BEHAVIOUR OF LINEAR ACOUSTIC WAVES

induced in a Conversion Target for e+ Production

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INTRODUCTION

Objective

We are all familiar with the heat load problem in conversion targets

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❒ Here, we will present an analytical solution, that can be easily used to calculate the stress induced in solid target.

❒ Also, the solution can serve as a litmus test for the simulation software used in stress analysis for conversion targets.

THE MODEL

THE MODEL IS BASED ON CONTINUUM MECHANICS AND COMPRISES OF:

conservation of Mass

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

where ρ: density and $\mathbf{v}$: velocity

Equation of Motion

Equation of Motion

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla P$$

where ρ: density,= **v**: velocity and P: Pressure

Equation of State

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$$P = \frac{\Gamma}{V} Q$$

where Γ:Güneisen Coefficient , Q: Energy, V: Volume and P: Pressure

By perturbing those three equations near equilibrium

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Linear Acoustic Waves (LAW) Equation

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Linear Acoustic Waves (LAW) Equation

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \nabla^2 P = \frac{\Gamma}{V_0} \frac{\partial^2 Q}{\partial t^2}$$

where V_0: Volume at Equilibrium / Initial state and c_s is the sound speed

ENERGY DEPOSITION

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where $Q(\mathbf{x})$ is the spatial distribution and

$$\delta(t) = \begin{cases} 1 & : t = 0 \\ 0 & : t > 0 \end{cases}$$

By using this assumption, we can reduced the LAW equation

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Homogeneous Partial Differential Equation

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Homogeneous Partial Differential Equation

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \nabla^2 P = 0 \qquad \text{for} \quad t > 0$$

CONDITIONS

Initial Conditions (at t=0)

Considering the instantaneous energy deposition assumption, the IC can be written as

$$P = \frac{\Gamma}{V_0} Q(\mathsf{x}) \tag{1}$$

and

$$\frac{\partial P}{\partial t} = 0 \tag{2}$$

Boundary Conditions

Dirichlet BC (DBC)

specify the value on the function on the surface

Or

Neumann BC (NBC)

specify the normal derivative of the function on a surface

Or

Robin BC is a combination of both DBC and NBC

GEOMETRY

For simplicity, we will consider 1 Dimensional case.

Basically, an approximation of a cylindrical solid Target, namely:

- ❒ A Thin Rod and
- ❒ Cylindrical Disc

Thin Rod with Dirichlet BC

Problem Set-up:

Partial DE:

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \frac{\partial^2 P}{\partial z^2} = 0$$

Initial Conditions:

$$P(z,0) = \frac{\Gamma}{V_0} Q(z) \qquad \text{and}$$

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = 0$$

Boundary Condition:

$$P(0,t) = 0 = P(L,t)$$

Analytical Solution of LAW Eqn in Thin Rod with Dirichlet BC

$$P(z,t) = \frac{2\Gamma}{LV_0} \sum_{n=1}^{\infty} \cos\left(\frac{\pi n}{L} c_s t\right) \sin\left(\frac{\pi n}{L} z\right) \times$$
$$\times \int_0^L Q(z) \sin\left(\frac{\pi n}{L} z\right) dz$$

Disc with Dirichlet BC

Problem Set-up:

Partial DE:

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} \right) = 0$$

Initial Conditions:

$$P(r, 0) = \frac{\Gamma}{V_0} Q(r) \qquad \text{and}$$

$$\left. \frac{\partial P}{\partial t} \right|_{t=0} = 0$$

Boundary Condition:

$$P(R, t) = 0$$

Analytical Solution of LAW Eqn in Cylindrical Disc with Dirichlet BC[1]

$$P(r,t) = \frac{2\Gamma}{V_0 R^2} \sum_{n=1}^{\infty} \frac{1}{J_1^2(\lambda_n)} J_0\left(\lambda_n \frac{r}{R}\right) \cos\left(\lambda_n \frac{c_s t}{R}\right) \times$$
$$\times \int_0^R \xi Q(\xi) J_0\left(\lambda_n \frac{\xi}{R}\right) d\xi$$

[1] λ_n: are positive zeros of Bessel function, $J_0(\lambda_n) = 0$

Disc with Neumann BC

Problem Set-up:

Partial DE:

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} \right) = 0$$

Initial Conditions:

$$P(r,0) = \frac{\Gamma}{V_0} Q(r) \qquad \text{and}$$

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = 0$$

Boundary Condition:

$$\left.\frac{\partial P}{\partial r}\right|_{r=R} = 0$$

Analytical Solution of LAW Eqn in Cylindrical Disc with Neumann BC[2]

$$P(r,t) = \frac{2\Gamma}{V_0 R^2}\left[\int_0^R Q(\xi)\xi d\xi + \sum_{n=1}^{\infty} \frac{1}{J_0^2(\lambda_n)} J_0\left(\lambda_n \frac{r}{R}\right) \times \right.$$
$$\left. \times \cos\left(\lambda_n \frac{c_s t}{R}\right) \int_0^R \xi Q(\xi) J_0\left(\lambda_n \frac{\xi}{R}\right) d\xi \right]$$

[2] λ_n: are positive zeros of Bessel function, $J_1(\lambda_n) = 0$

MULTI-BUNCH EFFECT

M Bunch Set-up

Let T_b be the time spacing between bunches and $P_*(x,t)$ be the pressure induced by a single bunch.

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2nd Bunch:

$$P_2(x,t)$$

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1st Bunch:

$$P_1(x,t) = P_*(x,t)$$

2nd Bunch:

$$P_2(x,t) = P_*(x,t) + P_1(x,T_b)$$

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3rd Bunch:

$$P_3(x,t)$$

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3rd Bunch:

$$P_3(x,t) = P_*(x,t) + P_2(x,T_b)$$

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2nd Bunch:

$$P_2(x,t) = P_*(x,t) + P_1(x,T_b) = P_*(x,t) + P_*(x,T_b)$$

3rd Bunch:

$$P_3(x,t) = P_*(x,t) + P_2(x,T_b) = P_*(x,t) + 2P_*(x,T_b)$$

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m-th Bunch:

$$P_m(x,t)$$

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2nd Bunch:

$$P_2(x,t) = P_*(x,t) + P_1(x,T_b) = P_*(x,t) + P_*(x,T_b)$$

3rd Bunch:

$$P_3(x,t) = P_*(x,t) + P_2(x,T_b) = P_*(x,t) + 2P_*(x,T_b)$$

$$\vdots$$

m-th Bunch:

$$P_m(x,t) = P_*(x,t) + (m-1) \times P_*(x,T_b)$$

Analytical Solution of LAW Eqn in Thin Rod with Dirichlet BC for Multi- Bunch Effect

$$P_m(z,t) = \frac{2}{L}\sum_{n=1}^{\infty}\cos\left(\frac{\pi n}{L}c_s t\right)\sin\left(\frac{\pi n}{L}z\right)\times$$
$$\times\left\{\sum_{i=1}^{m}\cos^{i-1}\left(\frac{\pi n}{L}c_s T_b\right)\right\}\times$$
$$\times\int_0^L P(z,0)\sin\left(\frac{\pi n}{L}z\right)dz$$

Analytical Solution of LAW Eqn in Cylindrical Disc with Dirichlet BC for Multi-Bunch Effect

$$P_m(r,t) = \frac{2}{R^2}\sum_{n=1}^{\infty}\frac{1}{J_1^2(\lambda_n)}J_0\left(\lambda_n\frac{r}{R}\right)\cos\left(\lambda_n\frac{c_s t}{R}\right)$$
$$\times\left\{\sum_{i=1}^{m}\cos^{i-1}(\lambda_n c_s T_b)\right\}\times$$
$$\times\int_0^L \xi P(\xi,0)J_0\left(\lambda_n\frac{\xi}{R}\right)d\xi$$

Analytical Solution of LAW Eqn in Cylindrical Disc with Neumann BC for Multi-Bunch Effect

$$
\begin{aligned}
P_m(r,t) = \frac{2}{R^2} \Bigg[& \sum_{n=1}^{\infty} \frac{1}{J_0^2\left(\lambda_n\right)} J_0\left(\lambda_n \frac{r}{R}\right) \cos\left(\lambda_n \frac{c_s t}{R}\right) \\
& \times \left\{ \sum_{i=1}^{m} \cos^{i-1}\left(\lambda_n c_s T_b\right) \right\} \times \\
& \times \int_0^L \xi P(\xi,0) J_0\left(\lambda_n \frac{\xi}{R}\right) d\xi + m \int_0^R \xi P(\xi,0) d\xi \Bigg]
\end{aligned}
$$

APPLICATION TO ILC

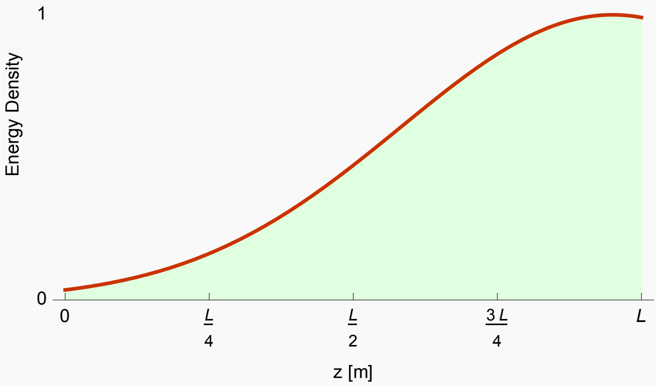
Parameters

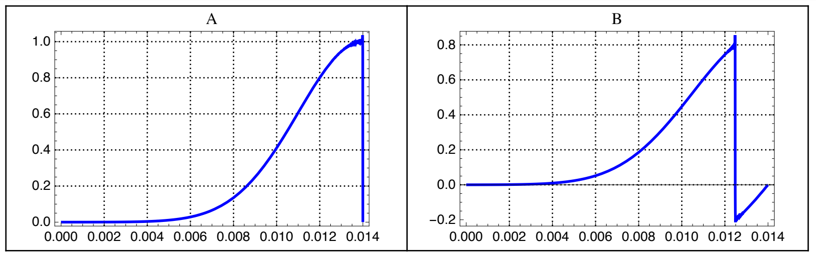
Table: *ILC Target Material and BeamParameters*

Parameters	Symbol	Units	Value
Target Thickness	L	m	0.014
Target Radius	R	m	0.01
Standard deviation	σ_z	m	0.003
Mean of the Distribution	B	m	0.014
beam spot size	r_0	m	0.003
Bunch per Train	N_b		1312
bunch spacing	T_b	ns	300
Peak Energy Density Dep.	$\frac{Q_0}{V_0}$	J/m^3	1.8×10^6
Grüneisen Coef.	Γ		1.262
Density	ρ	Kg/m^3	4430
Speed of sound	c_s	m/s	5072.833
Wheel Rotation Speed	v	m/s	100
Ultimate Tensile Strength	UTS	MPa	950

Thin Rod - Energy Distribution

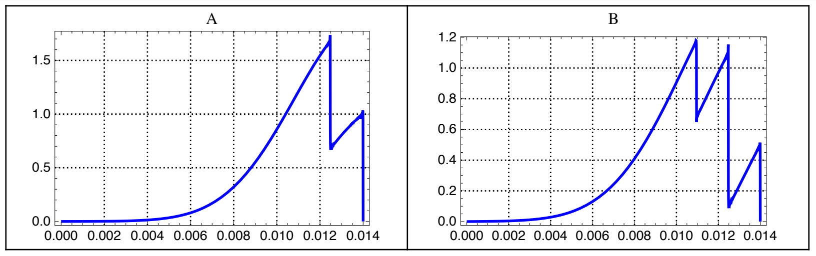
$$\text{Definition :} \qquad Q(z) = Q_0 \exp\left(-\frac{(z-B)^2}{2\sigma_z^2}\right)$$





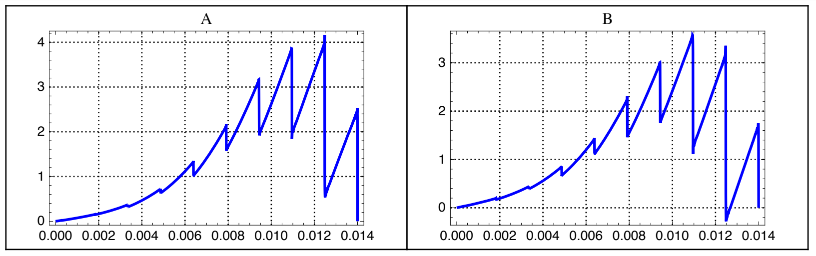
Plot of Pressure (normalised by 2.2 MPa) vs. z [m]: ILC Thin Rod Approximation with Dirichlet BC. (A) Spatial Distribution of the Pressure in Target for the 1st Bunch. (B) Spatial Distribution of the Pressure in Target, at time T_b, after the 1st bunch

Thin Rod with DBC



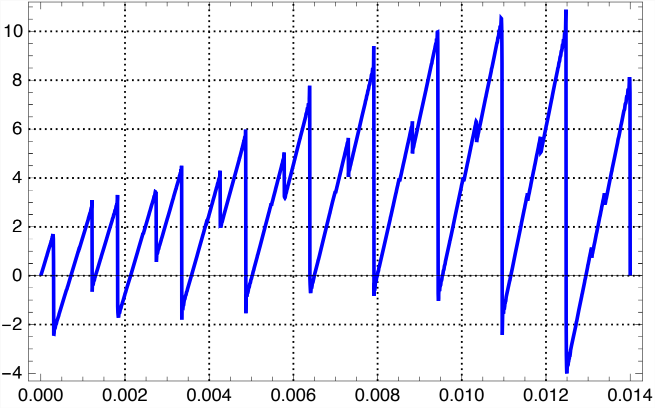
Plot of Pressure (normalised by 2.2 MPa) vs. z [m]: ILC Thin Rod Approximation with Dirichlet BC. (A) Spatial Distribution of the Pressure in Target for the 2nd Bunch. (B) Spatial Distribution of the Pressure in Target, at time T_b, after the 2nd bunch

Thin Rod with DBC



Plot of Pressure (normalised by 2.2 MPa) vs. z [m]: ILC Thin Rod Approximation with Dirichlet BC. (A) Spatial Distribution of the Pressure in Target for the 10th Bunch. (B) Spatial Distribution of the Pressure in Target, at time T_b, after the 10th bunch

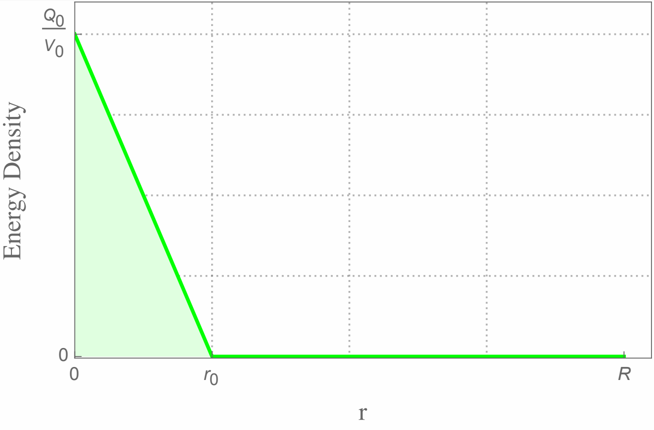
Thin Rod with DBC



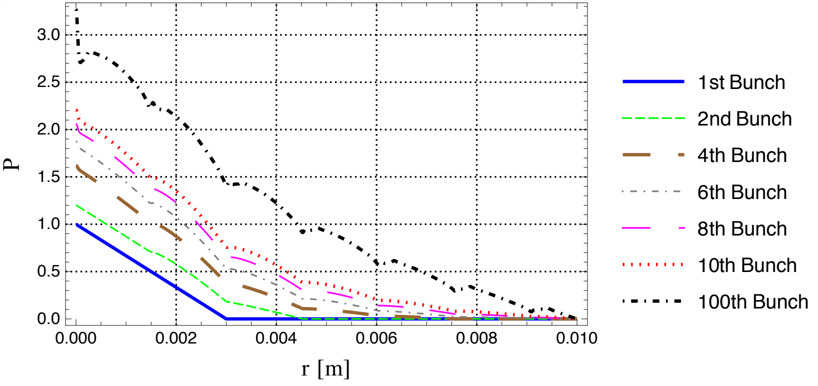
Plot of Pressure (normalised by 2.2 MPa) vs. z [m]: Spatial Distribution of the Pressure acoustic wave in Target for the 100th Bunch.

Cylindrical Disc - Energy Distribution

$$\text{Definition :} \qquad Q(r) = \begin{cases} Q_0\left(1 - \frac{r}{r_0}\right) & : 0 < r < r_0 \\ 0 & : r > r_0 \end{cases}$$

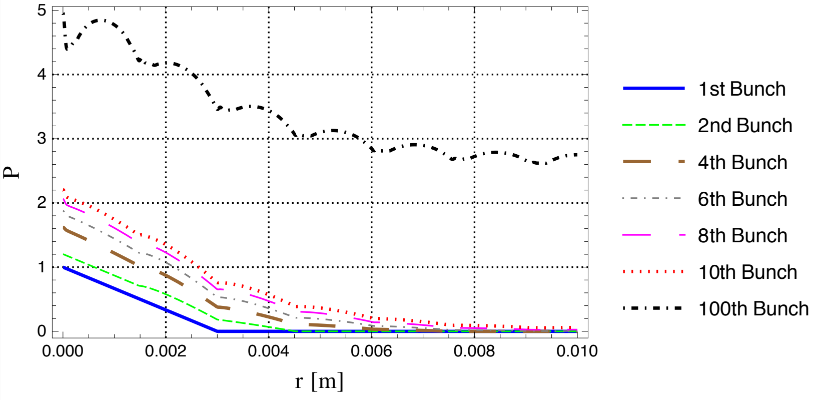


Cylindrical Disc with DBC



Plot of Pressure (normalised by 2.2 MPa) vs. r [m]: ILC Disc Approximation with Dirichlet BC.

Cylindrical Disc with NBC



Plot of Pressure (normalised by 2.2 MPa) vs. r [m]: ILC Disc Approximation with Neumann BC.

CONCLUSION

On a general note

1 The deposited energy density profile of the incident particle beam determines how the compressive waves will look like at the initial state. It also determines how the wave evolves in time.

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$$\left| \Gamma \frac{Q_0}{V_0} \right| \tag{3}$$

irrespective of time and boundary condition imposed.

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3 In the case of multiple bunches, the boundary condition, the geometry and the bunch spacing play a big role on the amplitude of the induced peak pressure. As a rule of thumb, one wants to keep the absolute value of:

$$\sum_{i=1}^{m} \cos^{i-1}(\lambda_n c_s T_b) \tag{4}$$

between zero and unity.

For ILC Conversion Target

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② For 100 consecutive bunches hitting the same spot on the target (by considering a worst case scenario when cooling and damping is not implemented) the peak stress induced is about 70 MPa.

Hence, no immediate damage of the target.

Thank You

For

listening

QUESTIONS?