Top Mass at Threshold - Experiment:
Impact of Theory Systematics

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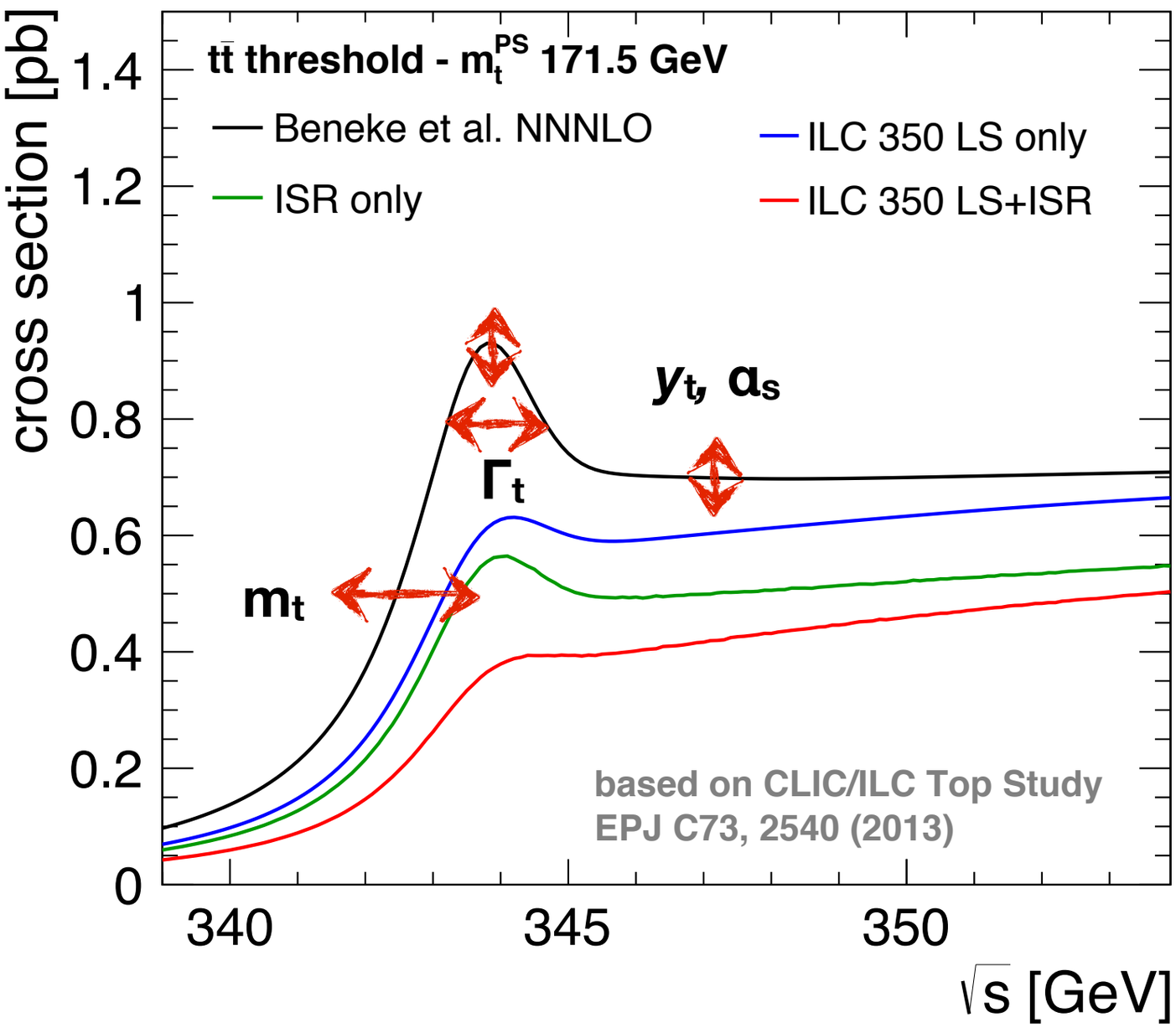
(Werner-Heisenberg-Institut)

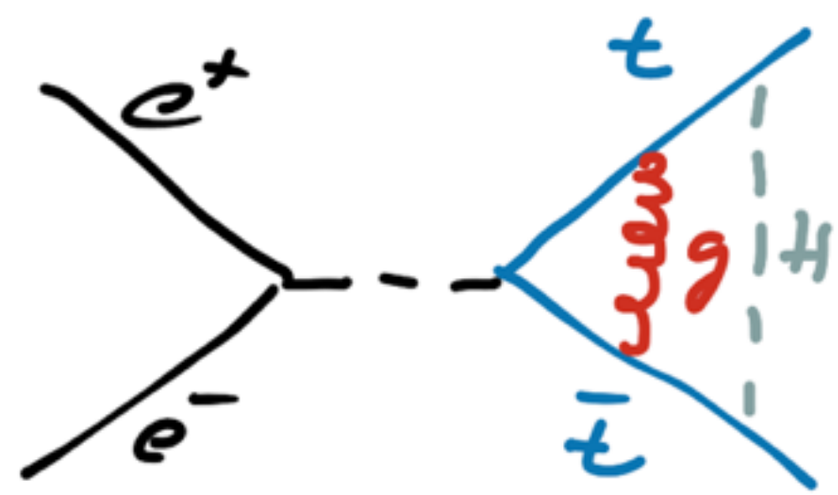
Outline

- Motivation & Systematics Status
- Sensitivity of the Top Threshold to various Parameters
- Impact of NNNLO QCD Scale Uncertainties
- Measuring the Top Mass with Scale Uncertainties incorporated in the Fit
- Optimising the choice of energies
- Summary

Threshold Scans: The Motivation



- The cross-section around the threshold is affected by several properties of the top quark and by QCD
  - Top mass, width, Yukawa coupling
  - Strong coupling constant



- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external α_s helps

Top Mass Uncertainties - Status

- A number of studies in Tesla, ILC, CLIC contexts: Expected statistical uncertainty **20 - 30 MeV** (for 100 fb^{-1})

- *Experimental* Systematics
  - Beam Energy: **~ 30 MeV** or lower
  - Non-ttbar background, selection efficiencies (assuming < 5% bgd uncertainty, ~ 0.5% knowledge on signal selection): **~ 15 MeV**
  - Luminosity Spectrum (studied for CLIC LS with reconstruction of spectrum via Bhabha scattering, scaling from 3 TeV studies, full study on the way): **~ 10 MeV**
  - Integrated luminosity (assuming full correlation point to point, 0.5% uncertainty): ~ **10 MeV**
  - Single top contamination: **< 30 MeV**

Top Mass Uncertainties - Status

- A key factor: *Theory systematics*
  - So far: Used naive estimates assuming 3% normalisation uncertainty on cross section - Impact on mass: **~ 55 MeV**

- Uncertainties from the *strong coupling*
  - When not included in the fit: ~ 3 MeV per 10^{-4} uncertainty on α_s: today **~ 18 MeV**
  - In addition: impact on the conversion from 1S / PS masses used at threshold to MSbar mass: today: **~ 50 MeV**
  - ➪ Discussed later in this session

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Today: Re-examine this number and put it on more solid footing

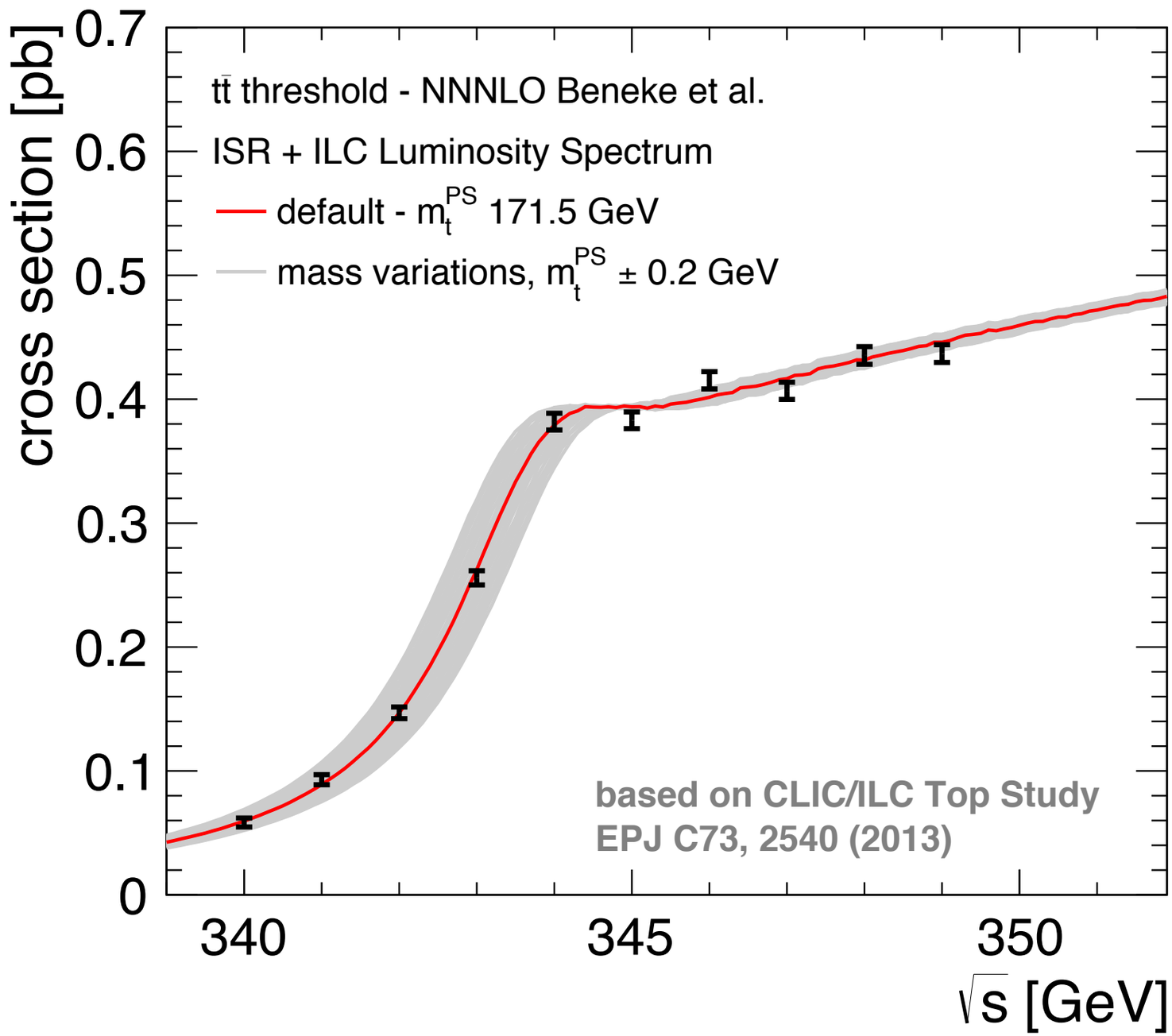
The Study

- Experimental details:
  - Based on CLIC / ILC top threshold study (EPJ C73, 2540 (2013)):
    - CLIC_ILD Detector model
    - Threshold simulated using efficiency & backgrounds from full simulations, signal scaled according to theory input
    - Assuming ILC TDR luminosity spectrum

- Theory input:
  - NNNLO QCD Theory calculations, using Mathematica program based on:
    - M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum, M. Steinhauser, Phys. Rev. Lett. 115, 192001 (2015)
  - Including NNNLO Higgs effects, NLO non-resonant EW contributions, NLO QED
    - M. Beneke, A. Maier, J. Piclum, T. Rauh, Nucl. Phys. B899, 180 (2015)

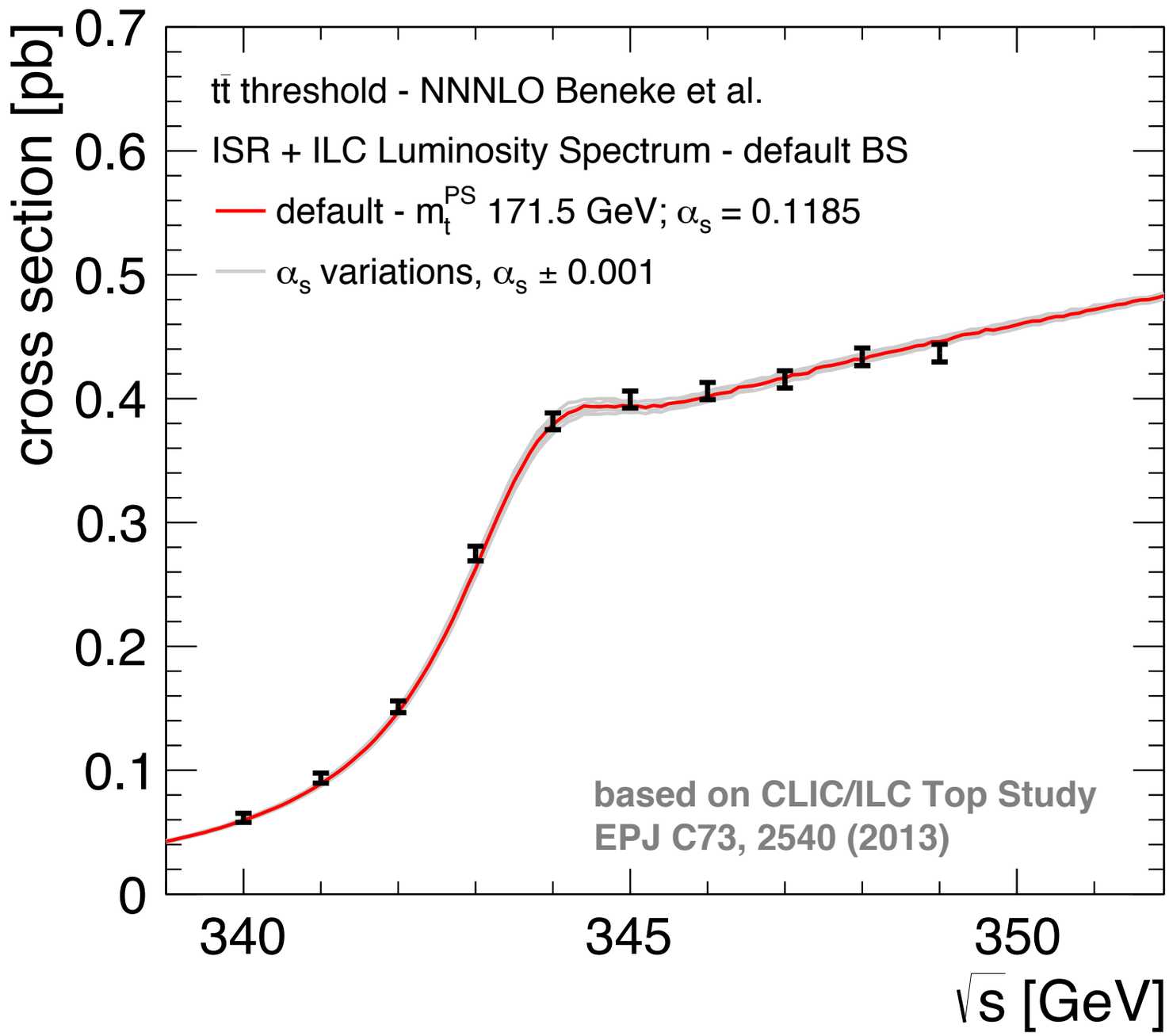
Thanks to Martin Beneke and Jan Piclum for sharing code and expertise!

Threshold Scan - Sensitivity to Mass Variations



- The assumption:
  10 x 10 fb^{-1}, points spaced by 1 GeV from 340 to 349 GeV

Threshold Scan - Sensitivity to α_s Variations



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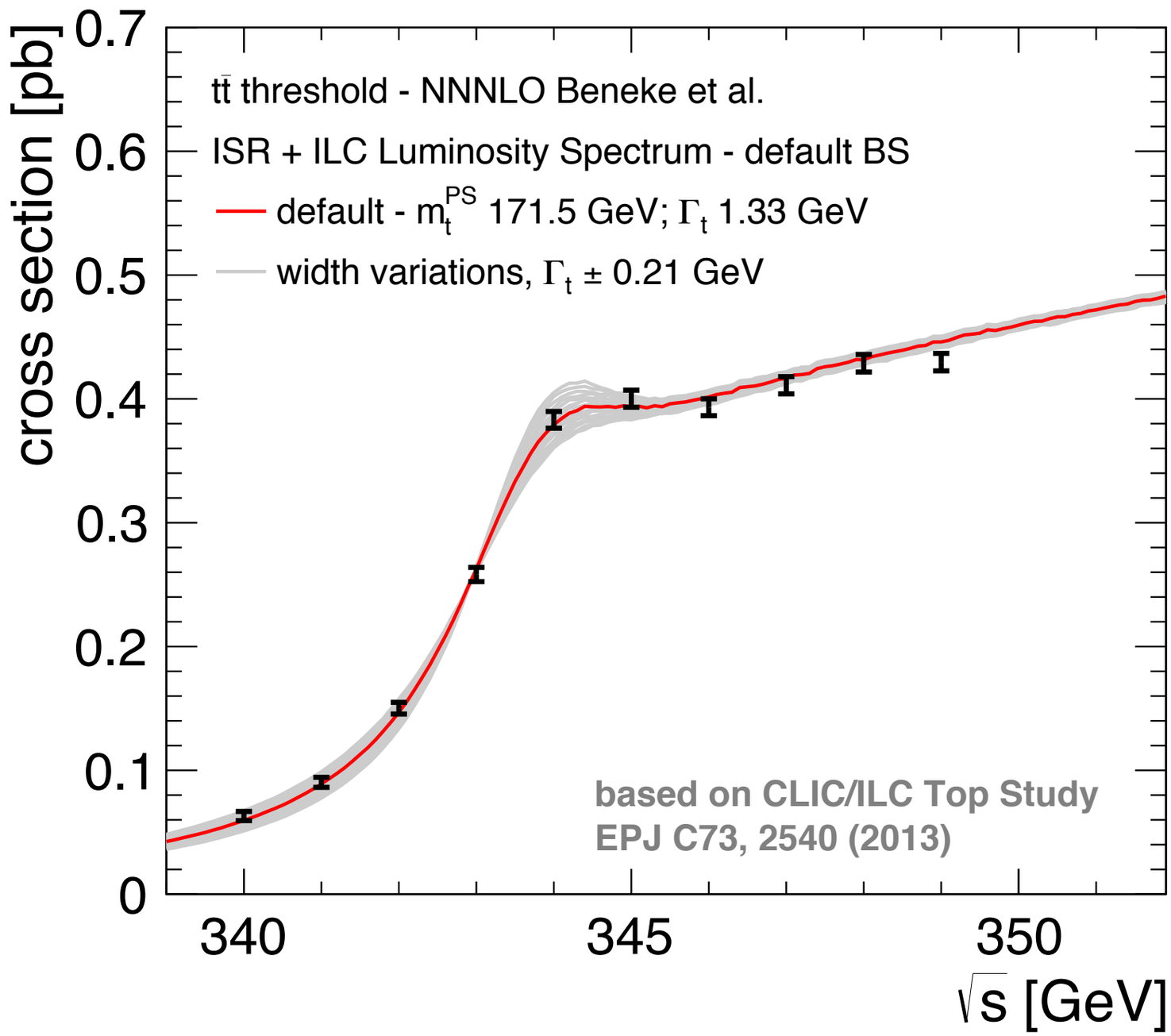
Threshold Scan - Sensitivity to Width Variations



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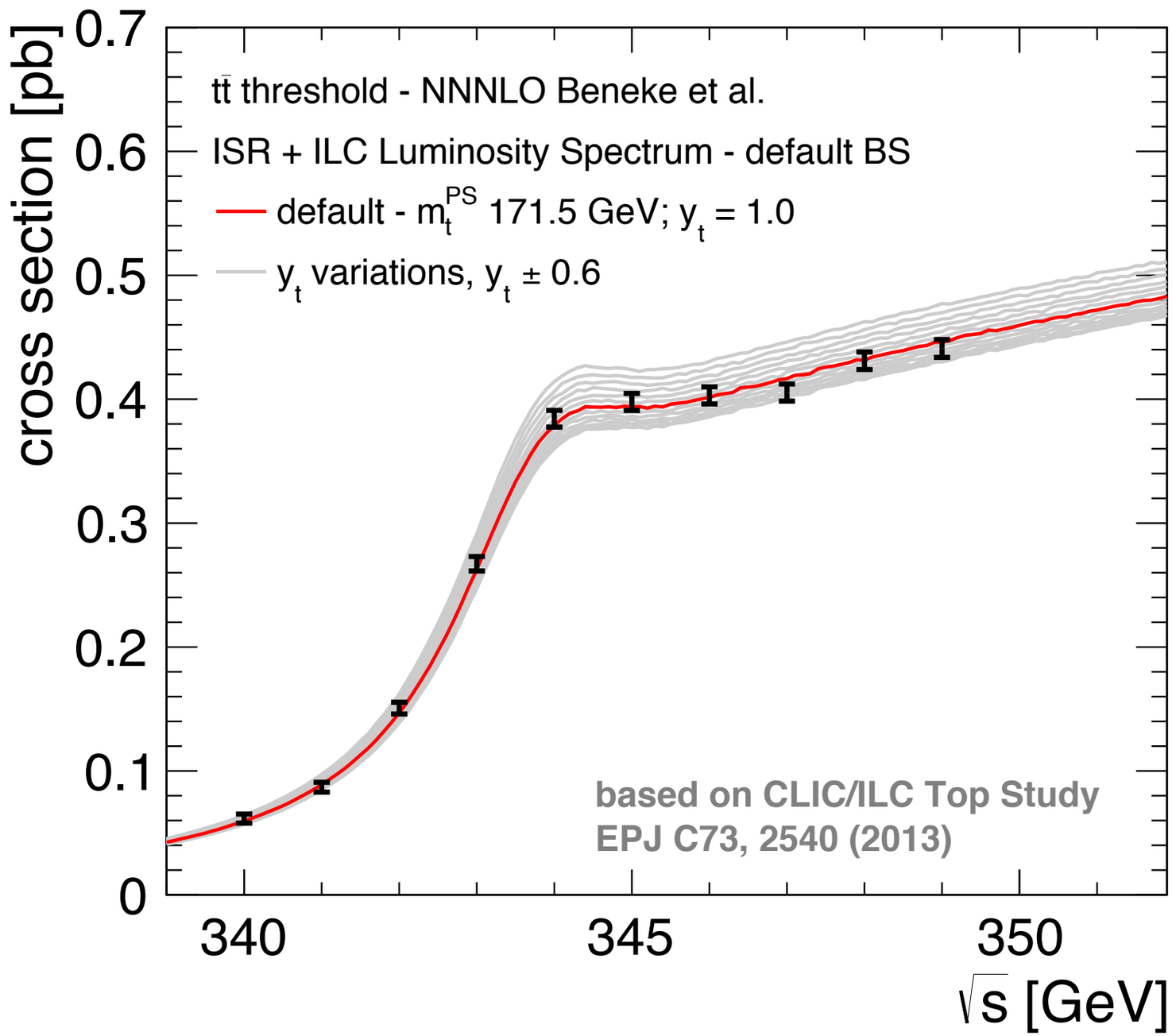
Threshold Scan - Sensitivity to Yukawa Variations



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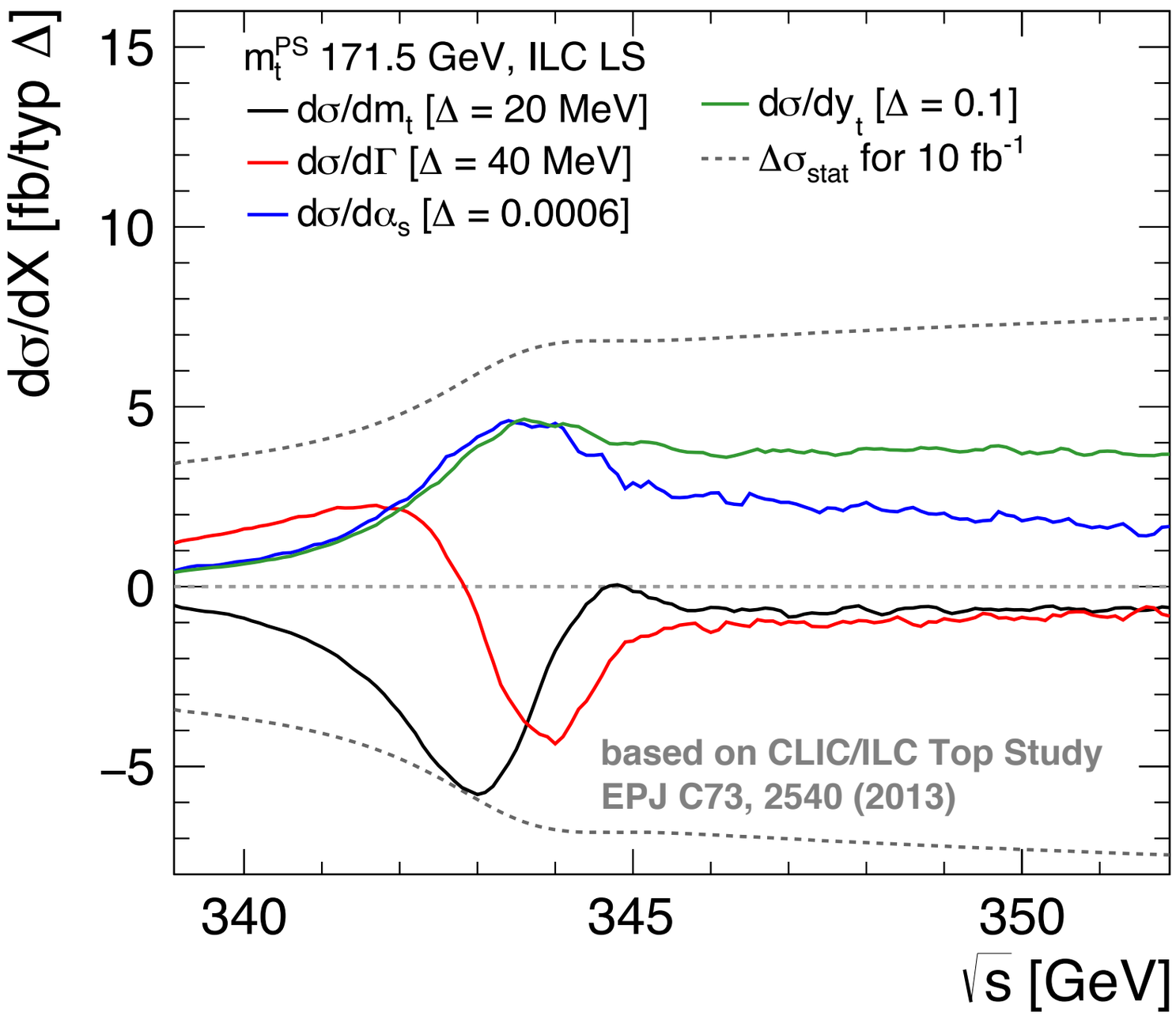
Threshold Scan - Sensitivity to Parameters



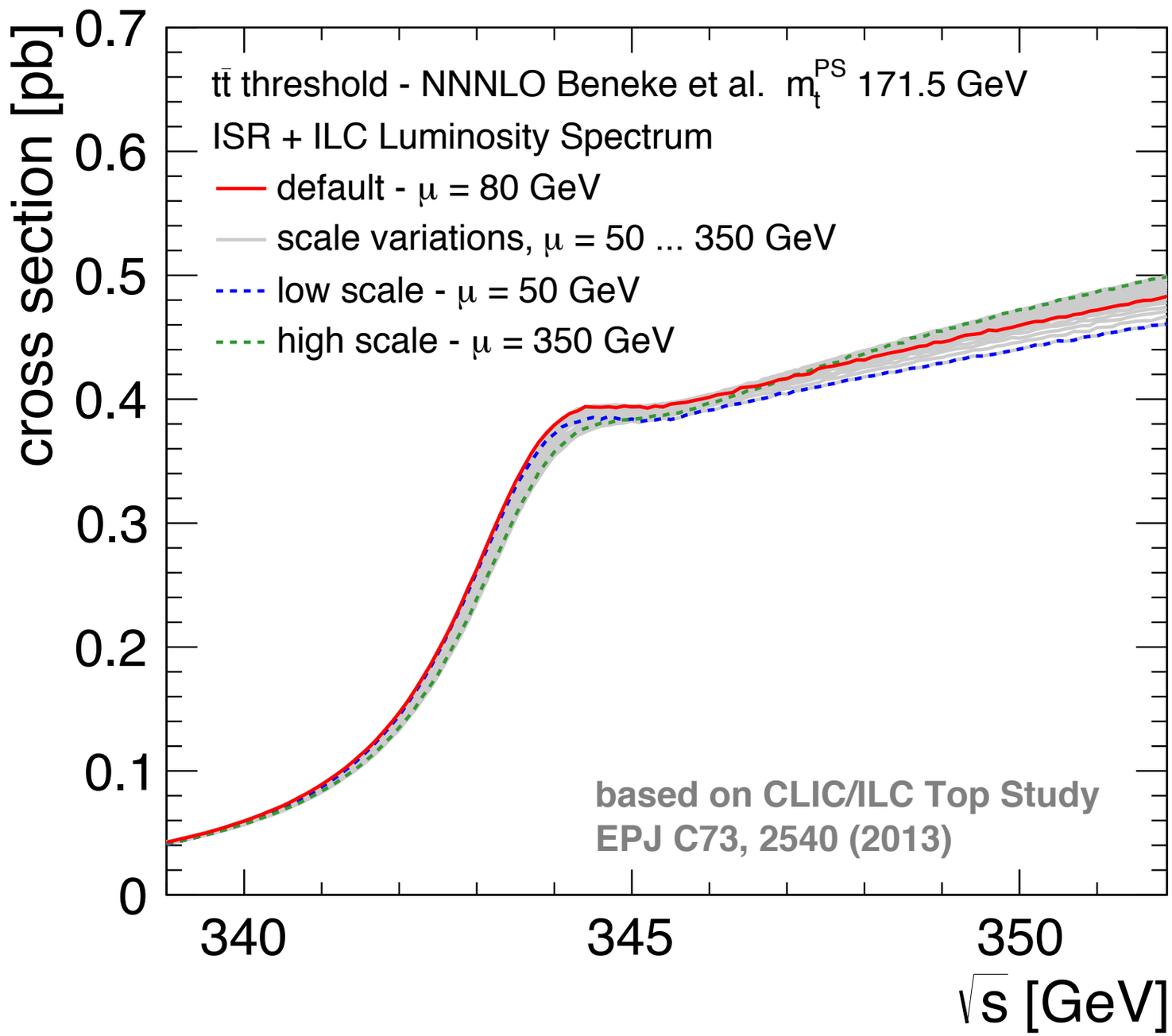
- Illustration of sensitivity: Variation of cross section for typical uncertainties assumed on parameters
  - typical LC stat uncertainty for m_t, Γ_t
  - WA for α_s
  - 10% for y_t

➪ Strong correlation between y_t and α_s

➪ Mass sensitivity maximum in steepest region of cross-section

➪ Width the only one changing sign

Impact of Scale Uncertainties on Threshold Scan



- Include scale variations in cross section calculation
  - Default scale: 80 GeV
  - Scales below 50 GeV lead to instable behavior - are not considered

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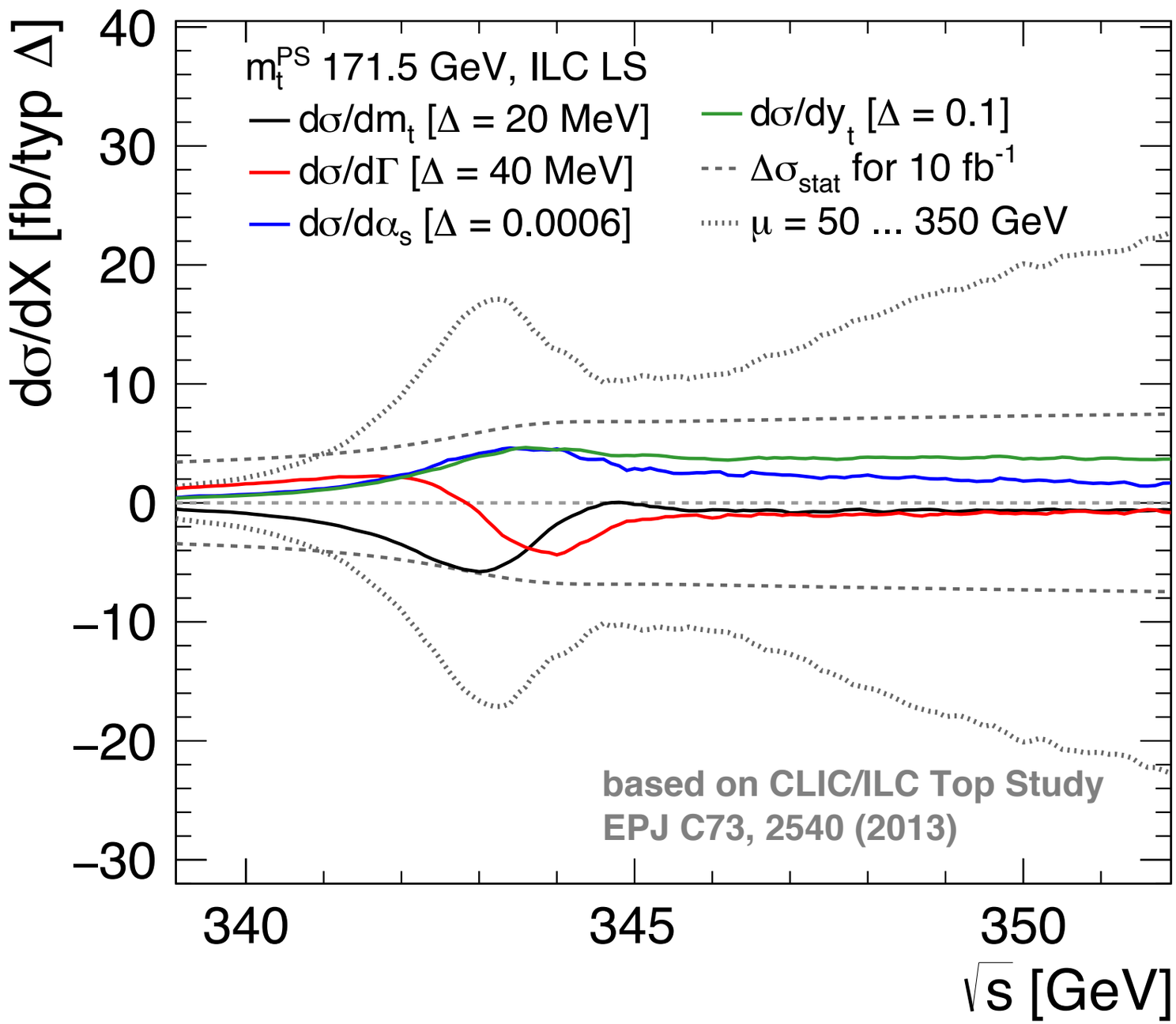
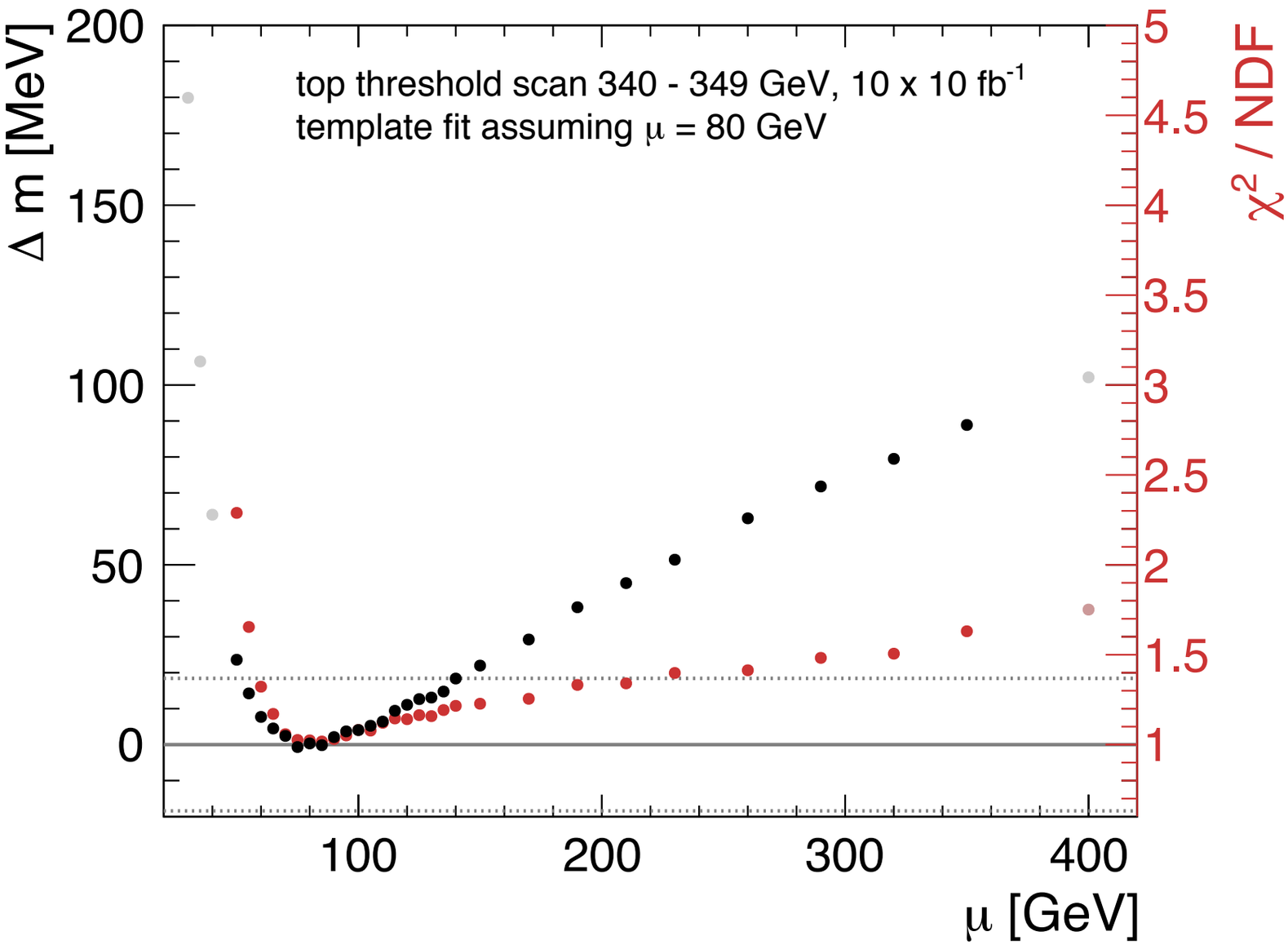


- Include scale variations in cross section calculation
  - Default scale: 80 GeV
  - Scales below 50 GeV lead to instable behavior - are not considered
- Substantial variations of cross section - beyond variations induced by parameters based on projected stat. uncertainties alone

Quantifying the Impact of Scale Uncertainties

- Studied the impact of scale uncertainties on the mass measurement in a threshold scan in two different ways:
  - Using the “standard” template fit: Different masses, all templates based on μ = 80 GeV
    - Look at impact of different scales taken as “true” cross section
    - Look at random point-to-point cross-section distribution within area defined by the full range of scales
    - Look at smooth interpolation within area defined by the full range of scales
    - ▸ Statistical uncertainty determined from width of the distribution of fitted mass for a large sample of trials
  - Using a template fit that incorporates scale uncertainties in the templates
    - ▸ Statistical uncertainty from many trials unreliable - “event-by-event” uncertainty determination

Scan of Scales with Default Fit



- Smallest mass reconstructed around default scale of μ = 80 GeV
  - Expected: Cross section maximal on rising edge
  - ‣ One-sided variation, up to **90 MeV**
- Chi2 gets worse for large deviations in scale - cross section curve deviates from expected behavior, in particular above threshold

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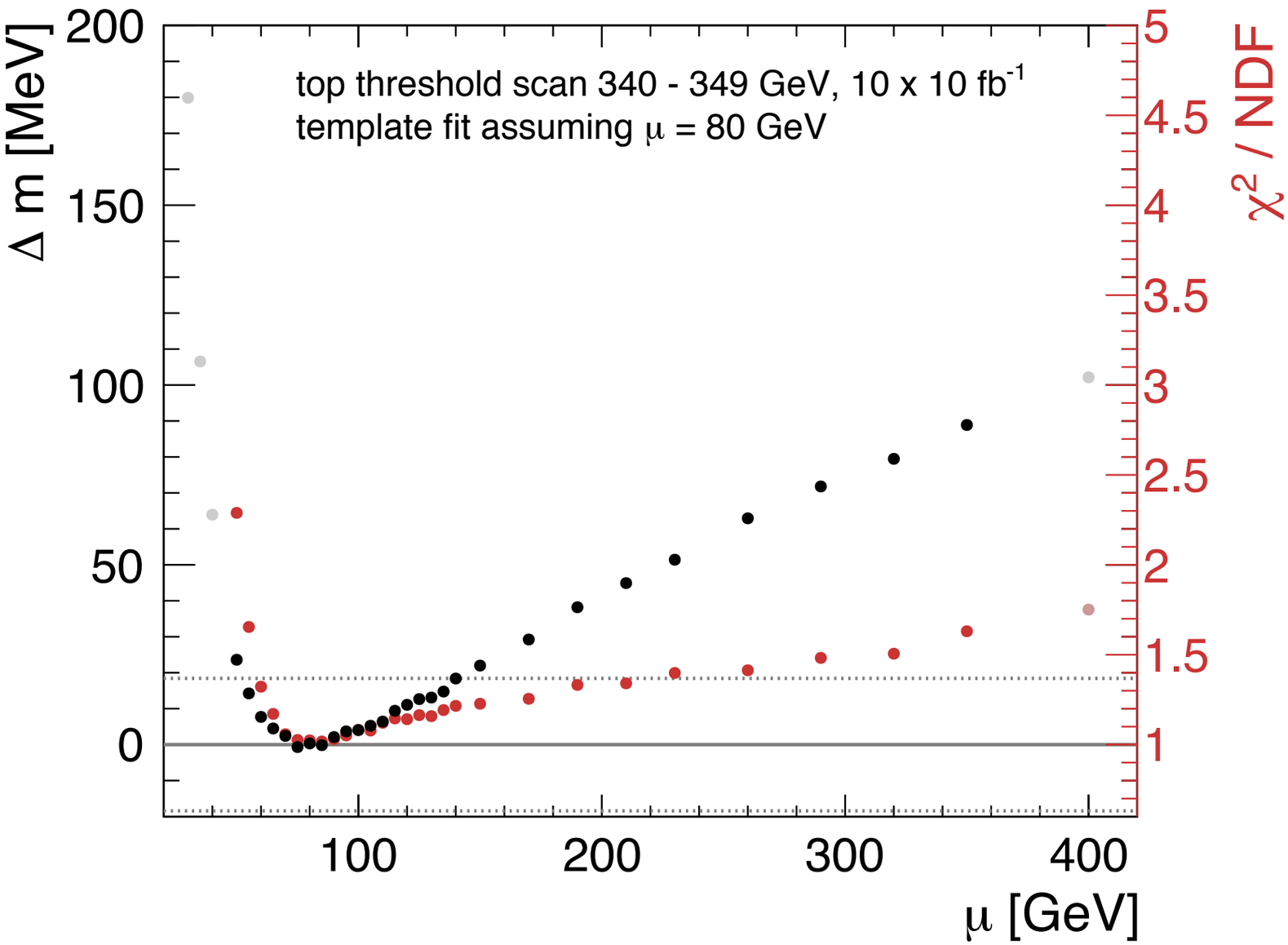

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With symmetrized variations: **45 MeV** uncertainty from NNNLO scale variations

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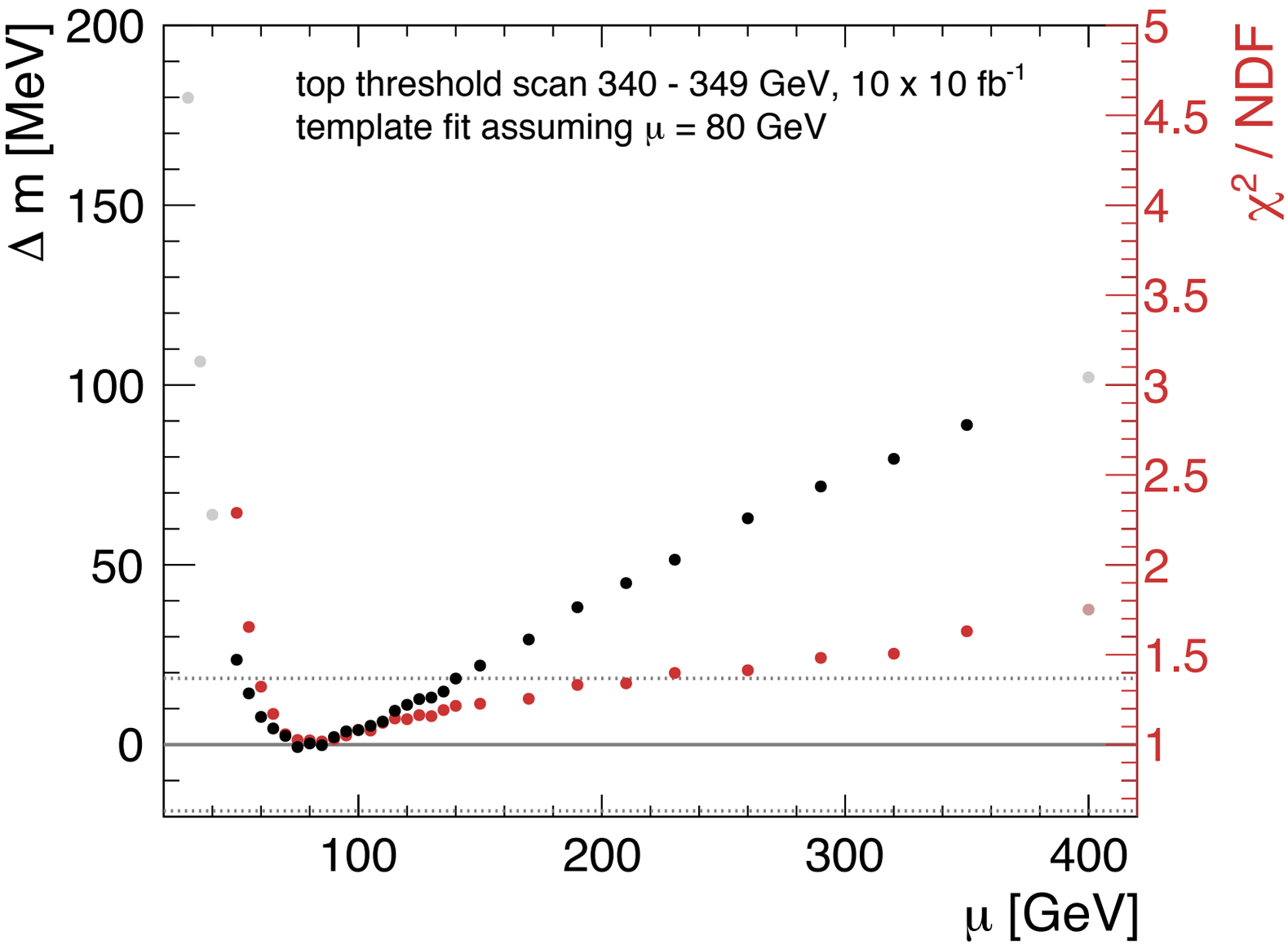

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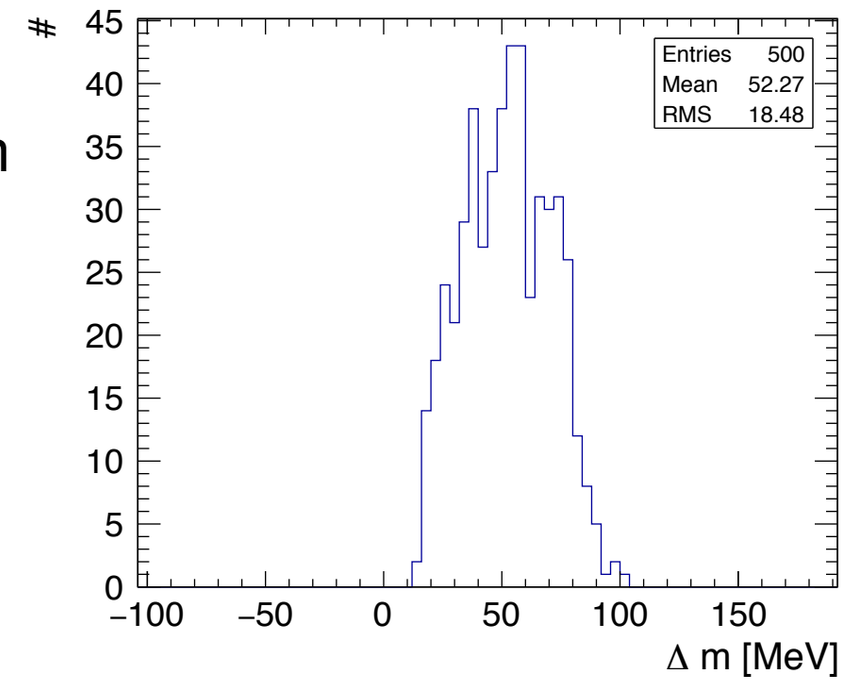
With symmetrized variations: **45 MeV** uncertainty from NNNLO scale variations

➪ But: Templates do not cover the range of variations above the threshold

Variations within Scale Uncertainties with Default Fit

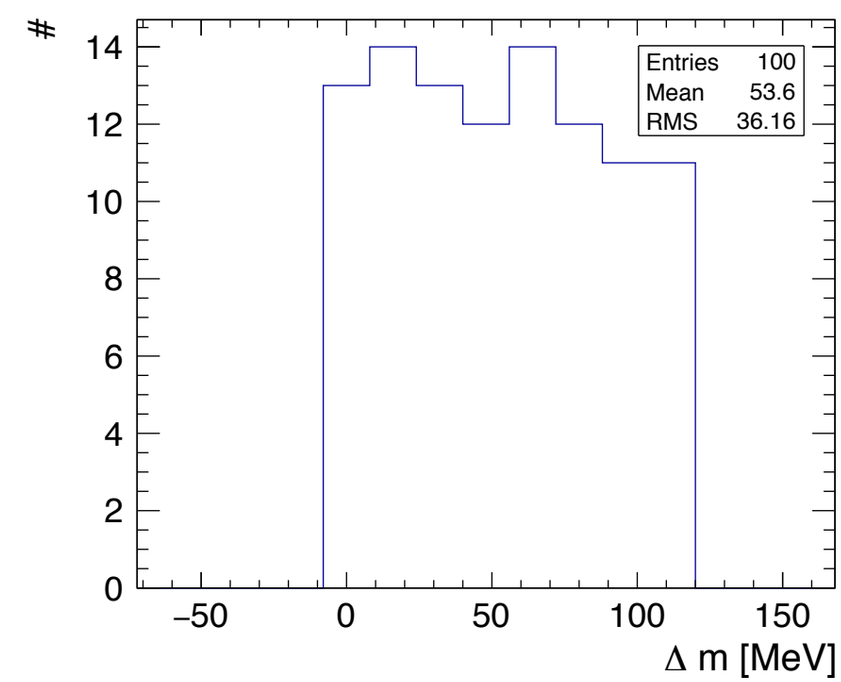
- Two scenarios (both unrealistic):
  - Assume cross section can vary between extremes given by the scale variations - with no correlation from point to point

    Variations in mass of ~ 80 MeV (RMS 20 MeV), mean bias of 50 MeV (expected!)



  - Assume cross section is always at the same point between the two extremes - scan from 0 (low envelope) to 1 (high envelope)

    Variations in mass of ~ 130 MeV (RMS 36 MeV), mean bias of 50 MeV (expected!)

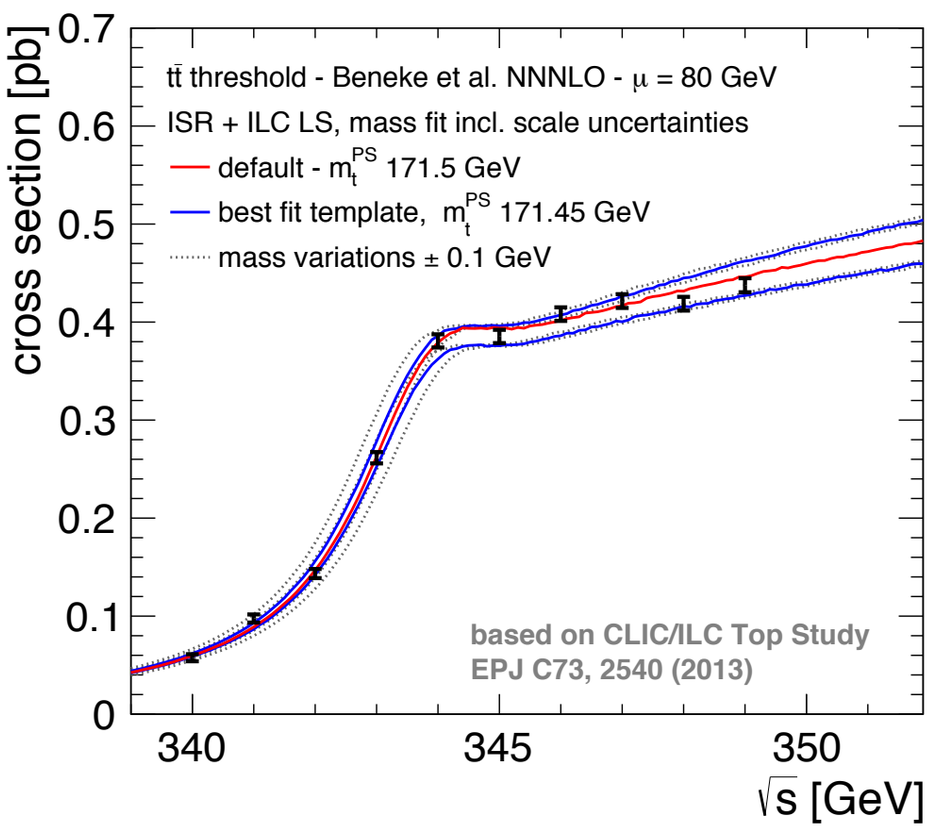


Template Fit with Scale Uncertainties

- More realistic scenario: Incorporate scale uncertainties in the template fit
  - Templates turn from lines to bands
  - ➪ Requires modification of χ^2 calculation in the template fit

    For each data point, there are two options:
    - point within the band of a given mass template: $\Delta\chi^2 = 0$
    - point outside of the band:

      $\Delta\chi^2$ = (distance to closest band edge)^2/(data uncertainty)^2



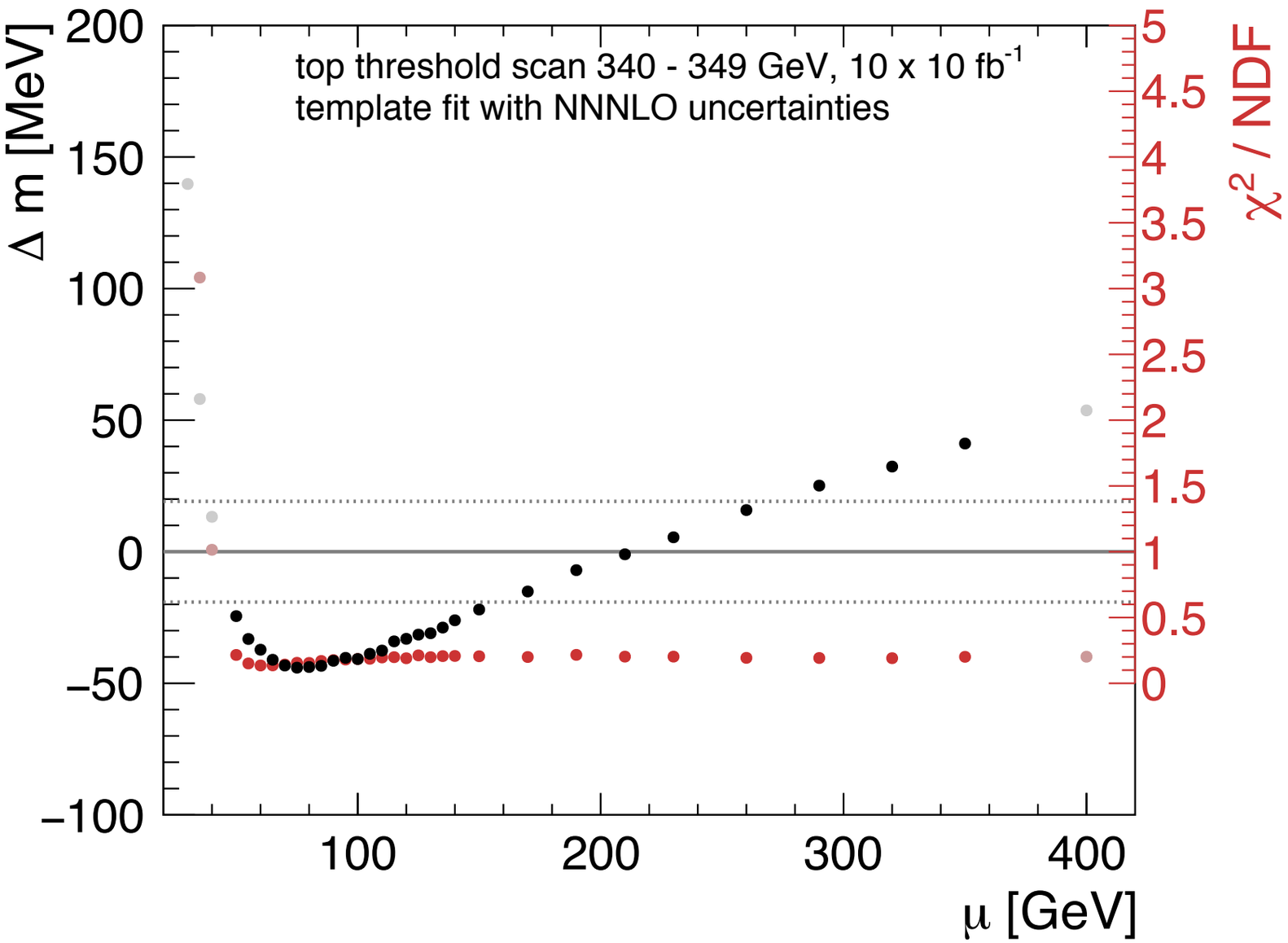
Mass determination as usual:

- Calculate χ^2 for all templates
- Fit a parabola to determine the minimum of the χ^2 distribution - This is the measured mass

Results for the "standard" input (μ = 80 GeV):

- Average fitted mass: 171.456 GeV (**44 MeV** low)
- Stat. uncertainty from width of distribution of many trials 19.4 MeV (only 1 MeV more than when fitting w/o scale uncertainty) - *unrealistic!*

Scan of Scales with Fit with Scale Uncertainties



- The variations are now symmetric around the input value (expected)
- Variations **± 45 MeV** - total variations identical to the default fit
- χ^2 values much lower: Many points do not contribute for best fit - large scales do not drive up χ^2 since all templates cover the large variations above threshold

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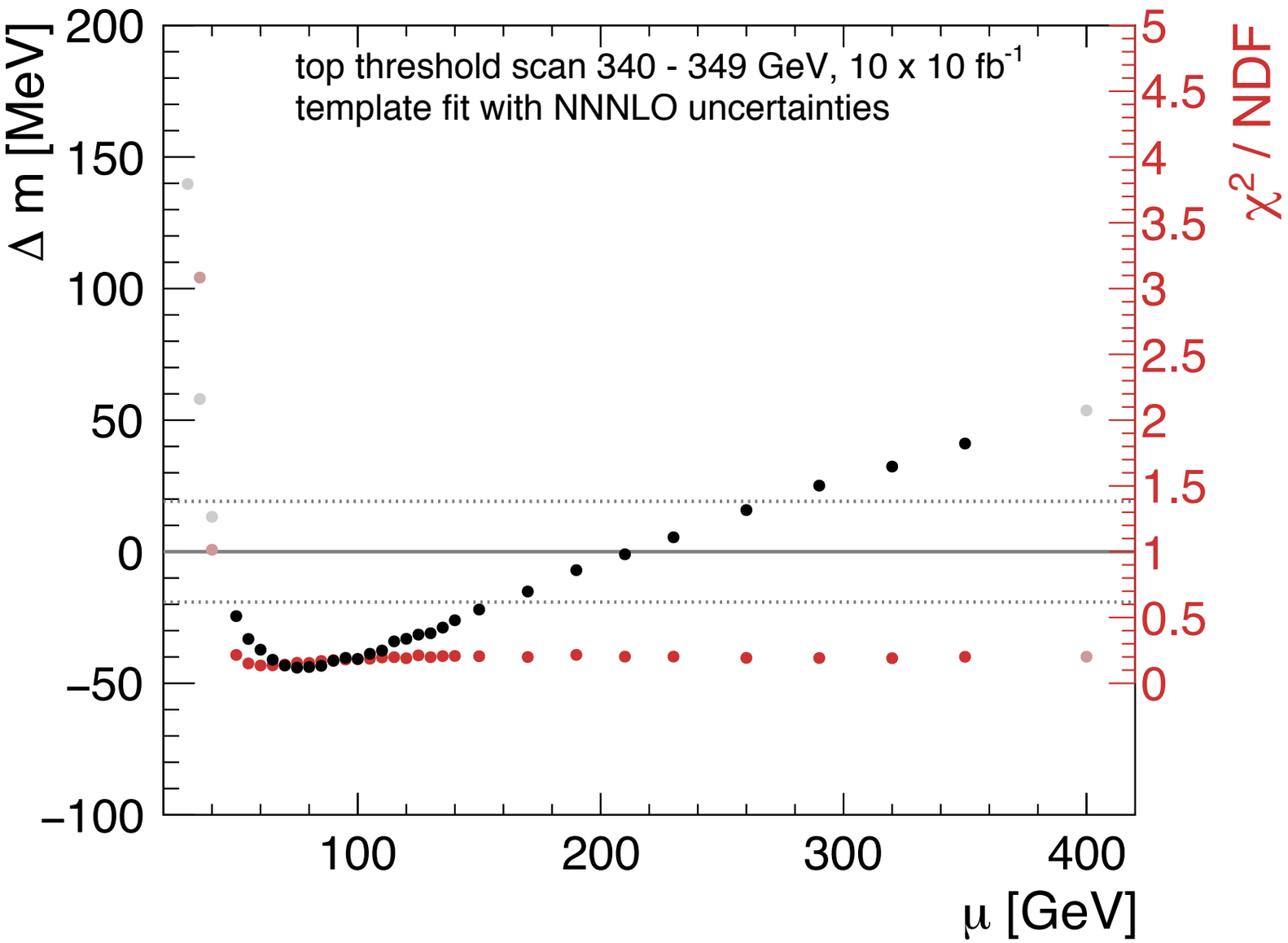

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As for default fit: **45 MeV** uncertainty from NNNLO scale variations

Template Fit with Scale Uncertainties - Results

- A realistic evaluation of the performance of the fit with scale uncertainties requires a different treatment of the fit uncertainty
  - Due to the bands as templates, the fit uncertainty for a given simulated scan depends on the distribution of the points
  - ➪ Determine fit uncertainty "event-by-event" by looking at mass for $\chi^2_{min} + 1$
    - In cases where the fitted $\chi^2_{min} < 0$, $\chi^2 = 1$ is taken to determine the $\pm 1\ \sigma$ mass values

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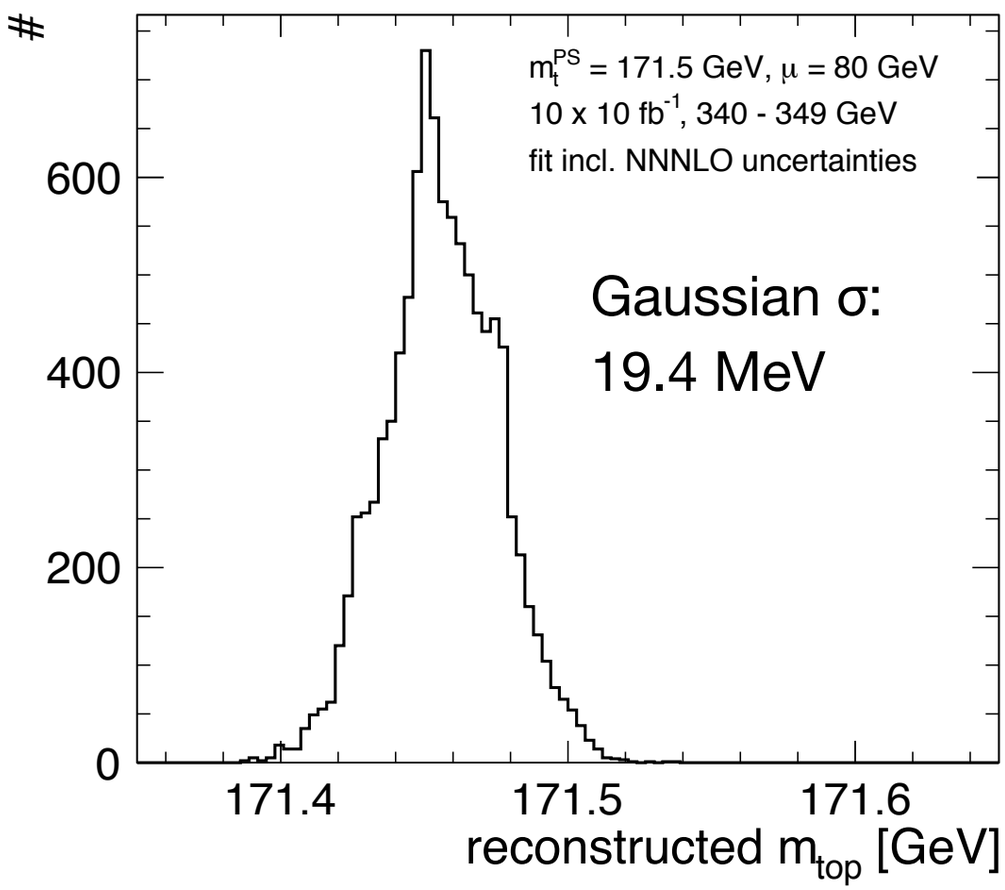

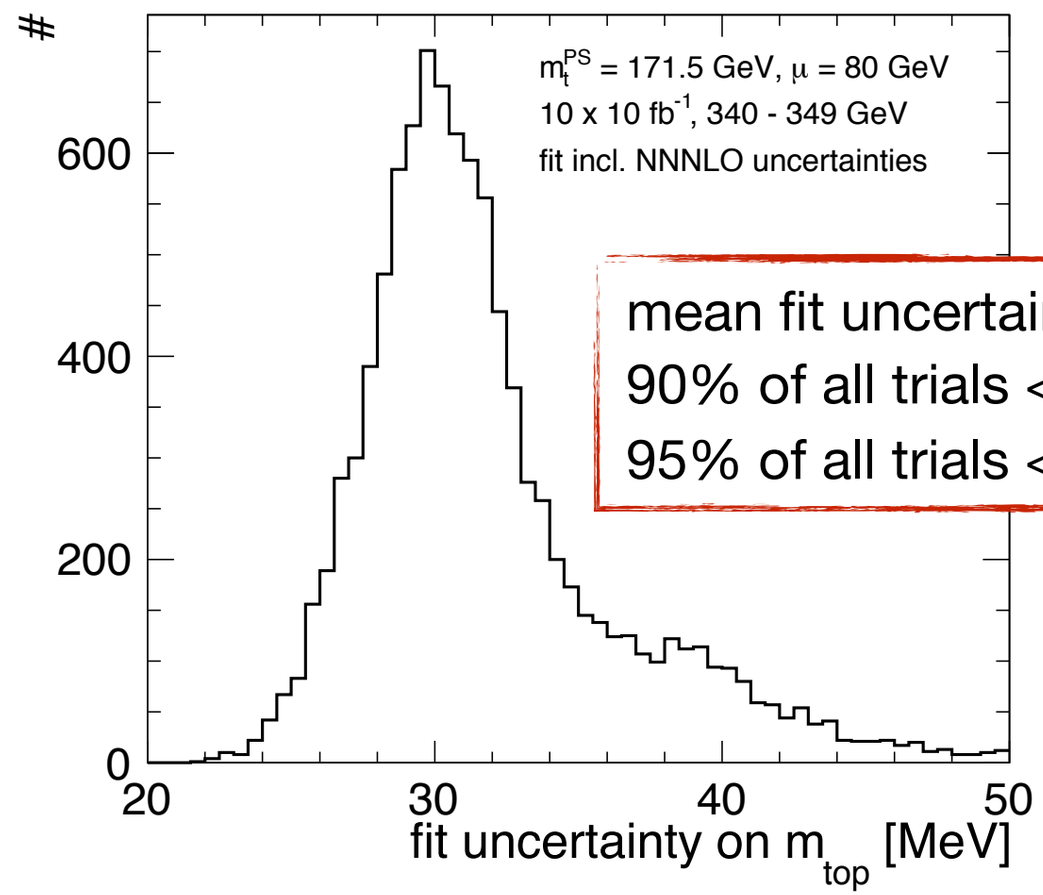

mean fit uncertainty: 31.9 MeV
90% of all trials < 38.5 MeV
95% of all trials < 41.5 MeV

Optimising Threshold Scans

Disclaimer: This is ongoing work, up to now a few quick checks with the new theory curves

Optimising Threshold Scans: First Ideas

- Ongoing study: How to optimise a threshold scan to provide the best results with the lowest integrated luminosity - and the least sensitivity to systematics

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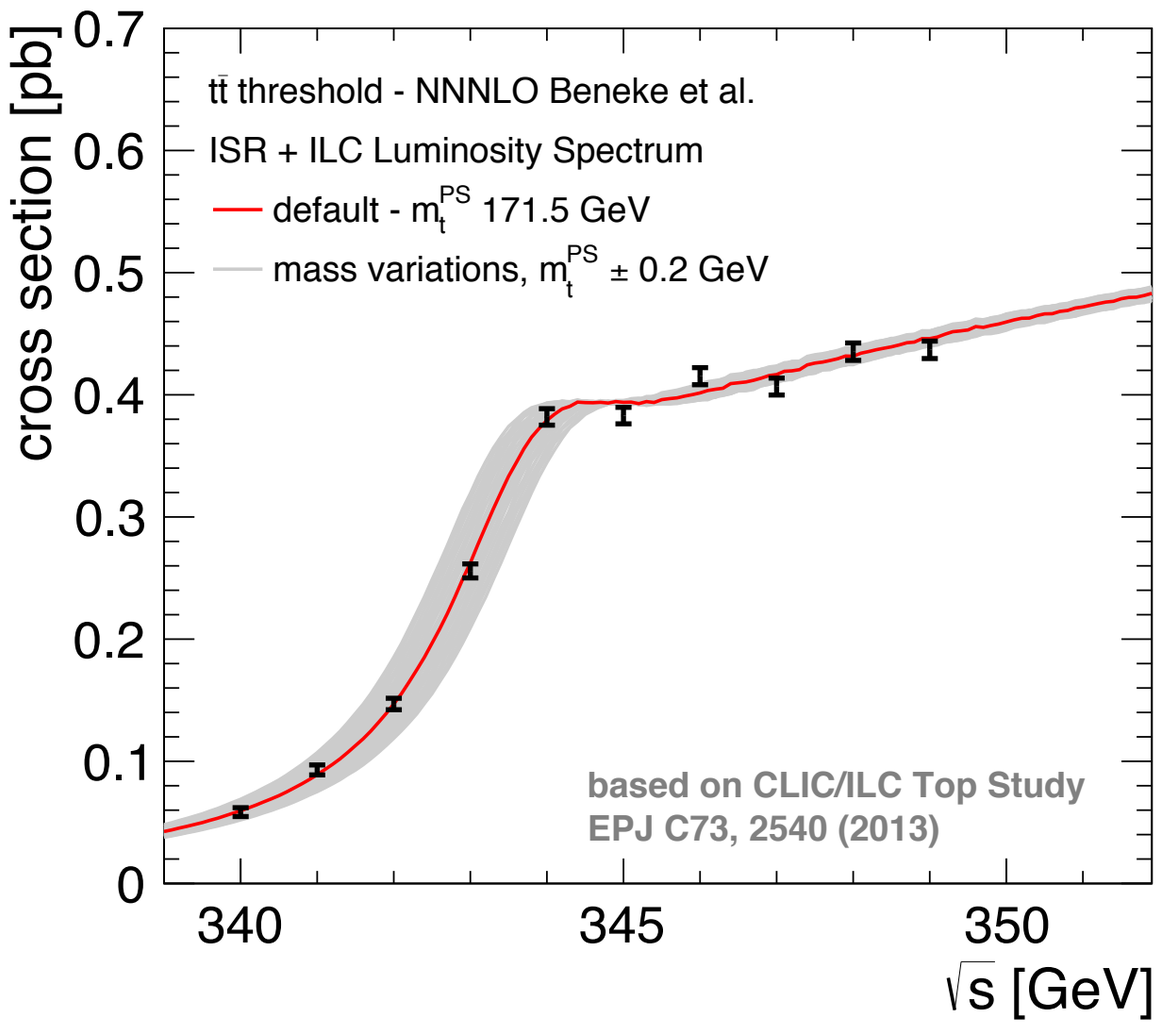

First step: The mass: Clearly some regions are much more sensitive than others…

Important constraint: Have to take expected mass precision (in PS scheme!) prior to ILC running into account

From LHC: Small uncertainty on "kinematic mass" by 2030, assume total uncertainty of 1 GeV

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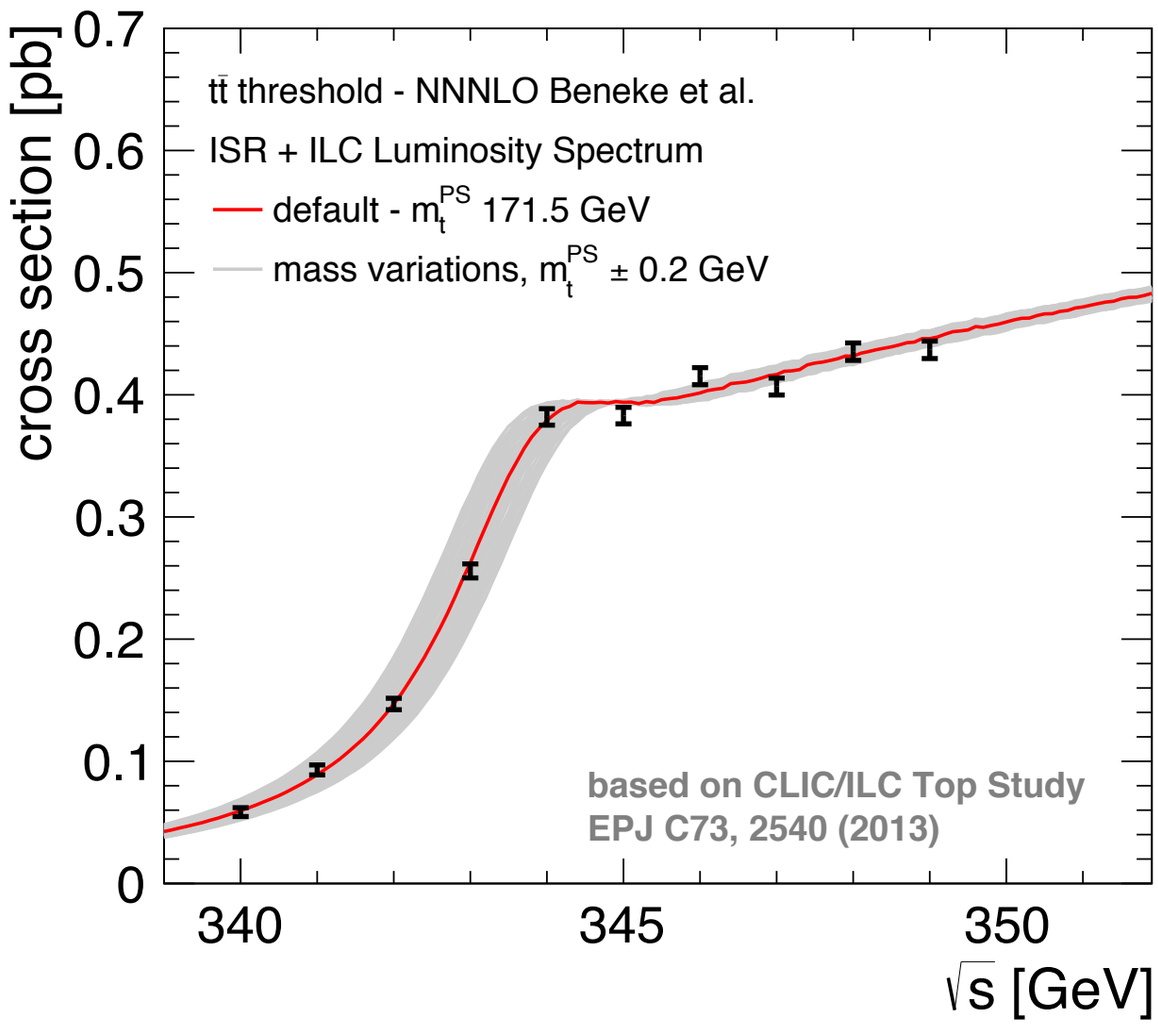

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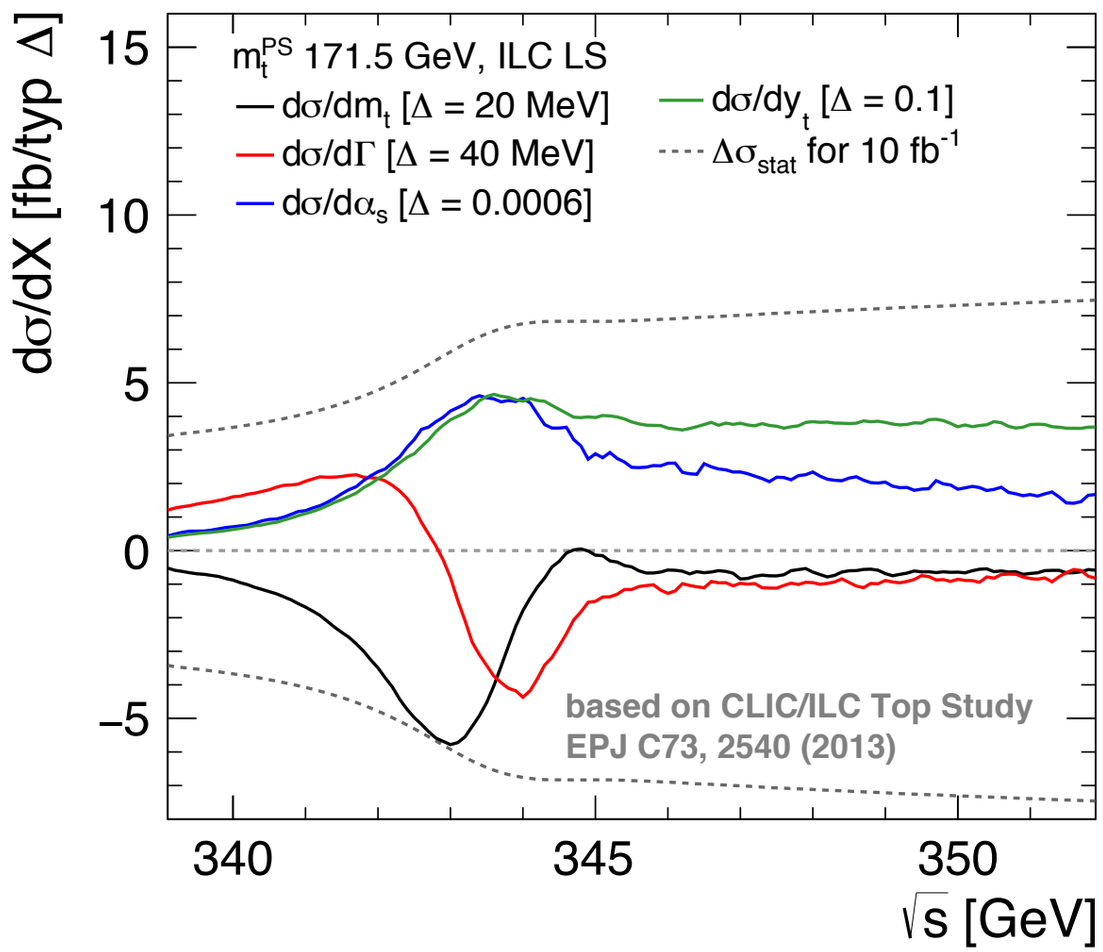
Then: iterative procedure to concentrate the luminosity at the most sensitive point

Picking the First Energy Point



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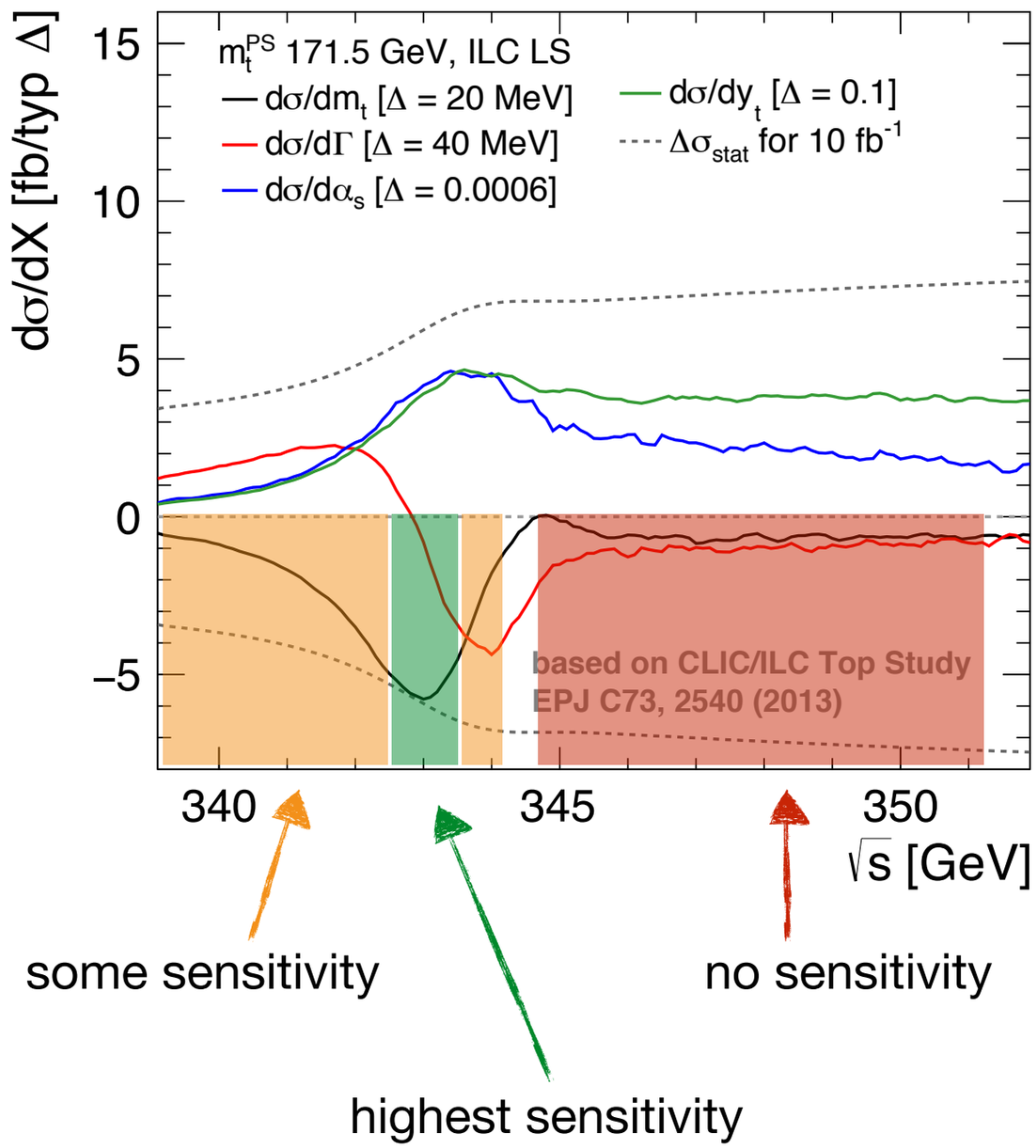

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- Avoid: Measuring at $E > 2 \times m_t^{PS} + 1.5$ GeV

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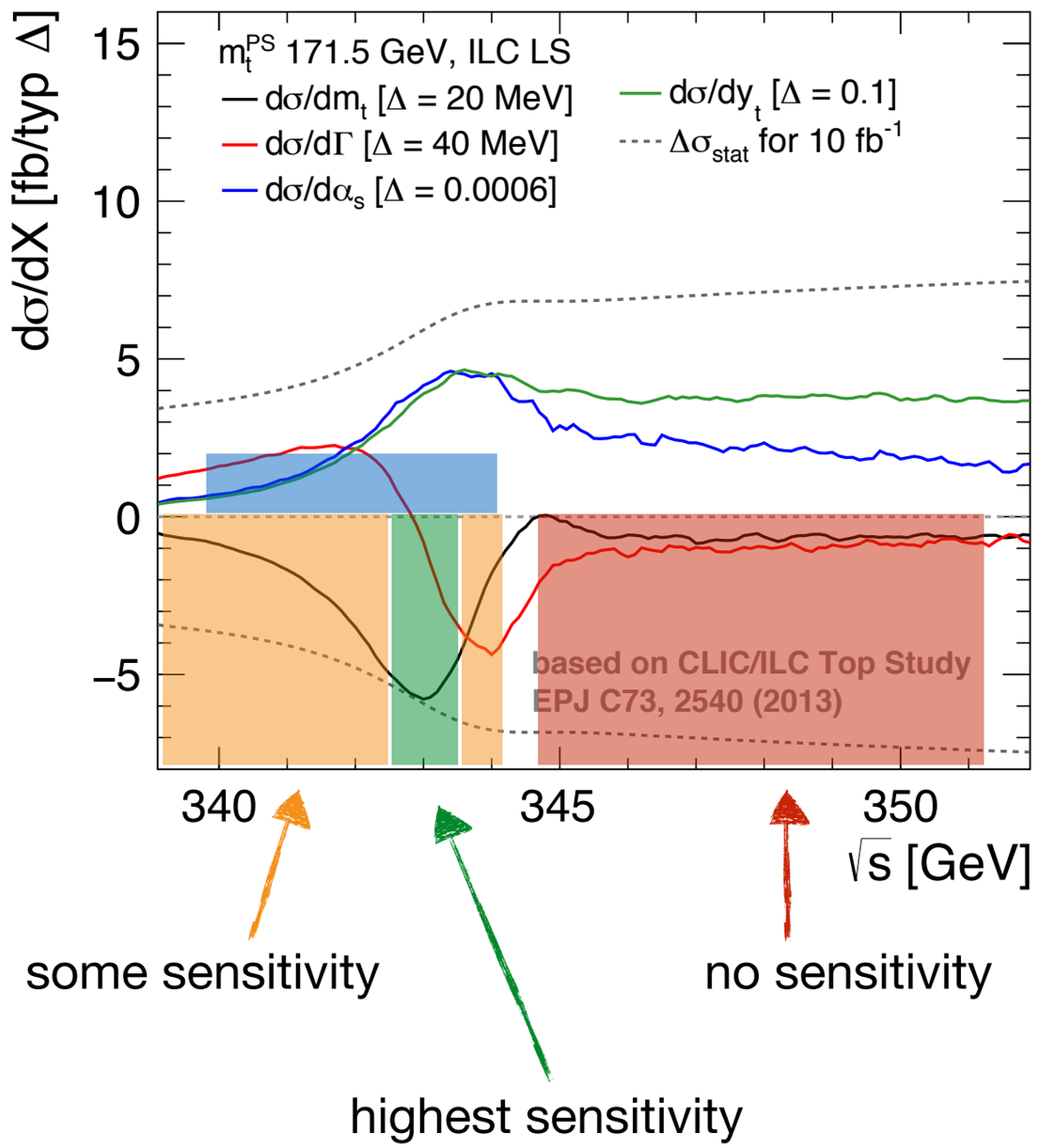


- Choose the first point with care: In all possible scenarios it has to be guaranteed that the measurement there will improve the m_t knowledge wrt LHC

- Ideal: Measure at E = 2 x m_t^{PS}
- Avoid: Measuring at E > 2 x m_t^{PS} + 1.5 GeV

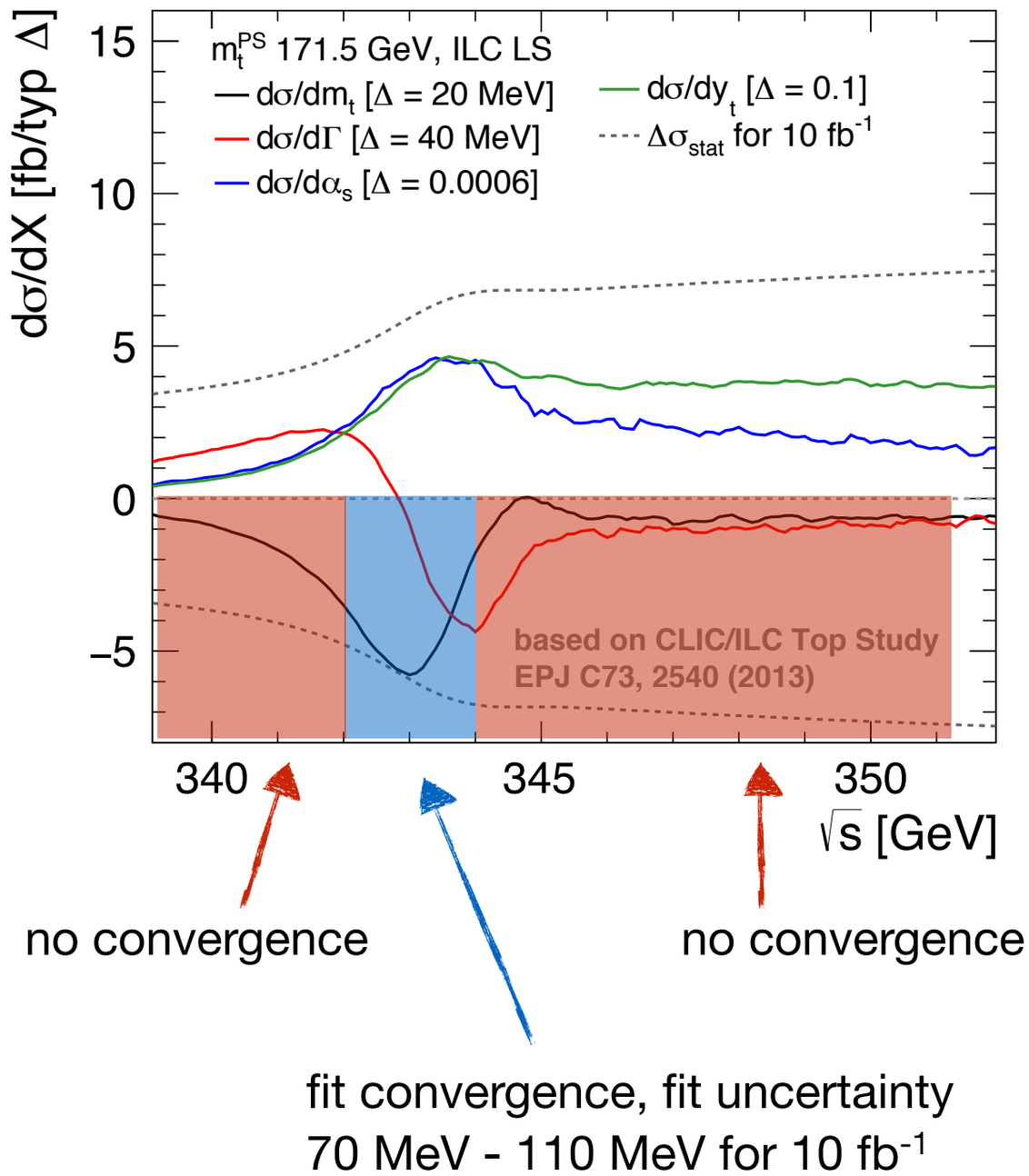
- With $\Delta m_t^{PS,LHC}$ = 1 GeV from LHC:
  "safe" starting point: E = 2 x $m_t^{PS,LHC}$ - 1 GeV

For true m_t^{PS} = 171.5 GeV the LHC-measured m_t^{PS} may be between 170.5 and 172.5 GeV

⇨ starting energy between 340 GeV and 344 GeV

⇨ always safe!

Reality Check: Including Scale Uncertainties



- With scale uncertainties included in the fit, the sensitivities are “diluted” - “safe zone” for single point m_t measurement shrinks to ± 1 GeV
- “Safe” single point measurement not possible when accounting for theory uncertainties

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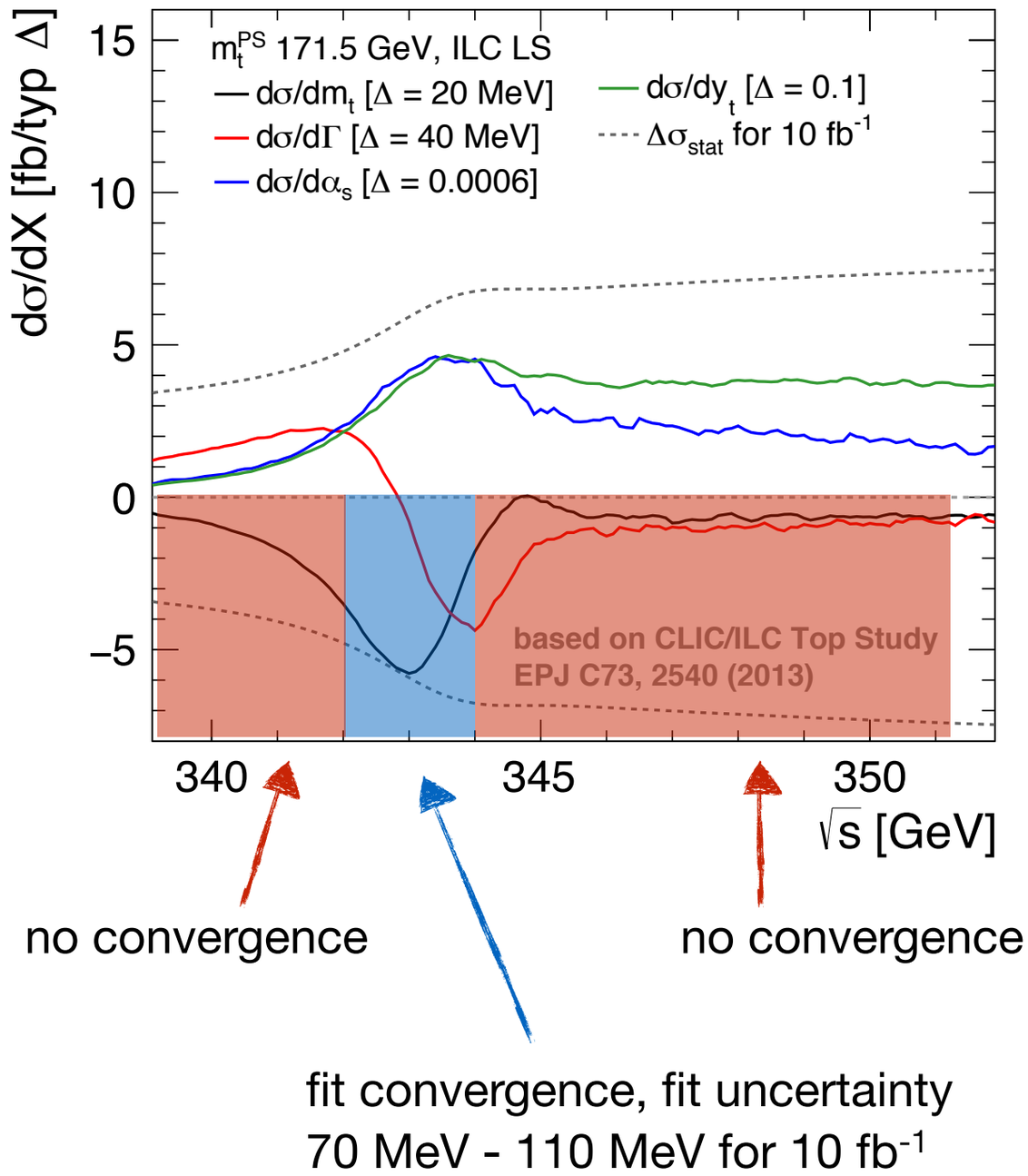

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- "Safe" single point measurement not possible when accounting for theory uncertainties

- Run an initial 2-point scan - spacing 2 GeV: 2 x $m_t^{PS,LHC}$ - 1.5 GeV; 2 x $m_t^{PS,LHC}$ + 0.5 GeV

For true m_t^{PS} = 171.5 GeV the LHC-measured m_t^{PS} may be between 170.5 and 172.5 GeV

➪ first energy between 339.5 GeV and 343.5 GeV
➪ second energy between 341.5 GeV and 345.5 GeV
➪ One energy point always safe!

Threshold Scan Strategies - Sensitivity

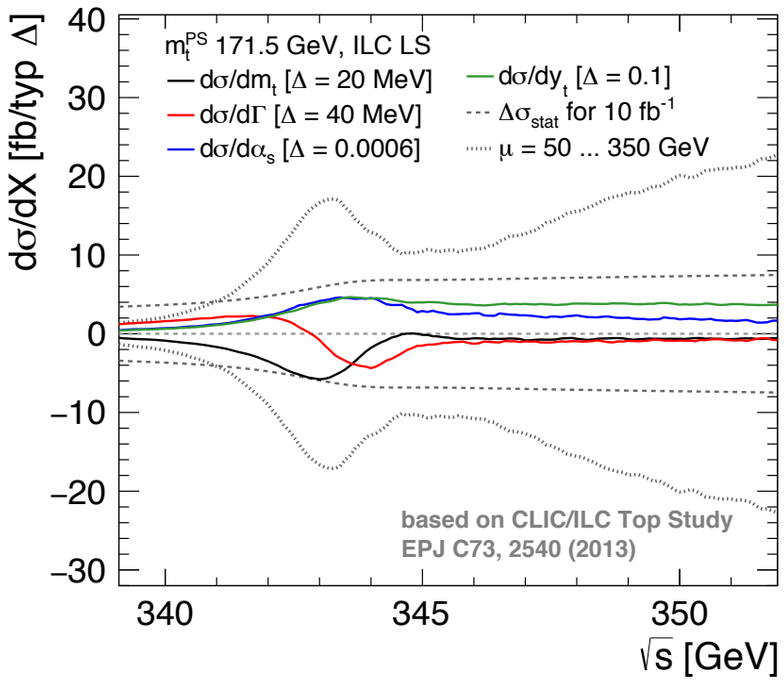
- Initial two-point scan to improve on LHC precision
  - In principle: 5 fb^{-1} / point sufficient:
    fit error between 75 and 120 MeV
  - Bias variations 30 MeV (on top of the ~ 45 MeV shift due to scale fit) - No problem

➪ After first scan can measure at point of maximum sensitivity (known within 300 MeV)
$E = 2 \times m_t^{PS}$

Scale uncertainty limit the usefulness of this strategy:
10 fb^{-1}: 69.0 MeV (fit)
20 fb^{-1}: 62.5 MeV (fit) ➪ Saturation of uncertainty
30 fb^{-1}: 60.0 MeV (fit)

Threshold Scan Strategies - Sensitivity

- Several points beneficial when fitting with uncertainties - No surprise:
  - ➪ Study multi-point scans!



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Three-point scan in most sensitive region:

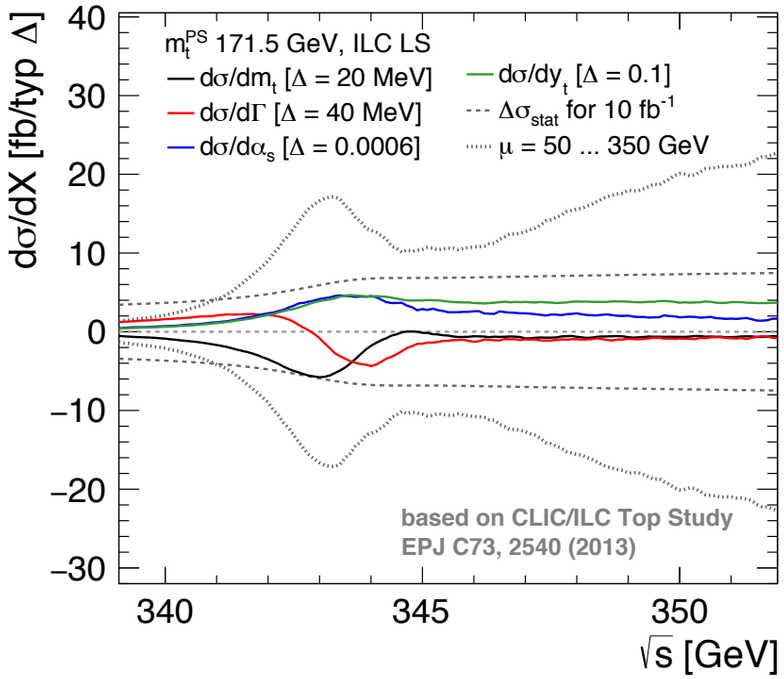
$E = 2 \times m_t^{PS}$, $E = 2 \times m_t^{PS} \pm 1$ GeV

5 fb^{-1}/point: 52 MeV (fit)

10 fb^{-1}/point: 45 MeV (fit)

20 fb^{-1}/point: 43 MeV (fit)

➪ Saturation of uncertainty at 10 fb^{-1} / point



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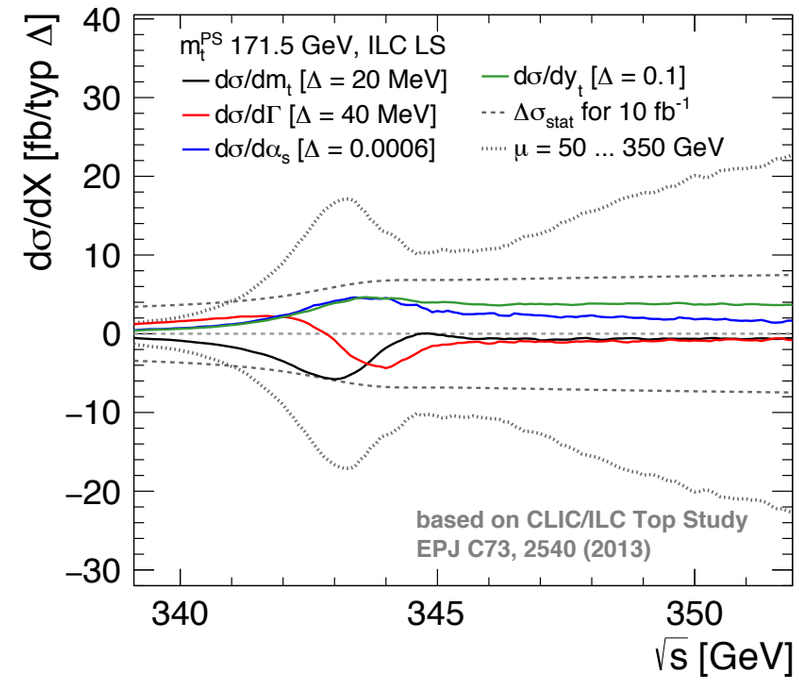

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Five-point scan in most sensitive region:

$E = 2 \times m_t^{PS}$, $E = 2 \times m_t^{PS} \pm 1$ GeV, $E = 2 \times m_t^{PS} \pm 2$ GeV

5 fb^{-1}/point: 44 MeV (fit)

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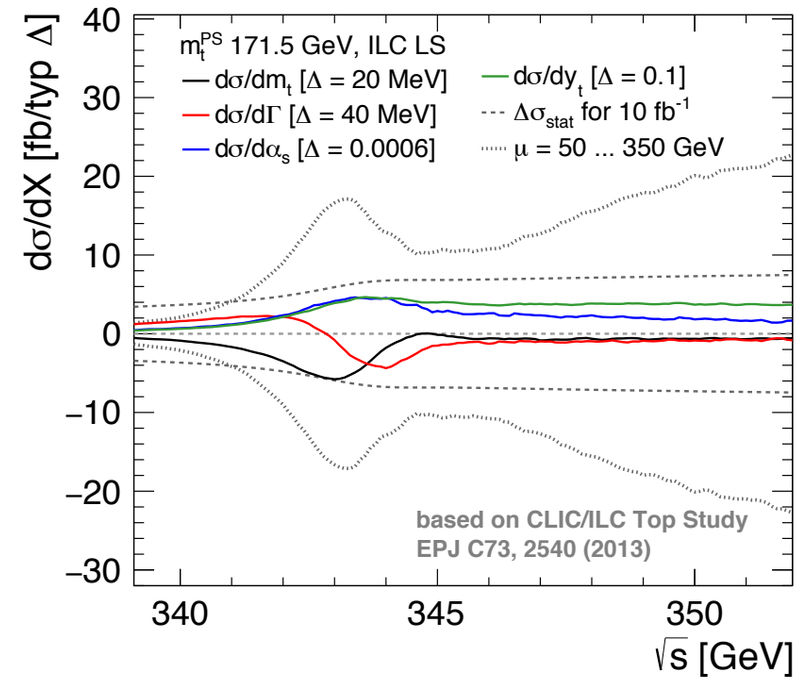

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10 fb^{-1}/point: 36 MeV (fit)

20 fb^{-1}/point: 32.5 MeV (fit)

➪ Saturation of uncertainty at 10 fb^{-1} / point

Default 10 point scan:

$E = 2 \times m_t^{PS} - 3$ GeV to $E = 2 \times m_t^{PS} + 6$ GeV

5 fb^{-1}/point: 40.5 MeV (fit)

10 fb^{-1}/point: 32 MeV (fit)

20 fb^{-1}/point: 28 MeV (fit)

Summary

- A scan of the ttbar threshold in e^+e^- collisions is the best method for a precise measurement of the top quark mass and other top properties
  - General assumption: The final precision may well be dominated by theoretical uncertainties

- For the first time: Incorporation of NNNLO QCD scale uncertainties in the experimental evaluation - Systematic uncertainty on the mass: **~45 MeV**

- Scale uncertainties can (and should!) be included in template fits of threshold scans
  - Results in a deterioration of the fit uncertainty: **~32 MeV** for 100 fb^{-1}
    NB: Uncertainty does not scale purely with statistics - full separation into different components TBD

- Energy choices of a threshold scans may be optimized - When taking uncertainties into account a “classical” 10 point scan may well be the best choice

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This study is far from complete: Extension to other top parameters planned!