Eddy Current Effects on the Target for e-Driven Source

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2015.04.24 ALCW15

The Issue

- The target for the e-Driven source is under the tail of the magnetic field of the flux concentrator
- Time scale of field variation is O(μs), much faster than for the undulator source
- Possible effects
  - Lorentz force to the target due to the eddy current and the transverse component of the magnetic field
  - Heating due to the eddy current

A Simple Model

- Infinite round plate
- Axisymmetric field **B**
  - Assume Gaussian form of radius σ_r
  - Slow time change

$$B_z = B_{z0} e^{-r^2/2\sigma_r^2} f(t), \qquad f(t) \text{ is normalized to } f_{max} = 1$$

- Skin depth effect

$$\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$$

- Induced electric field

$$E_\phi(r) = \frac{1}{r}\frac{d}{dt}\int_0^r r dr B_z$$
$$= B_{z0}\sigma_r^2\left[1 - e^{-r^2/2\sigma_r^2}\right] \times e^{-z/\delta}\frac{df}{dt}$$

- Induced current (σ: conductivity 1/Ωm)

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

- Assume the radial magnetic field

$$B_r = B_{r0}\frac{r}{\sigma_r}e^{-r^2/2\sigma_r^2}f(t)$$

- Lorentz force integrated over the target volume

$$F_z = -\pi\sigma\delta\sigma_r^3 B_{z0}B_{r0}\frac{df}{dt}f(t)$$

- Integration over time = 0 ($\int \dot{f} f dt = 0$)
- Assume f(t) grows linearly from t=0 to the maximum at t=τ and falls to 0 at t=2τ
- Impulse from t=0 to t=τ is

$$P \approx \frac{\sigma}{4} \delta \sigma_r^3 B_{z0} B_{r0}$$

- Heating

$$Q = \int_{-\infty}^{\infty} dt \int_0^{\infty} 2\pi r dr \int dz e^{-2z/\delta} \sigma \left[\frac{1}{r} B_{z0} \sigma_r^2 \left(1 - e^{-r^2/2\sigma_r^2} \right) \dot{f} \right]^2$$

$$= \pi \sigma \delta B_{z0}^2 \sigma_r^4 \int_{-\infty}^{\infty} (\dot{f})^2 dt \times \int_0^{\infty} \frac{dx}{x} (1 - e^{-x})^2, \qquad (x \equiv r^2/2\sigma_r^2)$$

$$\int (\dot{f})^2 dt = 2/\tau$$

- Cut the r integral at r=Mσ_r

$$\int_0^{\infty} \frac{dx}{x} (1 - e^{-x})^2 = 2 \log \frac{M}{2} + \gamma_E \qquad (M \gg 2)$$

- Heat deposit per pulse

$$Q \approx 2\pi \frac{\sigma \delta B_{z0}^2 \sigma_r^4}{\tau} [\log(M/2) + \gamma_E/2]$$

- Plug-in numbers

$$\sigma = 1/53\mathrm{n}\Omega \quad \tau = 6\mu\mathrm{s} \quad \omega \sim 2\pi/12\mu\mathrm{s} \quad \delta \approx 0.4\mathrm{mm}$$

B_{z0}=3T, B_{r0}=0.3T, M=3, σ_r=14mm

- Then,
  P = 0.005 Ns
  Q = 1.9J/pulse (190kW for 5Hz, 20 triplets)

Peter's estimation

$$(1)\qquad Q=\int_0^{\tau}\left[j(t)\right]^2 dt\int_0^{\infty}e^{-2z/\delta}\,dz\int_0^{r_m}2\pi r\,dr\,\sigma B_{z,0}^2\,\sigma_r^4\left(\frac{1-e^{-r^2/2\sigma_r^2}}{r^2}\right)^2$$

$$f(t)=\sin\frac{\pi}{\tau}t$$

$$(2)\qquad Q=\frac{\pi^3}{2\tau}\,\delta\,\sigma\,B_{z,0}^2\,\sigma_r^4\int_0^{r_m}\frac{\left(1-e^{-r^2/2\sigma_r^2}\right)^2}{r}\,dr$$

$$(3)\qquad Q=\frac{\pi^3}{2\tau}\,\delta\,\sigma\,B_{z0}^2\left(\frac{r_m}{2}\right)^4\left\{1-\frac{1}{3}\left(\frac{r_m}{\sigma_r}\right)^2+\frac{7}{96}\left(\frac{r_m}{\sigma_r}\right)^4\cdots\cdots\right\}$$

Q=150J/pulse (15kW)