Unitarity bounds in general 2HDM

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arXiv: 1509.06060 Phys. Lett. B, in press

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Introduction

Discovery of *h*(125) at LHC in 2012

- Existence of a scalar particle,
- Mass and measured couplings are consistent with the SM

Higgs sector remains unknown

- Most of extended Higgs sectors can also satisfy current data as well

Requirement of BSM

- Hierarchy Problem *SUSY, Dynamical Symmetry Breaking, Shift-Symmetry, …*
- BSM Phenomena *Baryon Asymmetry, Neutrino Masses, Dark Matter, …*

Higgs as a probe of new physics

- Shape of Higgs sector (multiplet structure, symmetry, scales, …) is related to BSM scenarios
- Essence of the Higgs particle is directly connected to a BSM paradigm

It is important to determine the structure of Higgs sector, which shall open the door to new physics BSM

Ways of approaching BSM

Assume new paradigms and evaluate phenomena

- Useful as benchmarks
- Based on each belief

Effective Theory Approaches with higher dim. operators

- General, but non-renormalizable
- Low energy theory may not be always the SM with one doublet scalar

Non-minimal Higgs models

- Renormalizable
- Alternative low energy effective theory
- Infinite kinds of extended models to be considered

2^{nd} Simplest Higgs models

Multiplet Structure (2^{nd} simplest Higgs models)

Φ_{SM}+Singlet, Φ_{SM}+Doublet (2HDM),

Φ_{SM}+Triplet, …

Additional Symmetry

Discrete or Continuous?

Exact or Softly broken?

Interaction

Weakly coupled or Strongly Coupled ?

Decoupling or Non-decoupling?

Note: 2^{nd} simplest Higgs models (HSM, 2HDMs, …) can be effective theories of more complicated Higgs sectors

Probing the *extended* Higgs sector

How we experimentally study non-minimal Higgs sectors?

- **Direct Searches of additional Higgs bosons (H, A, H^+, H^{++}, …)**

- **Indirect Searches by detecting deviations in various quantities**

  EW observables m_W, S, T, U, Zff, Wff', WWV, …

  ***h*(125) couplings** hWW, hZZ, $h\gamma\gamma$, hff, hhh, …

They will be precisely measured at future experiments

How we constrain extended Higgs sectors ?

Theoretical Bound

- Unitarity
- Triviarity
- Vacuum stability, …

Experimental bounds

- LEP, Tevatron direct searches
- LEP/SLC indirect searches
- LHC direct/indirect searches
- $b \to s\gamma$/g-2/EDM/…

This talk

- We discuss the bound from perturbative unitarity on the parameters of an extended Higgs sectors

  (a general 2HDM with CPV)

- Lee, Quigg, and Thacker showed that there is the upper bound on the mass of the SM Higgs

  Lee, Quigg, Thacker (1977)

- Now that the Higgs *h* was found, and we know the mass to be 125GeV

- What is the next?

- Bound on the mass of second lightest Higgs boson (the scale of new physics!)

  SK, Yagyu, arXiv: 1509.06060 (2015)

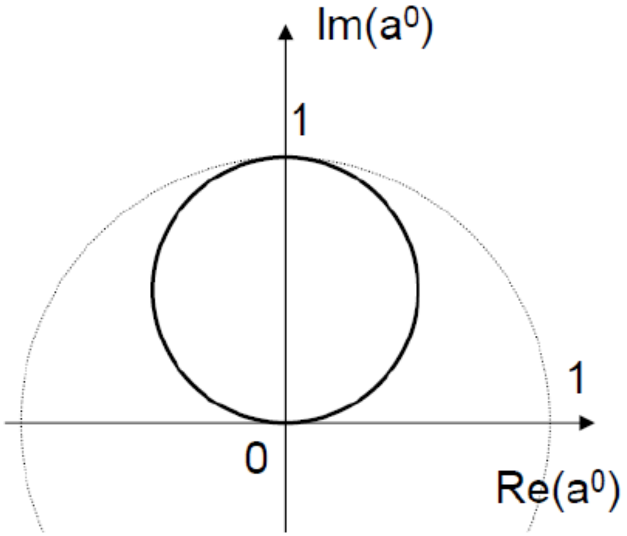
Unitarity in elastic scatterings

A S-Matrix is unitary

$$|S|^2 = 1$$

For 2→2 elastic scatterings,
S-wave amplitudes satisfy

$$\boxed{|a^0| = \mathrm{Im}\, a^0}$$



a^0 is on the circle for unitarity

Argand diagram

Perturbative Unitarity

If perturbation calculation is correct,
the tree-level result should be near the circle.

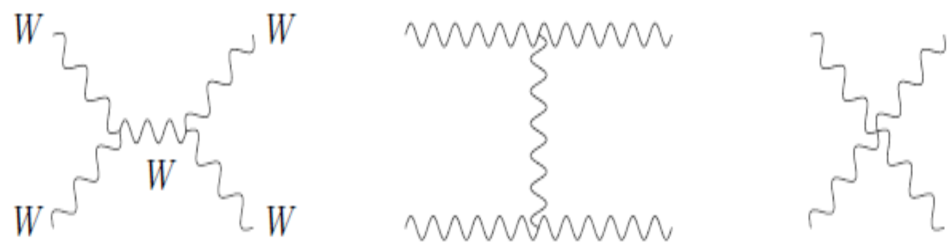
Perturbative Unitarity

Lee, Quigg, Thacker (1977)

$W_L^+ W_L^-$ Elastic Scattering $\qquad \epsilon_L^\mu = (p, 0, 0, E)$

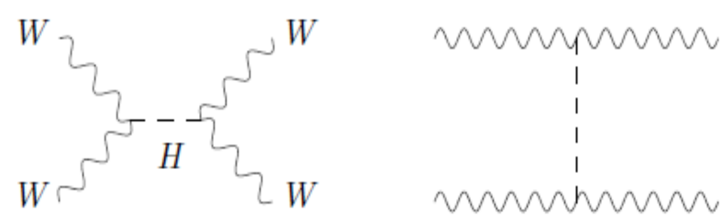
$$a^0(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \approx A\,E^4 + B\,E^2 + C \qquad (E \rightarrow \infty)$$

Unitarity Violation if *A, B* ≠ 0



A=0 by gauge symmetry

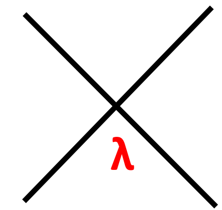
To make *B* = 0, diagrams mediated by a scalar field must be added



A Higgs field *h* is required to save unitarity

In the SM $\quad |C| < 1 \;\Rightarrow\; m_h < 1.2\ \mathrm{TeV}$

Perturbative Unitarity



m_h^2=2 λ v^2

Multi-Channel Unitarity $W_L^+W_L^-, Z_LZ_L, hh, Zh$

O(4): φ= (w_1, w_2, z, h)

$$a^0 \to -\frac{G_F m_h^2}{4\pi\sqrt{2}} \times \begin{pmatrix} 1 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & 0 \\ \frac{1}{\sqrt{8}} & \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{\sqrt{8}} & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$4 \otimes 4 = 1 \oplus 9 \oplus 6$$

(1)	+ 2ww + zz + hh
$(9)_{33} + (9)_{44}$	- 2ww + zz + hh
$(9)_{33} - (9)_{44}$	zz - hh
$(9)_{34}$	zh

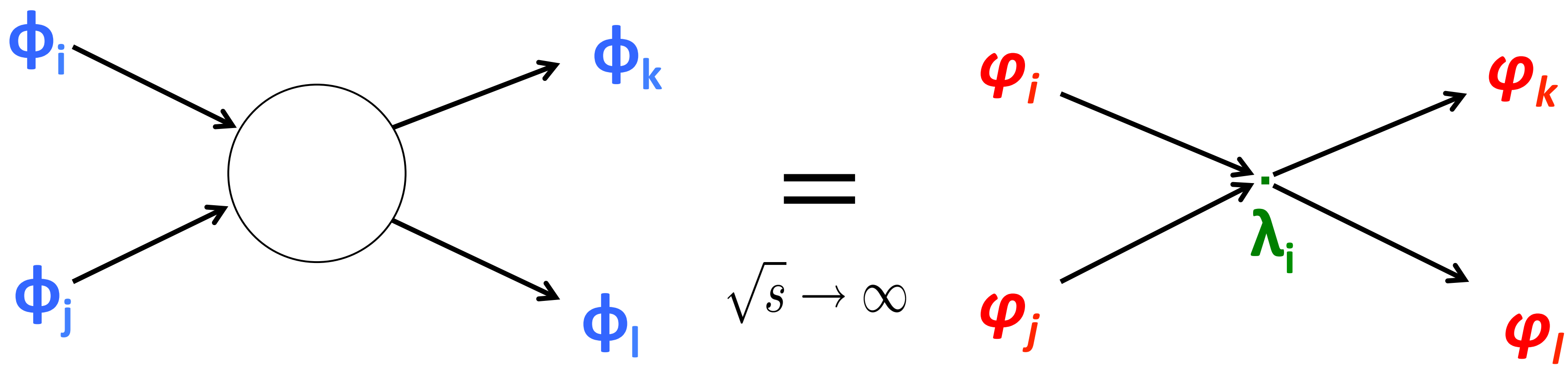
Eigenvalues $\left(\frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$$|a^0| < 1 \quad \Rightarrow \quad m_h < 1\ \mathrm{TeV}$$

Lee, Quigg, Thacker (1977)

Equivalence Theorem

Scattering amplitudes



$\phi = W_L^+, W_L^-, Z_L, h$

$\varphi = \underline{w^+, w^-, z,}\ h$

NG bosons

SU(2) $\sim$ O(4)
is restored

2HDM with softly broken Z_2

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)}$$
$$+\frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2$$
$$+\lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + (\text{h.c.})\right]$$

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

Φ_1 and $\Phi_2 \Rightarrow \underline{h, \quad H,}\quad A^0, H^\pm \oplus$ Goldstone bosons

↑ ↑ ↑charged

CPeven CPodd

Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \qquad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix}$$

$$\begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan\beta$$

$$m_h^2 = v^2 \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda)\sin^2\beta\cos^2\beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

M_{soft}: soft breaking scale

$M_{\text{soft}} \ \left(= \frac{m_3}{\sqrt{\cos\beta\sin\beta}}\right)$:

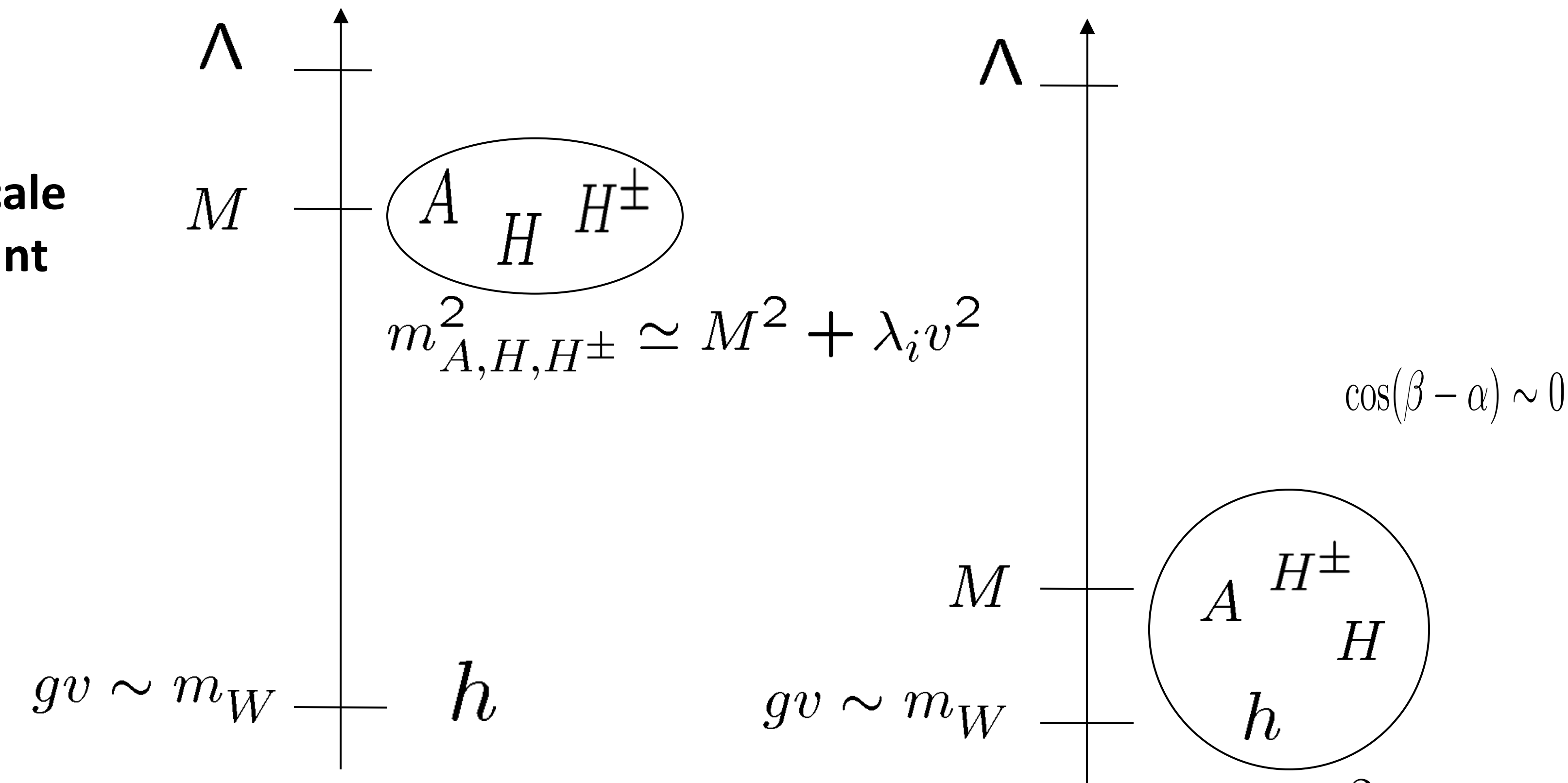
soft-breaking scale

of the discrete symm.

Decoupling/Non-decoupling

Λ: Cutoff

***M*: Mass scale irrelevant to VEV**



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{v^2}{M^2}\mathcal{O}^{(6)}$$

Effective Theory is the SM

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{nonSM}} + \frac{v^2}{\Lambda^2}\mathcal{O}^{(6)}$$

Effective Theory is an extended Higgs sector

Most general Higgs potential

$$V = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - (m_3^2\Phi_1^\dagger\Phi_2 + \text{h.c.})$$
$$+ \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right]$$
$$+ \left[\lambda_6|\Phi_1|^2\Phi_1^\dagger\Phi_2 + \lambda_7|\Phi_2|^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right],$$

$$\Phi_i = \begin{bmatrix} \omega_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{bmatrix}, \quad (i = 1, 2)$$

$$\left.\frac{\partial V}{\partial \varphi_a}\right|_0 = 0, \quad (\varphi_a = h_1,\ h_2,\ z_1,\ \text{and } z_2)$$

13 Independent parameters

v, m_h, m_{H2}, m_{H3}, m_{H+}, α, β, M, $|\lambda_6|$, $|\lambda_7|$, ϑ_5, ϑ_6, ϑ_7

Unitarity bound in the 2HDM

Kanemura, Kubota, Takasugi (1993)
Akeroyd, Arhrib, Naimi (2000)
Ginzburg, Ivanov (2005)
SK, Yagyu,arXiv: 1509.06060 (2015)

Not only m_h, but also many parameters

$v, m_h, m_H, m_A, m_{H+}, \alpha, \beta, M$ — 2HDM with softly broken Z_2 without CPV

$v, m_h, m_H, m_A, m_{H+}, \alpha, \beta, M, |\lambda_6|, |\lambda_7|$ — 2HDM without CPV

$v, m_h, m_{H2}, m_{H3}, m_{H+}, \alpha, \beta, M, |\lambda_6|, |\lambda_7|, \vartheta_5, \vartheta_6, \vartheta_7$ — 2HDM with CPV

Many more particles in extended Higgs, so that channels are more than *WW, ZZ, hh, hZ*

$w^+, w^-, z, h, H_2, H_3, H^+, H^-$

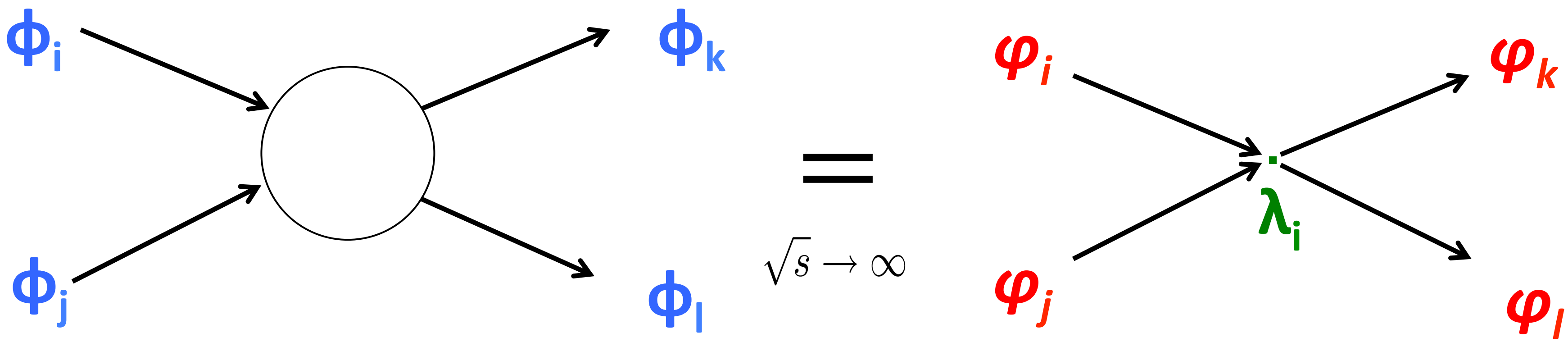
8 fields

14 two-body neutral channels

8 for singly charged (+)

3 for doubly charged

Equivalence Theorem



$\phi = W_L^+, W_L^-, Z_L,$
h, H_2, H_3, H^+, H^-

$\varphi = w^+, w^-, z,$
h, H_2, H_3, H^+, H^-

In 2HDMs, there is no O(4) any more

Ginzburg, Ivanov (2005)
SK, Yagyu,arXiv: 1509.06060 (2015)

Scattering Matrix

S-wave amplitude matrix can be brock-diagonalized using group theoretical classification and charge of each field

$$a_0^0 = \frac{1}{16\pi}\begin{pmatrix} X_{4\times4} & 0 & 0 & 0 \\ 0 & Y_{4\times4} & 0 & 0 \\ 0 & 0 & Z_{3\times3} & 0 \\ 0 & 0 & 0 & Z_{3\times3} \end{pmatrix}$$

$$X_{4\times4} = \left(\begin{array}{cc|cc} 3\lambda_1 & 2\lambda_3+\lambda_4 & \frac{3\lambda_6^R}{\sqrt{2}} & 3\sqrt{2}\lambda_6^I \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & \frac{3\lambda_7^R}{\sqrt{2}} & 3\sqrt{2}\lambda_7^I \\ \hline 3\sqrt{2}\lambda_6^R & 3\sqrt{2}\lambda_7^R & \lambda_3+2\lambda_4+3\lambda_5^R & 3\lambda_5^I \\ 3\sqrt{2}\lambda_6^I & 3\sqrt{2}\lambda_7^I & 3\lambda_5^I & \lambda_3+2\lambda_4-3\lambda_5^R \end{array}\right)$$

$$Y_{4\times4} = \left(\begin{array}{cc|cc} \lambda_1 & \lambda_4 & \frac{\lambda_6^R}{\sqrt{2}} & \sqrt{2}\lambda_6^I \\ \lambda_4 & \lambda_2 & \frac{\lambda_7^R}{\sqrt{2}} & \sqrt{2}\lambda_7^I \\ \hline \sqrt{2}\lambda_6^R & \frac{\lambda_7^R}{\sqrt{2}} & \lambda_3+\lambda_5^R & \lambda_5^I \\ \sqrt{2}\lambda_6^I & \frac{\lambda_7^I}{\sqrt{2}} & \lambda_5^I & \lambda_3-\lambda_5^R \end{array}\right)$$

$$Z_{3\times3} = \left(\begin{array}{cc|c} \lambda_1 & \lambda_5^R+i\lambda_5^I & \sqrt{2}(\lambda_6^R+i\lambda_6^I) \\ \lambda_5^R-i\lambda_5^I & \lambda_2 & \sqrt{2}(\lambda_7^R-i\lambda_7^I) \\ \hline \sqrt{2}(\lambda_6^R-i\lambda_6^I) & \sqrt{2}(\lambda_7^R+i\lambda_7^I) & \lambda_3+\lambda_4 \end{array}\right)$$

X,Y

If no CPV, **$4\times4 = 3\times3 + 1\times1$**

If Z_2 symmetry, **$4\times4 = 2\times2 + 2\times2$**

If Z_2 without CPV, **$4\times4 = 2\times2 + 1\times1 + 1\times1$**

In any case, we can analyze these matrices numerically.

SK, Yagyu,arXiv: 1509.06060 (2015)

Charged State Channels

$$a_0^+ = \frac{1}{16\pi}\begin{pmatrix} Y_{4\times4} & 0 & 0 \\ 0 & Z_{3\times3} & 0 \\ 0 & 0 & \lambda_3 - \lambda_4 \end{pmatrix}$$

$$a_0^{++} = \frac{1}{16\pi} Z_{3\times3}$$

Unitarity bound on the 2nd Higgs mass?

Can we have upper bounds for additional Higgses From perturbative unitarity?

In general, No!

There is the decoupling parameter *M* in the model, where SM limit is realized in the limit of $M \to \infty$

Unitarity gives constraints on λ $m_A^2 = M^2 - \lambda_5 v^2$

$$\lambda_5 = |m_A^2 - M^2|/v^2 < \text{const.}$$

No bound for m_A by taking $M \sim m_A$ $(\to \infty)$.

m_A is a free parameter

Deviation in κ_i^2 and the scale of BSM

- **Mass of the second Higgs boson is a free parameter**

- **Correlation with the SM-like *h* couplings**
  - **Structure of BSM (MSSM)**
  - **Unitarity and Vacuum Stability in general**

- **If $\kappa_V^2 < 1$ is observed by experiment, the upper bound on the scale of the second Higgs boson is obtained**

Future *h*(125)-coupling measurements

Facility	LHC	HL-LHC	ILC500	ILC500-up
$\sqrt{s}$ (GeV)	14,000	14,000	250/500	250/500
$\int \mathcal{L}dt$ (fb^{-1})	300/expt	3000/expt	250+500	1150+1600
κ_γ	$5-7\%$	$2-5\%$	8.3%	4.4%
κ_g	$6-8\%$	$3-5\%$	2.0%	1.1%
κ_W	$4-6\%$	$2-5\%$	0.39%	0.21%
κ_Z	$4-6\%$	$2-4\%$	0.49%	0.24%
κ_ℓ	$6-8\%$	$2-5\%$	1.9%	0.98%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%	0.60%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%	1.3%

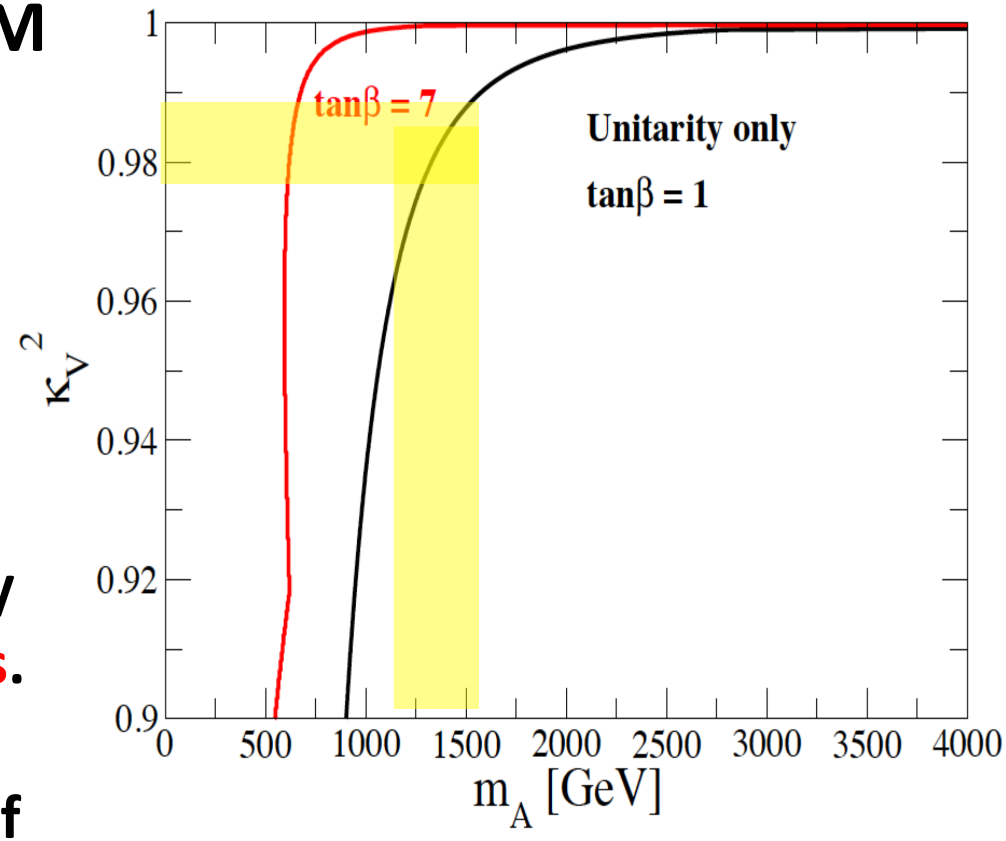
Snowmass Higgs Working Group Report 1310.8361

Unitarity bound in the 2HDM

$\kappa_V^2 = \sin^2(\beta-\alpha)$

If κ_V^2 is found to be less than 1, M cannot be taken to be infinity without taking large λ couplings.

The upper bound on the mass of the second Higgs is obtained by the unitarity constraint



SK, Yagyu,arXiv: 1509.06060 (2015)

Bound on M_{2nd} in the 2HDM with SBZ_2

Assuming that m_h (=125GeV) is the lightest,

$m_h < M_{2nd} < \ldots < M_{heaviest}$

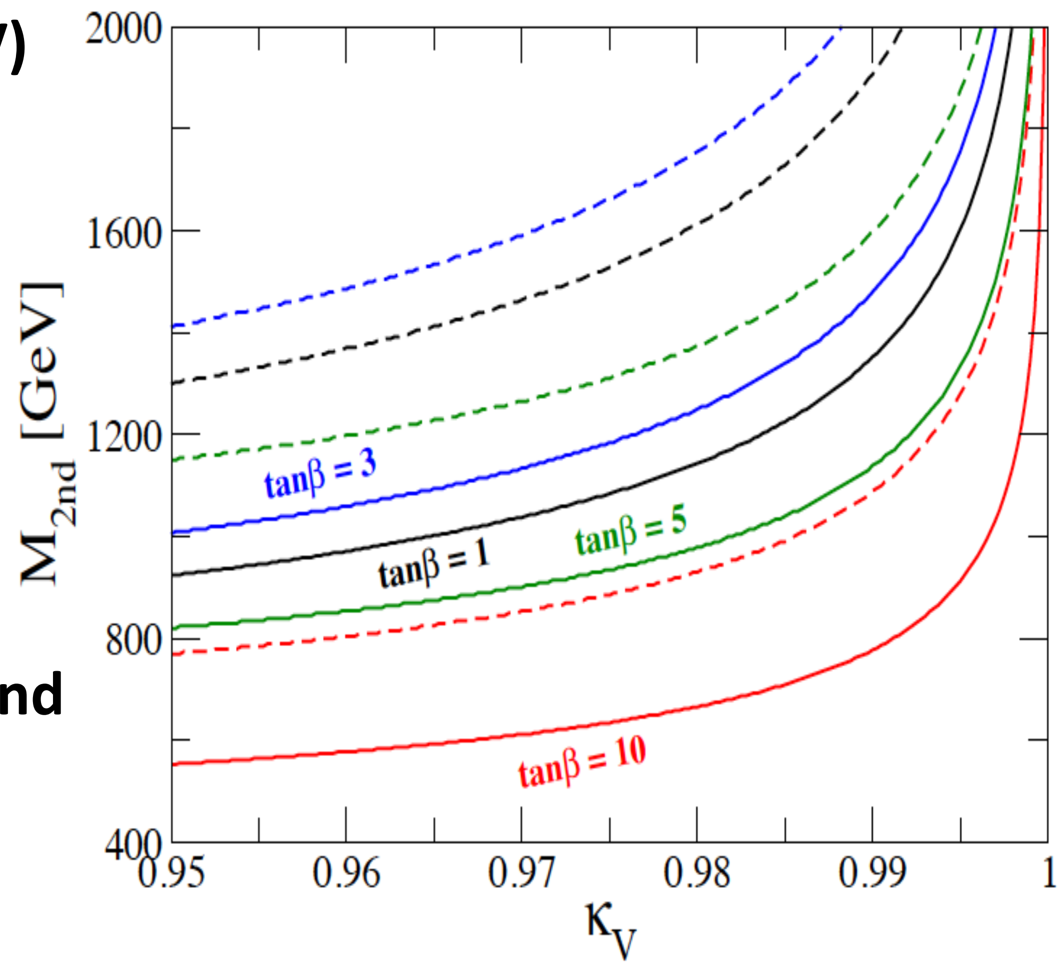
We can obtain bound on the mass M_{2nd} of the 2nd lightest Higgs by taking

$M_{2nd} = m_{H2} = m_{H3} = m_{H+}$

When $\kappa_V < 1$, the upper bound is obtained

For larger tanβ, the bound is stronger

(M scanned)

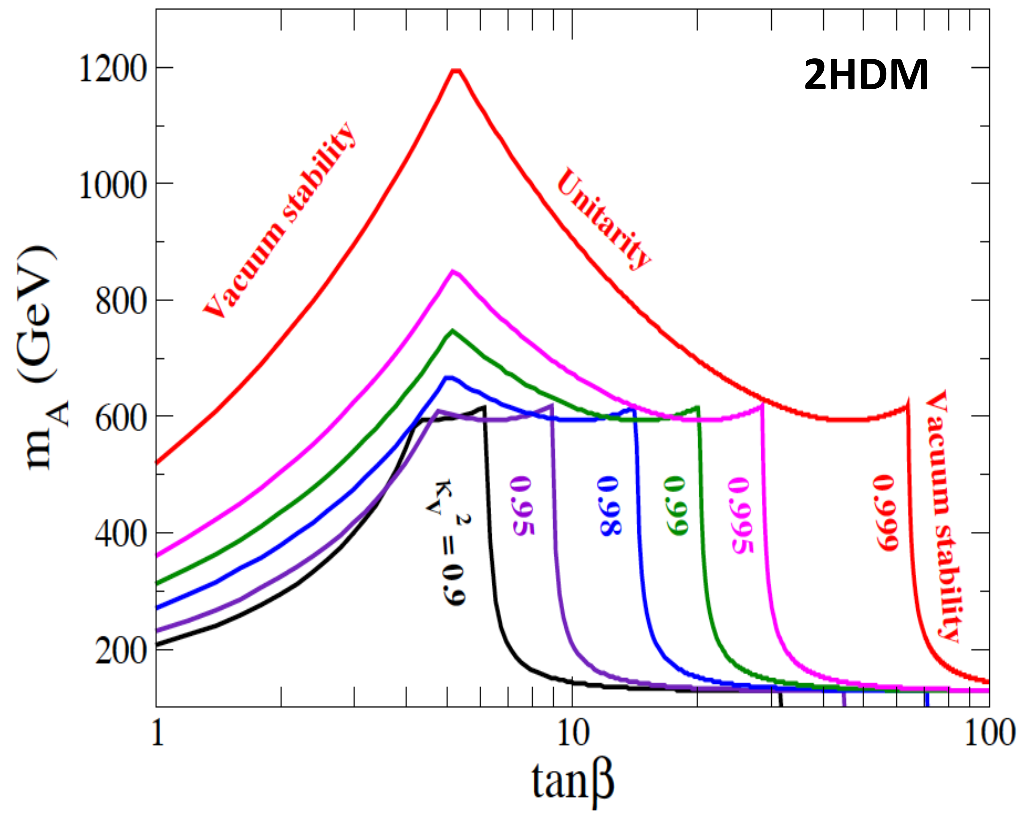


Theoretical upper bounds on the second Higgs mass, when $\kappa_V^2 < 1$

If measured κ_V^2 is slightly smaller than 1 (say, 0.99), the second Higgs must be lighter than 700 GeV.

Then, if no second Higgs is found below 700 GeV, the 2HDM is excluded

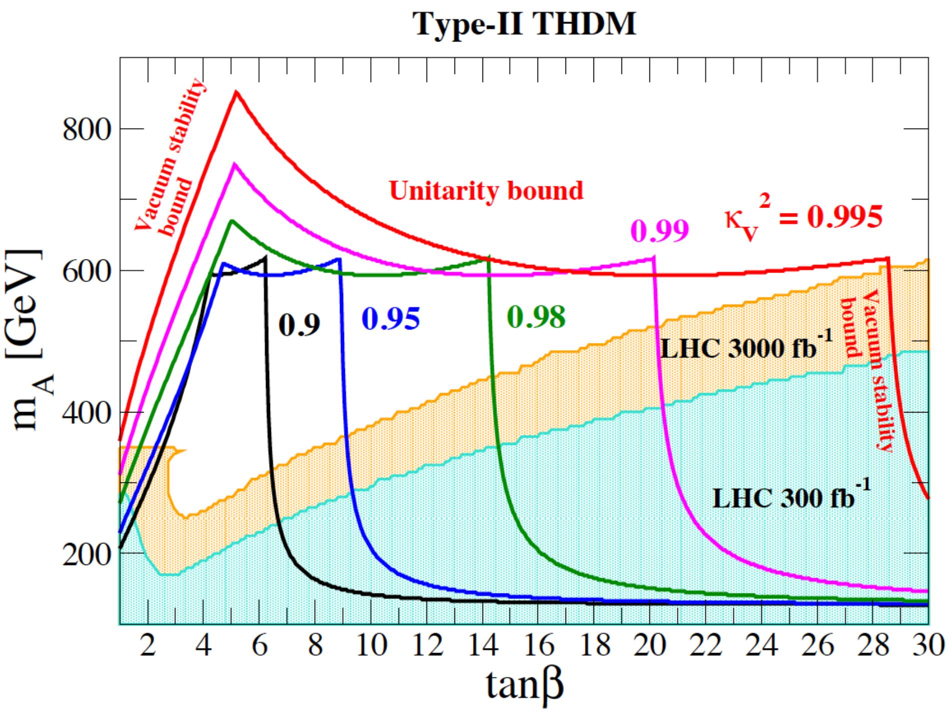
The rest possibility may be the Higgs singlet model, or other exotics

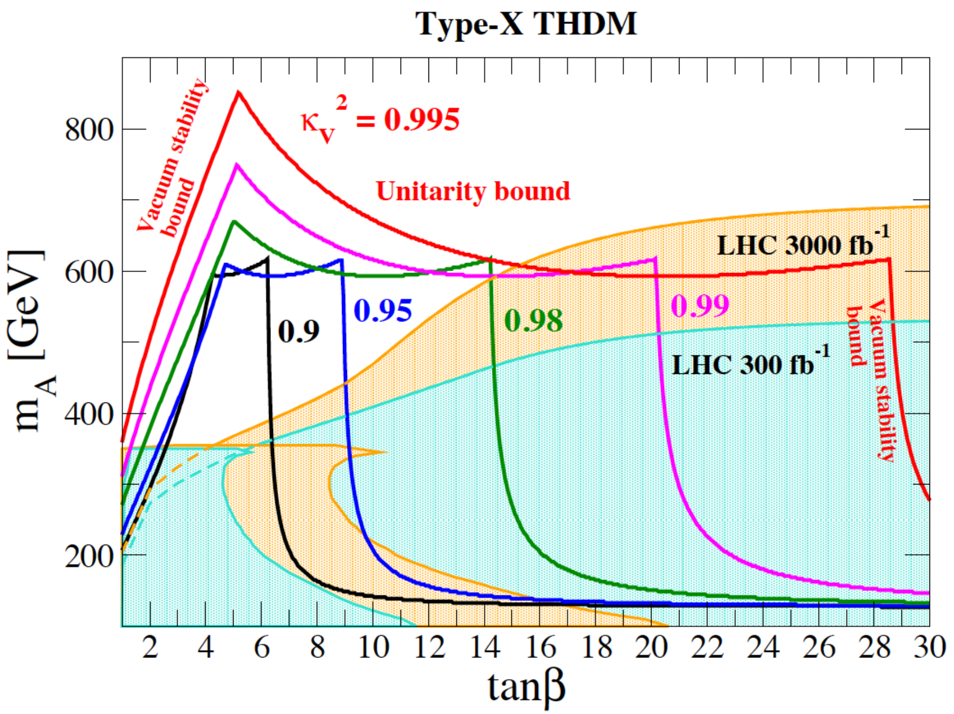


Precision determination of *hVV* coupling is very important

SK, Tsumura, Yagyu, Yokoya (2014)

LHC can search relatively large tanβ regions





Precision measurements at LCs can reach more than LHC direct searches, unless tanβ is too large.

SK, Yagyu,arXiv: 1509.06060 (2015)

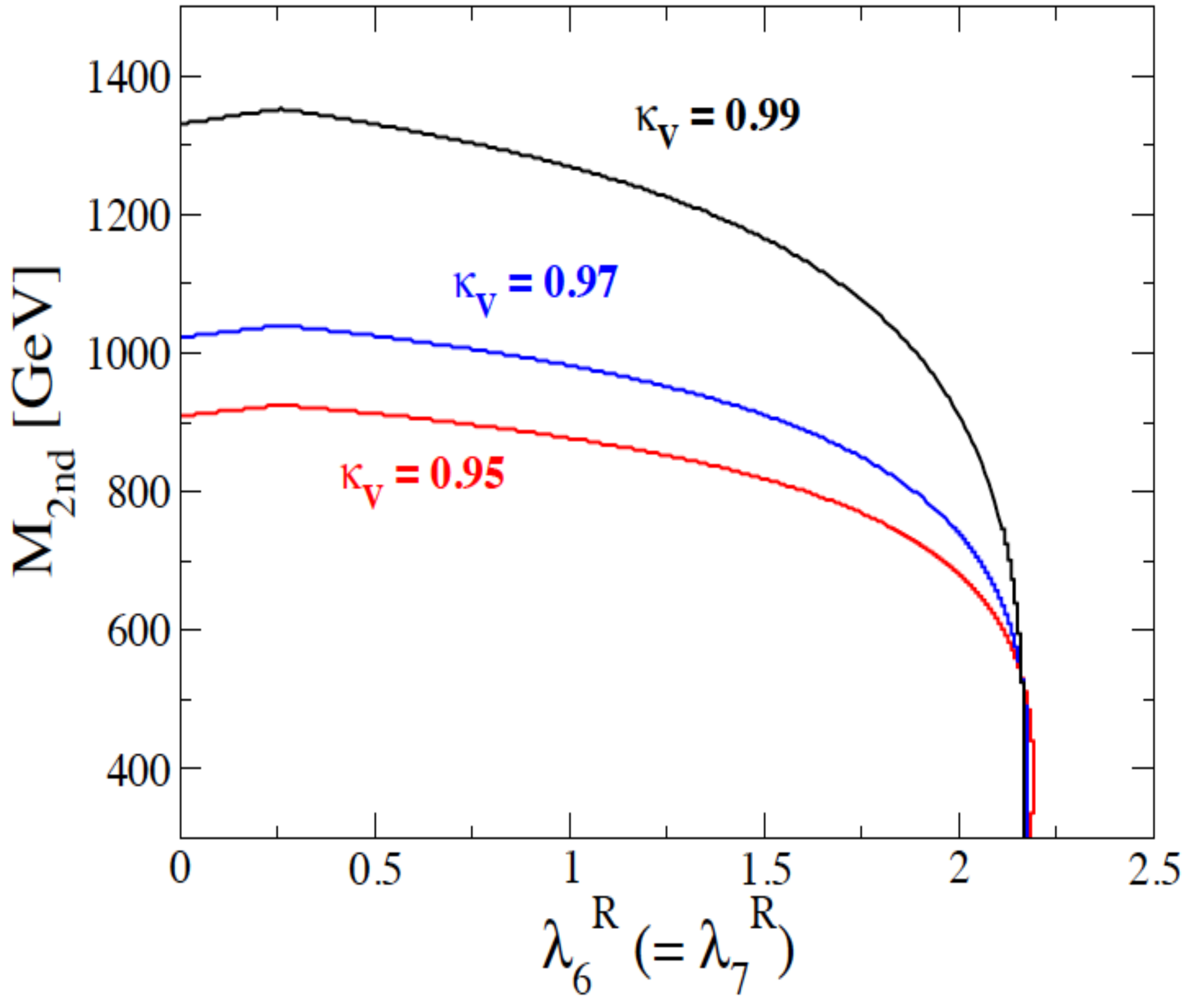
Bound on the general 2HDM without Z_2

Unitarity bound is stronger for larger λ_6^R (λ_7^R)

$\lambda_6^R = \lambda_7^R < 2.2$

M scanned

$\tan\beta$ scanned



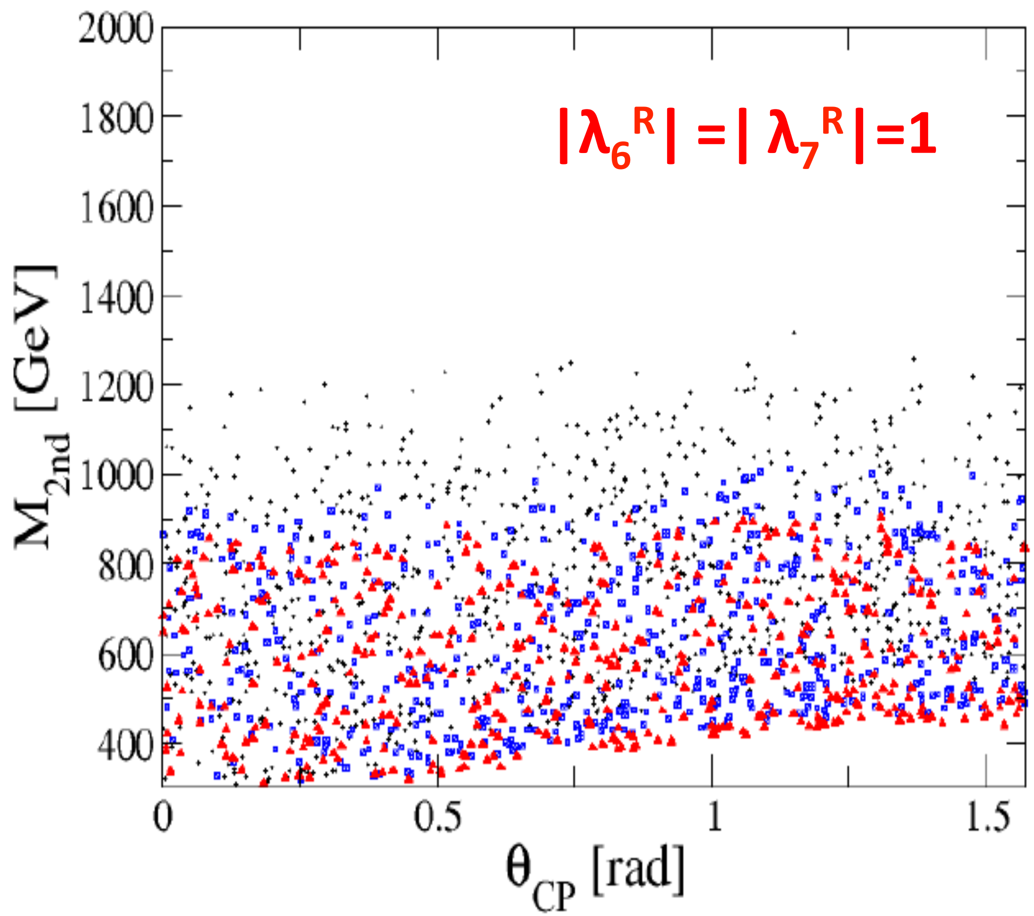
SK, Yagyu,arXiv: 1509.06060 (2015)

Unitarity bounds vs CP phases

Unitarity bound in the general 2HDM with CPV for given value of κ_V

κV = 0.98 ± 0.01
κV = 0.96 ± 0.01
κV = 0.94 ± 0.01

Non-zero θ_{CP} makes M_{2nd} slightly larger for fixed value of m_h



We can constrain parameter spaces of CPV 2HDM by unitarity in addition to the data from EDM etc.

Summary

- We discussed unitarity bound in the most general 2HDM with CPV
- Our study can be useful to constrain parameter spaces in all analyses in the general 2HDM with/without CPV
- When deviation in κ_V can be detected at ILC, we obtain **the upper bound on the mass of the 2nd lightest Higgs** (what ever it is), which can be beyond the reach of LHC.
- Precision measurement of the *hVV* coupling is very important.

Back up slides

Masses and couplings

$$\lambda_1 v^2 = M_{11}^2 + (M_{22}^2 - M^2)\tan^2\beta - 2M_{12}^2\tan\beta + \frac{1}{2}(\lambda_7^R\tan^2\beta - 3\lambda_6^R)\tan\beta,$$

$$\lambda_2 v^2 = M_{11}^2 + (M_{22}^2 - M^2)\cot^2\beta + 2M_{12}^2\cot\beta + \frac{1}{2}(\lambda_6^R\cot^2\beta - 3\lambda_7^R)\cot\beta,$$

$$\lambda_3 v^2 = M_{11}^2 - (M_{22}^2 + M^2) + 2M_{12}^2\cot 2\beta + 2m_{H\pm}^2 - \frac{1}{2}(\lambda_6^R\cot\beta + \lambda_7^R\tan\beta),$$

$$\lambda_4 v^2 = M^2 + M_{33}^2 - 2m_{H\pm}^2 - \frac{1}{2}(\lambda_6^R\cot\beta + \lambda_7^R\tan\beta),$$

$$\lambda_5^R v^2 = M^2 - M_{33}^2 - \frac{1}{2}(\lambda_6^R\cot\beta + \lambda_7^R\tan\beta).$$

$$M_{11}^2 = \tilde{m}_h^2 s_{\beta-\tilde{\alpha}}^2 + \tilde{m}_H^2 c_{\beta-\tilde{\alpha}}^2, \qquad M_{33}^2 = \tilde{m}_A^2.$$

$$M_{22}^2 = \tilde{m}_h^2 c_{\beta-\tilde{\alpha}}^2 + \tilde{m}_H^2 s_{\beta-\tilde{\alpha}}^2,$$

$$M_{12}^2 = (\tilde{m}_h^2 - \tilde{m}_H^2) s_{\beta-\tilde{\alpha}} c_{\beta-\tilde{\alpha}}.$$

FCNC Suppression

In multi-doublet model, FCNC appears at tree via Higgs mediation

2 Higgs doublet model with a (softly broken) symmetry:

to avoid FCNC, give different charges to Φ_1 and Φ_2

ex) Discrete sym. $\Phi_1 \rightarrow +\Phi_{1,} \quad \Phi_2 = -\Phi_2$

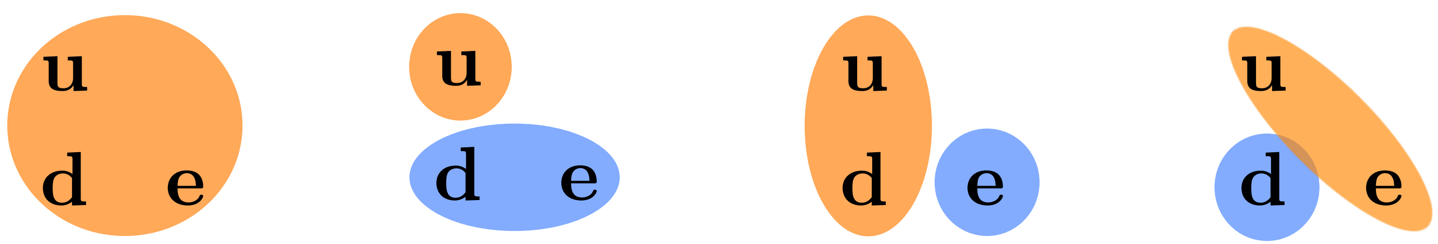
Each quark or lepton couples only one Higgs doublet

No FCNC at tree level

Barger, Hewett, Phillips

Classified by Z_2 charge assignment

Four Types of Yukawa coupling



Type-I **Type-II** **Type-X** **Type-Y**

Neutrino Philic etc SUSY etc Radiative seesaw etc

Higgs Multiplet Structure

If the Higgs sector contains more than one scalar bosons, possibility would be

- **SM + extra Singlets (NMSSM, B-L Higgs, ...)**
- **SM + extra Doublets (MSSM, CPV, EW Baryogenesis, Radiative Neutrino mass, …)**
- **SM + extra Triplets (Type II seesaw, LR models….)**
- **….**

Multiplet Structure of the Higgs sector gives an important hint for New Physics BSM

Future *h*(125)-coupling measurements

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κ_ℓ	$6-8\%$	$2-5\%$	1.9%	0.98%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%	0.60%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%	1.3%

Snowmass Higgs Working Group Report 1310.8361

Radiative Corrections

In future, the Higgs couplings will be measured with much better accuracies

Clearly, tree level analyses are not enough

Analysis with Radiative Corrections (including quantum effect of the 2nd Higgs/BSM particles) is necessary

Theoretical predictions at loop levels $\times$ **Precision measurements at future colliders**

New Physics !

Do not miss Mariko's talk

Fingerprinting the 2HDM (tree level)

$$\kappa_V \equiv \frac{g_{hVV(2HDM)}}{g_{hVV(SM)}} = \sin(\beta - \alpha)$$

$x = \cos(\beta-\alpha)$ **SM-like case: $|x| \ll 1$**

$$\kappa_V = 1 - (1/2)\,x^2 + \ldots$$

When a Fermion couples to Φ_2

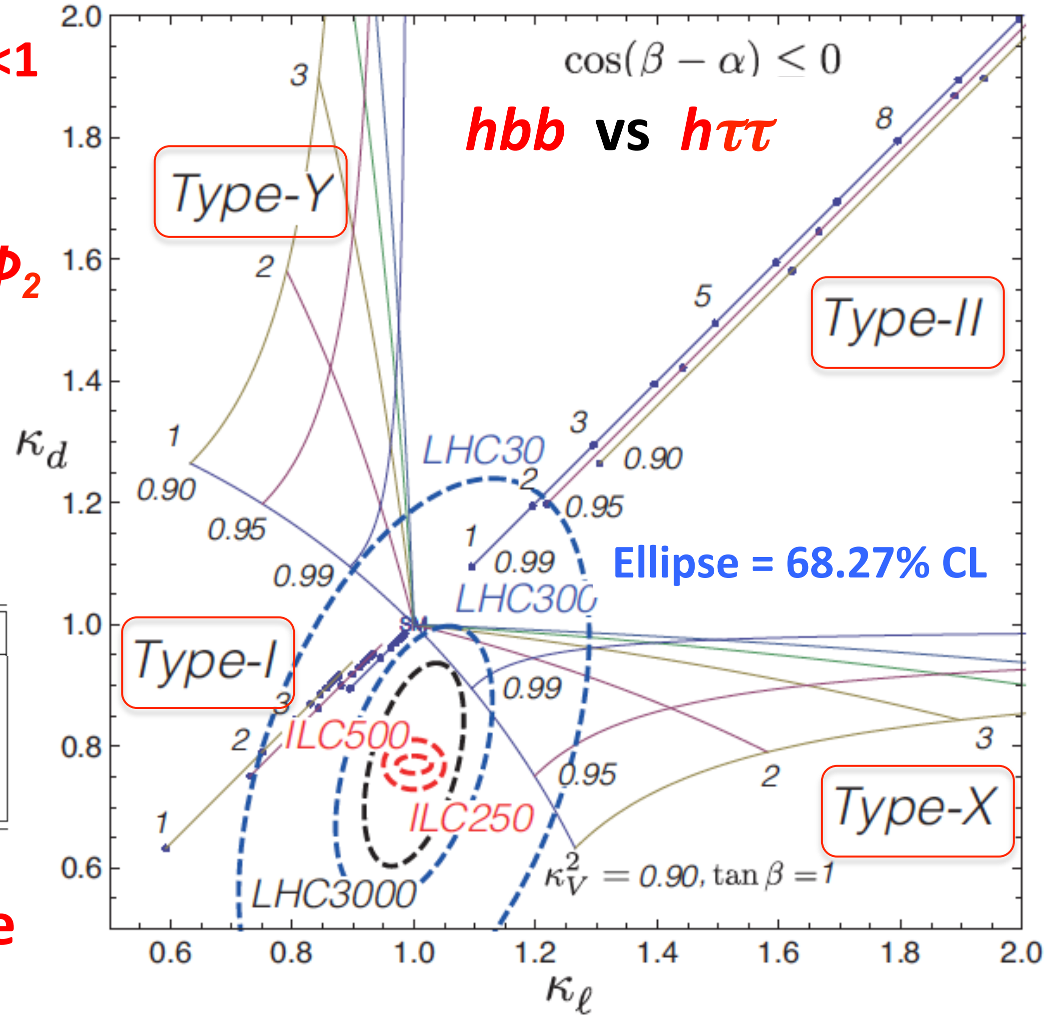
$$\kappa_f = 1 + \cot\beta\, x + \ldots$$

and if it couples to Φ_1

$$\kappa_f = 1 - \tan\beta\, x + \ldots$$

Model	μ	τ	b	c	t	g_V
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

SK, K. Tsumura, K. Yagyu, H. Yokoya 2014
ILC Higgs White Paper 2013



How does this result change with radiative corrections?