Strong Coupling Overview

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Introduction

In the absense of any sign of new physics the demands on the precision of theoretical predictions within the Standard Model are steadily increasing.

Parametric uncertainties of the parameters of the Standard Model (heavy quark masses, strong coupling, CKM matrix elements) play an important role in this context.

Strong coupling determinations are important:

- Precise predictions for observables sensitive to QCD corrections.
- Consistency of determinations from different observables reflects our understanding of QCD.

Overall endeavor characterized by:

- Increased awareness that proper and adequate estimate of theoretical uncertainties has highest priority.
- New and more precise analyses can lead to an increase in the uncertainties compared to earlier analyses.
- Existence of very precise, but mutually incompatible results enforces cross checks at all levels and critical view on how error estimates are carried out.



Introduction

Important observables where precision of strong coupling is important:

- All hadron collider quantities sensitive to production rates and mechanism or parton distribution functions.
- All observables where QCD corrections are large and required/intended precision is high.
- Higgs, vector bosons: production, hadronic decays
- Top quark pair threshold @ LC
- ttH @ LC
- ...



Introduction

Aim of this talk:

- Review major areas of α_s determinations following the new preliminary 2015 World Average.
- Status concerning most up-to-date results
- Assessment of possible next steps for improvement

Note:

I use a lot of information from the preliminary 2015 World Average in the PDG QCD review, but some statements I make reflect my own opinion and are not statements made in the World Average.

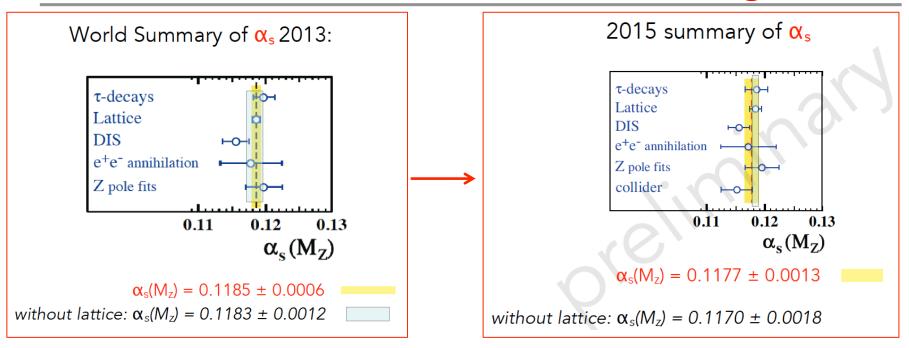
I use a lot of material presented in talks at the CERN Workshop on high-precision alpha_s measurements: from LHC to FCC-ee and thank the authors for their contributions.



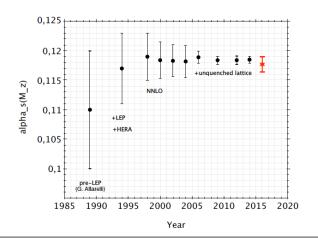
Outline

- Comments on (preliminary) 2015 World Average
- Lattice QCD
- Hadronic tau decay
- e⁺e⁻- annihilation (event shapes)
- DIS and pdf fits
- Hadron collider data (LHC)
- Electroweak precision fits
- Conclusions





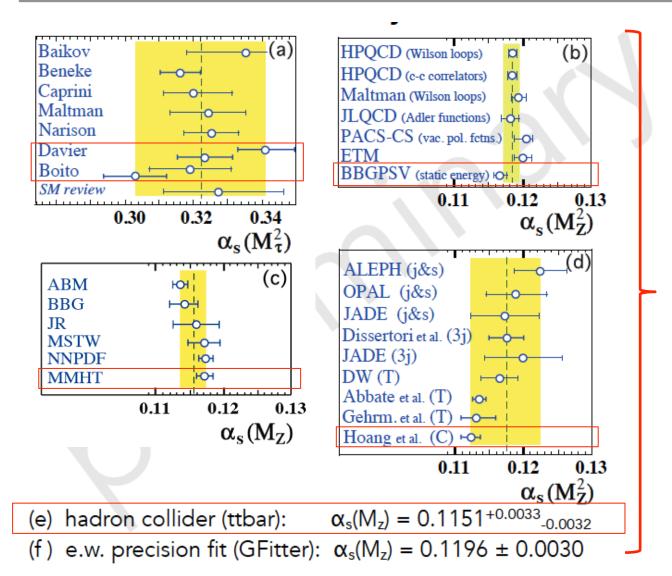
history of world average of α_s

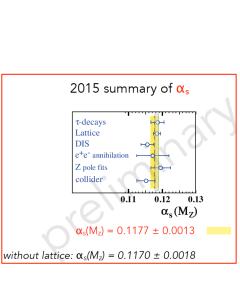


Error has gone up for the first time.

From S. Bethke @ Fcee WS







= new

Only at least NNLO based analyses shall be included!

wrap-up:

- new **preliminary** value of world $\alpha_s(M_z)$: = 0.1177 ± 0.0013
- change from 2013 value ($\alpha_s(M_z)=0.1185\pm0.0006$) mainly due to:
 - decreased weight (increased error) of lattice results
 - decreased central value from τ-decays
 - -result from new class (hadron collider, ttbar x-section), with only one published result, however known to be systematically low
- known but unresolved issues for almost all classes
- no convergence of issues in sight
 - however -
- even within conservative uncertainties, Asymptotic
 Freedom and in general, QCD is in excellent shape!



Averaging Methods:

- 5 classes of measurements, each pre-averaged
- all at least using NNLO QCD
- using two methods to determine (pre-)averages:
- "range averaging" average value with symmetric overall uncertainty that encompasses the central values of all individual α_s -results
- "χ² method"
 weighted average treating individual uncertainties as being uncorrelated and of Gaussian nature.

If overall χ^2 < 1/d.o.f., an overall correlation coefficient is introduced and adjusted such that χ^2 = 1/d.o.f.

If overall $\chi^2 > 1/d.o.f.$, all uncertainties are enlarged by a common factor such that $\chi^2 = 1/d.o.f.$

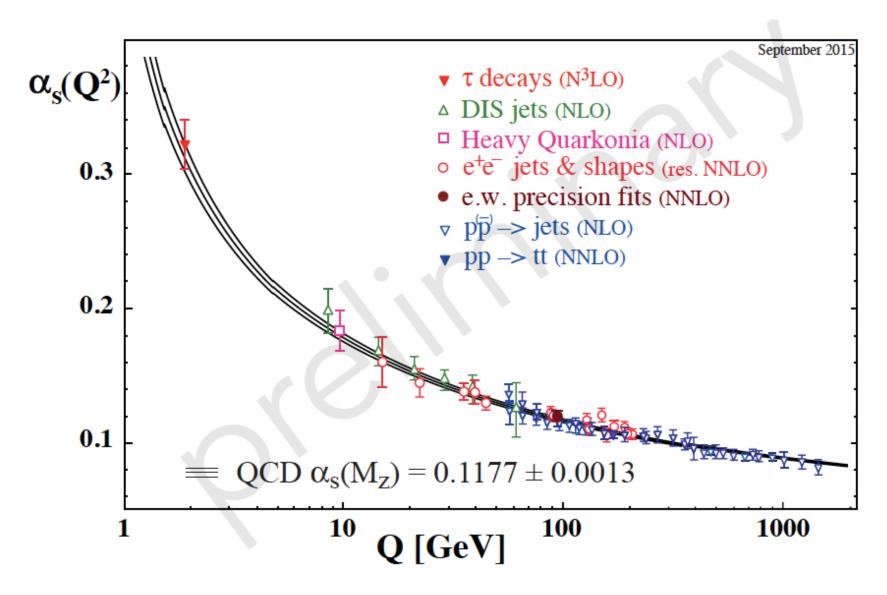
Intention:

For individual subfield in case there are large systematic differences in analyses based on same observable/data.

For avaraging in all other cases to account for possible (unknown) correlations and possible understimated systematic uncertainties.

From S. Bethke @ FCCee WS







Comments:

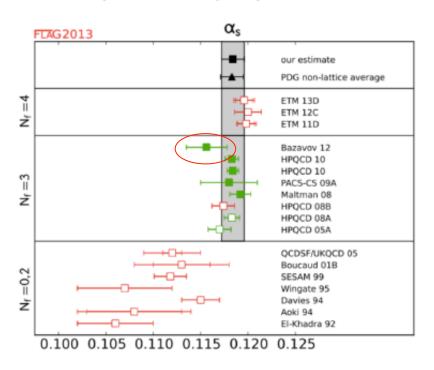
- Higher degrees of sophistication in a number of different strong coupling extraction methods (tau decay, event shapes) have revealed that there are issues that are not yet understood. This might happen for other methods as well.
- Status concerning most up-to-date results
- Assessment of next steps for improvement



Lattice QCD

summary from FLAG collaboration, 2013:

The importance of quality criteria is seen in our estimate of \(\Omega_{\text{strong}}\)



- FLAG estimate has conservative error (not all FLAG agrees)
- PDG total average takes all lattice results at face value
- PDG without lattice agrees with FLAG

$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.1184(12)$$



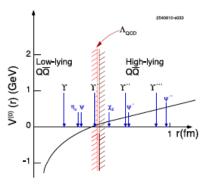


Lattice QCD

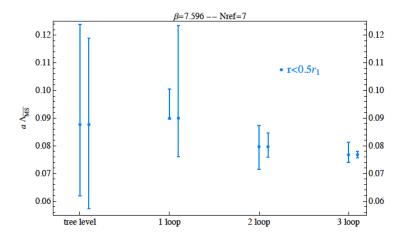
New: Static energy - Lattice (updated) vs. NNNLO QCD

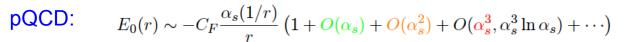
A. Bazavov, N. Brambilla, XGT, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D 86, 114031 (2012) [arXiv:1205.6155 [hep-ph]];

Phys. Rev. D 90, 074038 (2014) [arXiv:1407.8437 [hep-ph]]



From N. Brambilla et al., Eur. Phys. J. C71 (2011) 1534

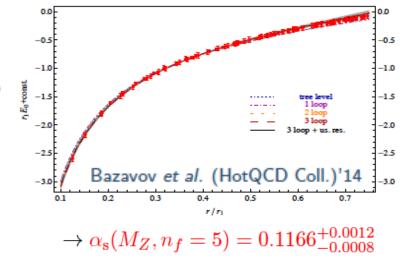




1-loop: Fischler'77 Billoire'80 2-loop: Peter'96'97 Schröder'98 Kniehl Penin Steinhauser Smirnov'01 3-loop Smirnov Smirnov Steinhauser'08'10 Anzai Kiyo Sumino'10

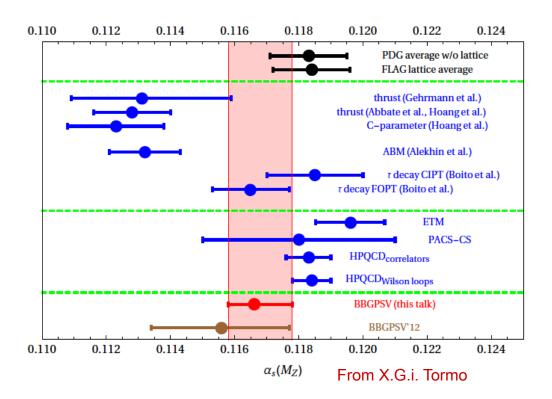
New Lattice + pQCD fit:

$$n_f = 2 + 1$$
 flavor QCD

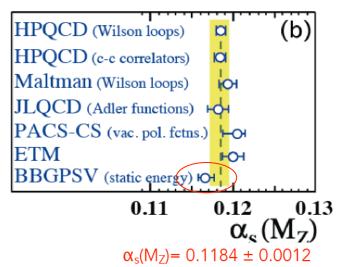


- Good pQCD convergence
- Many consistency checks

Lattice QCD



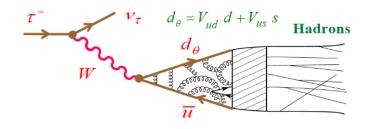
- New result somewhat low compared to other recent highprecision lattice determinations
- Indication for systematic deviation?

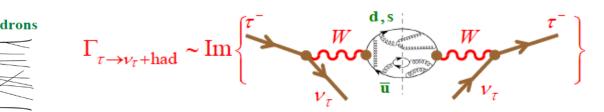


Ways to improve situation / go on:

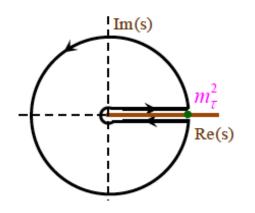
- Add more high-precision analyses based on other lattice-QCD calculations.
- Critical assessment of origin of differences / missing ingredients for proper error estimates.







$$R_{\tau} = \frac{\Gamma(\tau^{-} \to \nu_{\tau} + \text{had})}{\Gamma(\tau^{-} \to \nu_{\tau} e^{-} \overline{\nu_{e}})} = 12\pi \int_{0}^{1} dx \, (1 - x)^{2} \left[(1 + 2x) \, \text{Im} \, \Pi^{(1)}(x \, m_{\tau}^{2}) + \text{Im} \, \Pi^{(0)}(x \, m_{\tau}^{2}) \right]$$





$$x \equiv s/m_{\tau}^2$$

$$R_{\tau} = 6\pi i \oint_{|x|=1} dx (1-x)^{2} \left[(1+2x) \Pi^{(0+1)}(x m_{\tau}^{2}) - 2x \Pi^{(0)}(x m_{\tau}^{2}) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu) \left\langle O_D(\mu) \right\rangle}{\left(-s\right)^{D/2}}$$
 OPE

From T. Pich @ FCCee WS



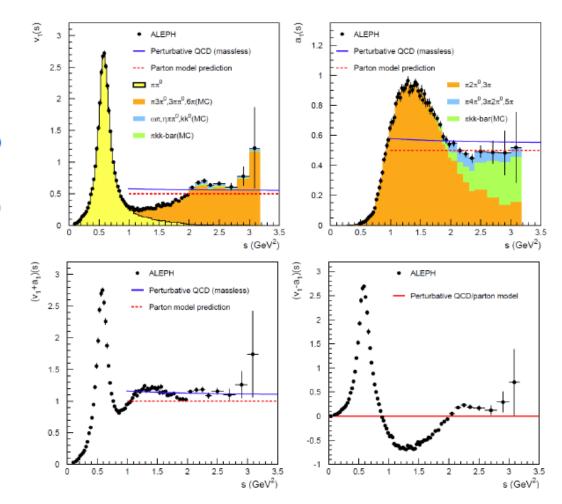
Experimental spectral functions



$$V_1(s) = 2\pi \operatorname{Im} \Pi_{ud,V}^{(0+1)}(s)$$

$$a_1(s) = 2\pi \operatorname{Im} \Pi_{ud,A}^{(0+1)}(s)$$

Better data needed







Theory Input

Perturbative
$$(m_q=0)$$

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} K_n \left(\frac{\alpha_s(-s)}{\pi} \right)^n$$

$$K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101 \quad , \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08



$$\delta_{p} = \sum_{n=1}^{\infty} K_{n} A^{(n)}(\alpha_{s}) = a_{\tau} + 5.20 a_{\tau}^{2} + 26 a_{\tau}^{3} + 127 a_{\tau}^{4} + \cdots$$

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) \left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_{\tau}^n + \cdots \qquad ; \qquad a_{\tau} = \frac{\alpha_s(m_{\tau})}{\pi}$$

Power Corrections

$$\Pi_{\text{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2} \sum_{n \ge 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4 \left\langle O_4 \right\rangle \approx \frac{2\pi}{3} \left\langle 0 \left| \alpha_{\rm S} G^{\mu\nu} G_{\mu\nu} \left| 0 \right\rangle \right.$$

$$\delta_{\text{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx \ (1 - 3x^2 + 2x^3) \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{\left(-x m_{\tau}^2\right)^n} = -3 \frac{C_6 \langle O_6 \rangle}{m_{\tau}^6} - 2 \frac{C_8 \langle O_8 \rangle}{m_{\tau}^8}$$

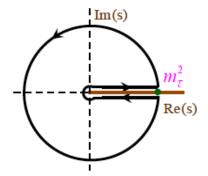
Suppressed by m_{τ}^{6} [additional chiral suppression in $C_{6}\langle O_{6}\rangle^{V+A}$]



Fixed-order vs Contour-improved perturbation theory

$$\delta_{P} = \sum_{n=1}^{\infty} K_{n} A^{(n)}(\alpha_{S}) = \sum_{n=0}^{\infty} r_{n} a_{\tau}^{n}$$
CIPT FOPT

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) a_\tau (-x m_\tau^2)^n$$



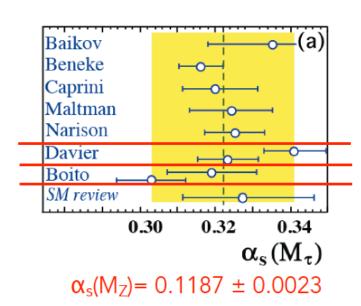
$$a(-s) \simeq \frac{a_{\tau}}{1 - \frac{\beta_1}{2} a_{\tau} \log\left(-s/m_{\tau}^2\right)} = \frac{a_{\tau}}{1 - i\frac{\beta_1}{2} a_{\tau} \phi} = a_{\tau} \sum_{n} \left(i\frac{\beta_1}{2} a_{\tau} \phi\right)^n \qquad ; \qquad \phi \in [0, 2\pi]$$

FOPT expansion only convergent if $\alpha_{\tau} < 0.14$ (0.11) [at 1 (3) loops]

Experimentally $\alpha_{\tau} \approx 0.11$

- pQCD for tau decay at the borderline of convergence between FOPT and CIPT
- Both approaches do not seem to converge to the same perturbative result.
- Scale variation gives smaller error estimate than using FOPT vs. CIPT.
- Whether FOPT or CIPT is "better" might not be made clearer by even having the next order in pQCD. Nothing a priori wrong with either FOPT or CIPT.





 Systematic difference FOPT vs CIPT should be taken as the current theoretical error.

New results:

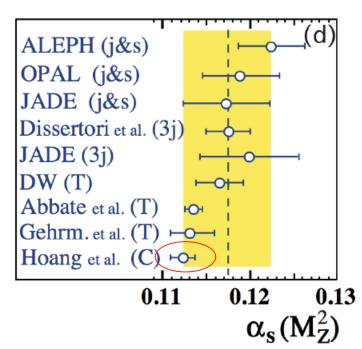
Davier et al., Eur.Phys.J. C74 (2014) 3, 2803 Boito et al., Phys.Rev. D91 (2015) 3, 034003

Ways to improve situation / go on:

- Next order pQCD might help to reduce discrepancy, but not clear whether it really will.
- New insights into the structure of non-perturbative corrections and/or renormalon effects.



e⁺e⁻-Annihilation



- Contains among the most precise individual determinations, but has largest uncertainty in the world average, due to very small values for the strong coupling.
- Two classes of observables combined:
 - event shapes
 - (3) jet rates (NNLO, but still larger errors)
- Large spread in results from event shapes.

$-> \alpha_s(M_Z) = 0.1174 \pm 0.0051$

New result:

AHH, Kolodrubetz, Mateu, Stewart; PRD 91 (2015) 9, 094018

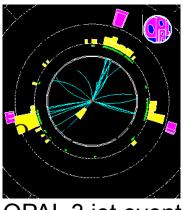
I explain what leads to the widely different results for the event shapes analyses concentrating mostly on our new C-parameter analysis.



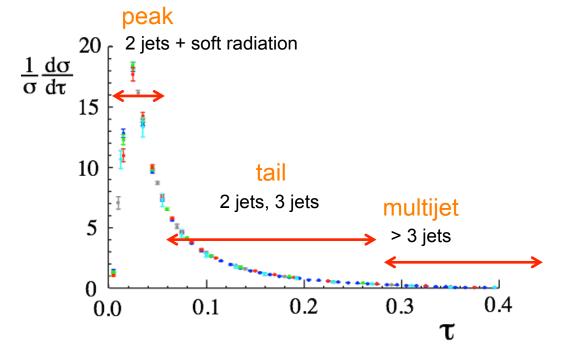
Classic method for determining $\alpha_s(M_z)$ Single-variable jet distributions

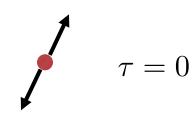
e.g. Thrust
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p_i} \cdot \hat{n}|}{\sum |\vec{p_i}|}$$

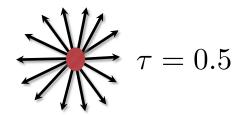
C-parameter
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p_i}| |\vec{p_j}| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p_i}|\right)^2}$$



OPAL 3 jet event



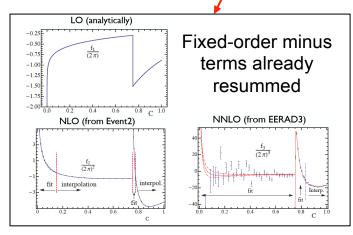


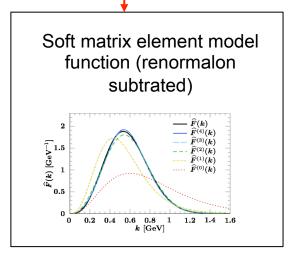




Anatomy of theory prediction (SCET, but similar in other approaches)

$$\left(\frac{d\sigma}{dC}\right) = \int d\ell \left[\left(\frac{d\sigma}{dC}\right)_{\text{part}}^{\text{sing}} \left(C - \frac{\ell}{Q}\right) + \left(\frac{d\sigma}{dC}\right)_{\text{part}}^{\text{nonsing}} \left(C - \frac{\ell}{Q}\right) \right] S^{\text{mod}}(\ell, \Delta(R))$$





$$\left(\frac{d\sigma}{dC}\right)_{\mathrm{part}}^{\mathrm{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J\left(\frac{QC}{6} - \ell - \ell', \mu_J, \mu_S\right) J_\tau(Q\ell', \mu_J) S_{\mathrm{C}}(\ell - \Delta, \mu_S)$$

- Factorization: hard vs. collinear vs. soft
- Resummation of large logarithm up to NNNLL order

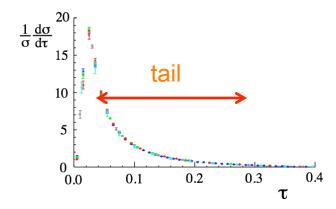


Anatomy of theory prediction (SCET, but similar in other approaches)

$$\left(\frac{d\sigma}{dC}\right) = \int d\ell \left[\left(\frac{d\sigma}{dC}\right)_{\text{part}}^{\text{sing}} \left(C - \frac{\ell}{Q}\right) + \left(\frac{d\sigma}{dC}\right)_{\text{part}}^{\text{nonsing}} \left(C - \frac{\ell}{Q}\right) \right] S^{\text{mod}}(\ell, \Delta(R))$$

For $C \gg \Lambda_{QCD}/Q$, in the tail region, the soft model function can be expanded in an OPE.

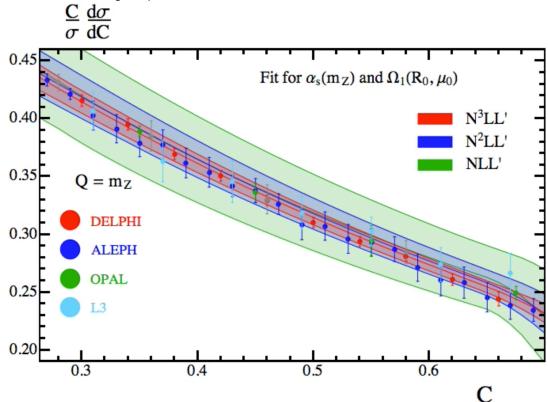
Only two fit parameters: α_s and Ω_1^C



- Two ways to determine non-perturbative matrix element Ω_1 :
 - From Monte-Carlo (parton-level vs. hadron-level output): classic method
 - From global fits to data: "analytic power corrections"



C-parameter Tail $\alpha_s\text{-}\Omega_1$ Global Fit



- Different behavior of fits with increase order
- Good convergence
- Very good agreement at N³LL + O(α_s ³) with renormalon subtraction.

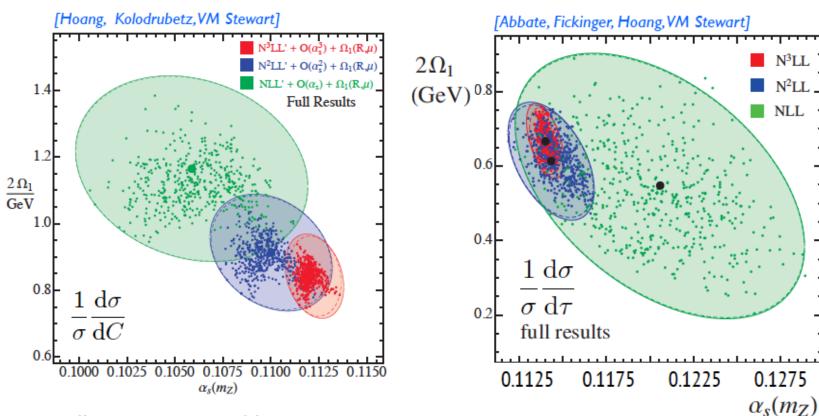


C-parameter Tail α_s - Ω_1 Global Fit

Thrust Tail α_s - Ω_1 Global Fit

 N^3LL

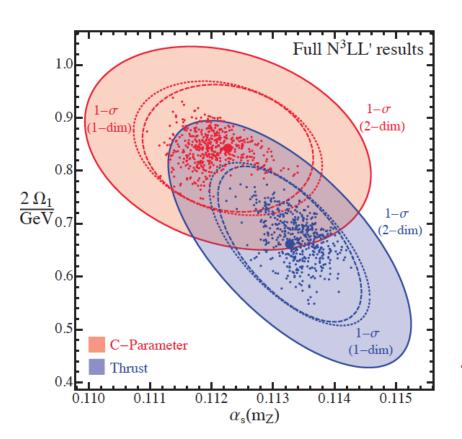
NLL



- Different behavior of fits with increase order
- Good convergence
- Very good agreement at N³LL + O(α_s ³) with renormalon subtraction.
- Error dominated by theory uncertainty (particularly pQCD)



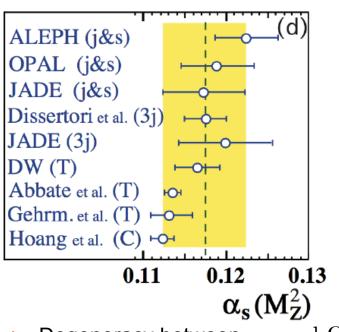
C-parameter versus Thrust Tail Global Fit



AHH, Kolodrubetz, Mateu, Stewart; PRD 91 (2015) 9, 094018

Thrust and C-parameter results are fully compatible concerning fit of non-perturbative matrix element Ω_1 .





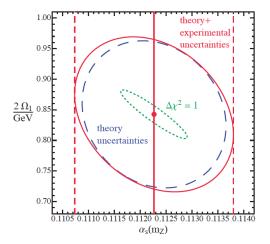
NNLO pQCD input & Non-perturbative corrections from Monte-Carlo estimate.

Problem: Monte-Carlo parton level has only LO/LL precision.

NNLO –QCD input & Non-perturbative corrections global fits to experimental data

WHY DIFFERENT?





$$\frac{\Omega_1}{41.26 \,\text{GeV}} = 0.1221 - \alpha_s(m_Z)$$

We would obtain

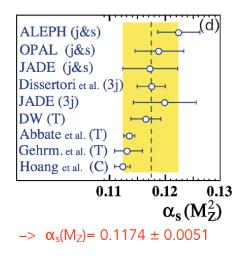
$$\alpha_s(m_Z) \sim 0.118 \text{ for } \Omega_1 \sim 0.170 \text{ GeV}$$

Degeneracy is lifted by fitting to data from different Q values.

Monte-Carlo method to determine Ω_1 leads to substantially smaller values than fits to data.



e⁺e⁻-Annihilation



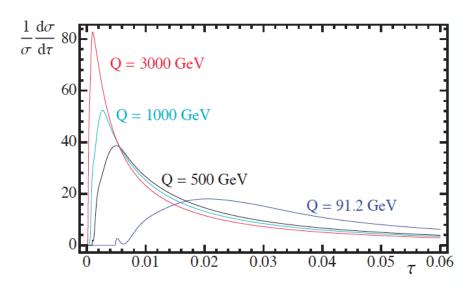
Ways to improve situation / go on:

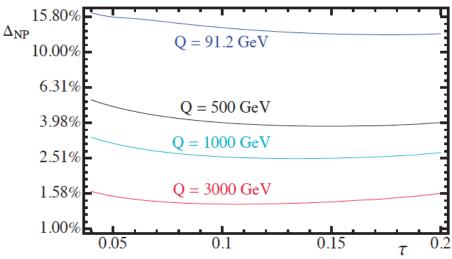
- Do NNNLL + matched NNLO analyses for other event shapes: e.g. heavy jet mass, broadening. A lot of work due to new systematics in summation and non-perturbative effects. (work in progress, heavy stuff)
- Computation of next order: very hard + not likely to remove discrepancies.
- High-precision low-Q data from B factories (off Y(4S) resonance). Data is existing, but has never been published. Would add a very important constraint that might affect the outcome of α_s - Ω_1 combined fits.



What can a future lepton collider help?

What would a precise measurement of event shapes at higher Q values contribute?





- Event accumulate in very small region at small values.
- High precision needed.
- Background tricky (γγ)

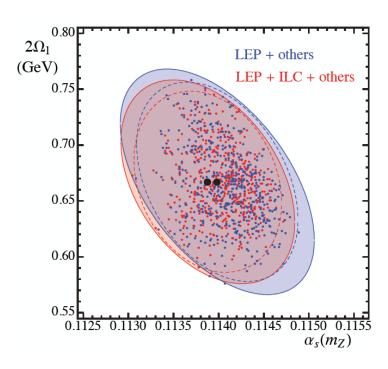
- Non-perturbative effects decrease with Q
- At some point smaller than experimental uncertainty and negligible!!



Strong Coupling from a Future Lepton Collider

What would a precise measurement of event shapes at higher Q values contribute?

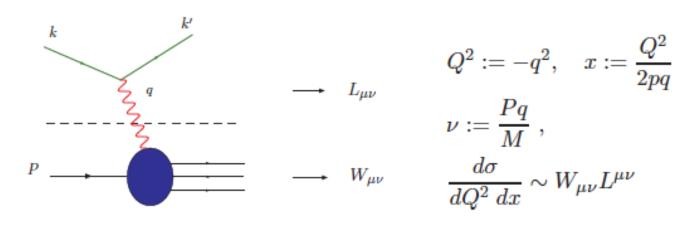
Exercise: Make up fictitious ILC data at 500 GeV, with assumed 1% statistical and 1% systematical uncertainties. Repeat fits.



- Limited impact concerning precision because high-energy uncertainties blown up in the evolution down to the Z mass
- Nevertheless EXTREMELY important for lifting degeneracy between α_s and Ω_1 and for checking for consistency of Ω_1 .

V. Mateu, AHH; unpublished





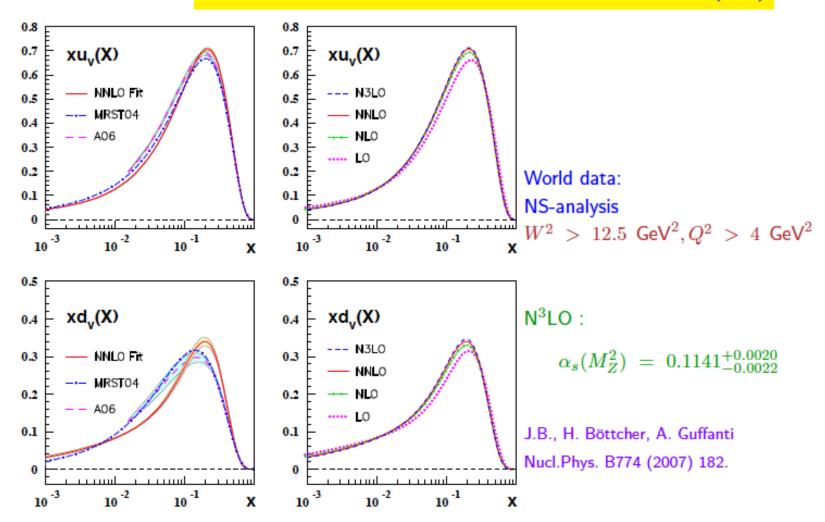
$$\begin{split} W_{\mu\nu}(q,P,s) &= \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s[J_{\mu}^{em}(\xi), J_{\nu}^{em}(0)]P, s \rangle \\ &= \frac{1}{2x} \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) F_L(x,Q^2) \\ &+ \frac{2x}{Q^2} \left(P_{\mu}P_{\nu} + \frac{q_{\mu}P_{\nu} + q_{\nu}P_{\mu}}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu} \right) F_2(x,Q^2) \end{split}$$

Structure Functions: $F_{2,L}$ contain light and heavy quark contributions \Longrightarrow Further Inclusion of Collider Data: DY (W^{\pm}, Z) , $t\bar{t}$, jets.

From J. Blümlein @ Fcee WS



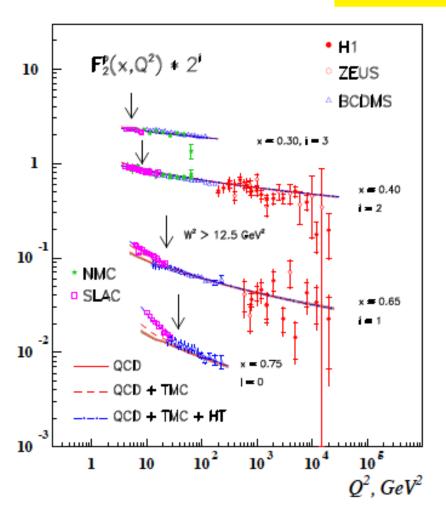
World Data Analysis: Valence Distributions (NS)

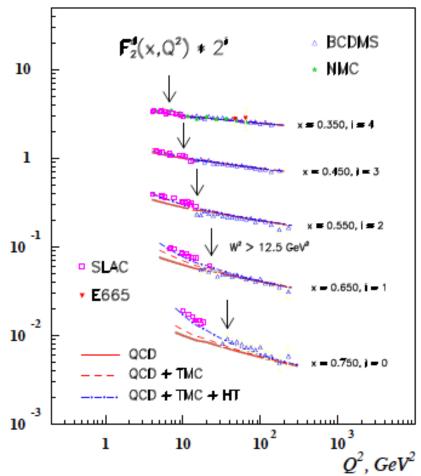


From J. Blümlein @ Fcee WS



Valence Distributions



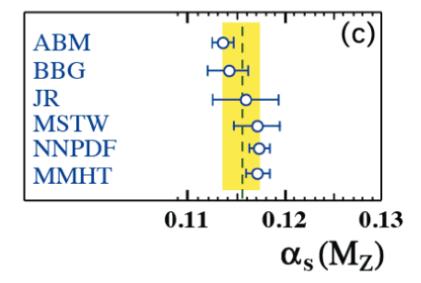


From J. Blümlein @ Fcee WS



- determination of parton densities from DIS; QCD in NNLO (up to N³LO);
- MSTW/NNPDF/MMHT: include hadron collider jet data (in order to constrain gluon at large x)

n.b.: all use similar (sub-)sets of data



 $-> \alpha_s(M_Z) = 0.1154 \pm 0.0020$ (same as in 2013)

From S. Bethke @ Fcee WS



Situation characterized by:

- Highly sophisticated analyses (at mostly NNLO) based on slightly differing experimental and theoretical input.
- The origin of the numerical differences in the strong coupling from the different analyses are known, but come from many sources (data sets used, parametrization of pdfs, theory input,...)
- There is no agreement among the groups about the optimal prescription.
 Each one claims they are correct.
- More higher-order calculations will affect individual analyses but unlikely to reduce overall discrepancies.

Ways to improve situation / go on:

Agreement among different groups about an optical procedure.



- With the common use of NLO-Monte-Carlos many pQCD-affected observables can be used to constrain / extract the strong coupling.
- Most of these observables mainly aim at quality control of our understanding of QCD (and of the more accurate NLO-Monte-Carlos).
 - Inclusive jet production cross sections
- Ratios of cross sections for certain number of jets
- Angular correlations
- Jet masses
- Top production
- •





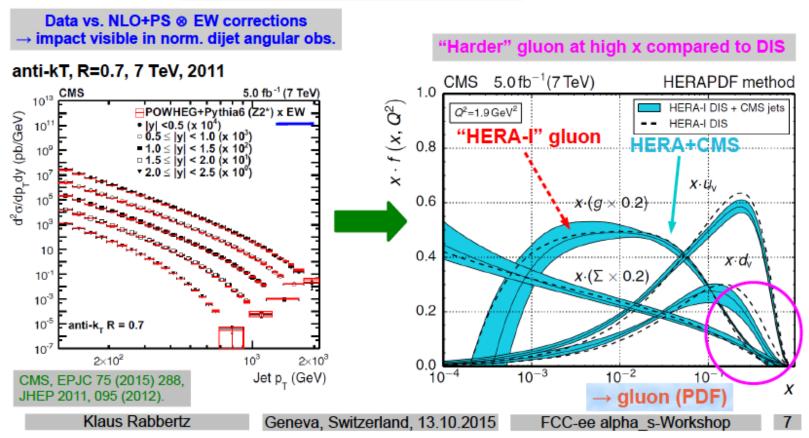
Inclusive Jets + α_s & PDFs



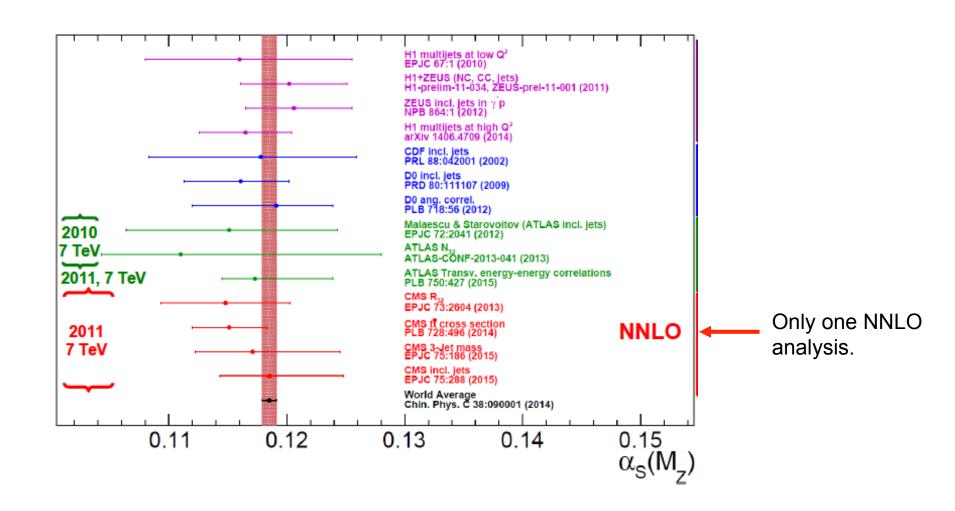
Simultaneous fit of α_s & PDFs possible combining HERA 1 DIS & CMS jet data using HERAFitter Tool

$$Q = p_{\mathrm{T,jet}}$$

$$rac{d^2\sigma}{dp_Tdy} \propto lpha_s^2$$







From K. Rabbertz @ Fcee WS



- Field just at the beginning.
- Correlation to (gluon) pdf shape intrinsic to many observables.
- Only NNLO based analyses reliable.

Ways to improve situation / go on:

 More NNLO analyses to get better overview of how convergent the results are.



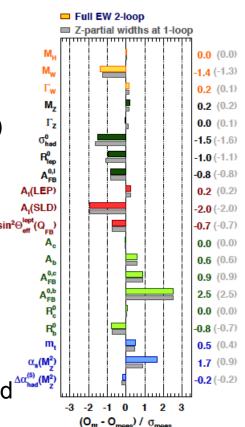
Electroweak Precision Fits

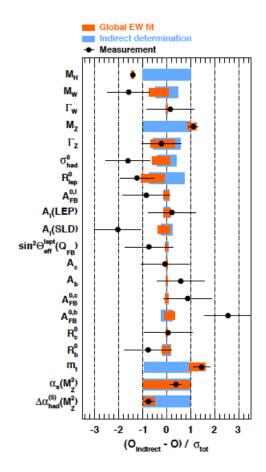
- Global fit on LEP/SLD Z-pole precision observables.
- Strong coupling dependence through NNLO (partial NNNLO)
 QCD corrections to

$$\Gamma_Z$$
 and σ_{had}

$$\alpha_s(M_z) = 0.1196 \pm 0.0030$$

 Error completely dominated by experimental uncertainty:
 Only more precise data can lead to reduction of error.





Gfitter, 1407.3792v2



Conclusions

- Many determinations of $\alpha_s(m_Z)$ with percent-level precision from different fields.
- Results made possible by man advances in pQCD, resummations, lattice QCD, high precision data, field theoretic developments.
- But many high precision determination are in contradiction (if the the uncertainties are interpreted strictly), which indicates that there are still, many issues that are not well understood (missing theory as well as missing data).
- A number of new high-precision determination has lead to an increase of the error in the world average.
- Most discrepancies do not seem to be resolvable by computations of the next orders in pQCD although individual determination might get more precise.
- Future analyses should not only aim for another increase in precision, but also for possible systematic effects and apply conservative error estimates

