Higher order QCD correction to quark mass relations

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DESY

in collaboration with

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Motivation – Quark mass definitions

- pole mass
- $\overline{\mathrm{MS}}$ mass
- PS mass [Beneke '98]
- 1S mass [Hoang,Smith,Stelzer,Willenbrock '99]
- kinetic mass [Bigi,Shifman,Uraltsev,Vainstein '97]
- ...

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1. Choose quark mass definition in theory calculation
2. Extract mass in chosen scheme by comparing with experiment
3. Convert to common mass scheme to compare the results

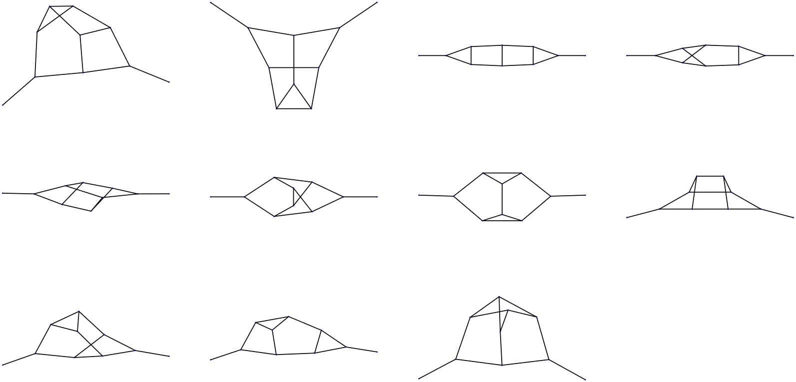
Setup of the calculation

- Need to calculate mass renormalization constant Z_m^{OS} by calculating four-loop on-shell integrals
- Together with the renormalization constant in the $\overline{\mathrm{MS}}$-scheme $Z_m^{\overline{\mathrm{MS}}}$ [Chetyrkin '97; Larin, van Rittbergen, Vermaseren '97]
  we get

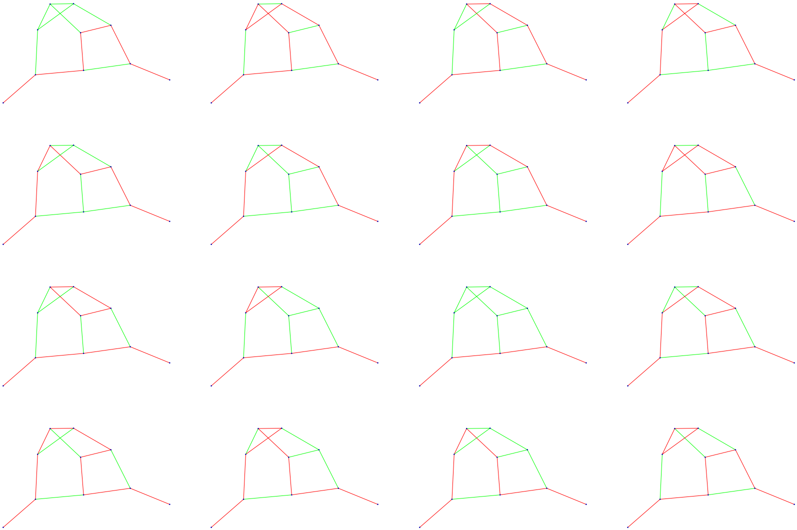
$$\left.\begin{array}{l} m_{\mathrm{bare}} = Z_m^{\mathrm{OS}} M \\ m_{\mathrm{bare}} = Z_m^{\overline{\mathrm{MS}}} m \end{array}\right\} \Rightarrow m = M \frac{Z_m^{\mathrm{OS}}}{Z_m^{\overline{\mathrm{MS}}}}$$

  - 1-loop [Tarrach'81]
  - 2-loop [Gray,Broadhurst,Grafe,Schilcher'90]
  - 3-loop [Chetyrkin,Steinhauser'99; Melnikov, v. Ritbergen'00; Marquard,Mihaila,Piclum,Steinhauser'07]

Diagrams – Prototypes



Diagrams – Dressed



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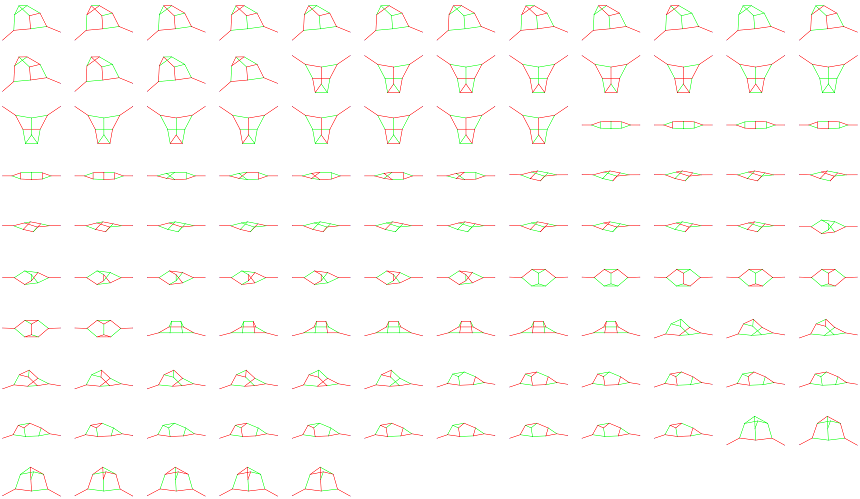

$\overline{\text{MS}}$–on-shell relation at four-loop order

$\overline{\text{MS}} \rightarrow$ on-shell

$$\begin{aligned} M_t &= m_t \Big(1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.375\,\alpha_s^3 \\ &\quad + (8.49 \pm 0.25)\,\alpha_s^4\Big) \\ &= 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm 0.005 \text{ GeV} \end{aligned}$$

small remaining error of about 3% due to numerical integration of the master integrals using FIESTA [A. Smirnov]

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small remaining error of about 3% due to numerical integration of the master integrals using FIESTA [A. Smirnov]

$$M_b = m_b \Big(1 + 0.4244\,\alpha_s + 0.9401\,\alpha_s^2 + 3.045\,\alpha_s^3 + (12.57 \pm 0.38)\,\alpha_s^4\Big)$$
$$= 4.163 + 0.401 + 0.201 + 0.148 + 0.138 \pm 0.004 \text{ GeV}\,.$$

Threshold mass schemes

- Potential-subtracted mass

$$m^{\mathrm{PS}} = M - \delta m(\mu_f)$$

$$\delta m(\mu_f) = -\frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{\mathrm{d}^3 q}{(2\pi)^3} V(\vec{q}) = \mu_f \frac{C_F \alpha_s}{\pi}(1 + \alpha_s \ldots)$$

need static potential @ three loops

[Smirnov,Smirnov,Steinhauser '09; Anzai,Kiyo,Sumino '09]

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- 1S mass

$$m^{\mathrm{1S}} = M + \frac{1}{2} E_1^{\mathrm{pt}}$$

$$E_1^{\mathrm{pt}} = -\frac{C_F^2 M \alpha_s^2}{8}(1 + \alpha_s \ldots)$$

need binding energy @ three loops

[Beneke et al]

Convergence and error estimate

input #loops	$m^{\rm PS} =$ 171.792	$m^{\rm 1S} =$ 172.227	$m^{\rm RS} =$ 171.215	
1	165.097	165.045	164.847	
2	163.943	163.861	163.853	1-2 GeV

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4 ($\times$1.03)	163.637	163.637	163.637	6 MeV

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half the 4-loop contribution $\{22, 4, 10\}$
3% uncertainty $\equiv$ 6 MeV
$\Big\} \Rightarrow \{23, 7, 11\}$MeV error

Final result for PS and 1S mass scheme

$$\frac{\bar{m}_t(\bar{m}_t)}{\text{GeV}} = 163.643 \pm 0.023 + 0.074\Delta_{\alpha_s} - 0.095\Delta_{m_t}^{\text{PS}}$$

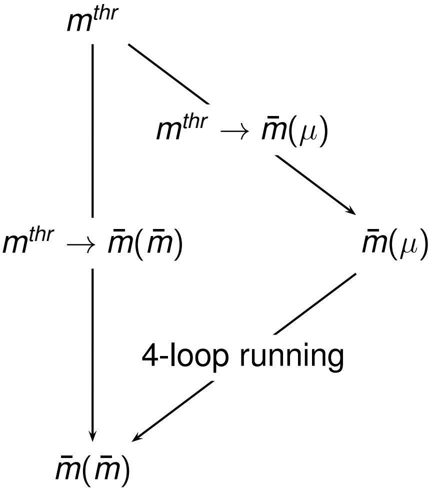
$$\frac{m_t(m_t)}{\text{GeV}} = 163.643 \pm 0.007 + 0.069\Delta_{\alpha_s} - 0.096\Delta_{m_t}^{\text{1S}}$$

$$\Delta_{m_t}^{\text{PS}} = \frac{171.792\,\text{GeV} - m_t^{\text{PS}}}{0.1}$$

$$\Delta_{m_t}^{\text{1S}} = \frac{172.227\,\text{GeV} - m_t^{\text{1S}}}{0.1}$$

$$\Delta_{\alpha_s} = \frac{(0.1185 - \alpha_s(M_Z))}{0.001}$$

$m^{\mathrm{thr}} \to \bar{m}(\bar{m})$



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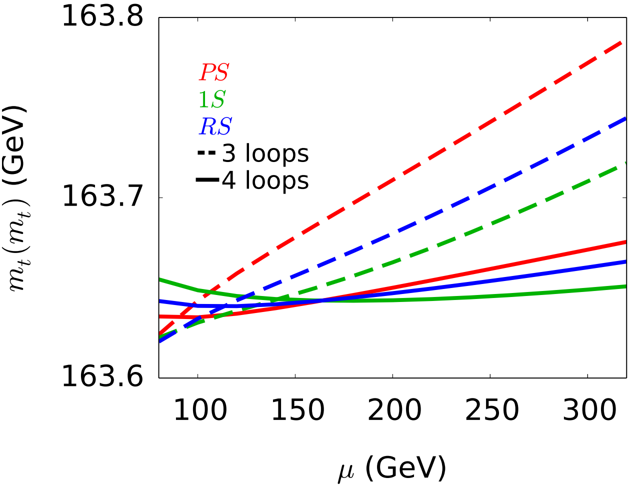

$\Rightarrow$ compatible error estimate

Conclusions

- Calculated the $\overline{\text{MS}}$–on-shell relation to four-loop order
- Threshold masses can be related to the $\overline{\text{MS}}$ mass with about a 20 MeV error
- Scheme conversion no limitation for the measurement of the top-quark mass at a linear collider