- Mass definition to be used depends on scale of the respective functions (→profile functions) !!

μ ≥ m: MSbar mass (n_l+1)

$$\bar{m}(\mu) = m_{\rm pole} - \bar{m}(\mu) \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \ln^k \frac{\mu}{\bar{m}}$$

→ usual MSbar RG-evolution

μ < m: R-scale short-distance mass (n_l)

- Jet mass: from bHQET jet function
- MSR mass: derived from MSbar mass coefficients

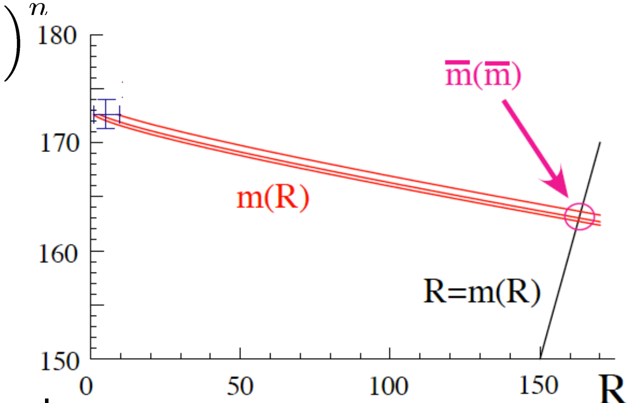
Jain, Scimemi, Stewart 08

Jain, Scimemi, Stewart, AH 08

$$m(R) = m_{\rm pole} - \delta m(R) \qquad \delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n$$

$$R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}$$

$$m(R_1) - m(R_0) = \int_{R_0}^{R_1} \frac{dR}{R}\, R\,\gamma^R[\alpha_s(R)]$$



μ_m~ m: matching: → pert. renormalons-free relation through pole mass

VFN Scheme: Primary Massive Quarks

Status:

NNLL+NLO:

All one-loop pQCD corrections known analytically.
All evolution equations known to the required level.

NNNLL+NNLO:

- ✓ 2-loop pQCD corrections available (Oleari, Nason + Rodrigo): but not analyzed yet
- ✓ 2-loop BHQET jet function
- ✓ All evolution equation known to the required level.

- ○ 2-loop SCET jet function
- ○ 2-loop threshold matching corrections

Full NNNLL+NLO probably available next year.

Application to data:

Tagged bottom event shape distribution data are on tape (JADE, OPAL), but have not been analyzed yet !!!   This could be done now with modern methods !

MC vs. QCD: Primary Bottom Production

Preliminary !!

Denahdi, AHH, Mateu

Compare MC with QCD (SCET, summation, hadronization effects) @ NNLL+NLO for Thrust

- Take central values for α_s and Ω_1 from our earlier NNLL thrust analysis for data on all-flavor production (=massless quarks)

$$\alpha_s(M_Z) = 0.1192 \pm 0.006$$
$$\Omega_1 = 0.276 \pm 0.155$$

- Compare with Pythia (m_b^{Pythia}=4.8 GeV) for consistency and mass sensitivity

- Which mass does m_b^{Pythia}=4.8 GeV correspond to for a field theoretic bottom mass?

Abbate,Fickinger, AHH, Mateu, Stewart 2010

order	$\bar{\Omega}_1$ ($\overline{\text{MS}}$)	Ω_1 (R-gap)
NLL$'$	0.264 ± 0.213	0.293 ± 0.203
NNLL	0.256 ± 0.197	$\boxed{0.276 \pm 0.155}$
NNLL$'$	0.283 ± 0.097	0.316 ± 0.072
N^{3}LL	0.274 ± 0.098	0.313 ± 0.071
N^{3}LL$'$ (full)	0.252 ± 0.069	$\mathbf{0.323 \pm 0.045}$
N^{3}LL$'_{(\text{QCD}+m_b)}$	0.238 ± 0.070	0.310 ± 0.049
N^{3}LL$'_{(\text{pure QCD})}$	0.254 ± 0.070	0.332 ± 0.045

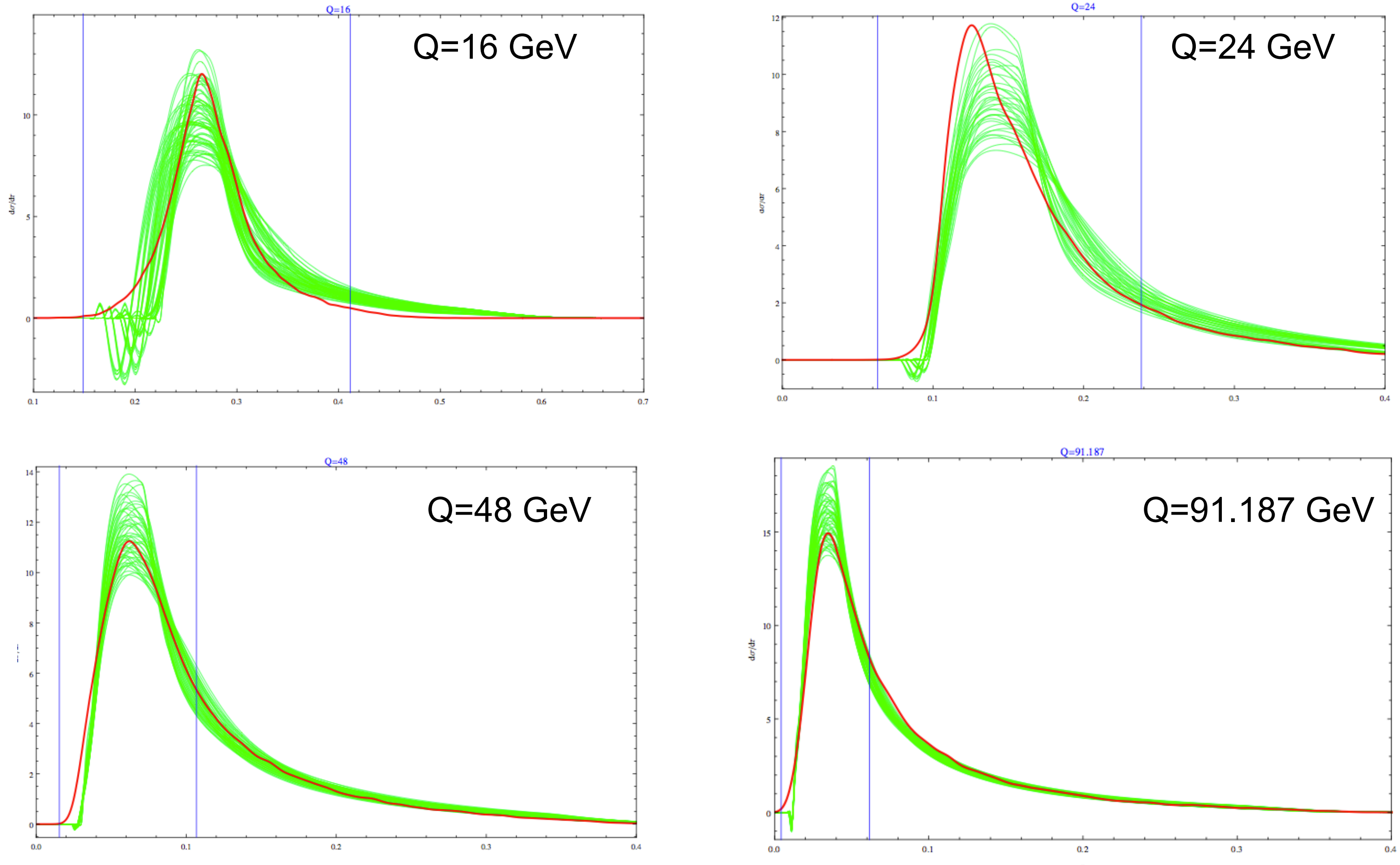
order	$\alpha_s(m_Z)$ (with $\bar{\Omega}_1^{\overline{\text{MS}}}$)	$\alpha_s(m_Z)$ (with Ω_1^{Rgap})
NLL$'$	0.1203 ± 0.0079	0.1191 ± 0.0089
NNLL	0.1222 ± 0.0097	$\boxed{0.1192 \pm 0.0060}$
NNLL$'$	0.1161 ± 0.0038	0.1143 ± 0.0022
N^{3}LL	0.1165 ± 0.0046	0.1143 ± 0.0022
N^{3}LL$'$ (full)	0.1146 ± 0.0021	$\mathbf{0.1135 \pm 0.0009}$
N^{3}LL$'_{(\text{QCD}+m_b)}$	0.1153 ± 0.0022	0.1141 ± 0.0009
N^{3}LL$'_{(\text{pure QCD})}$	0.1152 ± 0.0021	0.1140 ± 0.0008

MC vs. QCD: Primary Bottom Production

Preliminary !! (no fit yet) all NNLL+NLO

Pythia: $m_b^{\text{Pythia}} = 4.8\ \text{GeV}$

QCD calc.: $\overline{m}_b(\overline{m}_b) = 4.2\ \text{GeV}$ $\alpha_s(M_Z) = 0.1192$ $\Omega_1 = 0.276\,\text{GeV}$



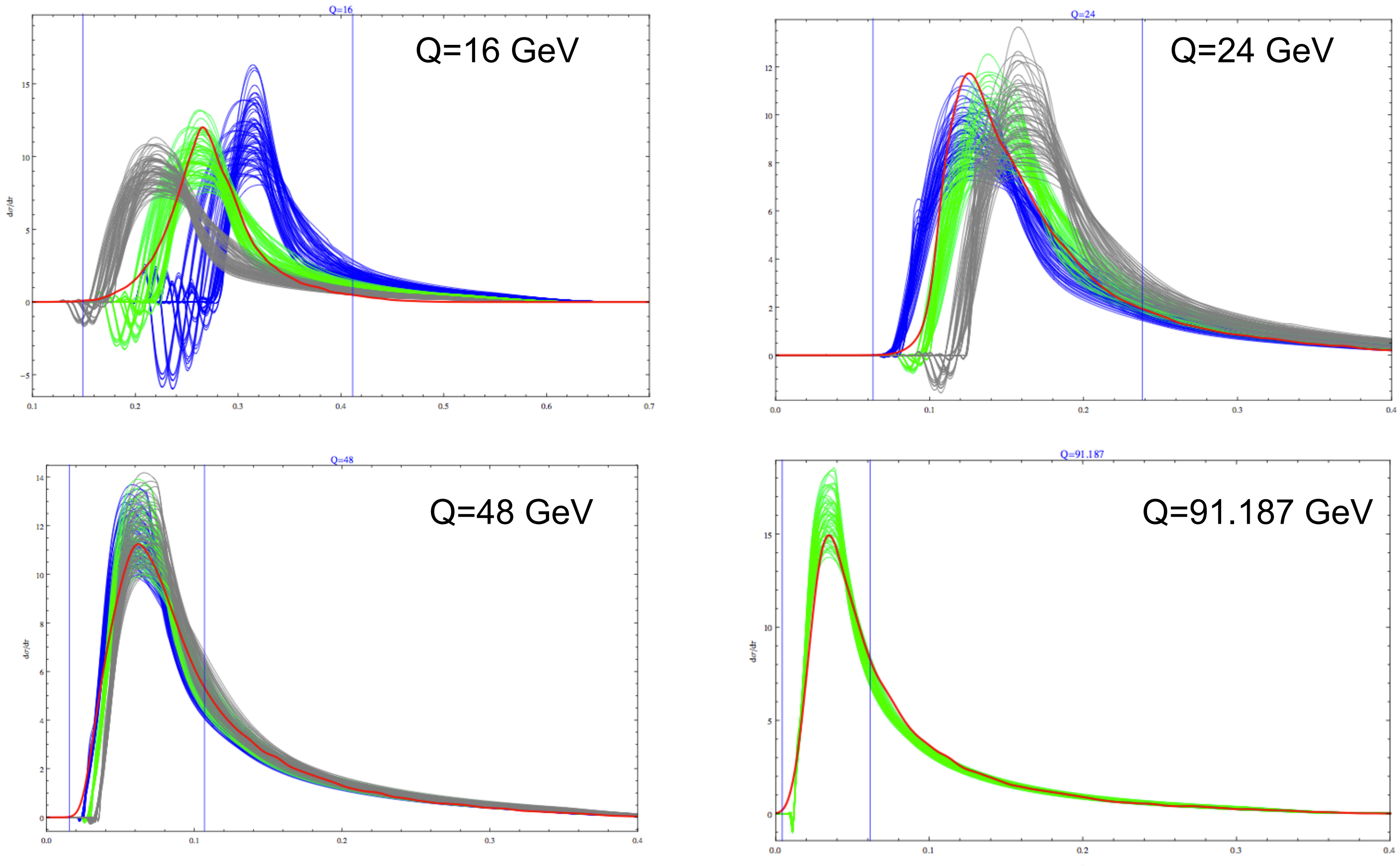
MC vs. QCD: Primary Bottom Production

Preliminary !! (no fit yet)   all NNLL+NLO

Pythia: $m_b^{\mathrm{Pythia}} = 4.8\ \mathrm{GeV}$

QCD calc.: $\overline{m}_b(\overline{m}_b) = 3.7, 4.2, 4.7\,\mathrm{GeV}$

Mass sensitivity for 0.1 < m/p_T < 0.3.



Conclusions

→ The MC top mass parameter has the status of a hadronic parameter and is therefore not a field theoretic mass definition

→ As long as we don't know more there is an uncertainty of about 1 GeV one needs to add when relating the MC mass to a low-scale field theory mass.

→ Suitable field theory mass definition in this context: e.g. MSR mass (R=1-3 GeV)

→ Using the pole mass in this context might be still ok for some applications (e.g. total cross section @ LHC), but will inevitably cause problems for other cases.

→ It is possible to relate the MC top mass to a field theoretic mass by fits of QCD calculations at the hadron level to MC output for very mass sensitive quantities.

→ QCD calculations for boosted top jet invariant masses allows to quantify this relation in a reliable manner (further work necessary for final answer).

→ Fully massive thrust using a VFNS for final state inclusive jets.

→ Upcoming:

- Analysis for top quarks
- C parameter, heavy jet mass, inv. mass distr. @ NNLL
- DIS for massive quarks @ large x
- pp → tt+X (2-jettiness) @ NLL → NNLL possible, NNNLL need NNLO full. Diff.

Counting Rules

LL NLL NNLL NNNLL

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s(\alpha_s \ln)^k + \alpha_s^2(\alpha_s \ln)^k + \ldots$$

		cusp	non-cusp	matching	alphas	Classic Counting
standard counting	LL	1	–	tree	1	LLA
	NLL	2	1	tree	2	NLLA
	NNLL	3	2	1	3	NNLLA + LLO
	$\mathrm{N^3LL}$	4^{pade}	3	2	4	NNNLLA + NLO
primed counting emphasizes fixed order	LL'	1	–	tree	1	LLA
	NLL'	2	1	1	2	NLLA + LLO
	NNLL'	3	2	2	3	NNLLA + NLO
	$\mathrm{N^3LL}'$	4^{pade}	3	3	4	NNNLLA + NNLO

Theory error from Padé estimate of Γ_3^{cusp}