



$e^+ e^- \rightarrow b l^+ \nu_l \bar{b} l^- \bar{\nu}_l$   
Top couplings to  $Z, \gamma$  ( $W, H$ )  
(later)



LCWS2015 | International Workshop on Future Linear Colliders  
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Whistler BC Canada

TRIUMF

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LINÉAIRE

November 4<sup>th</sup> 2015

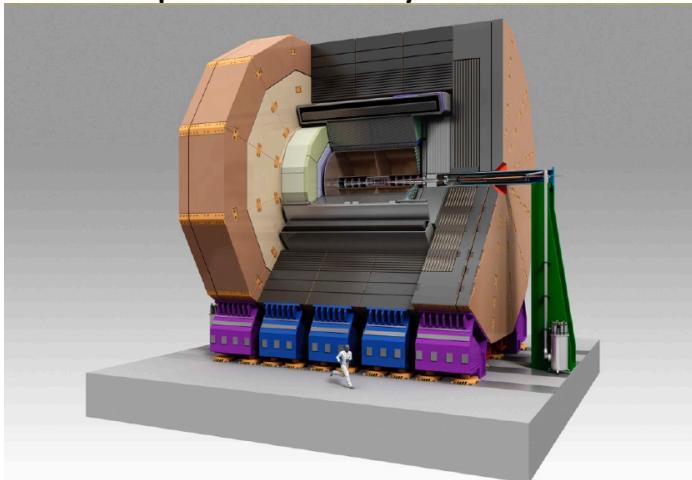


Top couplings to  $Z, \gamma$

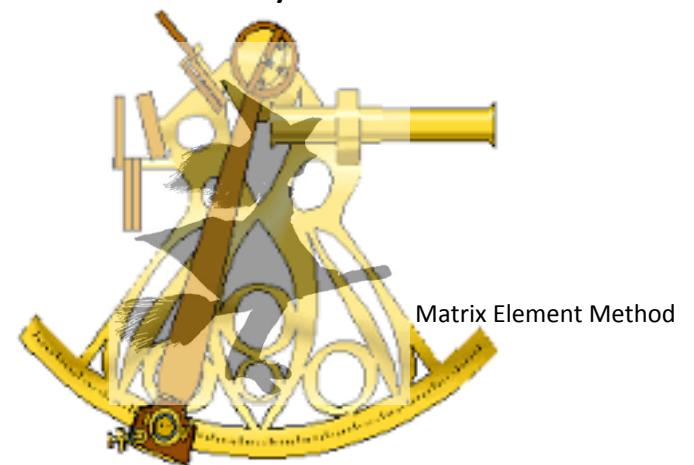
at sub-percent precision level



experimental systematics



theoretical systematics



This work

# Full kinematical reconstruction using pure leptonic final states

The diagram illustrates a particle interaction process. It starts with an incoming electron ( $e^-$ ) and positron ( $e^+$ ). They annihilate at a vertex labeled  $\gamma, Z^0$  into a top quark ( $t$ ) and an anti-top quark ( $\bar{t}$ ). The top quark decays into a muon ( $\mu^-$ ) and an anti-neutrino ( $\bar{\nu}_\mu$ ). The anti-top quark decays into a muon ( $\mu^+$ ) and a neutrino ( $\nu_\mu$ ). The muons and neutrinos are shown as yellow vertical bars.

**To have a full kinematical information, we try to use the fully leptonic final state:**

Angles for mu+, mu- and b, anti-b  
Energies of mu+ and mu-  
Energies of b and anti-b

Reconstruction of top polar/azimuthal angles

3 sigma contours

**Yes, we can!**

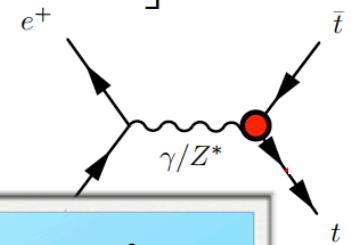
$\cos \theta_t^{\text{reco}} - \cos \theta_t^{\text{true}}$  (in ILC frame)

Rough idea of bottom energy is enough to distinguish two solutions!

# LO Lagrangian for $t\bar{t}$ production

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[ V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

$$\begin{aligned}\mathcal{M}(e_L \bar{e}_R \rightarrow t_L \bar{t}_R)^{\gamma/Z} &= c_L^{\gamma/Z} [F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 + \cos \theta) e^{-i\phi} \\ \mathcal{M}(e_L \bar{e}_R \rightarrow t_R \bar{t}_L)^{\gamma/Z} &= c_L^{\gamma/Z} [F_{1V}^{\gamma/Z} + \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 - \cos \theta) e^{-i\phi}\end{aligned}$$



- \* Using distribution of the events in the 9 dimensional phase space, we fit 10 form factors ( $\text{Re}[F_{1V}^V, F_{1A}^V, F_{2A}^V, F_{2A}^V]$  and  $\text{Im}[F_{2A}^V]$  where  $V=\gamma,Z$ ) !

**more precisely, we fit deviations with respect to SM values.**

$$\mathcal{M}(e_R \bar{e}_L \rightarrow t_R \bar{t}_R)^{\gamma/Z} = c_R^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} - \beta F_{2A}^{\gamma/Z})] \sin \theta e^{i\varphi}$$

where  $\beta^2 = 1 - 4m_t^2/s$ ,  $\gamma = \sqrt{s}/(2m_t)$  and the overall factors  $c_{L/R}^{\gamma/Z}$  are:

$$c_L^\gamma = -1, \quad c_R^\gamma = -1, \quad c_L^Z = \left( \frac{-1/2 + s_w^2}{s_w c_s} \right) \left( \frac{s}{s - m_Z^2} \right), \quad c_R^Z = \left( \frac{s_w^2}{s_w c_s} \right) \left( \frac{s}{s - m_Z^2} \right)$$

where  $s_w = \sin \theta_w$  and  $c_w = \cos \theta_W$ , with  $\theta_W$  being the weak mixing angle.

$$F_{1V}^{\gamma,SM} = -\frac{2}{3}, \quad F_{1A}^{\gamma,SM} = 0, \quad F_{1V}^{Z,SM} = -\frac{1}{4s_w c_w} \left( 1 - \frac{8}{3} s_w^2 \right), \quad F_{1A}^{Z,SM} = \frac{1}{4s_w c_w}, \quad F_{2V}^\gamma = Q_t(g-2)/2$$

Single-parameter fit

$$\chi^2(r) = -2 \left( \sum_{e=1}^N \ln \left( 1 + r \omega_r + r^2 \tilde{\omega}_r \right) - N_0(r \Omega_r + r^2 \tilde{\Omega}_r) \right)$$

Quadratic dependence in "r"

$\delta F_{2V}^Z$   
(as an example)

$r$  theoretical parameter  
 $\rightarrow \omega_r, \tilde{\omega}_r$  kinematical expressions

$$N_0 = \int f_{\text{SM}}(\omega_r, \tilde{\omega}_r) d\omega_r d\tilde{\omega}_r$$

$$\Omega_r = \langle \omega_r \rangle_{\text{SM-LO}} = \int \omega_r f_{\text{SM}}(\omega_r, \tilde{\omega}_r) d\omega_r d\tilde{\omega}_r$$

$$\tilde{\Omega}_r = \langle \tilde{\omega}_r \rangle_{\text{SM-LO}} = \int \tilde{\omega}_r f_{\text{SM}}(\omega_r, \tilde{\omega}_r) d\omega_r d\tilde{\omega}_r$$

$f_{\text{SM}}$  includes selection efficiency

Multi-parameter fit

$$\chi^2(\vec{r}) = -2 \left( \sum_{e=1}^N \ln \left( 1 + \sum_i r_i \omega_i + \sum_{ij} r_i r_j \tilde{\omega}_{ij} \right) - N_0 \left( \sum_i r_i \Omega_i + \sum_{ij} r_i r_j \tilde{\Omega}_{ij} \right) \right)$$

# Numerical analysis: SM LO result

Statistical uncertainties and correlation with the SM LO  
(no fit, but analytical expressions)

$\text{Re } \delta\tilde{F}_{1V}^\gamma$	$\text{Re } \delta\tilde{F}_{1V}^Z$	$\text{Re } \delta\tilde{F}_{1A}^\gamma$	$\text{Re } \delta\tilde{F}_{1A}^Z$	$\text{Re } \delta\tilde{F}_{2V}^\gamma$	$\text{Re } \delta\tilde{F}_{2V}^Z$	$\text{Re } \delta\tilde{F}_{2A}^\gamma$	$\text{Re } \delta\tilde{F}_{2A}^Z$	$\text{Im } \delta\tilde{F}_{2A}^\gamma$	$\text{Im } \delta\tilde{F}_{2A}^Z$
0.0037	-0.18	-0.09	+0.14	+0.62	-0.15	0	0	0	0
	0.0063	+.14	-0.06	-0.13	+0.61	0	0	0	0
		0.0053	-0.15	-0.05	+0.09	0	0	0	0
			0.0083	+0.06	-0.04	0	0	0	0
				0.0105	-0.19	0	0	0	0
					0.0169	0	0	0	0
						0.0068	-0.15	0	0
							0.0118	0	0
								0.0069	-0.17
									0.0100

% level

(17.5 k events) 500 GeV&500 fb<sup>-1</sup> Polarization 50/50 between ±80% and ±30%  
Probing New Physics using top quark polarization in the e+e- → t̄t process at future Linear Colliders arXiv:1503.04247

\*Our result (at tree level) shows that the 10 form factors can be fitted simultaneously at less than a percent precision !

\*Indeed, the matrix element method (with full kinematical reconstruction) is a powerful tool!

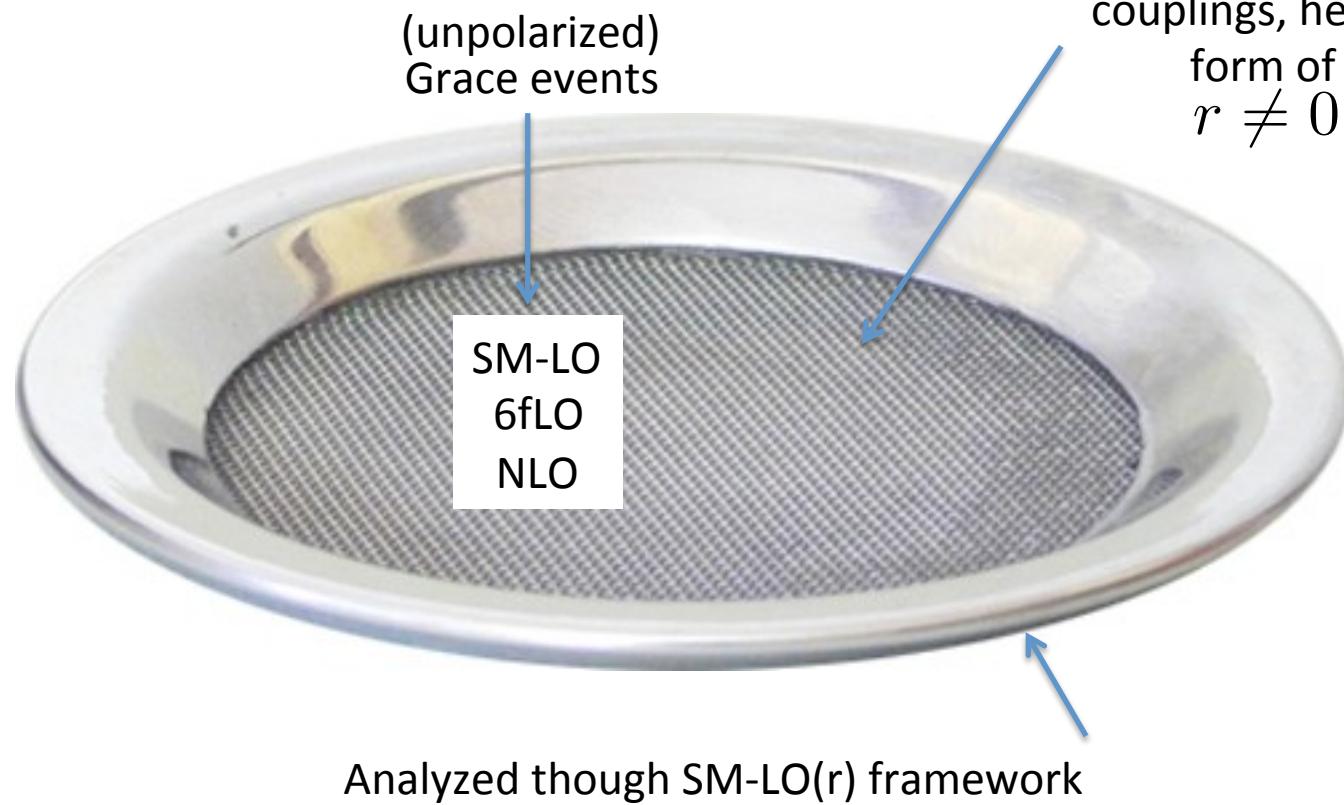


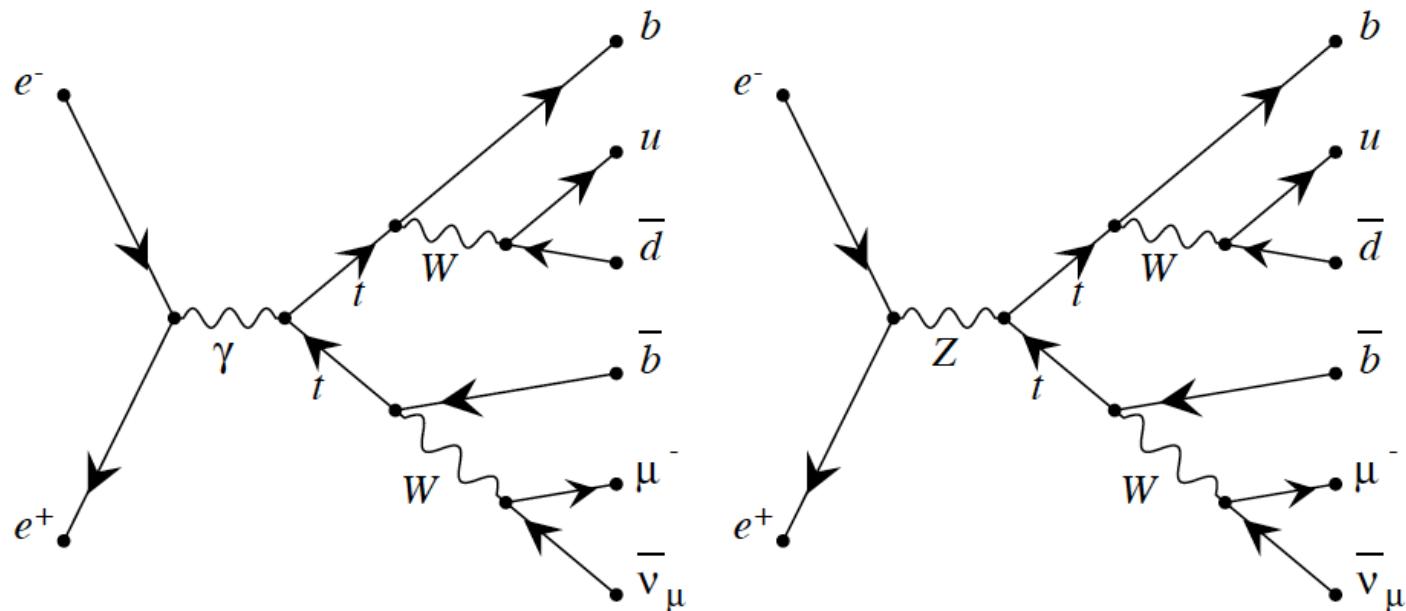
Our target is to study NLOew corrections from both experimentalist and theorist viewpoints:  
How precisely can ILC unfold NLOew corrections to unravel New Physics ?

New developments since arXiv:1503.04247 :

Diamond sieve

Goal is to detect deviation from SM couplings, here in form of  $r \neq 0$

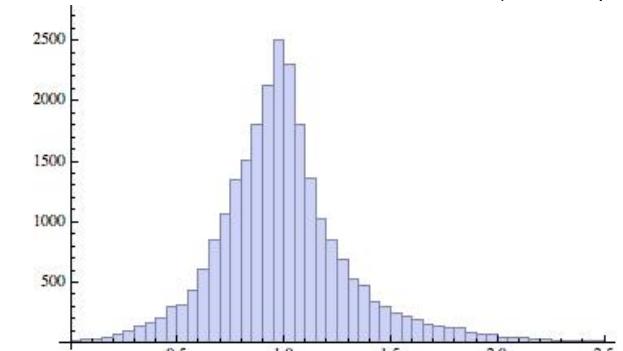




$$| (t^\uparrow \bar{t}^\uparrow + t^\uparrow \bar{t}^\downarrow + t^\downarrow \bar{t}^\uparrow + t^\downarrow \bar{t}^\downarrow)_\gamma + (t^\uparrow \bar{t}^\uparrow + t^\uparrow \bar{t}^\downarrow + t^\downarrow \bar{t}^\uparrow + t^\downarrow \bar{t}^\downarrow)_Z |^2$$

## LO-Data

(100's different components in  $|\mathcal{M}|^2$ )  
Very rich structure in 9 dimensions

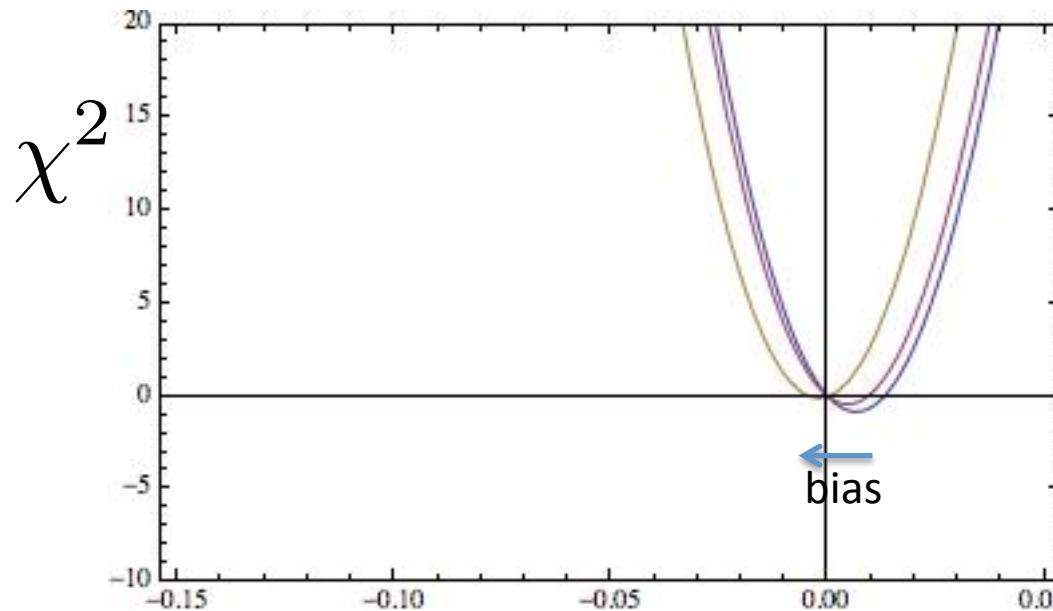


$$\frac{|M_{\text{full}}|^2}{|M_{\text{no interference}}|^2}$$

## LO Top and W widths + wrong b-jet

$\chi^2_{\text{rec}}$  : cut to keep 90% of events

Fit with top and W masses free to vary  
reconstructed masses within 10 GeV of nominal values  
overall 85% selection efficiency

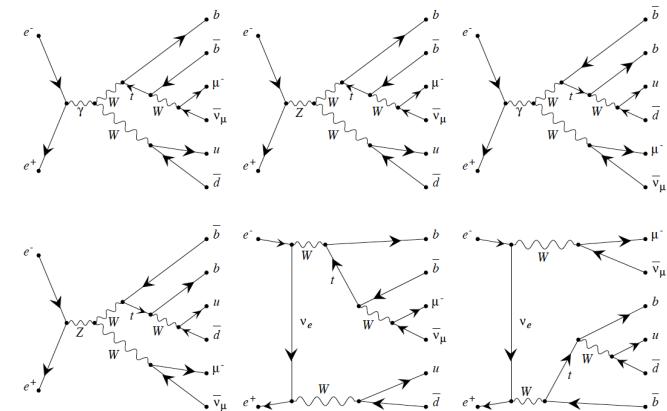
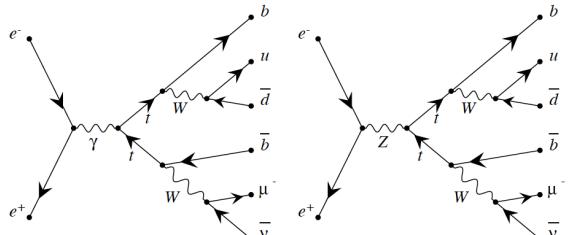


if only  $\chi^2$  cut : bias close to 1%

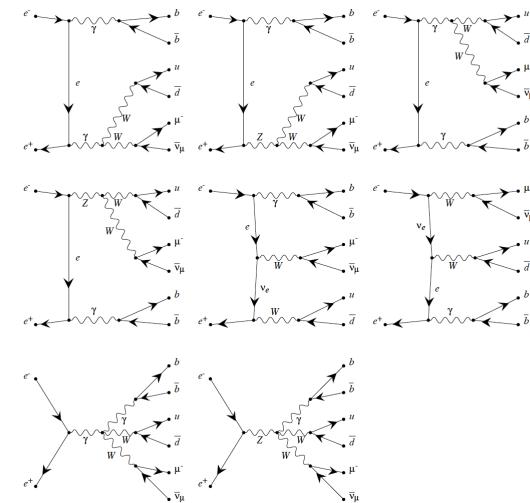
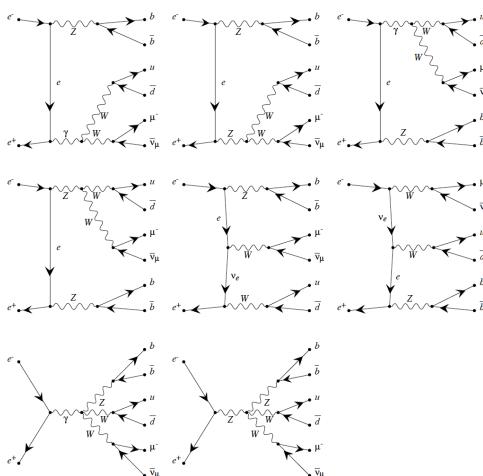
$\delta F_{2V}^Z$

Within 85% cut, bias appear to be well below the percent level



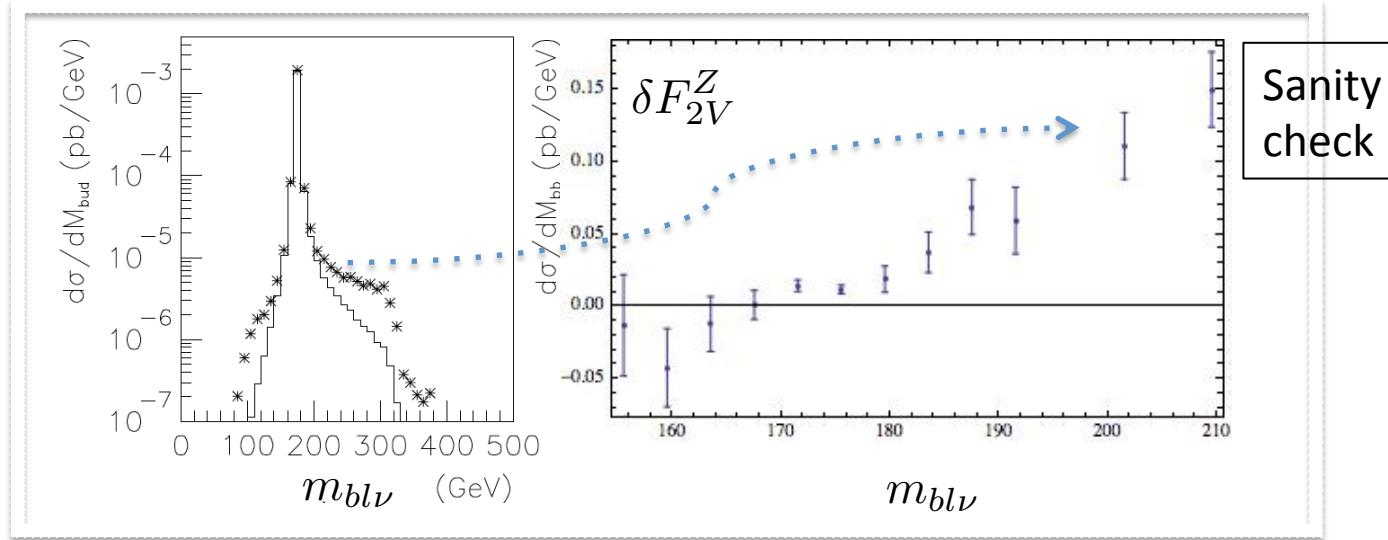


# 6fLO-Data



6fLO

$$\delta F_{2V}^Z = 0.0134 \pm 0.0022 \text{ (10}^5 \text{ events)}$$



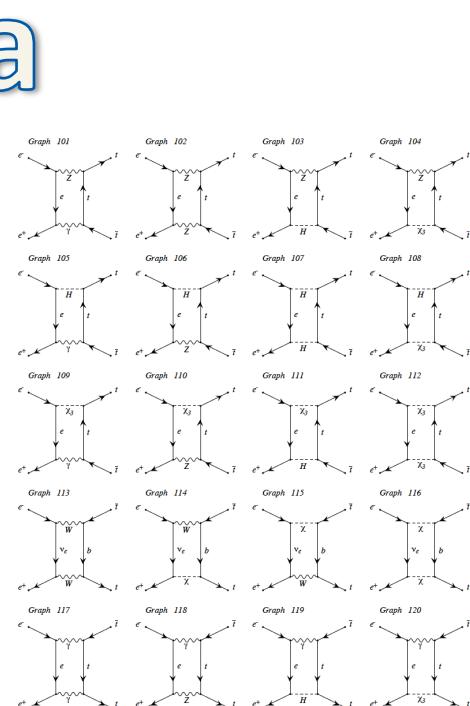
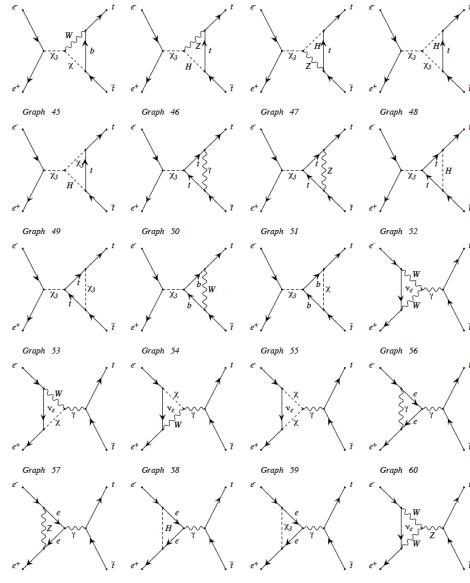
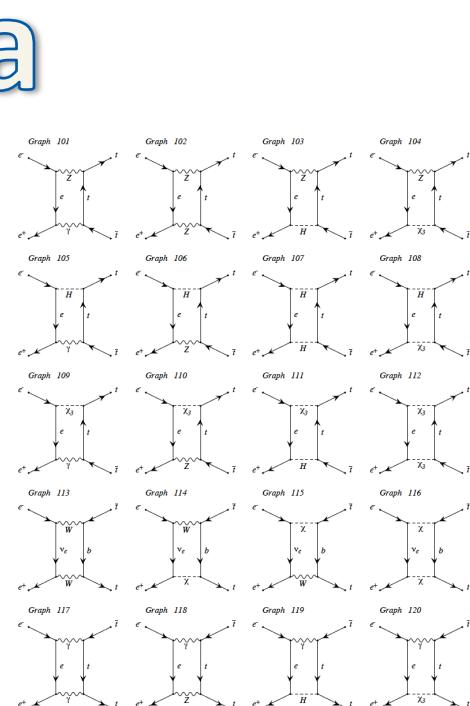
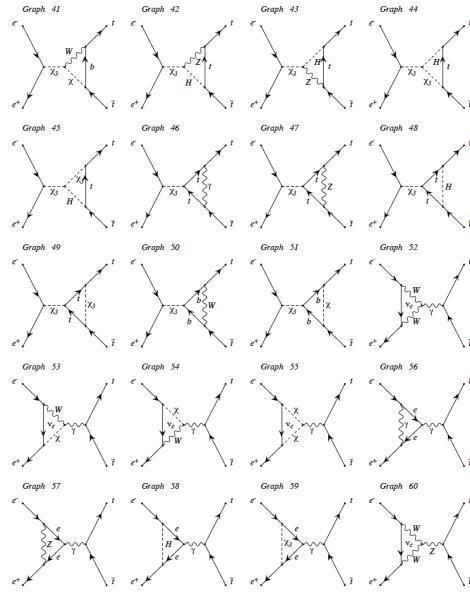
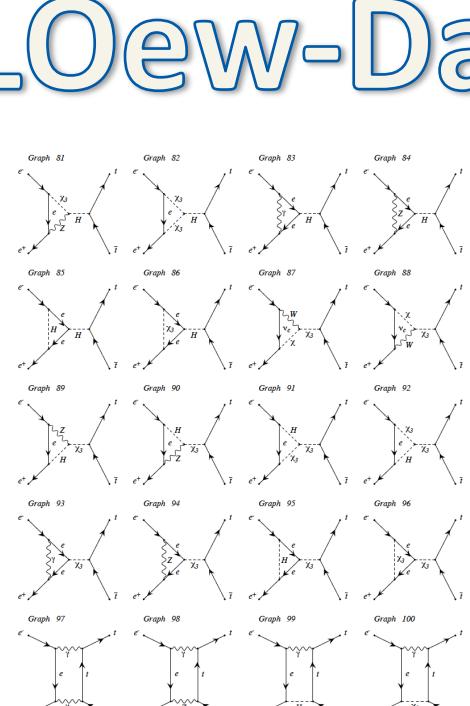
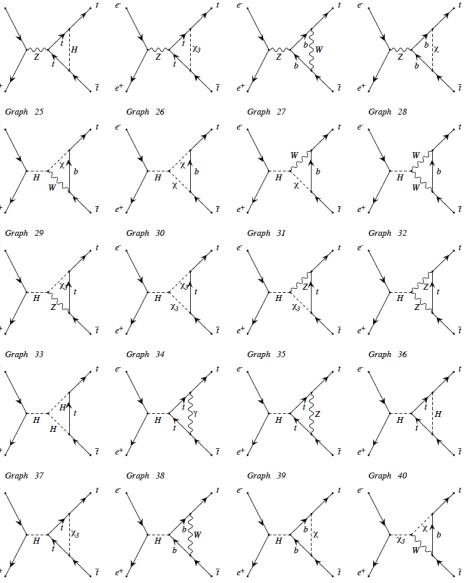
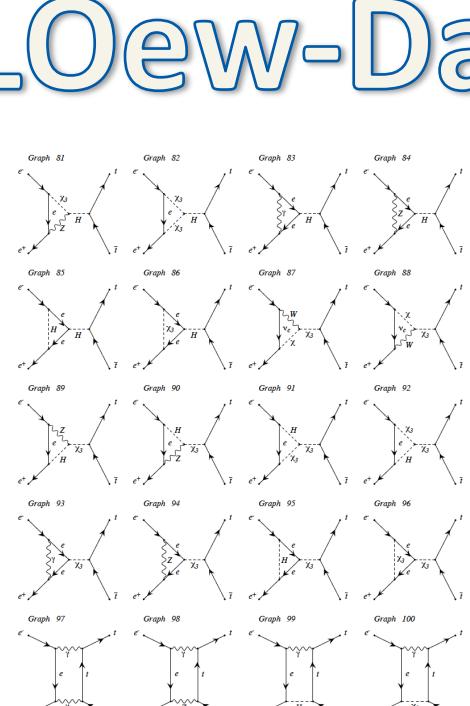
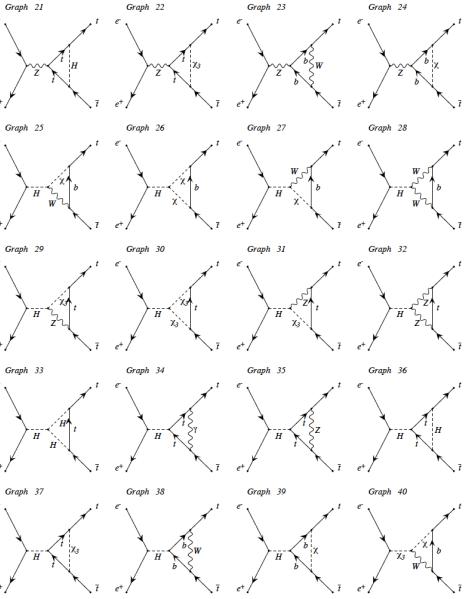
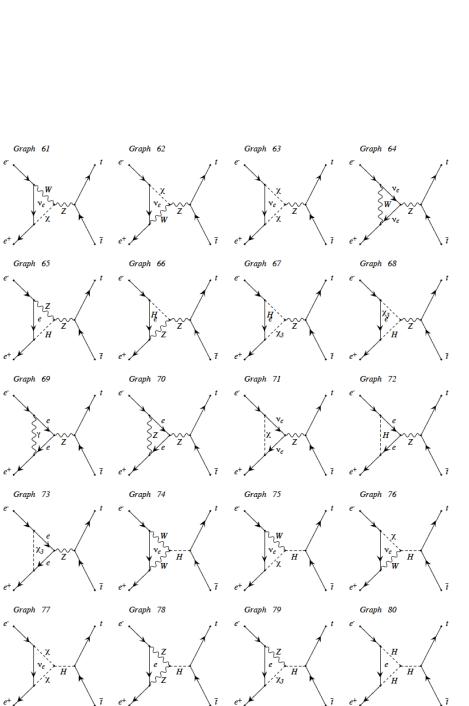
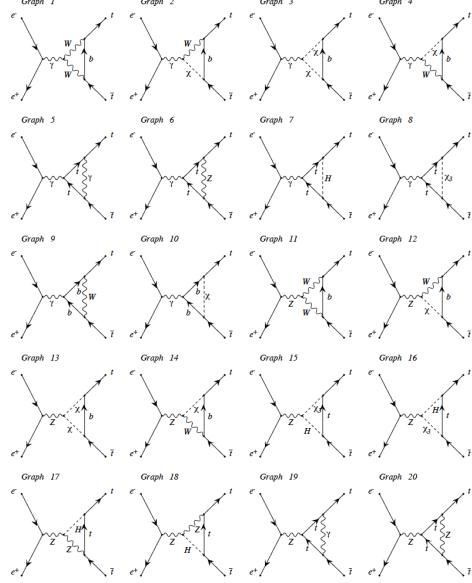
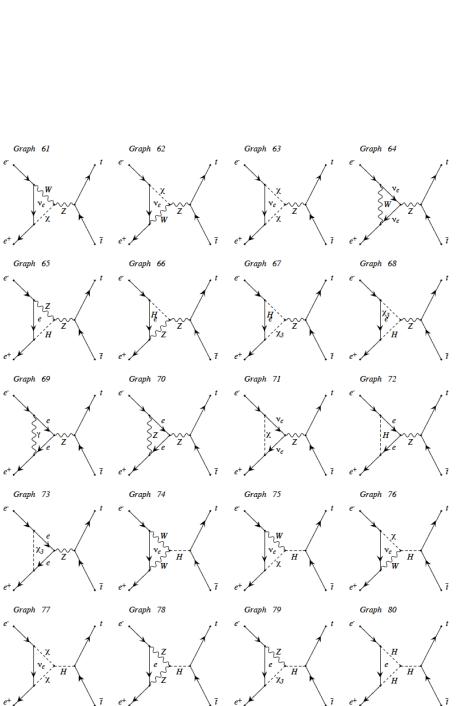
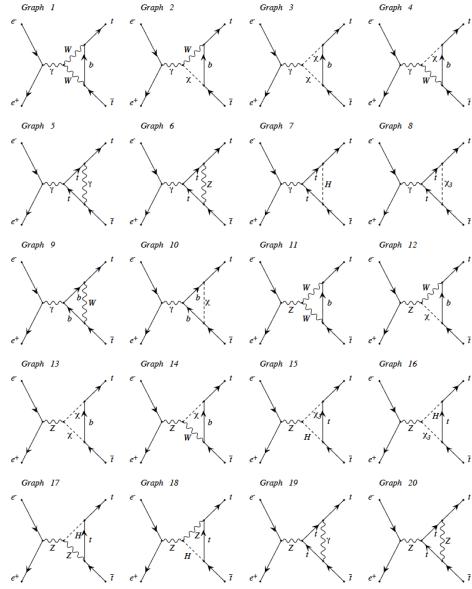
10 parameter fit : the effect is diluted ( $4 \cdot 10^4$  events)

$\chi^2 = 38.9$  with respect to SM-LO: a 4.3 sigma effect.

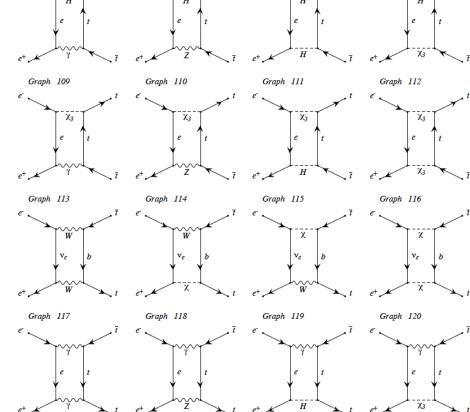
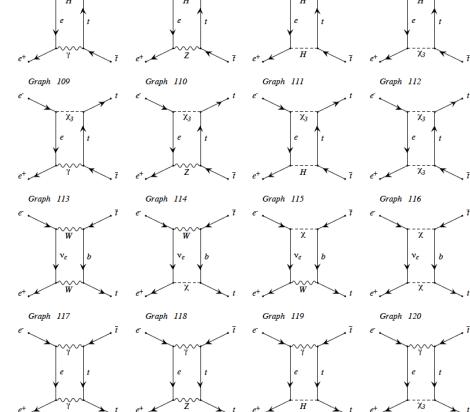
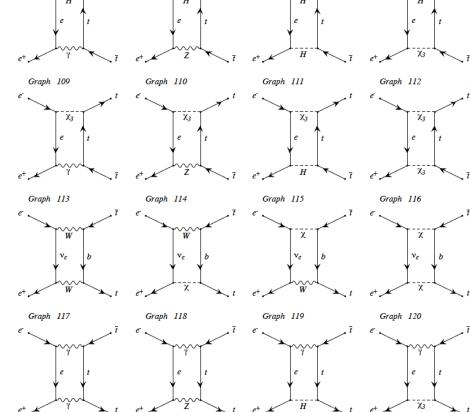
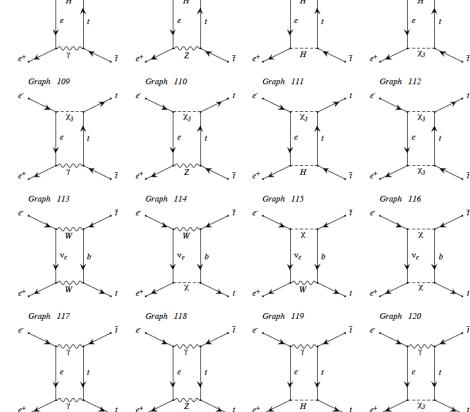
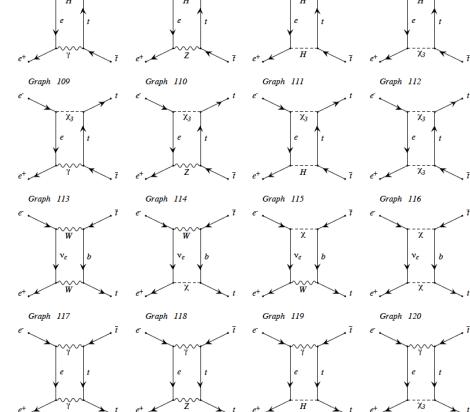
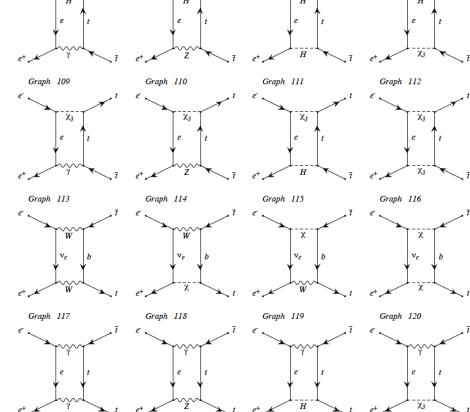
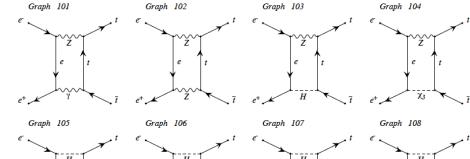
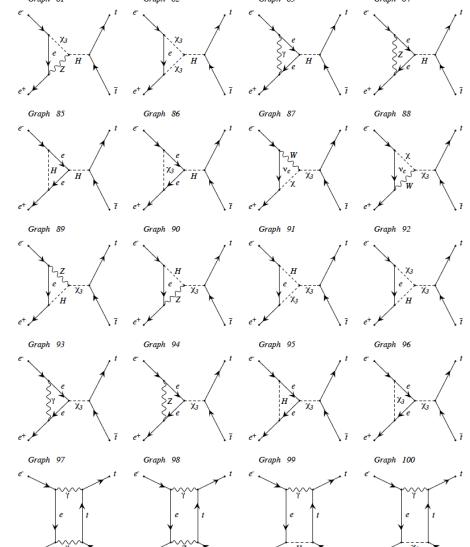
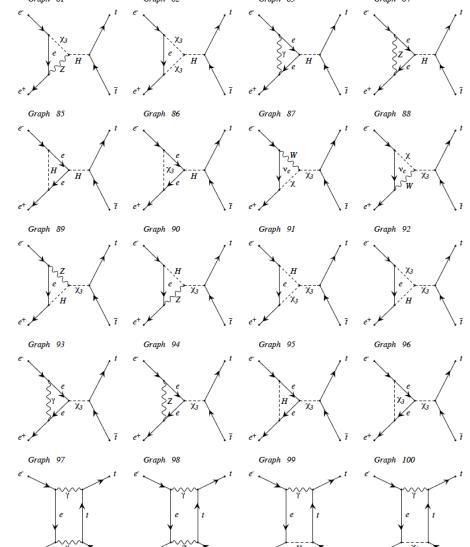
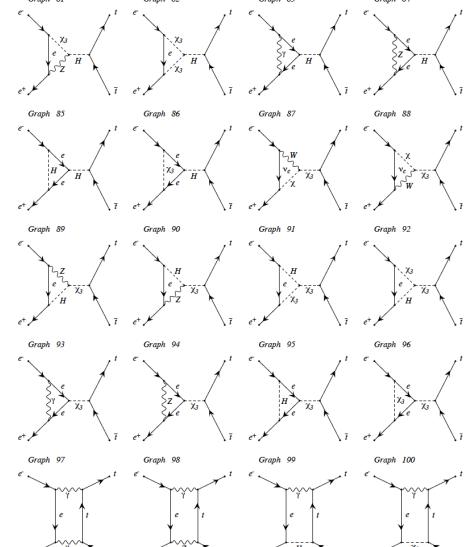
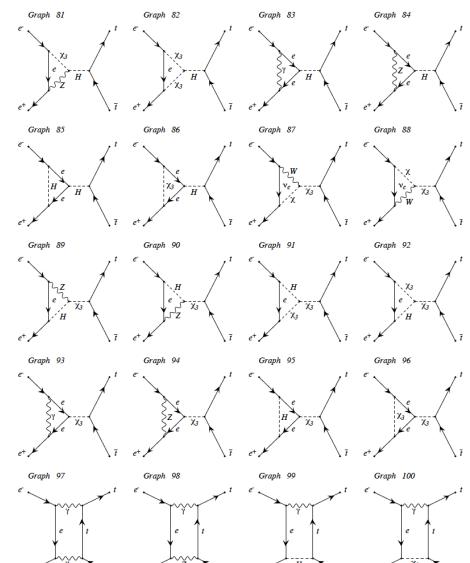
$\text{Re } \delta \tilde{F}_{1V}^\gamma$	$\text{Re } \delta \tilde{F}_{1V}^Z$	$\text{Re } \delta \tilde{F}_{1A}^\gamma$	$\text{Re } \delta \tilde{F}_{1A}^Z$	$\text{Re } \delta \tilde{F}_{2V}^\gamma$	$\text{Re } \delta \tilde{F}_{2V}^Z$	$\text{Re } \delta \tilde{F}_{2A}^\gamma$	$\text{Re } \delta \tilde{F}_{2A}^Z$	$\text{Im } \delta \tilde{F}_{2A}^\gamma$	$\text{Im } \delta \tilde{F}_{2A}^Z$
-0.0082	+0.0215	-0.0066	-0.0098	-0.0138	+0.0008	+0.0062	-0.0082	+0.0024	+0.0012
0.0028	0.0059	0.0043	0.0059	0.0091	0.0151	0.0051	0.0116	0.0082	0.0073

Six Fermion effect is small : not a serious issue  
(but detectable) (too)





# NLOew-Data



# NLO

$$\delta F_{2V}^Z = -0.1070 \pm 0.0050 \quad (23k \text{ events})$$

10-parameter fit

$\text{Re } \delta\tilde{F}_{1V}^\gamma$	$\text{Re } \delta\tilde{F}_{1V}^Z$	$\text{Re } \delta\tilde{F}_{1A}^\gamma$	$\text{Re } \delta\tilde{F}_{1A}^Z$	$\text{Re } \delta\tilde{F}_{2V}^\gamma$	$\text{Re } \delta\tilde{F}_{2V}^Z$	$\text{Re } \delta\tilde{F}_{2A}^\gamma$	$\text{Re } \delta\tilde{F}_{2A}^Z$	$\text{Im } \delta\tilde{F}_{2A}^\gamma$	$\text{Im } \delta\tilde{F}_{2A}^Z$
+0.0107	-0.0124	+0.0475	+0.1072	+1.1445	+0.0539	+0.0109	+0.0869	-0.3148	+0.1221
0.0044	0.0094	0.0070	0.0101	0.0293	0.0207	0.0105	0.0217	0.0236	0.0151

The  $\chi^2$  of SM-LO is 1184 units above the minimum,

Fake CP violation  
(understood)

10 fits each with a single parameter

$\text{Re } \delta\tilde{F}_{1V}^\gamma$	$\text{Re } \delta\tilde{F}_{1V}^Z$	$\text{Re } \delta\tilde{F}_{1A}^\gamma$	$\text{Re } \delta\tilde{F}_{1A}^Z$	$\text{Re } \delta\tilde{F}_{2V}^\gamma$	$\text{Re } \delta\tilde{F}_{2V}^Z$	$\text{Re } \delta\tilde{F}_{2A}^\gamma$	$\text{Re } \delta\tilde{F}_{2A}^Z$	$\text{Im } \delta\tilde{F}_{2A}^\gamma$	$\text{Im } \delta\tilde{F}_{2A}^Z$
+0.0150	-0.1126	+0.0685	+0.1008	-0.0379	+0.4642	-0.0052	+0.0802	-0.2067	+0.0349
0.0027	0.0064	0.0066	0.0092	0.0096	0.0173	0.0085	0.0329	0.0252	0.0127

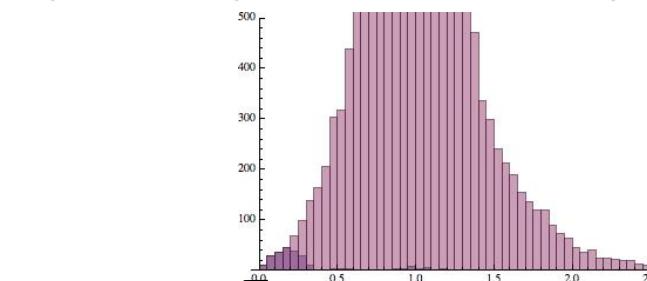
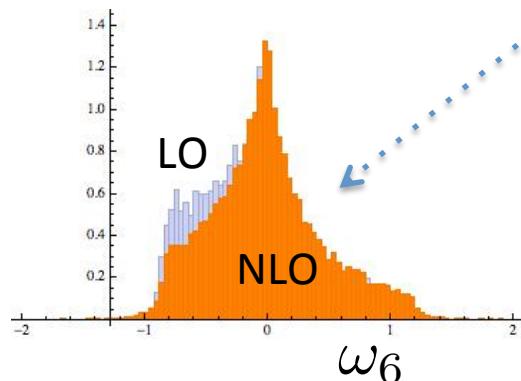
A part of the (huge) effect stems from very large  $|\tilde{\omega}_{ij}|$  ← outliers

$$\chi^2(\vec{r}) = -2 \left( \sum_{e=1}^N \ln \left( 1 + \sum_i r_i \omega_i + \sum_{ij} r_i r_j \tilde{\omega}_{ij} \right) - N_0 \left( \sum_i r_i \Omega_i + \sum_{ij} r_i r_j \tilde{\Omega}_{ij} \right) \right)$$

10-parameter fit

$$|\tilde{\omega}_{ij}| \leq 10 \quad \leftarrow \text{Outliers removed (shouldn't)}$$

$\mathcal{R}\text{e } \delta\tilde{F}_{1V}^\gamma$	$\mathcal{R}\text{e } \delta\tilde{F}_{1V}^Z$	$\mathcal{R}\text{e } \delta\tilde{F}_{1A}^\gamma$	$\mathcal{R}\text{e } \delta\tilde{F}_{1A}^Z$	$\mathcal{R}\text{e } \delta\tilde{F}_{2V}^\gamma$	$\mathcal{R}\text{e } \delta\tilde{F}_{2V}^Z$	$\mathcal{R}\text{e } \delta\tilde{F}_{2A}^\gamma$	$\mathcal{R}\text{e } \delta\tilde{F}_{2A}^Z$	$\mathcal{I}\text{m } \delta\tilde{F}_{2A}^\gamma$	$\mathcal{I}\text{m } \delta\tilde{F}_{2A}^Z$
+0.0131	-0.0094	+0.0592	+0.0924	-0.0176	+0.4416	-0.0071	+0.0916	-0.0326	+0.0243
0.0049	0.0105	0.0077	0.0108	0.0136	0.0262	0.0083	0.0211	0.0210	0.0133
+0.0107	-0.0124	+0.0475	+0.1072	+1.1445	+0.0539	+0.0109	+0.0869	-0.3148	+0.1221



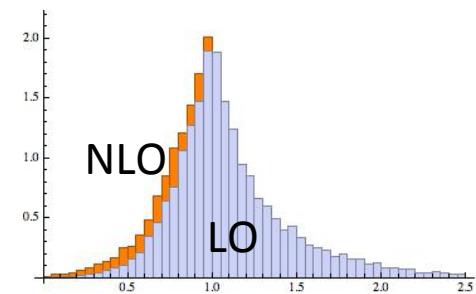
$$\frac{|\mathcal{M}_{\text{full}}|^2}{|\mathcal{M}_{\text{no interference}}|^2}$$

But

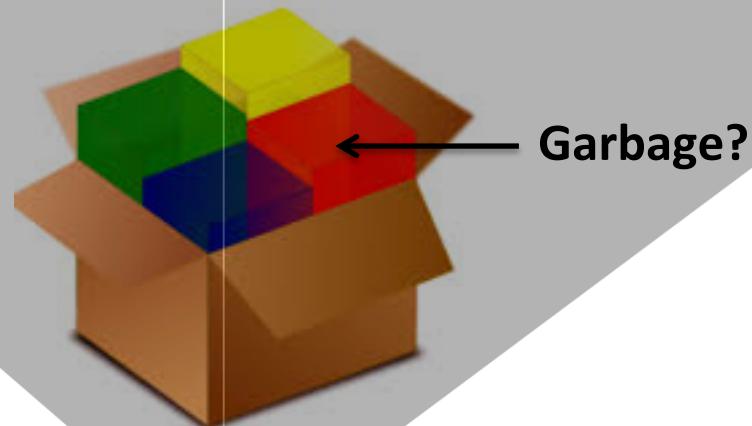
Goodness of fit

?????????????

Is SM-LO(r) able to describe NLO-Data  
Can NLO effects be absorbed into form factors



# Goodness of Fits



Garbage?

$r_i$  as given by the fit

$$f_{\text{LO}}(\vec{r}) = (1 + \sum_i r_i \omega_i + \sum_{ij} r_i r_j \tilde{\omega}_{ij}) f_{\text{SM}}(\omega, \tilde{\omega})$$

$$\langle \omega_k \rangle = \Omega_k + \sum_i r_i \Omega_{ik} + \sum_{ij} r_i r_j \tilde{\Omega}_{ij;k}$$

$$\langle \tilde{\omega}_{kl} \rangle = \tilde{\Omega}_{kl} + \sum_i r_i \tilde{\Omega}_{kl;i} + \sum_{ij} r_i r_j \tilde{\Omega}_{ijkl}$$

↑  
Not explicitly present in the Likelihood

10-parameter fit



Using these moments, one can take the fit by surprise and quantify the goodness of the fits.

$$\Omega_k = \int \omega_k f_{\text{SM}}(\omega, \tilde{\omega})$$

$$\Omega_{ik} = \int \omega_i \omega_k f_{\text{SM}}(\omega, \tilde{\omega})$$

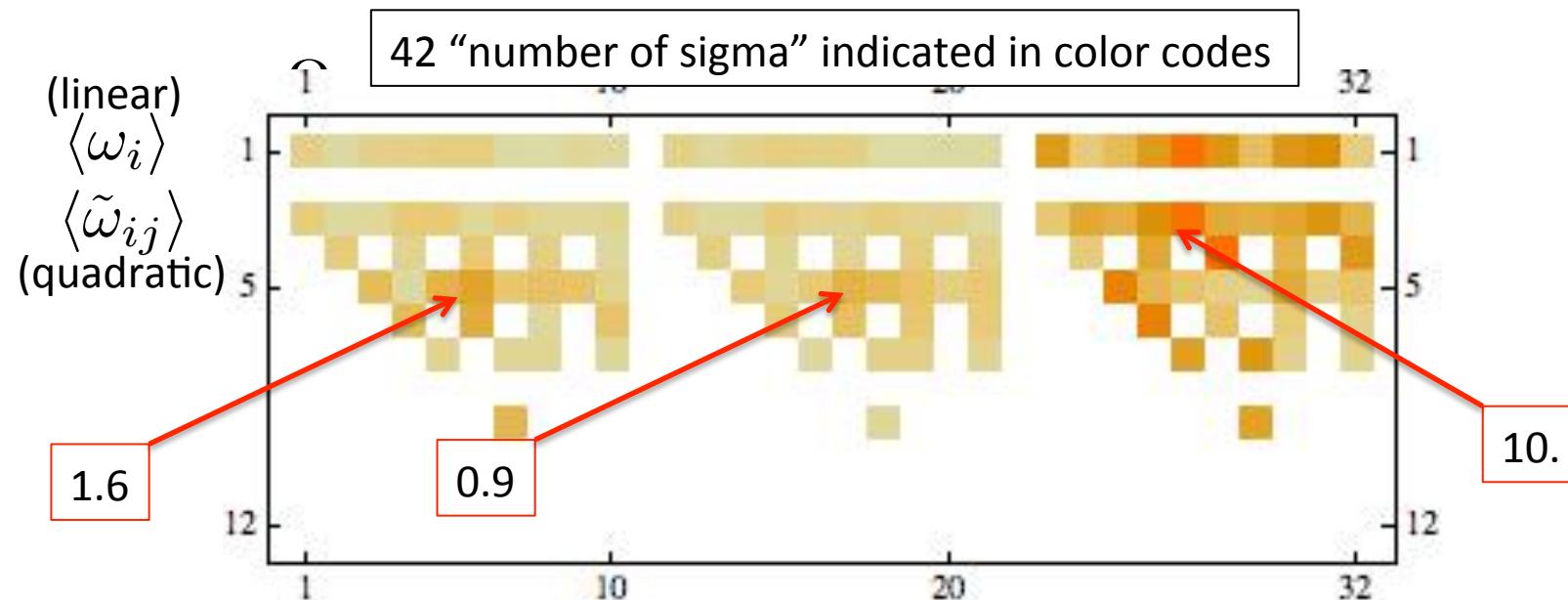
$$\tilde{\Omega}_{ij;k} = \int \tilde{\omega}_{ij} \omega_k f_{\text{SM}}(\omega, \tilde{\omega})$$

$$\tilde{\Omega}_{kl} = \int \tilde{\omega}_{kl} f_{\text{SM}}(\omega, \tilde{\omega})$$

$$\tilde{\Omega}_{ijkl} = \int \tilde{\omega}_{ij} \tilde{\omega}_{kl} f_{\text{SM}}(\omega, \tilde{\omega})$$

# Goodness of fit

(scaled to 10k events)



LO



6fLO



NLO



# Conclusion

$$e^+ e^- \rightarrow b l^+ \nu_l \bar{b} l^- \nu_l$$

appears promising as a very sensitive tool  
to search for new physics



NLOew analyzed through SM-LO(r) framework  
 $r \neq 0$  , but **Badness** of fit !



**Need to move to rNLOew framework**

Many things to do, not to mention ... detector effects.