Ideas for a sextupole-free final focus system

LCWS15 – Whistler, Canada

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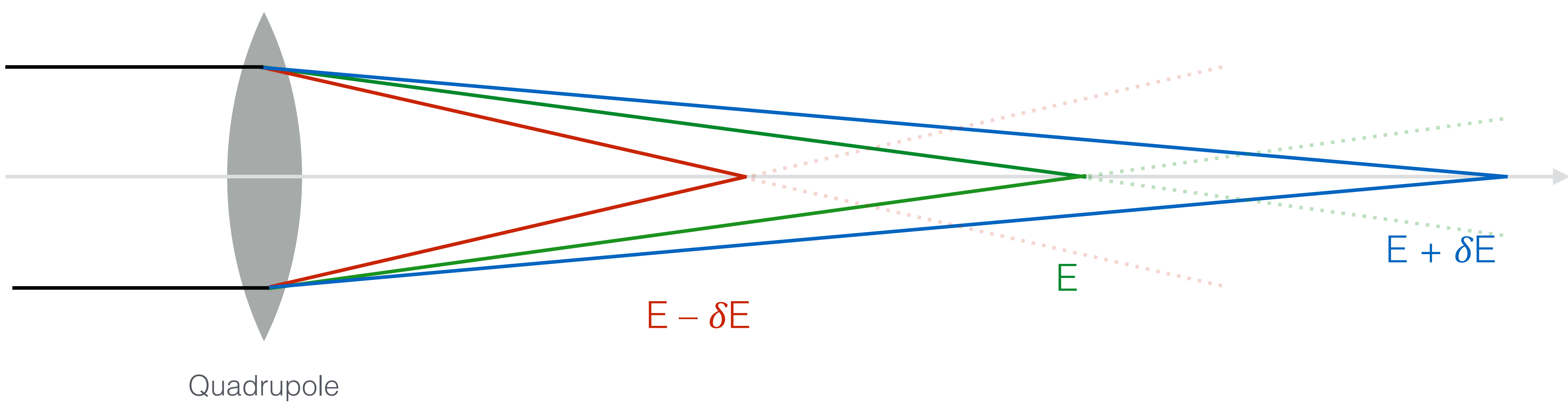
In short

- Chromaticity as a fundamental problem
- Conventional solution: Sextupoles + dipoles (non-linear lattice)
- Proposed solution: Quadrupoles only (linear lattice)
- Some definitions and clarifications
- Methodology
- Applications: PWFA interstage and Final Focus System

The chromaticity problem

- Quadrupole kicks are energy dependent: $\Delta x' \sim \frac{\frac{\partial B}{\partial x}}{E}$
- Chromaticity: **Energy dependent focusing.**
- Small spot sizes (β ≪ drift length) + energy spread = Large chromaticity
- Examples:
  - Final Focus Systems: Demagnify large beam to very small beam at IP.
  - LWFA/PWFA: Catch small and highly diverging beam exiting plasma.



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Conventional solution: Sextupoles + Dipoles

- **Disperse beam** in dipoles
  - \+ **position dependent focusing** of sextupoles
  - ⇒ **Energy dependent focusing**
  - ⇒ 1st order **chromaticity correction**

- However, introduces **new problems**:
  - – Dipoles ⇒ **synchrotron radiation**
  - – **Dispersion** from dipoles (must be cancelled)
  - – **Non-linear** terms from sextupoles (must be cancelled)

Sextupole B-fields:

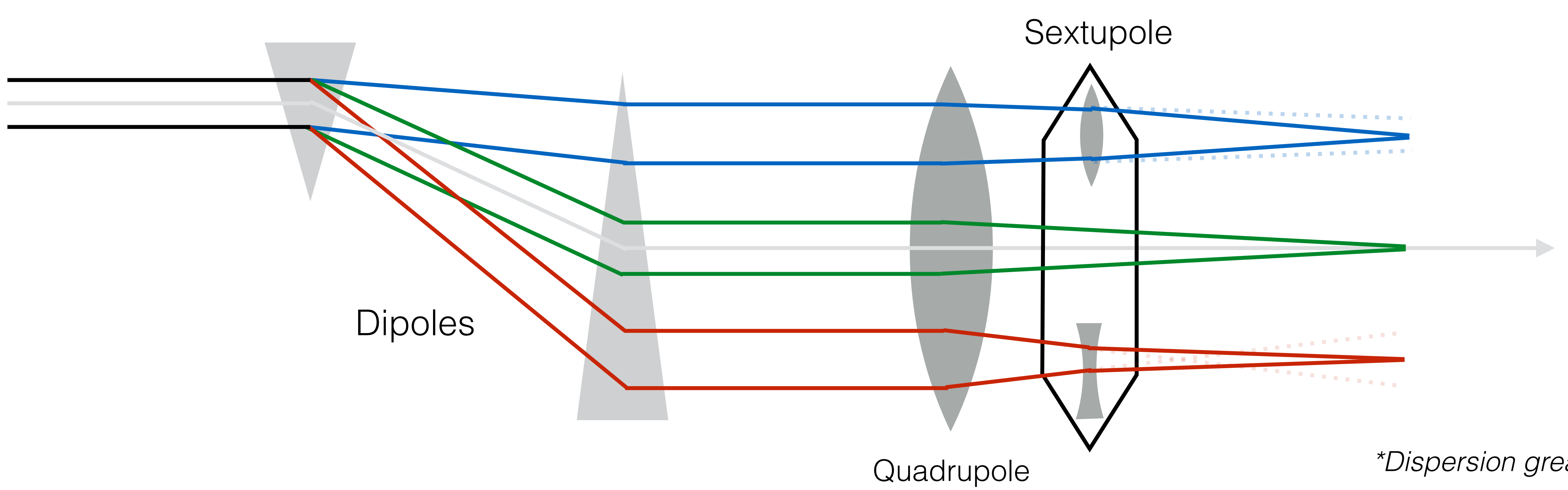
$$B_x \sim xy + \delta D_x y$$

$$B_y \sim \frac{1}{2}(x^2 - y^2) + x\delta D_x + \frac{1}{2}\delta^2 D_x^2$$

Non-linear geometric terms

Linear chromatic terms
CORRECT CHROMATICITY

Non-linear chromatic term



**Dispersion greatly exaggerated*

Proposed solution: Quadrupoles only

- Our target:

  To cancel chromaticity with only quadrupoles.

- Chromaticity can *in principle* be cancelled to any order in energy offset.

- Chromaticity will be **cancelled globally, not locally**.

- **"Why bother?"**
  - No dipoles ⇒ **No energy loss / energy spread** from SR (in dipoles)
    ⇒ **No dispersion** to cancel
  - No sextupoles ⇒ **No non-linear terms** to cancel

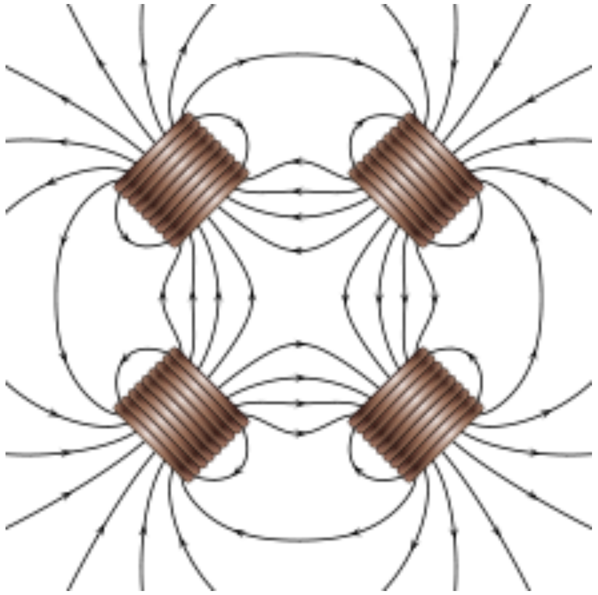
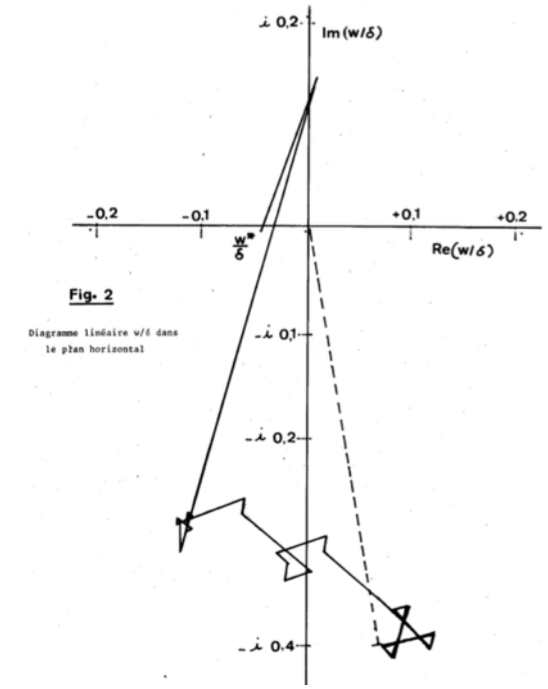


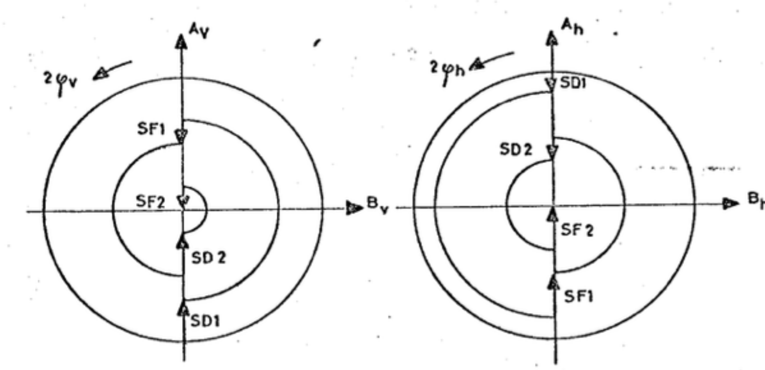
Image source:
https://commons.wikimedia.org/wiki/File:VFPt_quadrupole_coils_1.svg

Literature review

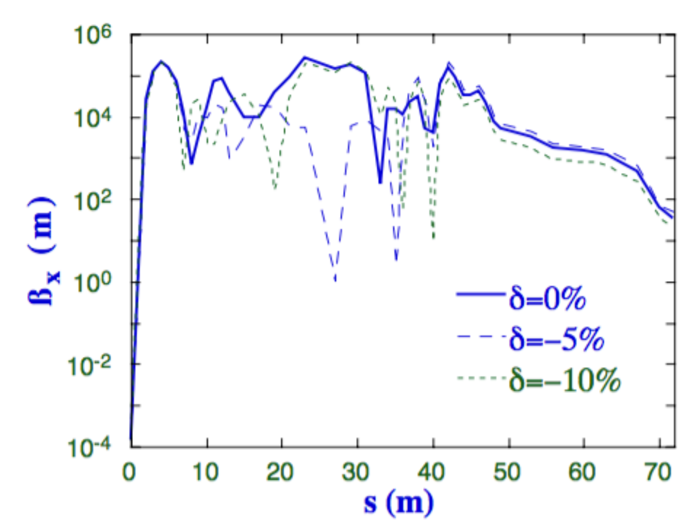
- **“Strategie pour la correction de chromaticite”, H. Zyngier (1977):**
  Effects of sextupoles on chromaticity, and how to use them for correction. Defines complex number framework for chromaticity (w).



- **“Linear optics for improved chromaticity correction”, B. W. Montague (1979):**
  Redefines framework for chromaticity (W-function, A, B, etc). Discusses LEP as an example. Only discussing sextupole correction.



- **“New Final Focus Concepts at 5 TeV and Beyond”, F. Zimmermann (1998):**
  Discusses unfavourable sextupole scalings, suggests a sextupole-free final focus system. Based on multiple ultra-low energy spread (10^{-5}) beams combined at IP after linear lattice.



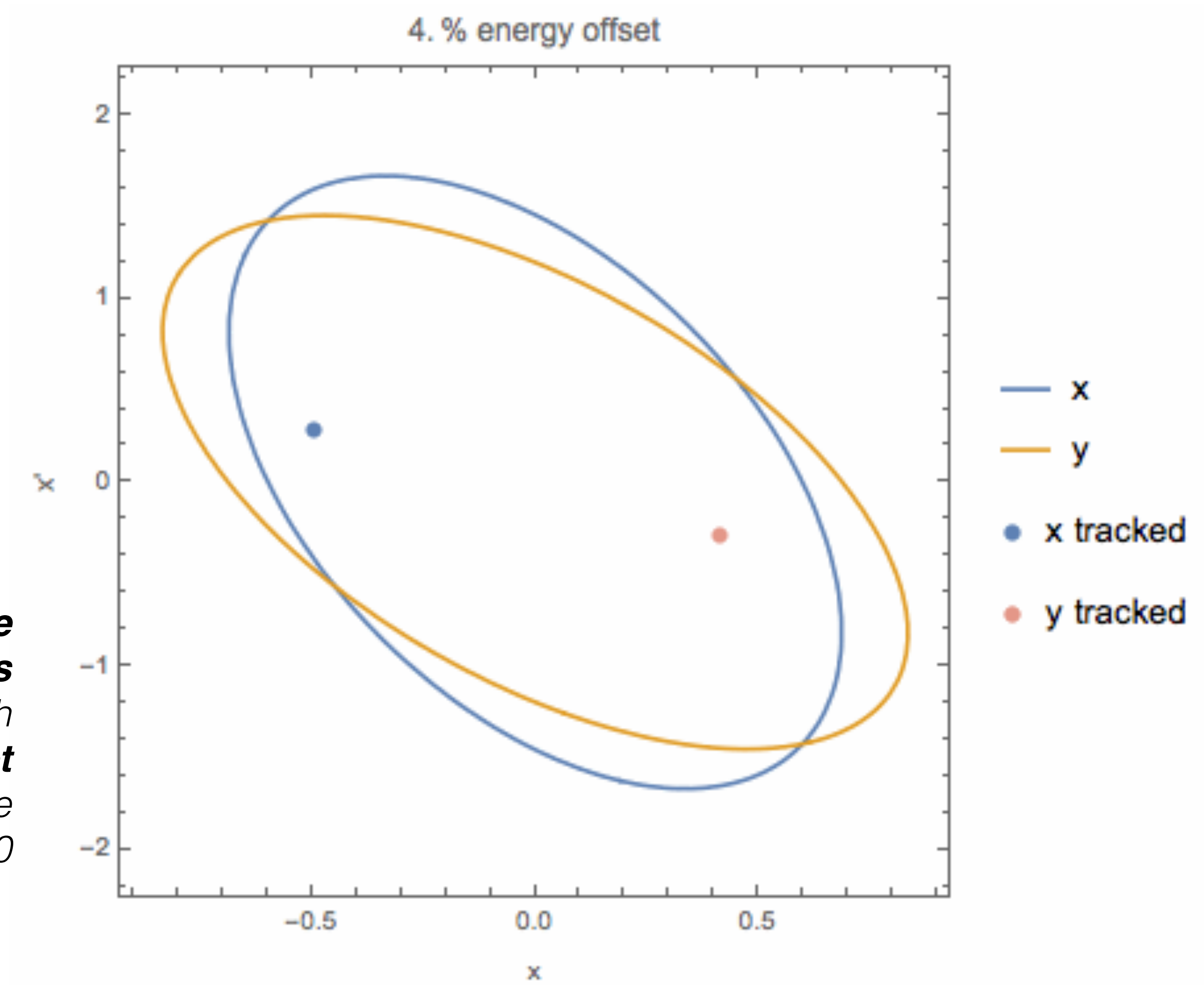
Distinction: "Phase chromaticity" vs. "Twiss chromaticity"

- "Phase chromaticity": $$\xi = \frac{1}{2\pi}\frac{\partial \mu}{\partial \delta} = \frac{\partial}{\partial \delta}\left(\frac{1}{2\pi}\int \frac{1}{\beta} ds\right) \qquad \delta = \Delta p / p$$
- "Twiss chromaticity"
  aka. chromatic amplitude: $$W = \sqrt{A^2 + B^2} \qquad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta} \qquad B = \frac{1}{\beta}\frac{\partial \beta}{\partial \delta}$$
- Sometimes confused in literature.
- Different use cases:
  - When phase advance is important (e.g. circular accelerators) ⇒ use ξ
  - When spot size/divergence is important (e.g. final focus) ⇒ use W
- "Phase chromaticity" **(ξ) requires sextupoles** for cancellation.
- "Twiss chromaticity" **(W) does not require sextupoles** (although they can be used).

Evolution in phase space

- Question: **How can α, β be constant (to 1st order) when the phase advance is not?**

Phase space ellipse
and ***tracked particles***
with
varying energy offset
for a linear lattice
with Wx=Wy=0



- Ellipse (α, β) stagnates (W = 0) around nominal energy ($\delta = 0$).

- Single particle phase advance (μ) varies with energy (ξ ≠ 0) around nominal energy ($\delta = 0$) .

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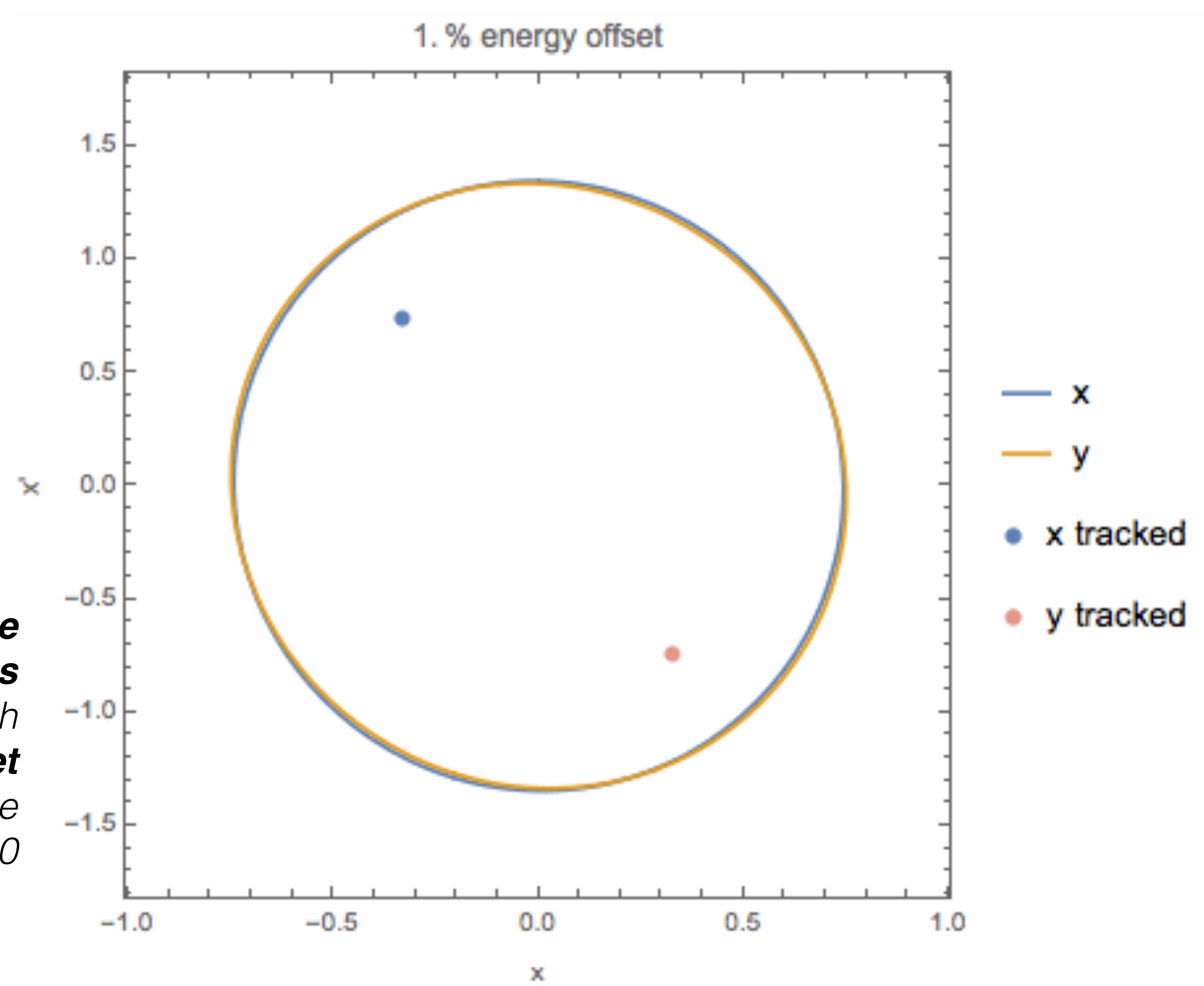

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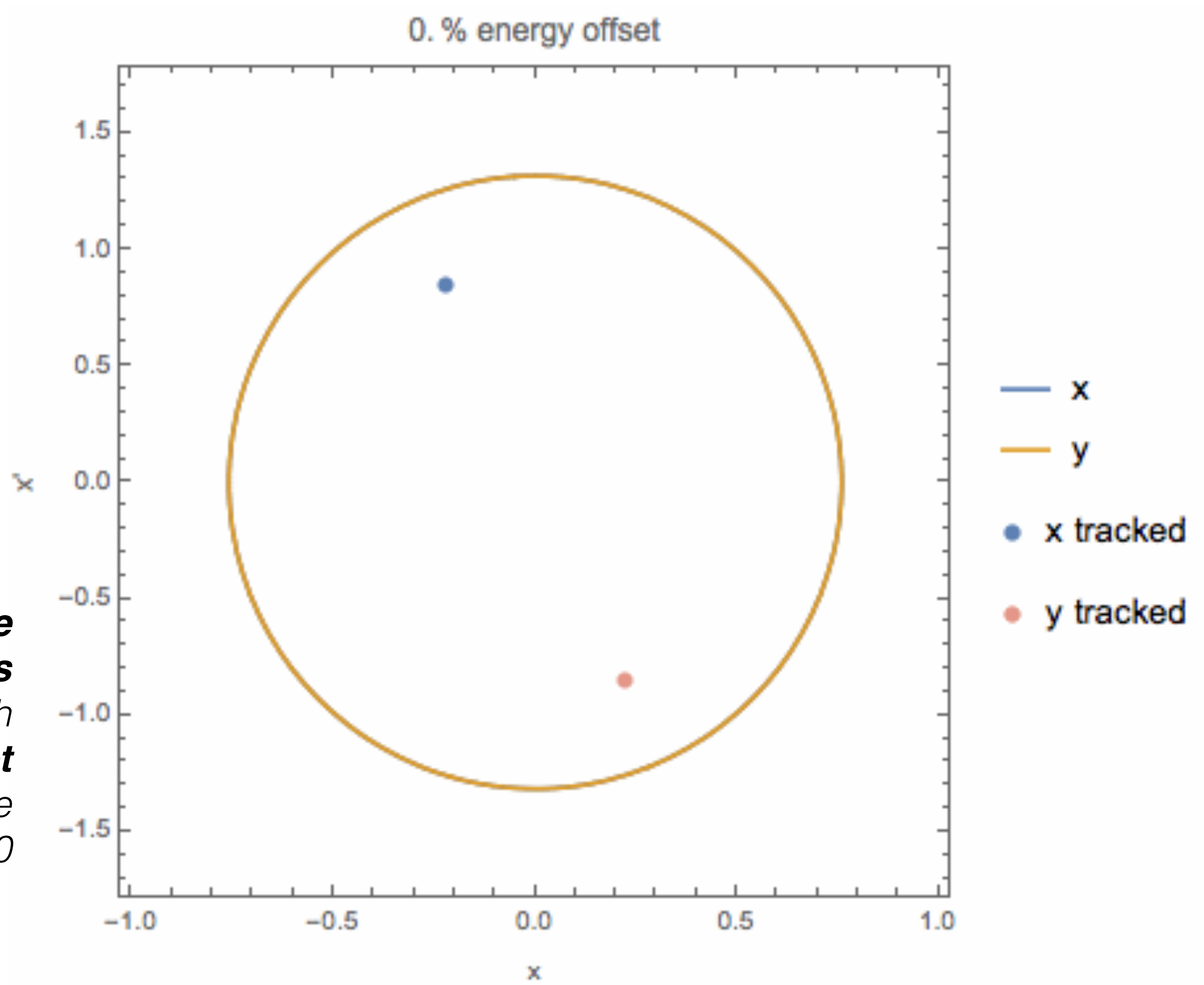

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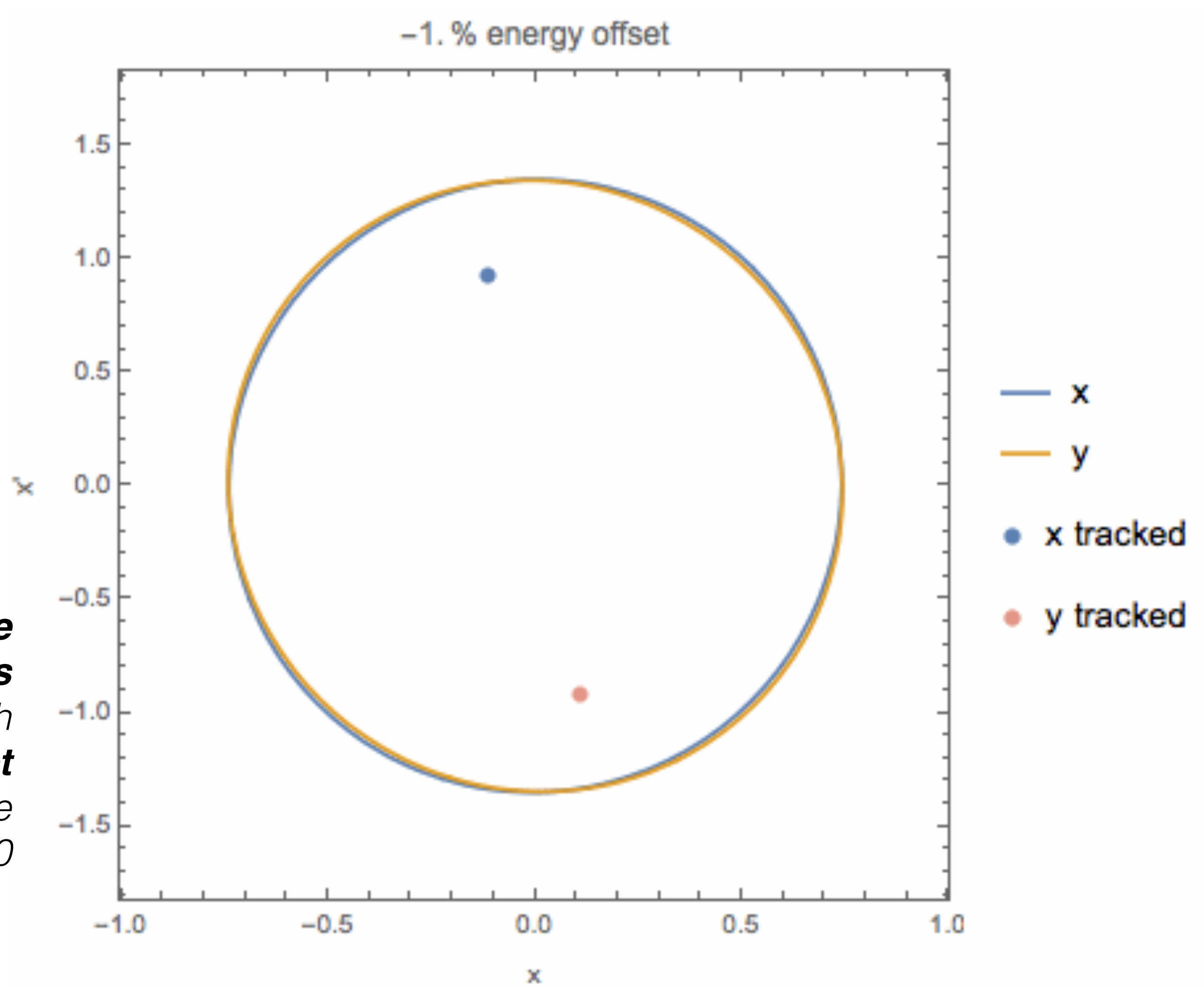

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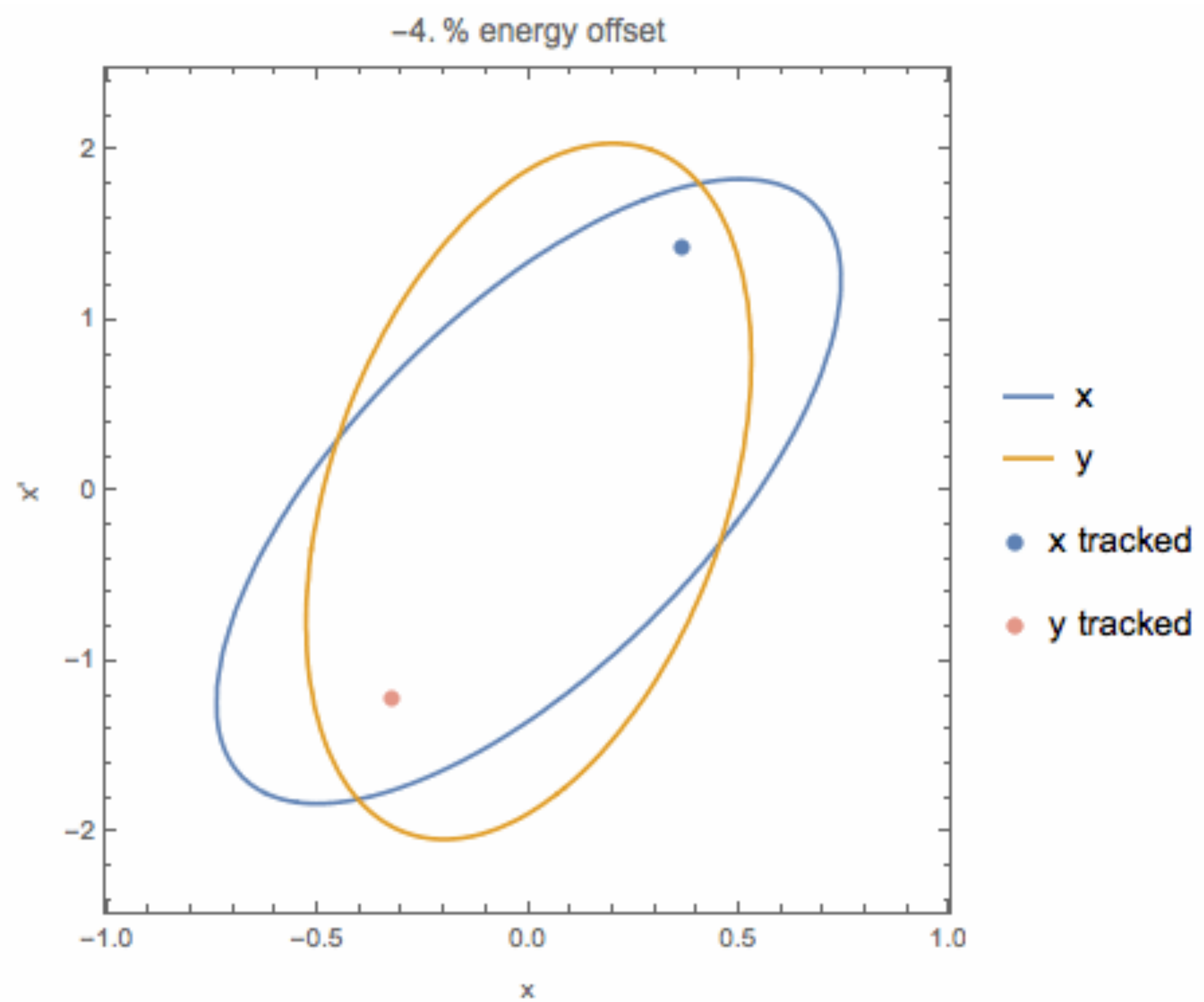

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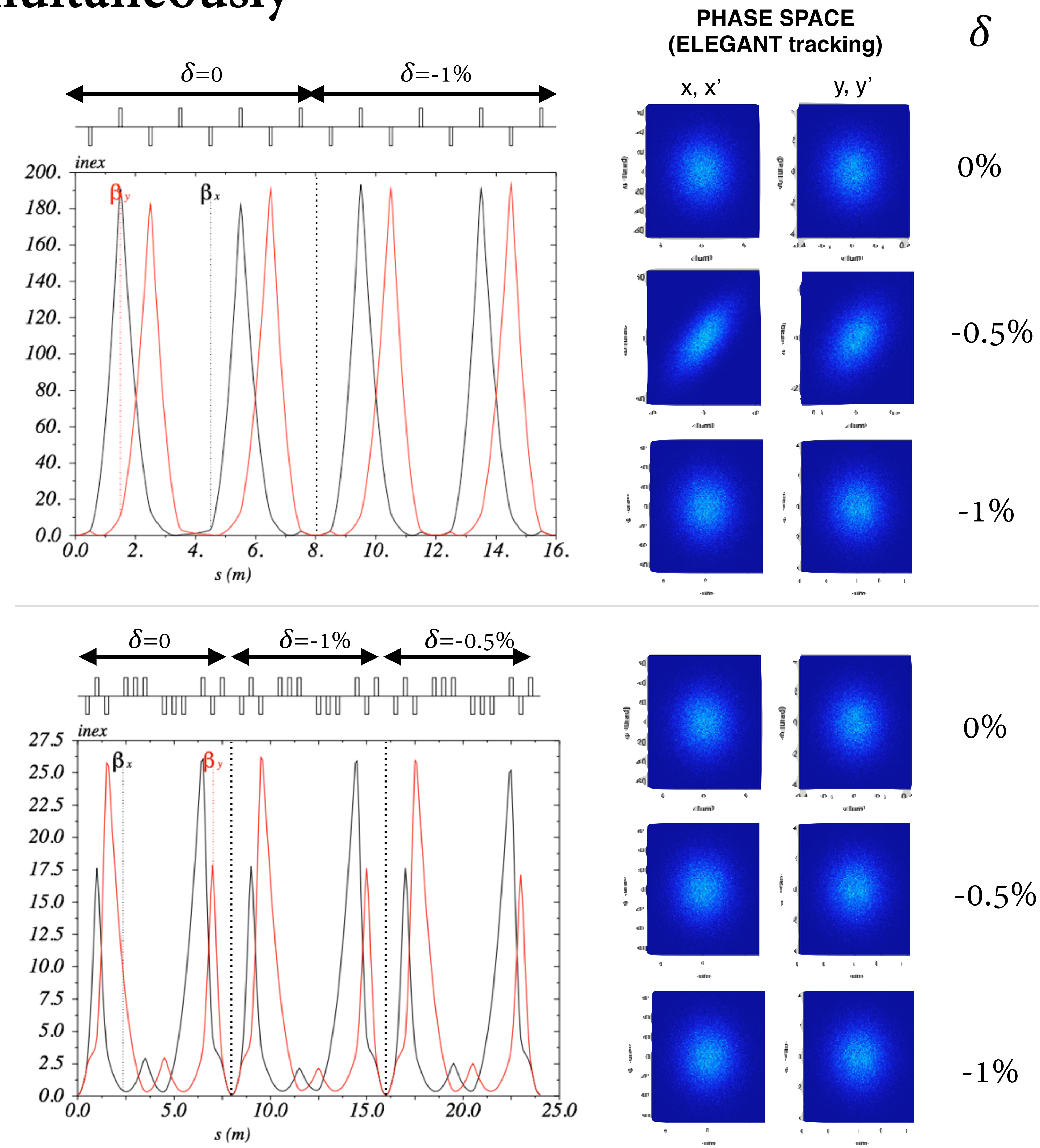
Matching multiple energies simultaneously

- How do we find such lattices?
- Question: Is possible to match **two different energies**?
- Answer: YES
- Next question: Is it possible to match **three different energies**?
- Again: YES
- If matched energies are close:
  2 δ's: No 1st order chromaticity.
  3 δ's: No 1st + 2nd order chromaticity.
  …



Chromatic expansions

- Expand α, β, μ for energy offset δ:
- Goal: Shaping $\alpha(\delta)$, $\beta(\delta)$ around $\delta = 0$.
- New constraints (1st order chromaticity correction):
  $$\partial\beta_x/\partial\delta = \partial\alpha_x/\partial\delta = \partial\beta_y/\partial\delta = \partial\alpha_y/\partial\delta = 0$$
  ⇒ Need 4 more degrees of freedom (quads/drifts).
- Chromaticity cancellation to nth order:
  $4n$ constraints, $4n$ degrees of freedom.

$$\beta(\delta) = \beta + \frac{\partial\beta}{\partial\delta}\delta + \frac{1}{2}\frac{\partial^2\beta}{\partial\delta^2}\delta^2 + O(\delta^3)$$

W

$$\alpha(\delta) = \alpha + \frac{\partial\alpha}{\partial\delta}\delta + \frac{1}{2}\frac{\partial^2\alpha}{\partial\delta^2}\delta^2 + O(\delta^3)$$

ξ

$$\mu(\delta) = \mu + \frac{\partial\mu}{\partial\delta}\delta + \frac{1}{2}\frac{\partial^2\mu}{\partial\delta^2}\delta^2 + O(\delta^3)$$

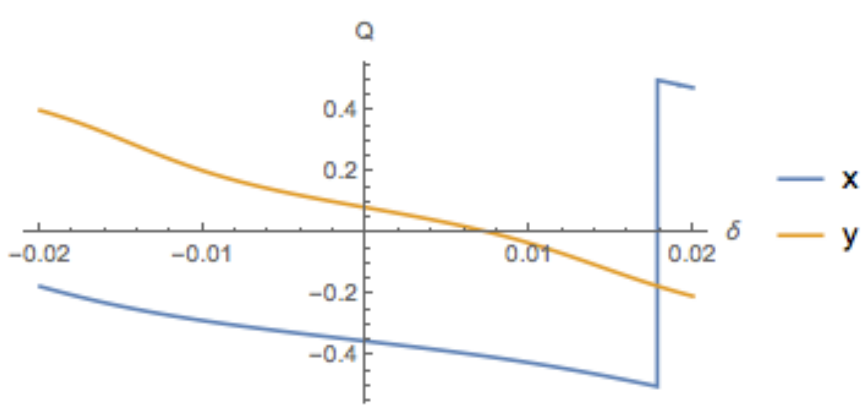
$\beta x \to 2. + 5.46058\,\delta + 2.63326\,\delta^2 + 198.664\,\delta^3 - 396.606\,\delta^4 - 1001.45\,\delta^5 + O[\delta]^6$

$\beta y \to 2. - 3.62135\,\delta + 16.6763\,\delta^2 - 124.297\,\delta^3 + 461.756\,\delta^4 - 884.161\,\delta^5 + O[\delta]^6$

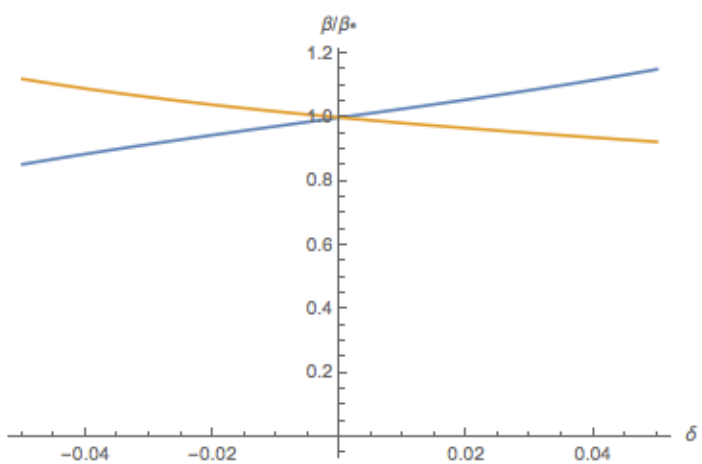
$\alpha x \to -1.25231\times10^{-9} - 0.526819\,\delta + 14.8658\,\delta^2 - 66.8473\,\delta^3 - 173.571\,\delta^4 + 2095.26\,\delta^5 + O[\delta]^6$

$\alpha y \to 6.60882\times10^{-10} - 1.73329\,\delta + 6.72818\,\delta^2 - 52.9228\,\delta^3 + 413.489\,\delta^4 - 1789.01\,\delta^5 + O[\delta]^6$

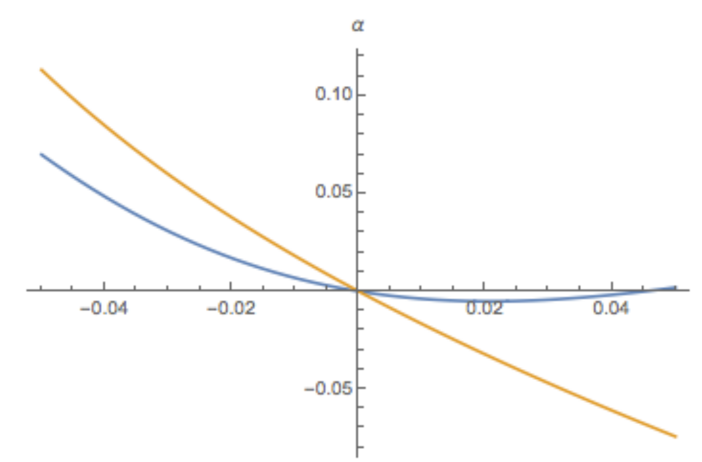
Example (no chromatic correction) **:**



$\mu(\delta)/2\pi$



$\beta(\delta)/\beta_0$



$\alpha(\delta)$

Methodology (simplified)

1. **Define lattice** with variable quads and drifts ({k}, {d}). Compute R-matrix.
2. **Rewrite all k → k/(1+δ)**. (Including non-variable k's).
3. **Express α, β in terms of R**-elements. Will be complicated functions of δ.
4. **Do chromatic δ-expansion** of $\alpha(\delta)$, $\beta(\delta)$ to obtain $\partial\beta/\partial\delta$, $\partial\alpha/\partial\delta$, $\partial^2\beta/\partial\delta^2$, etc.
5. **Solve/minimize system** of n equations (constraints) for n or more variables (quad strengths, drift lengths).
6. If hard to solve, **employ symmetries** to reduce n.

$$R(\{k\}, \{d\})$$

$$R(\{k/(1+\delta)\}, \{d\})$$

$$\alpha(R), \beta(R) \rightarrow \begin{matrix} \alpha(\delta, \{k\}, \{d\}) \\ \beta(\delta, \{k\}, \{d\}) \end{matrix}$$

$$\alpha(\delta) = \alpha_0 + \delta\, \partial\alpha/\partial\delta + O(\delta^2)$$
$$\beta(\delta) = \beta_0 + \delta\, \partial\beta/\partial\delta + O(\delta^2)$$

$$\alpha_0 = 0,\ \beta_0 = \text{const}$$
$$\partial\alpha/\partial\delta = \partial\beta/\partial\delta = 0, \text{ etc.}$$

$$\Rightarrow \{k\}, \{d\}$$

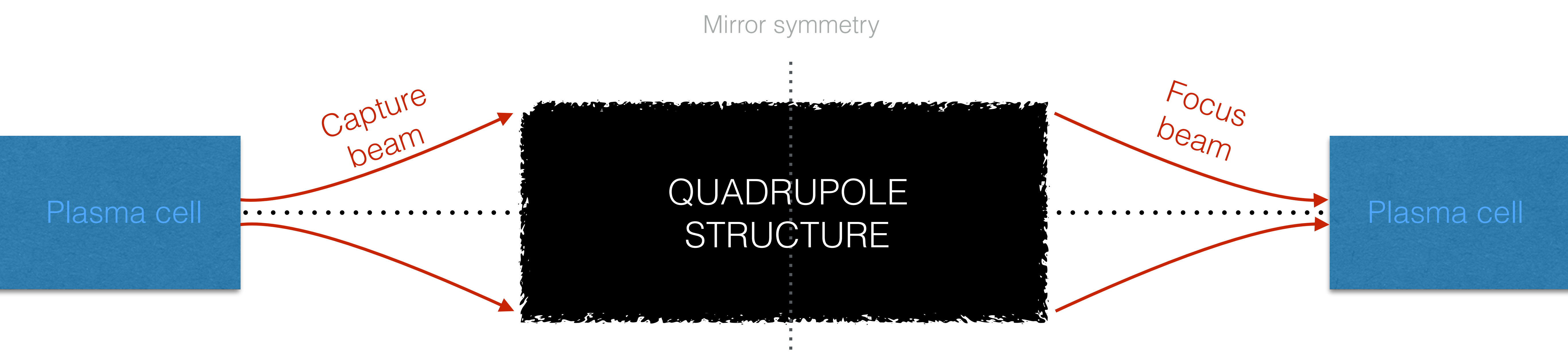
⇒ Just a "higher order" beta-matching.

Application: PWFA interstage

- Plasma wakefield accelerator interstage:
  Capture small and diverging beam exiting plasma stage, **focus into the next**.

- Motivation for considering linear lattices:
  - Must be corrected in **short distance** to maximize acceleration gradient.
  - Length must **scale well with energy** (sextupoles require dipoles/dispersion).

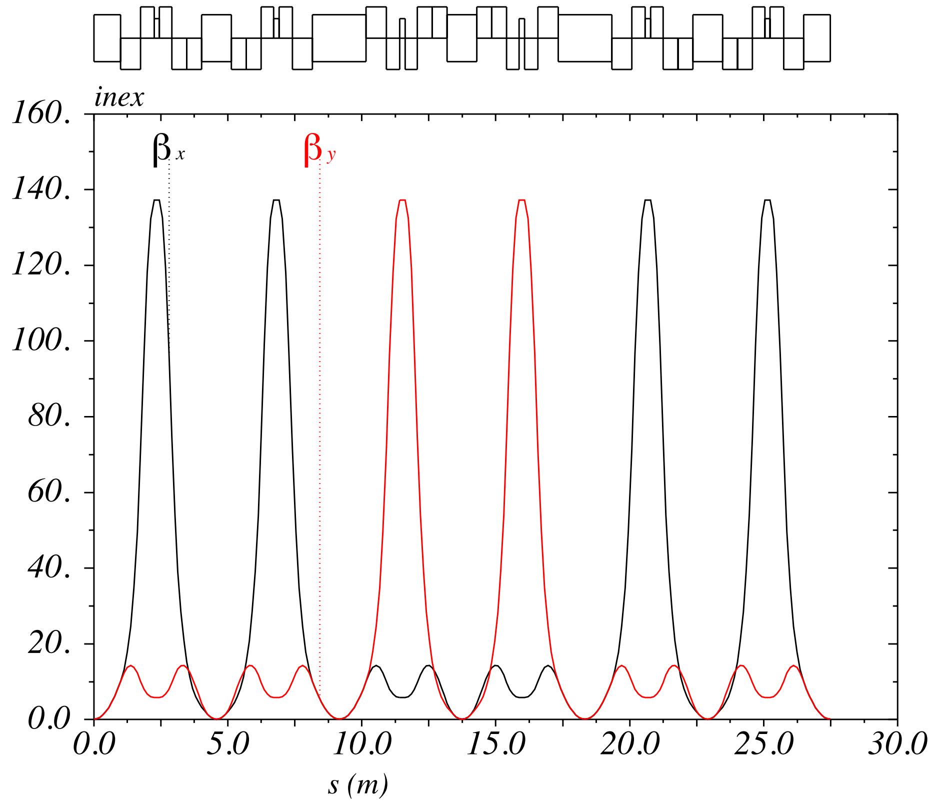
- **Simplified** due to **symmetry** (factor 2 less constraints).



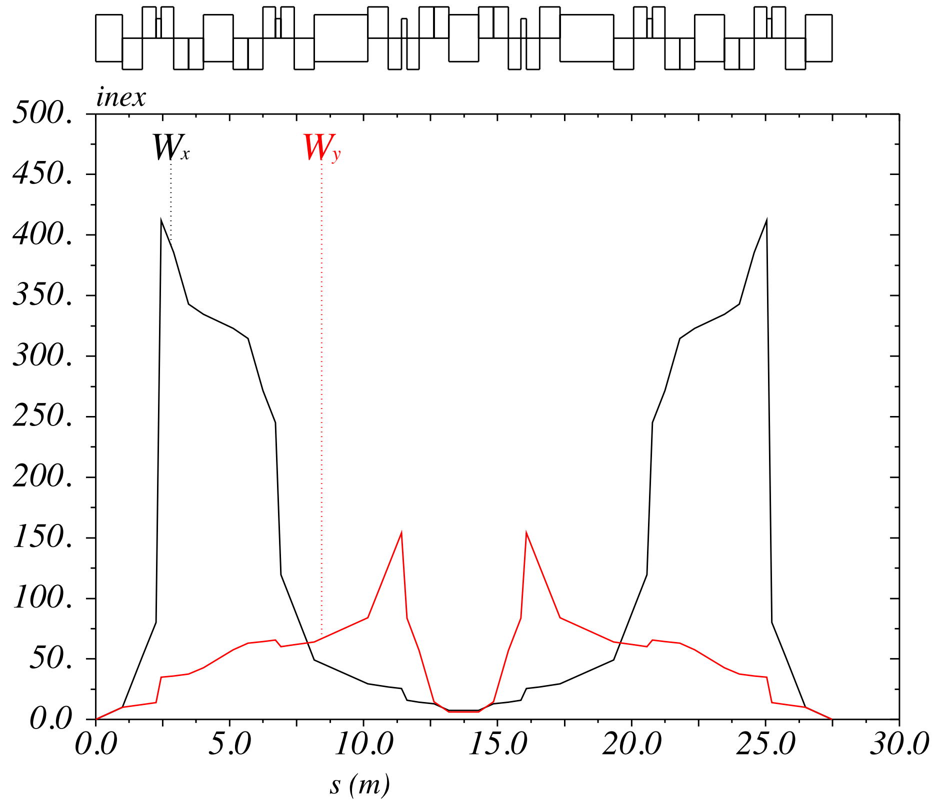
Application: PWFA interstage

- Emittance preserving optics using **sextupoles and 3 minus identity transforms.**
- **Long, complicated lattice** with dipoles/quadrupoles/sextupoles (can possibly be shortened).
- Dipoles needed for dispersion: **SR scaling is worse than for quads only.**

Beta functions



W-functions
(chromatic amplitude)

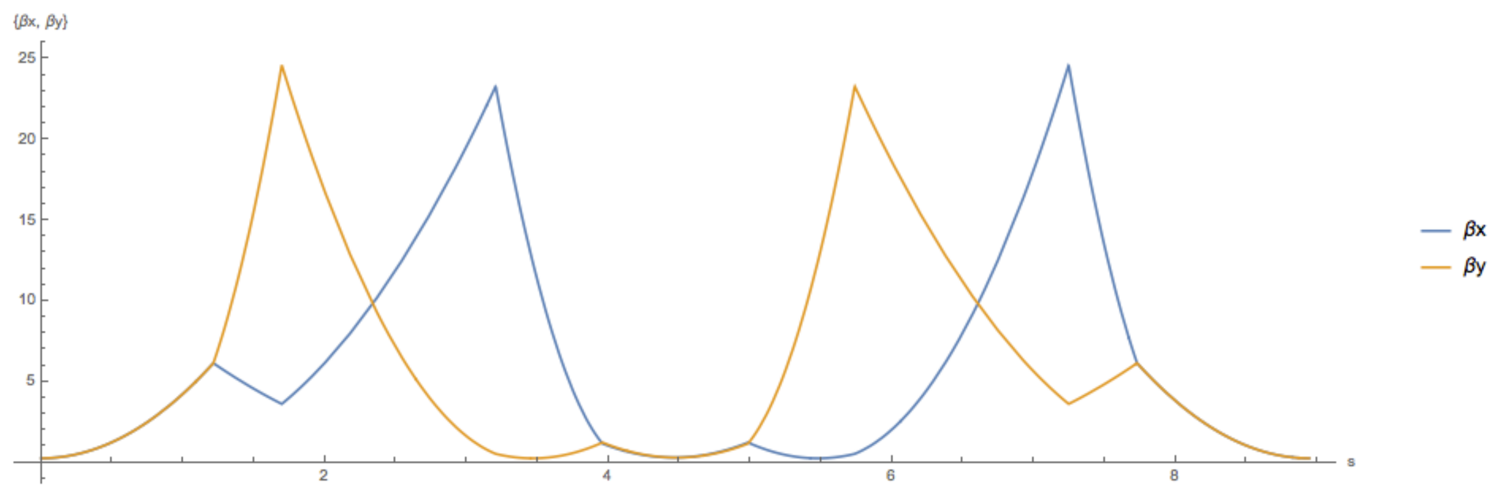


- This solution requires stronger sextupoles than currently manufacturable. Not a conceptual show-stopper.

Application: PWFA interstage

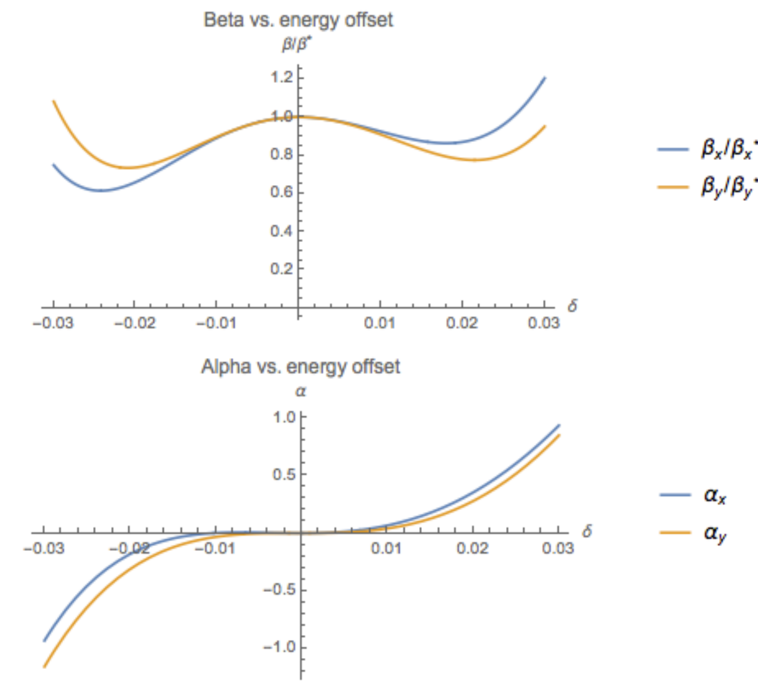
Note: linear lattices so far using thin quads (but physics is same).

- 8 quads (4 d.o.f.): **cancel chromaticity to 1st order.**

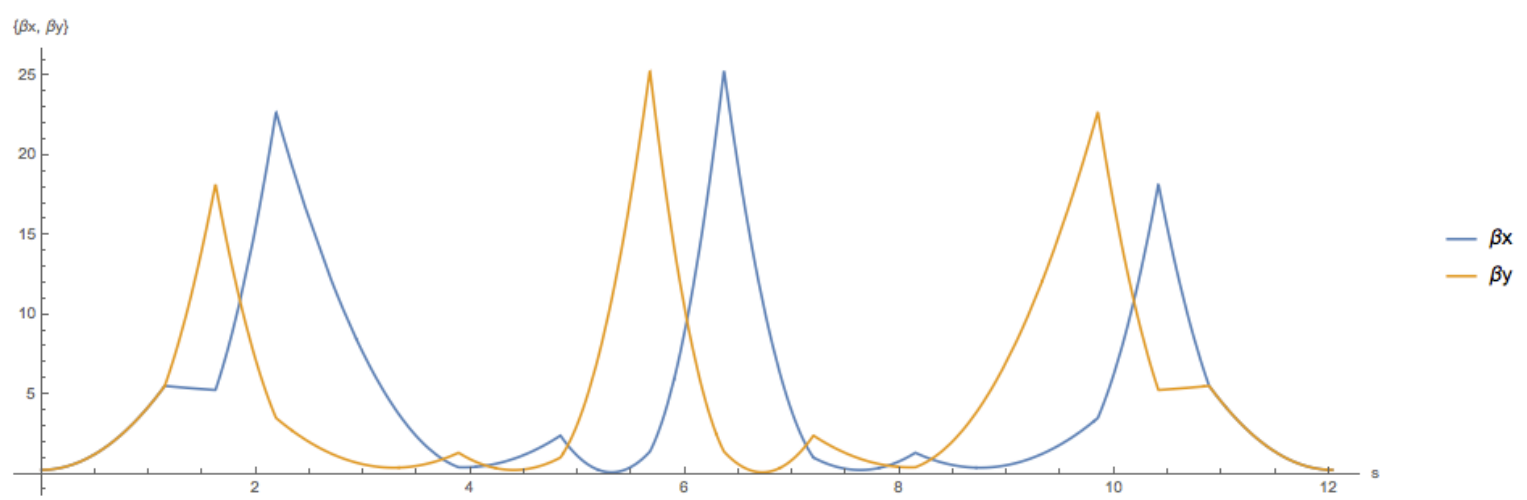


For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y) = (0.000781,\ 0.000782)$

For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y) = (0.0141,\ 0.0142)$

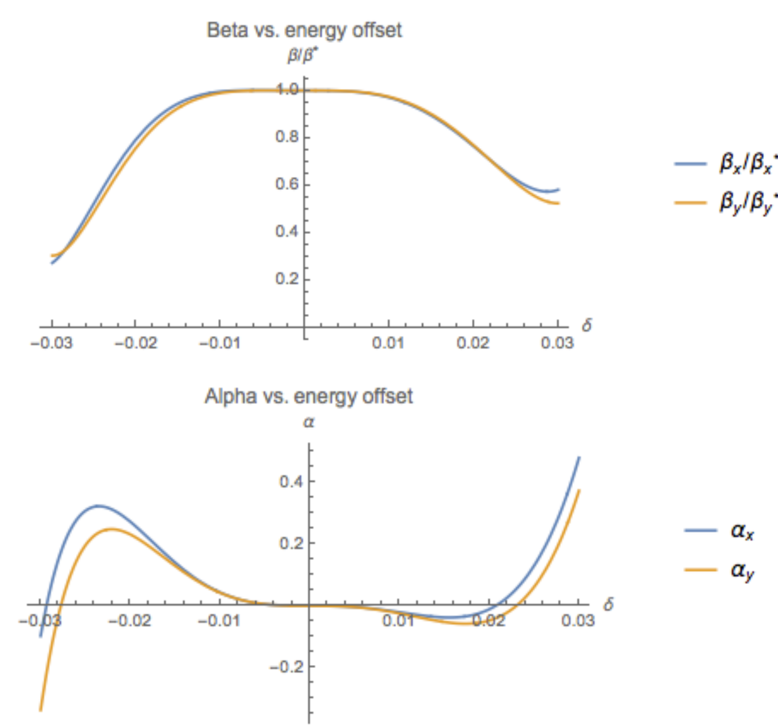


- 12 quads (6 d.o.f.): **cancel chromaticity to 2nd order.**



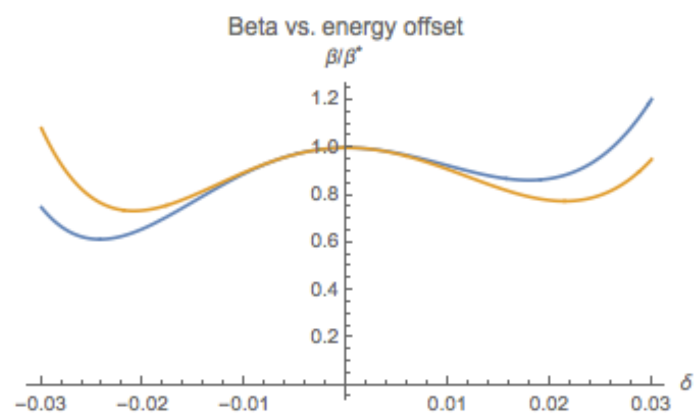
For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y) = (0.000186,\ 0.000186)$

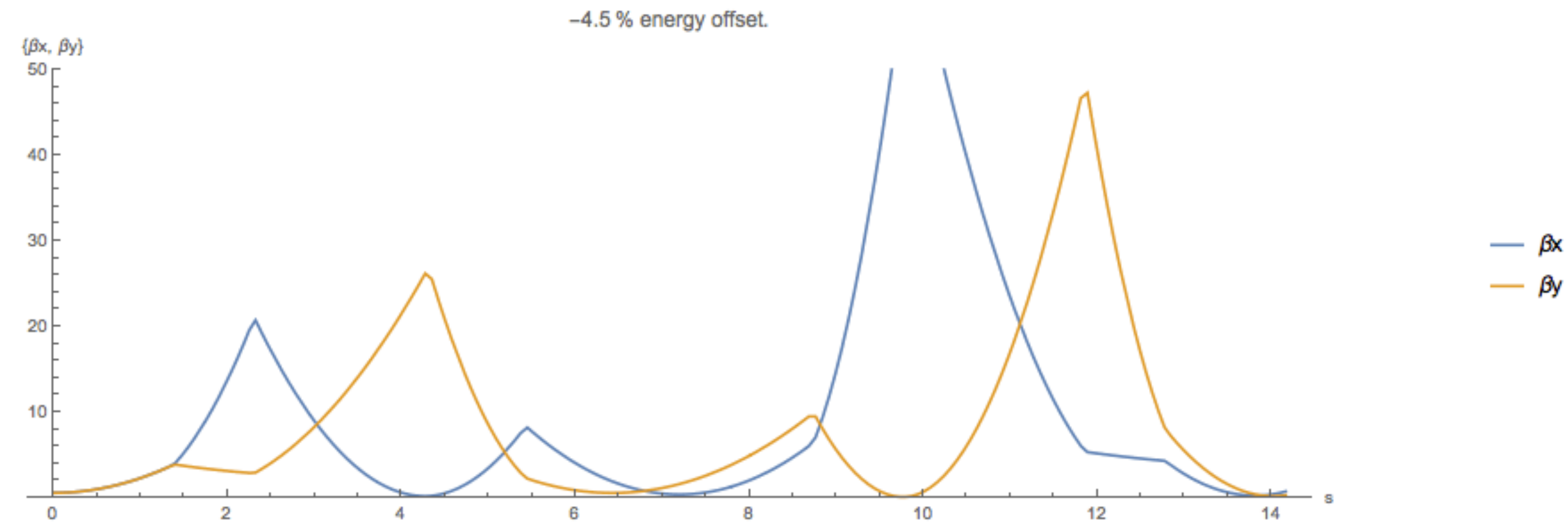
For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y) = (0.0119,\ 0.0119)$

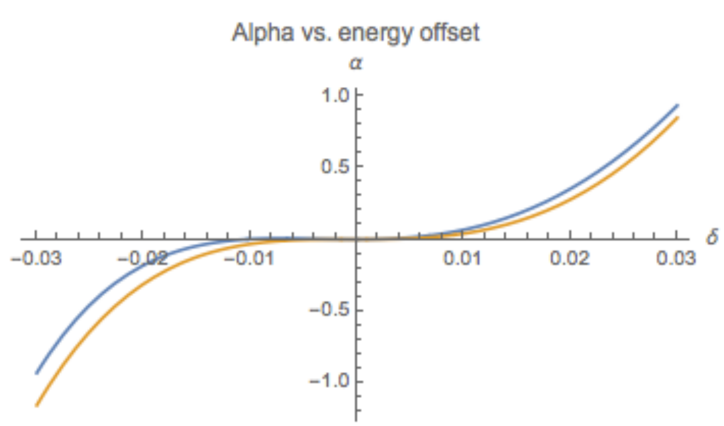


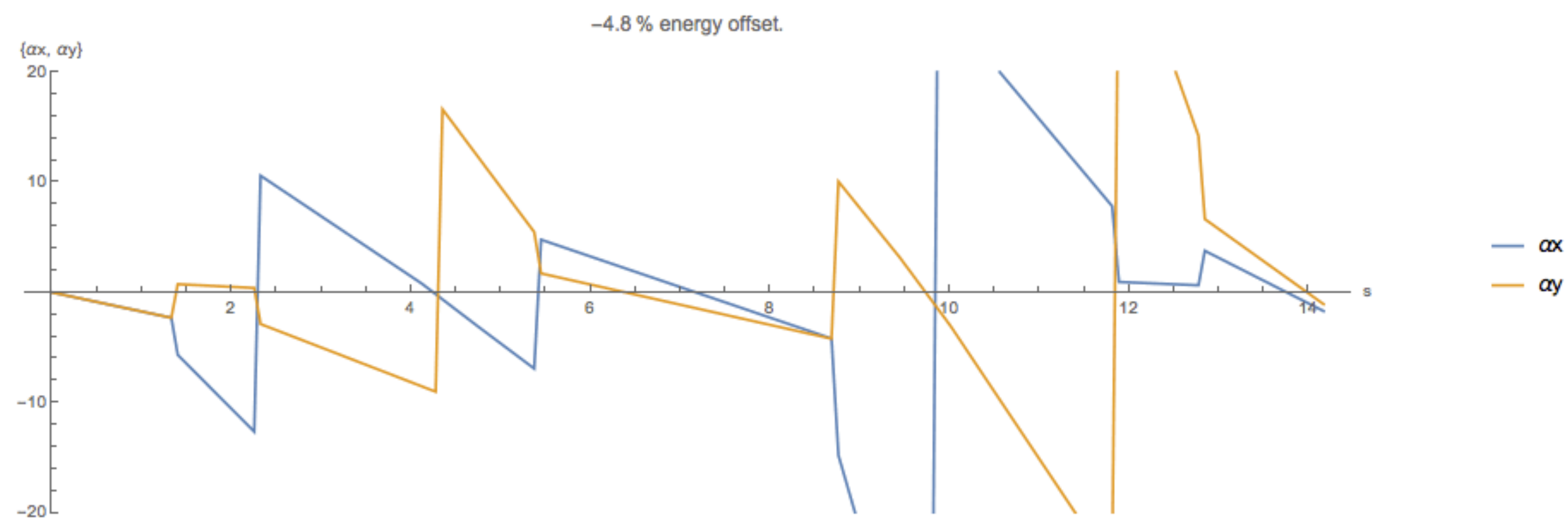
Evolution of α and β functions

- Lattice constructed such that an **error in focusing** is **corrected by** a subsequent **“inverse error” in focusing.**









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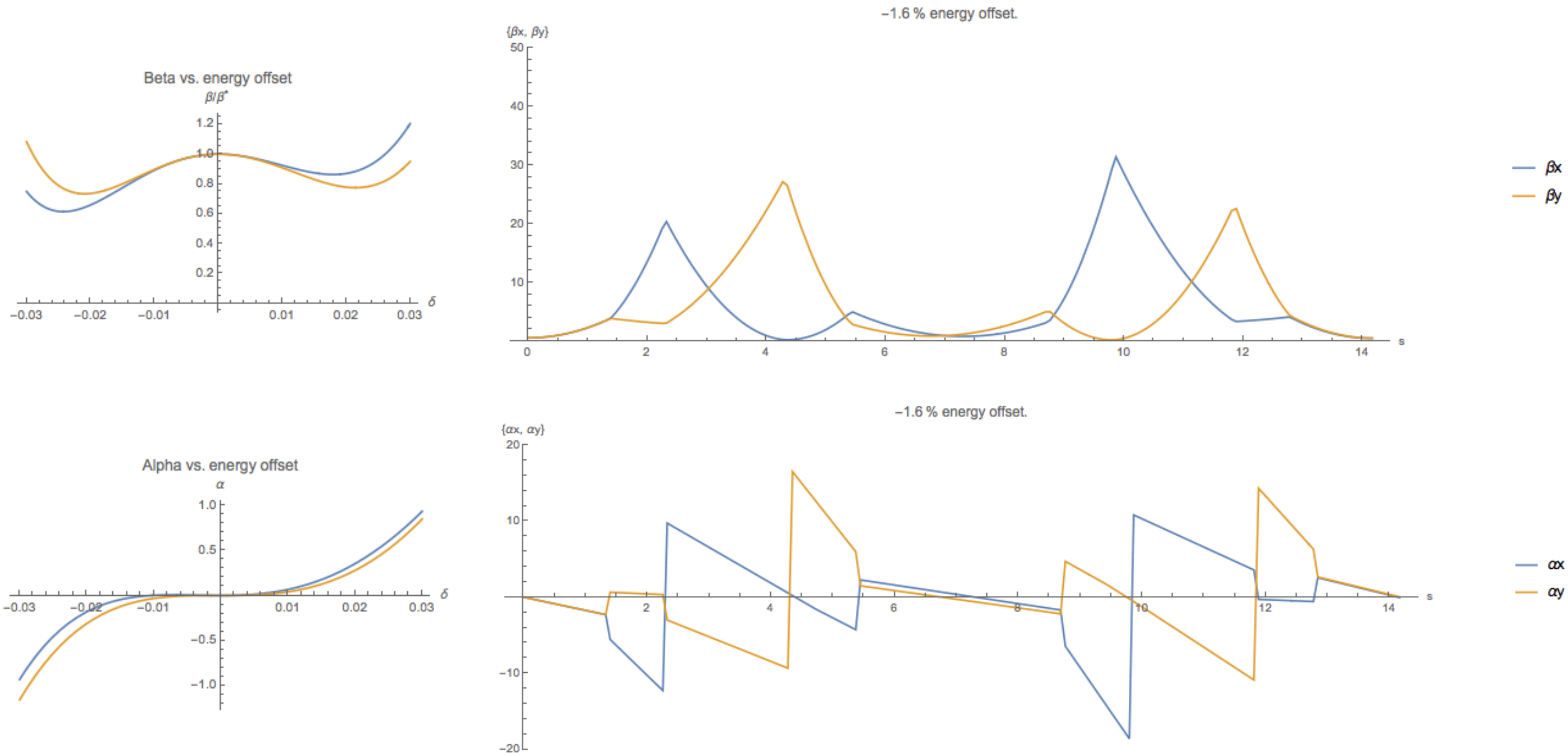

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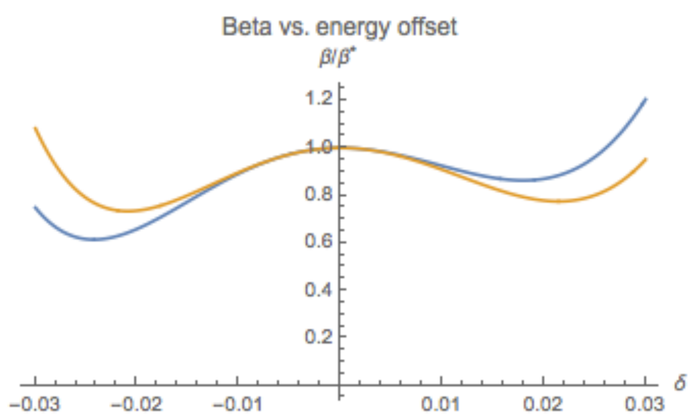

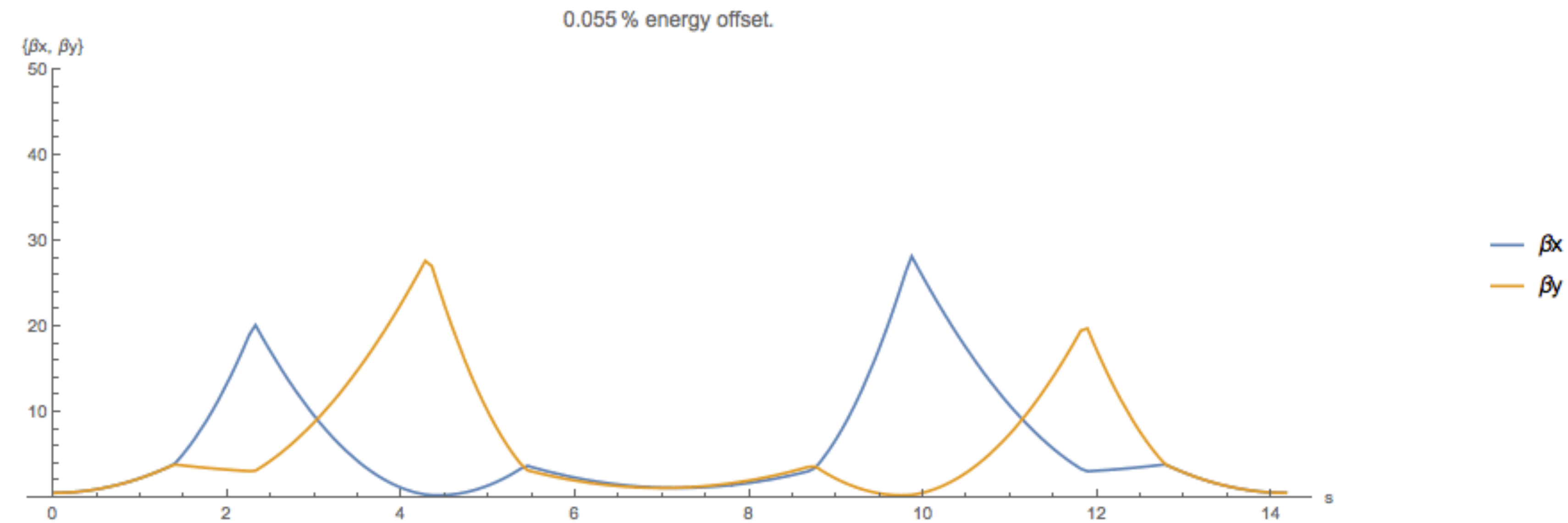

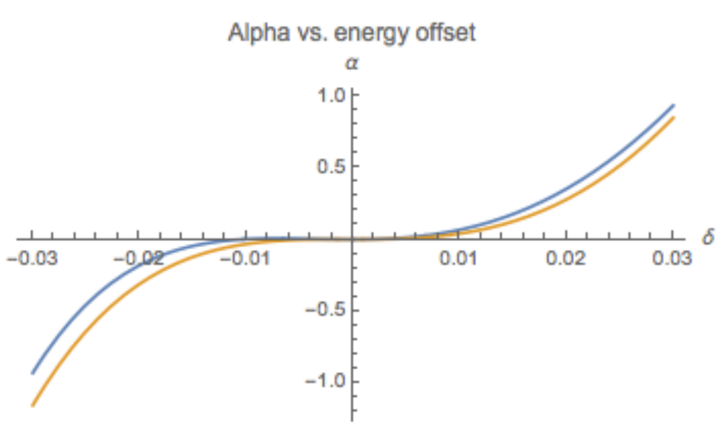

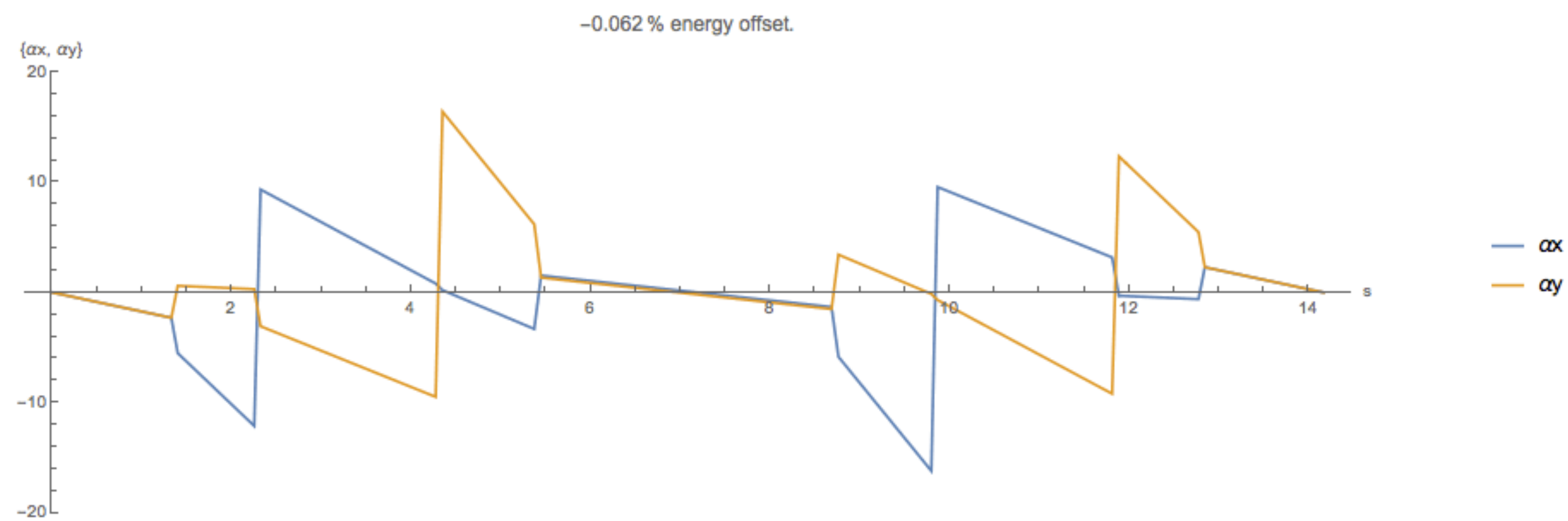

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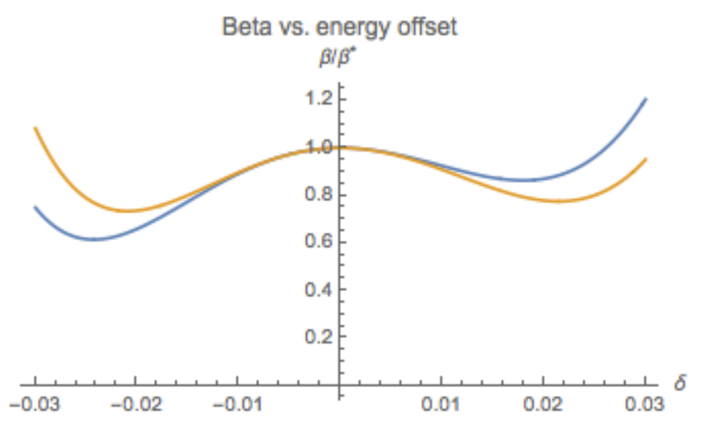

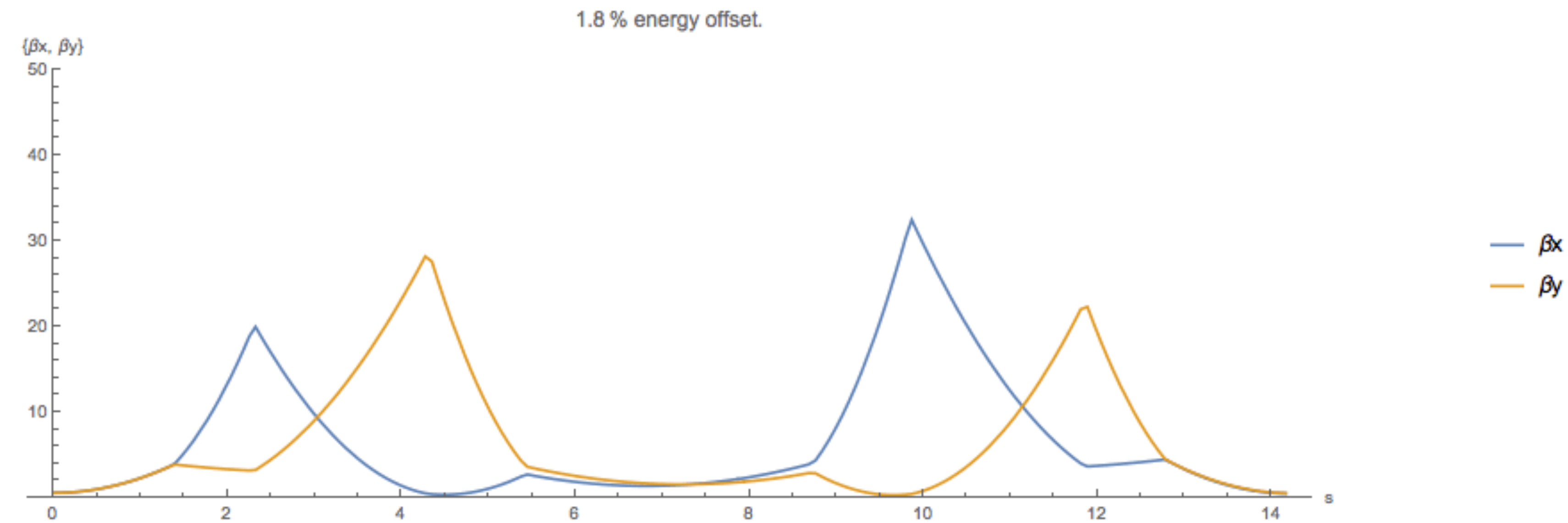

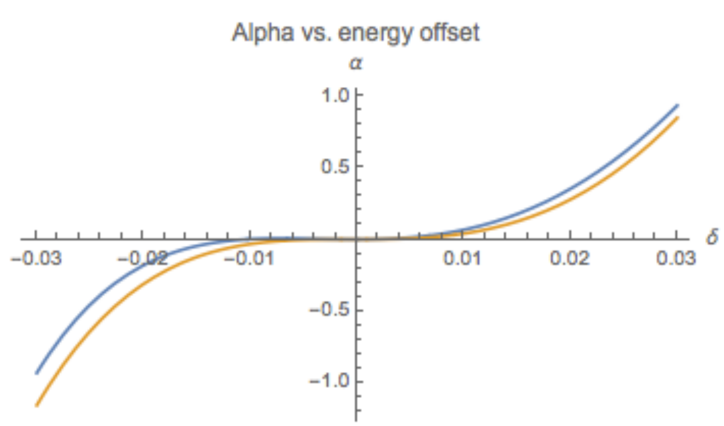

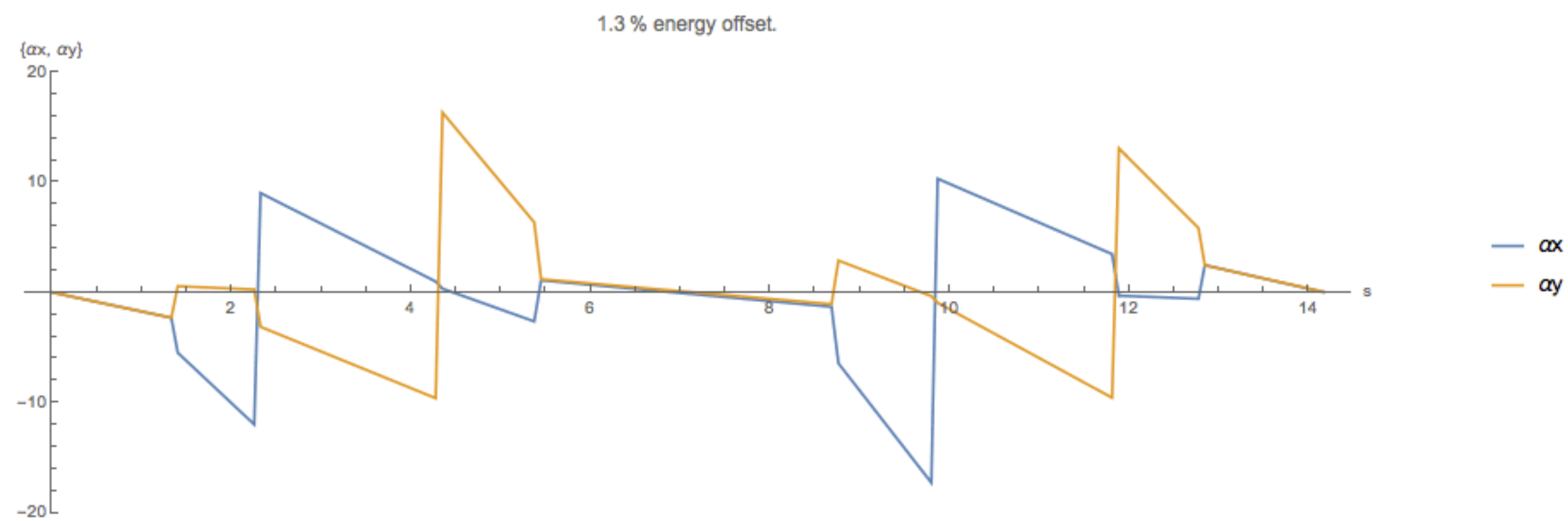

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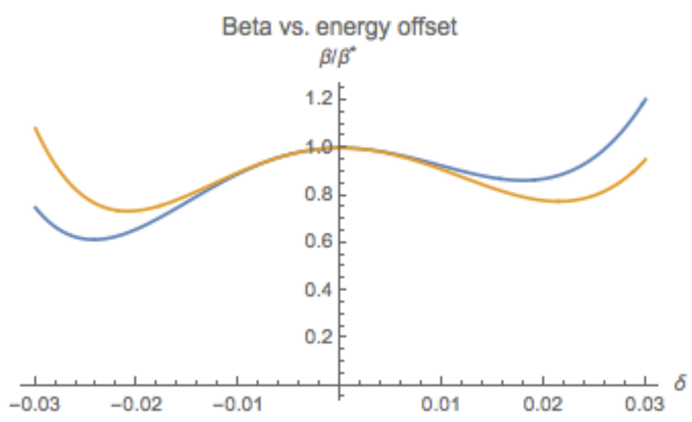
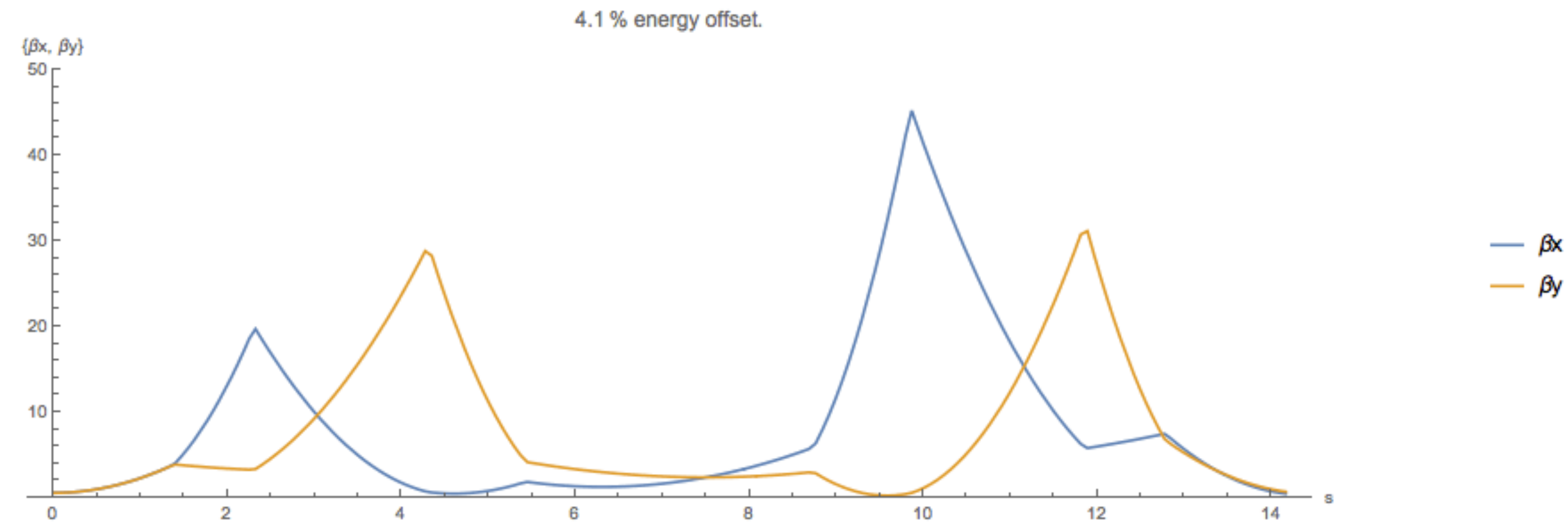
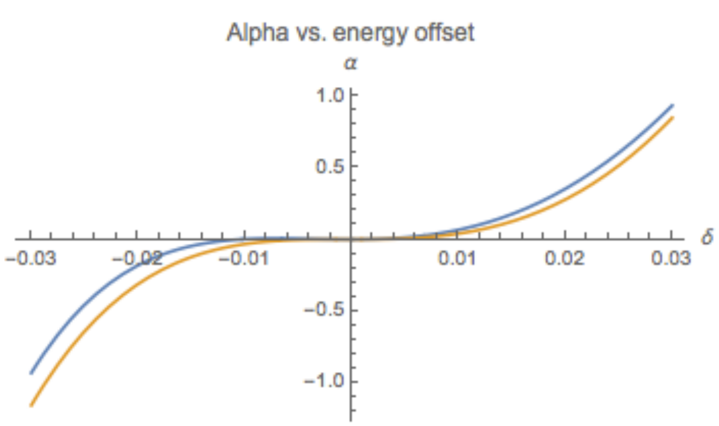
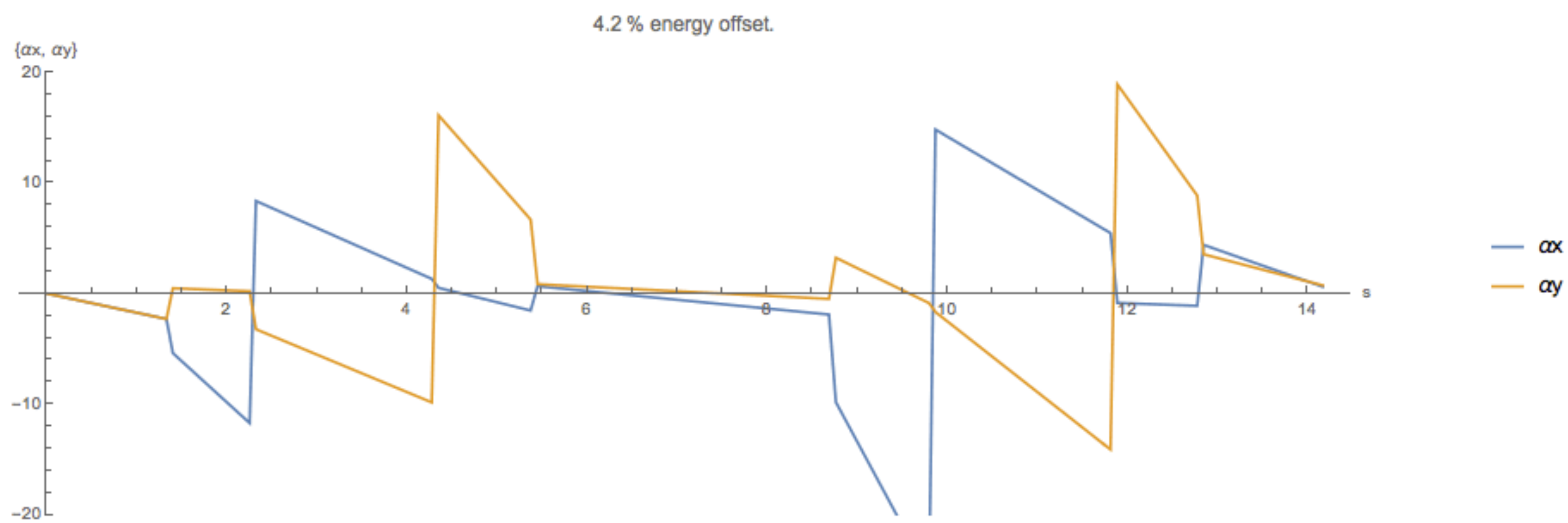




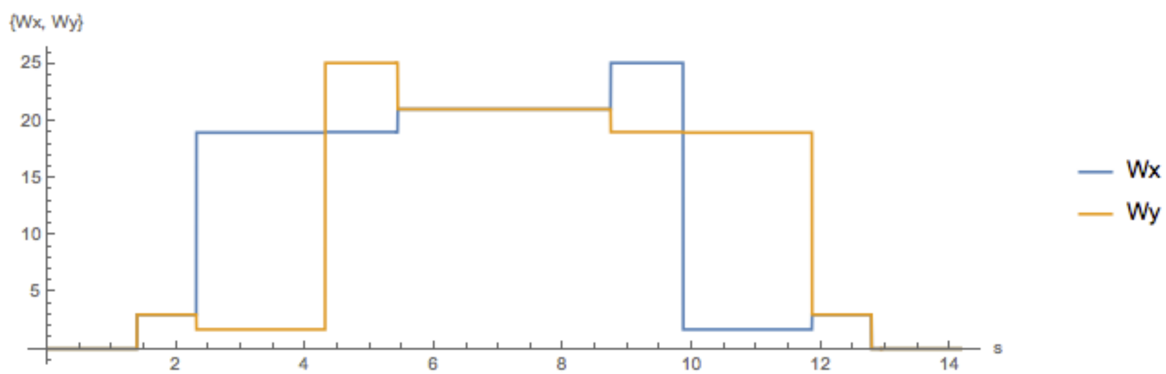




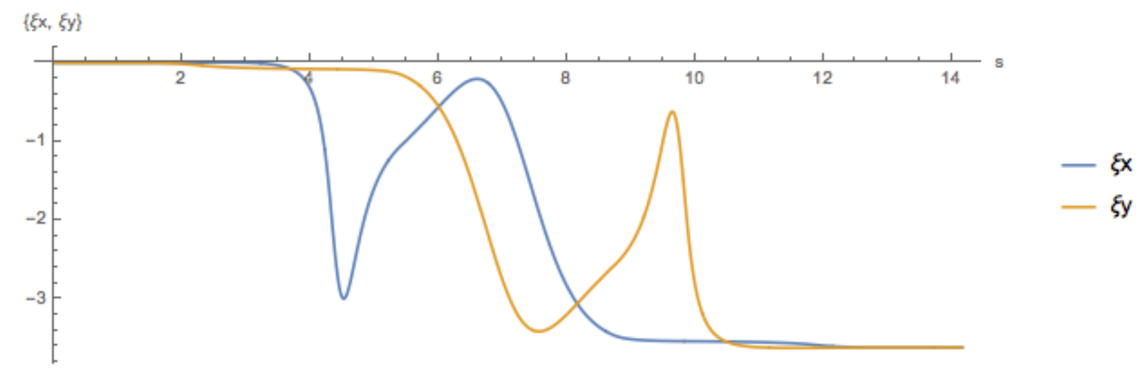
Evolution of chromaticity along beam line

- *In drifts:* Chromatic vector rotates with twice phase advance (2μ): $w(s) = W(s)e^{i2\mu(s)}$

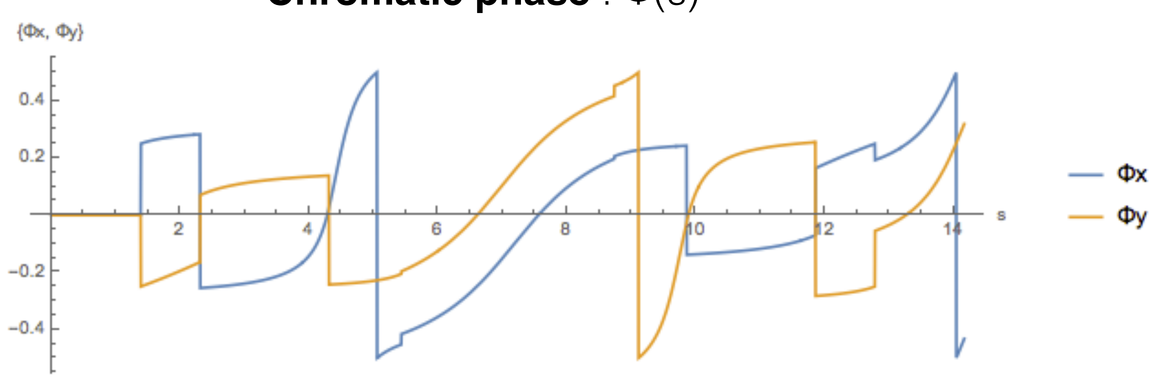
Chromatic amplitude : W(s)



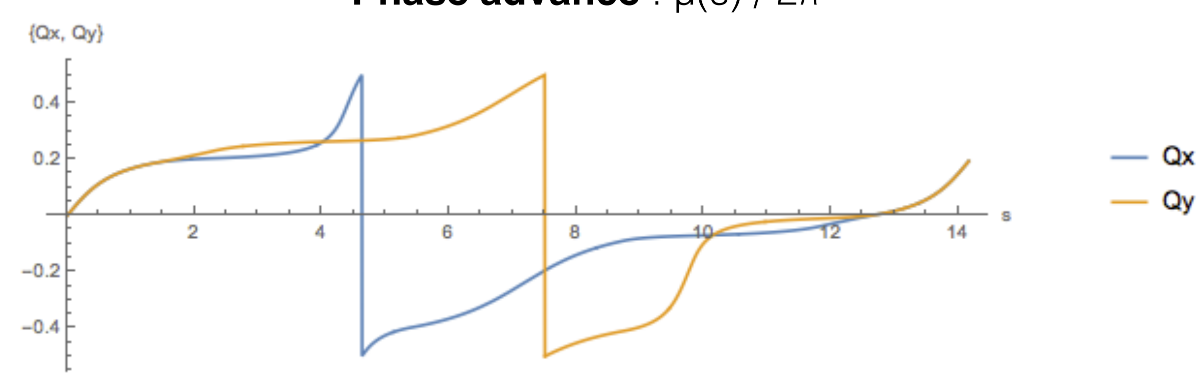
Phase chromaticity : ξ(s)



Chromatic phase : Φ(s)



Phase advance : μ(s) / 2π



- *Corollary:* **Two periods of 90° phase advance will cancel chromaticity (1st order).**
  Create ΔW ⇒ rotate Φ = π to -ΔW ⇒ add another ΔW ⇒ **W = 0**

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Emittance growth in linear lattices

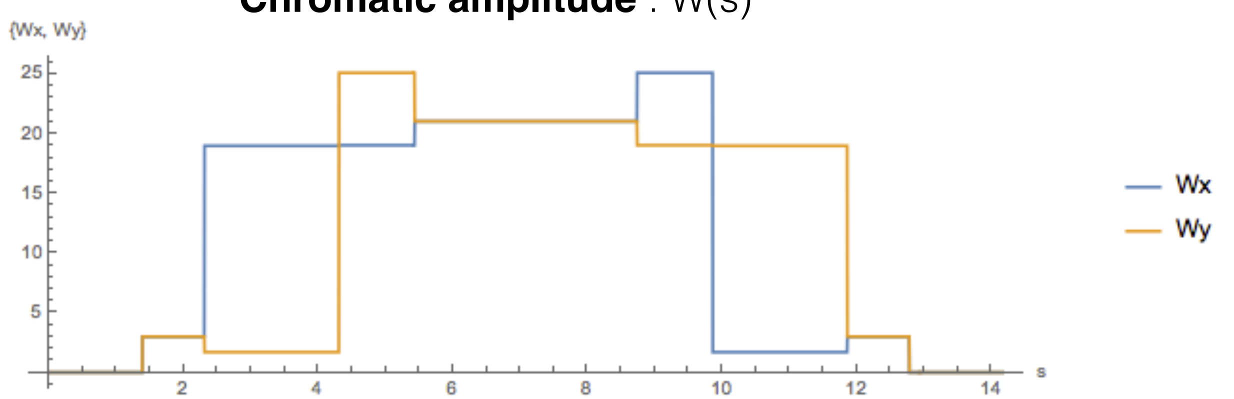
- For a *gaussian energy distribution* in a linear lattice:

$$\frac{\Delta\epsilon^2}{{\epsilon_0}^2} = W^2{\sigma_E}^2 + O({\sigma_E}^4)$$

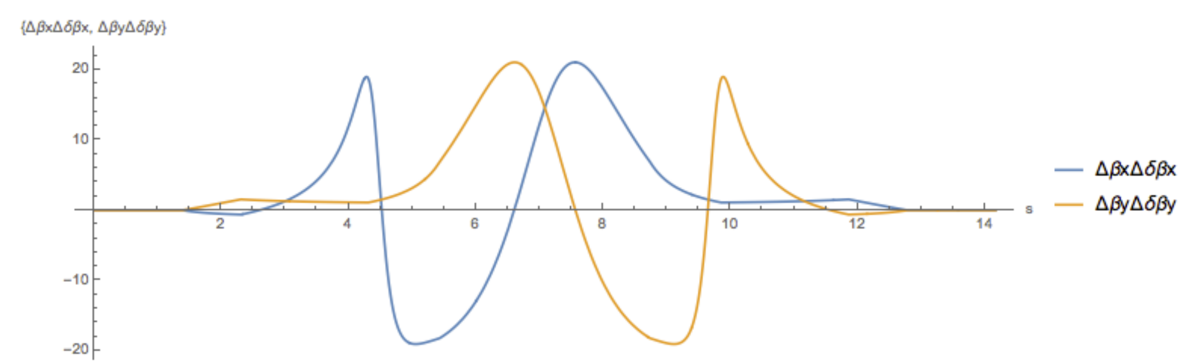
ϵ : *emittance* σ_E : rms energy spread

- Linear lattices $\Rightarrow$ emittance of each energy slice preserved (in drifts and quads).
- W conserved in drifts $\Rightarrow$ projected emittance preserved in drifts.

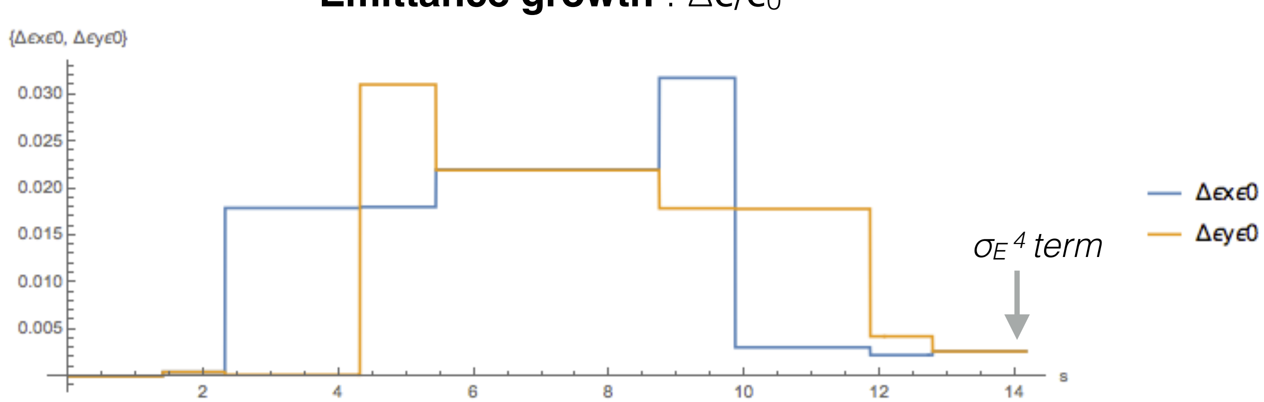
Chromatic amplitude : W(s)



(Normalized) **Chromatic β-derivative** : ∂β/∂δ / β0

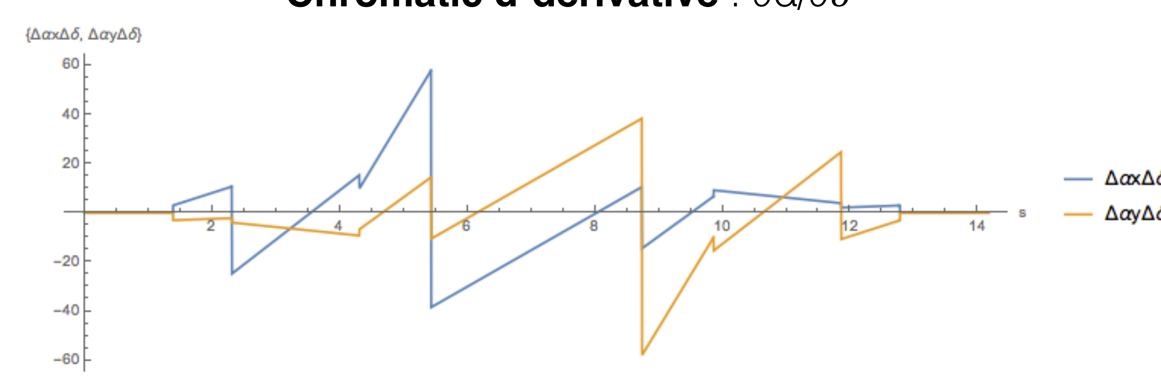


Emittance growth : Δϵ/ϵ₀



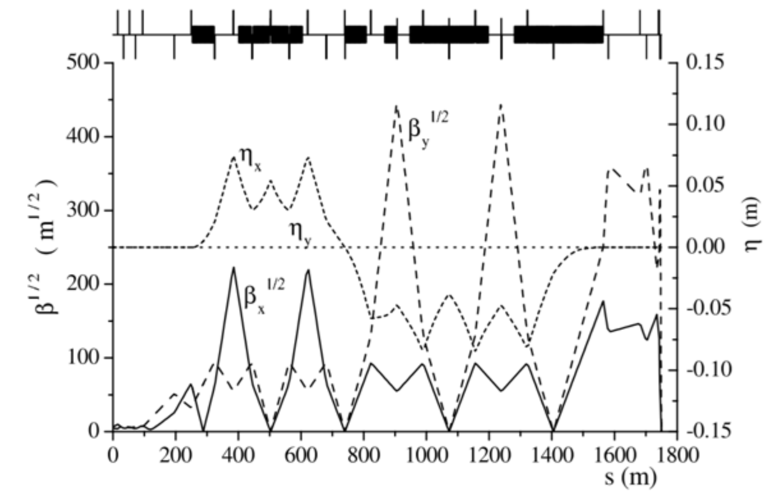
(1% rms energy spread)

Chromatic α-derivative : ∂α/∂δ

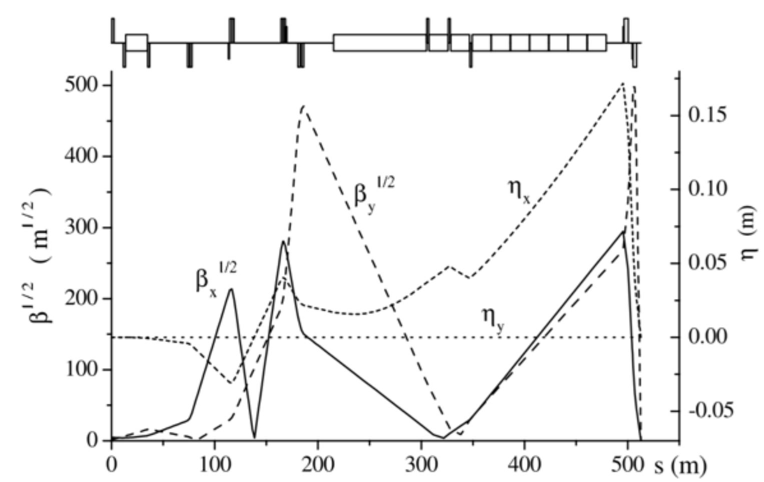


Application: Final Focus

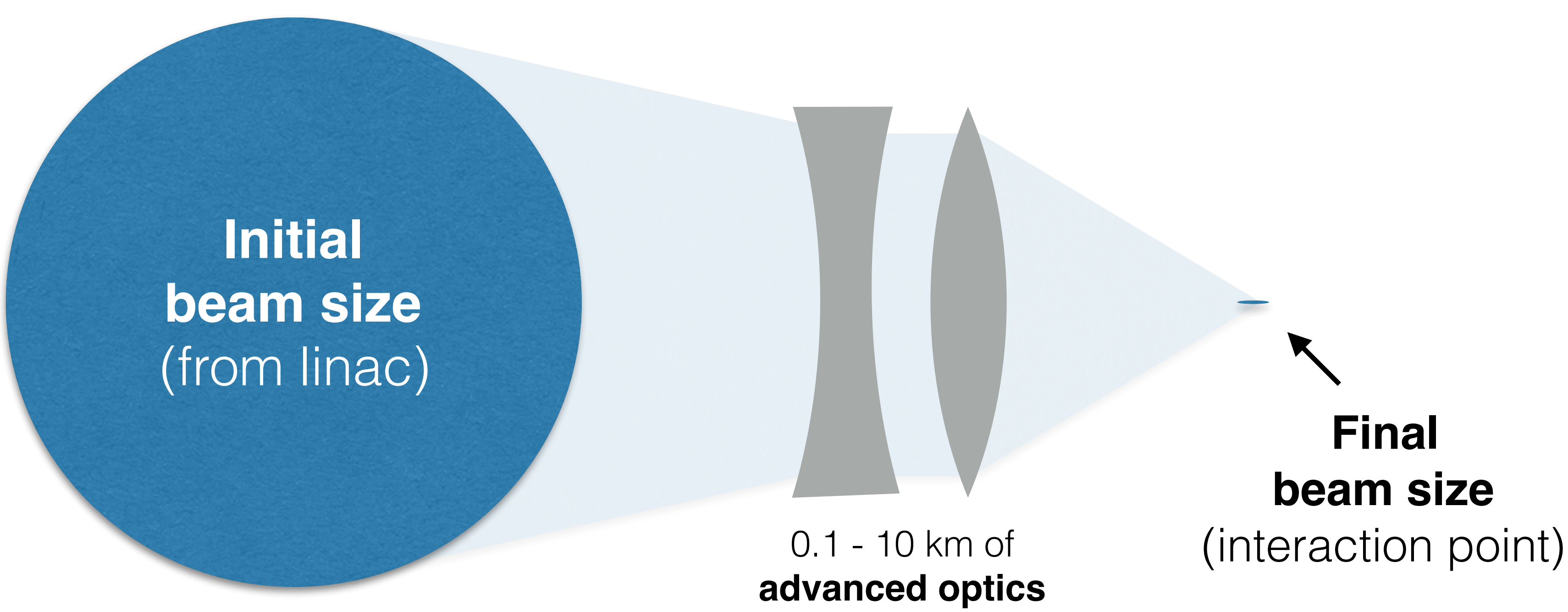
- FFS goal: **Greatly demagnify the beam size** (β) to increase luminosity, **while preserving spot size** for a certain spread of energies.
- Current best FFS design: **Raimondi & Seryi (2000)***.
- Make attempt: Apply linear lattice chromatic correction to a final focus.
- *Bad news*: **No symmetries** (harder to find solution)
  Good news: **Only care about spot size** and not emittance at IP (only one constraint per chromatic order).



Traditional FFS: global correction (very long).



New FFS: local correction (6x shorter).



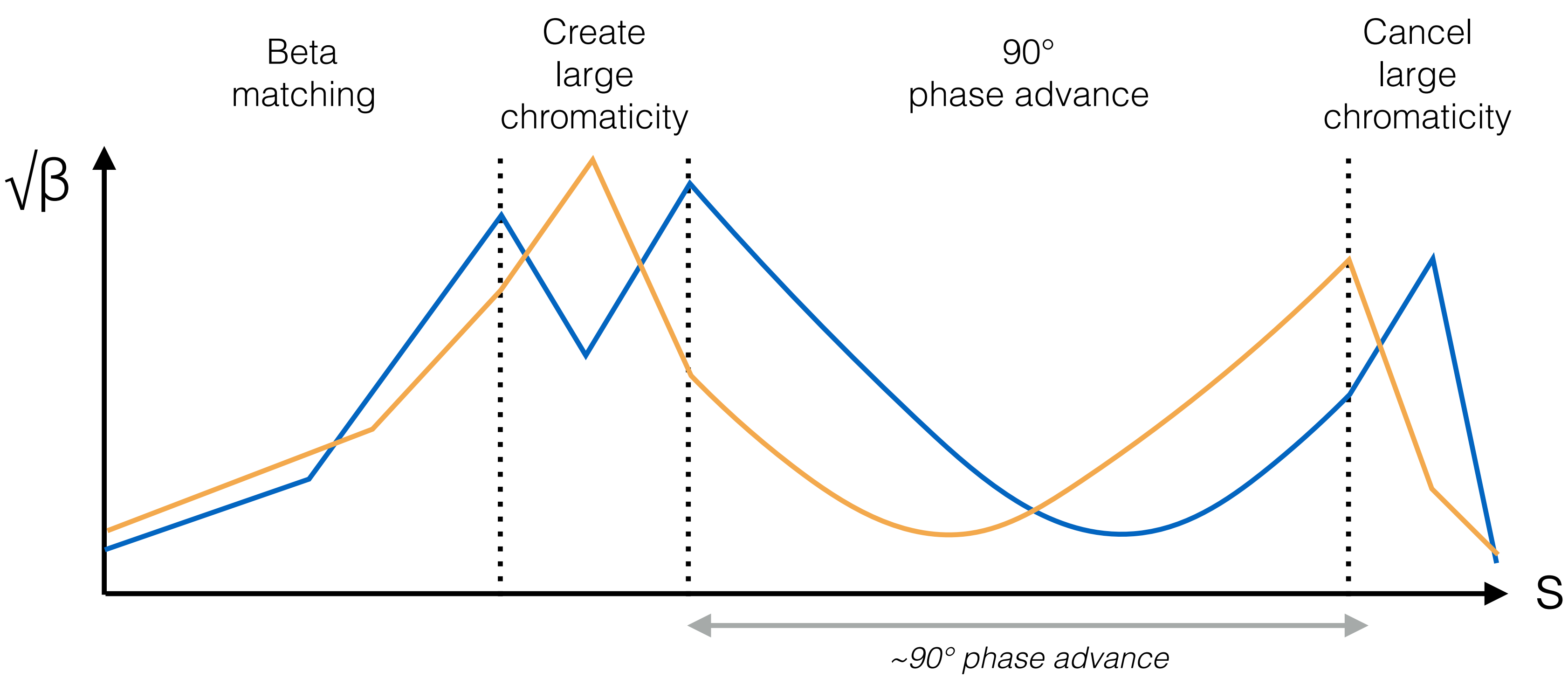
* *"Novel Final Focus Design for Future Linear Colliders"*, *P. Raimondi & A. Seryi (2000)*

Application: Final Focus

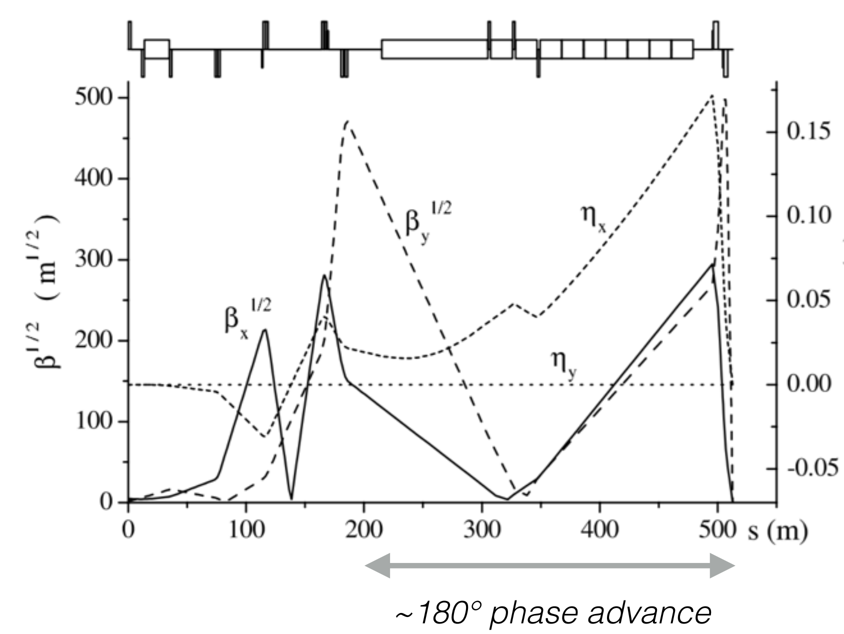
- A **plethora of solutions** exist. Tricky to find good solutions due to:
  - Large number of constraints/degrees of freedom.
  - Scale difference inherent in problem (large demagnification)

- Method:
  1. Find good solution for low demagnification.
  2. Using this solution as initial guess, increase demagnification slightly.
  3. Repeat point 2, hoping that length/energy acceptance scales well.

- **Solutions seem to show similar features:**

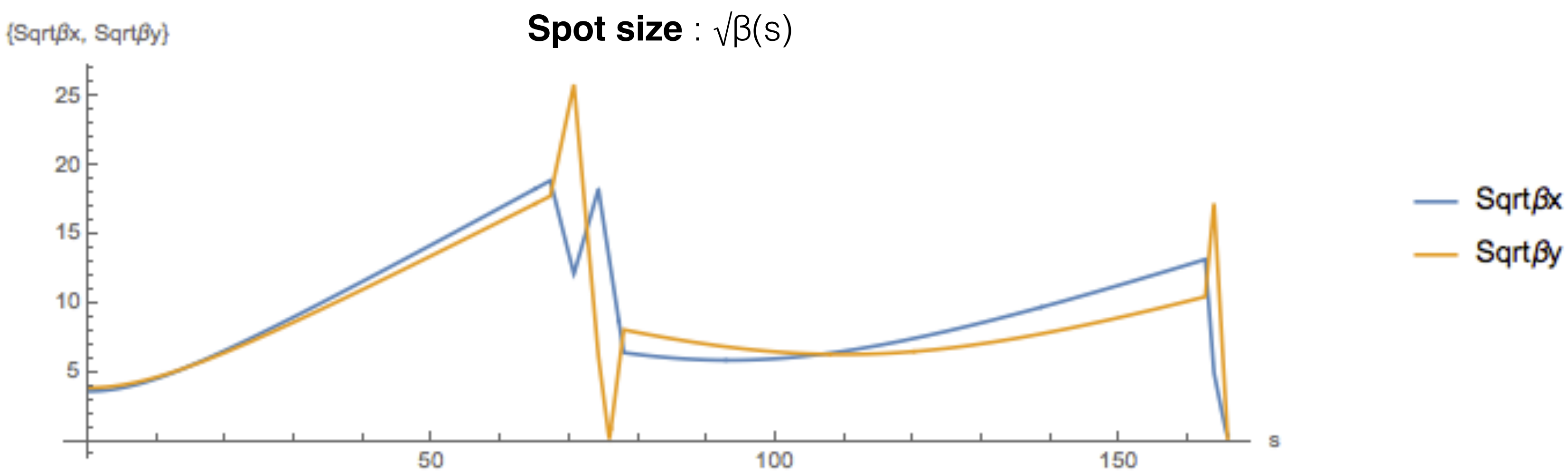


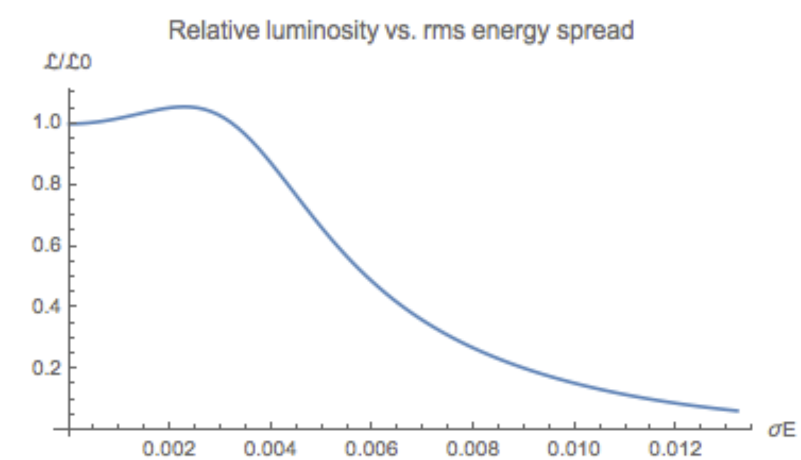
Looks similar to Raimondi/Seryi FFS

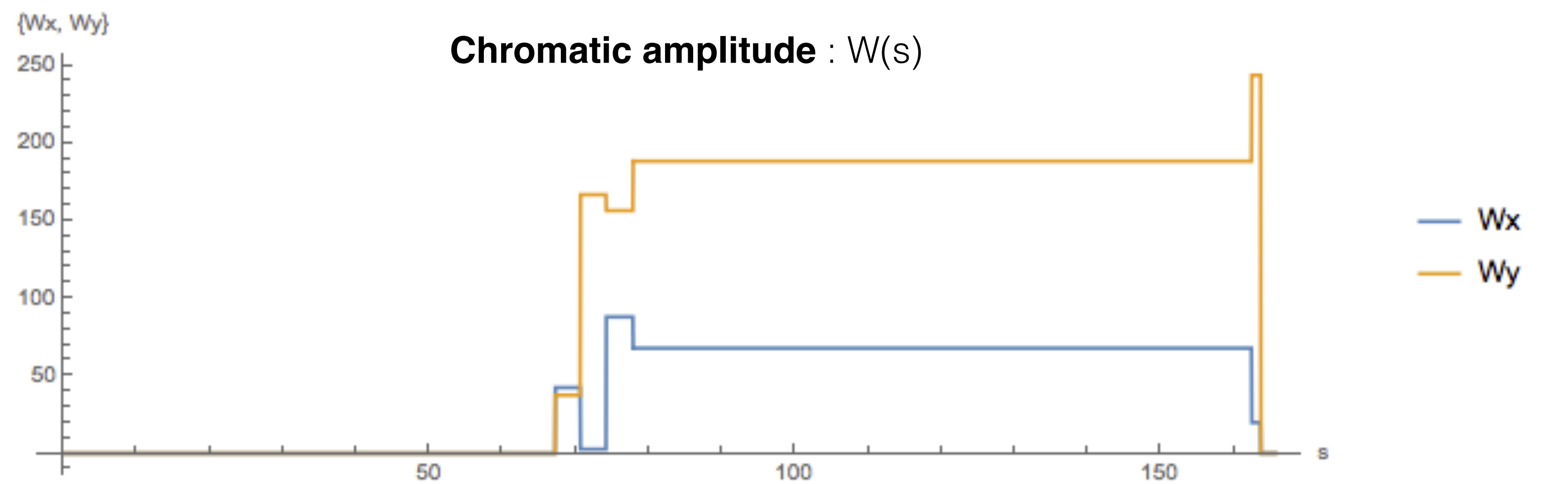


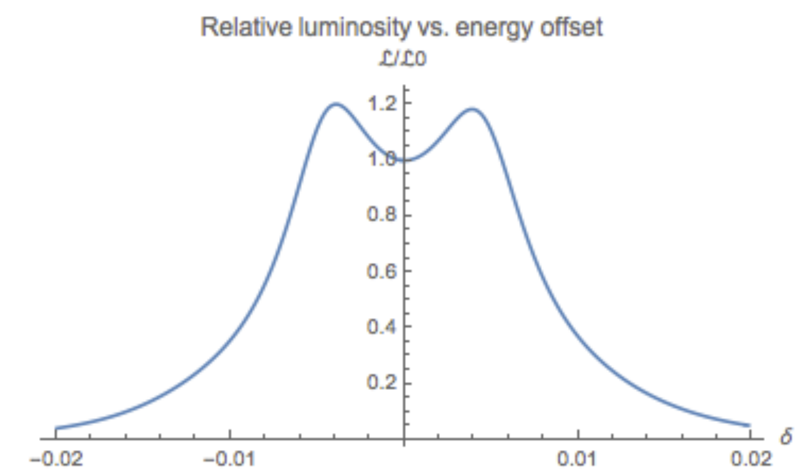
Application: Final Focus

- Parameters: **L^* = 2 m**, β_{linac} = (13, 15) m, **β^* = (16, 1.3) cm**, β-demag. = **(80, 1125) x**
- Solution: Lattice length **L_{FF} = 166 m**, Energy acceptance **σ_E = ~0.5%**





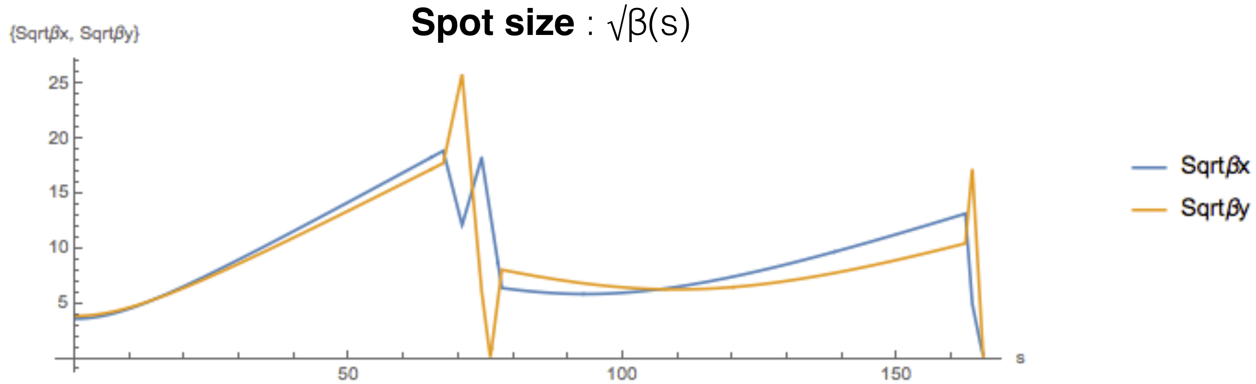




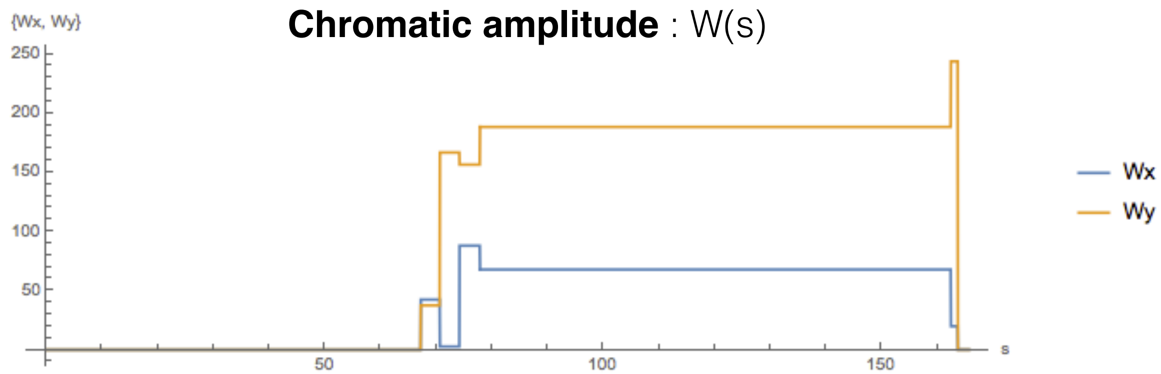
- Tracked in ELEGANT for verification. Everything linear ⇒ **emittance preserved for every energy slice.**

Application: Final Focus

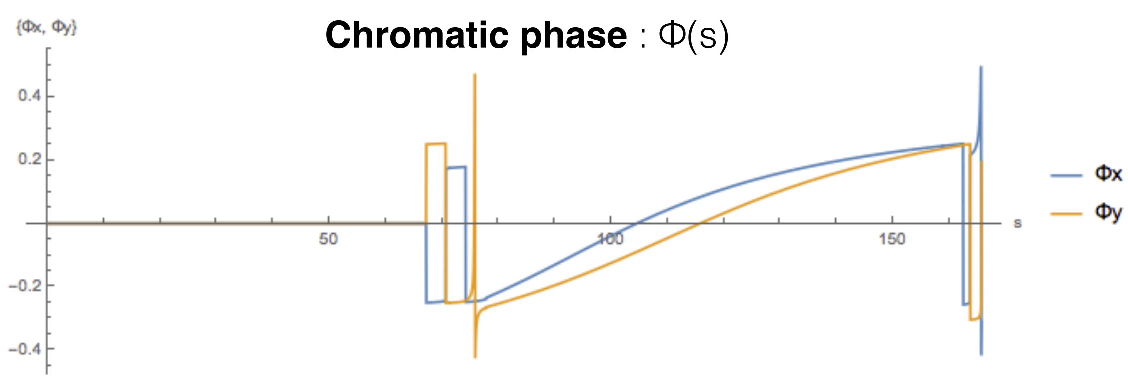
Spot size : $\sqrt{\beta(s)}$



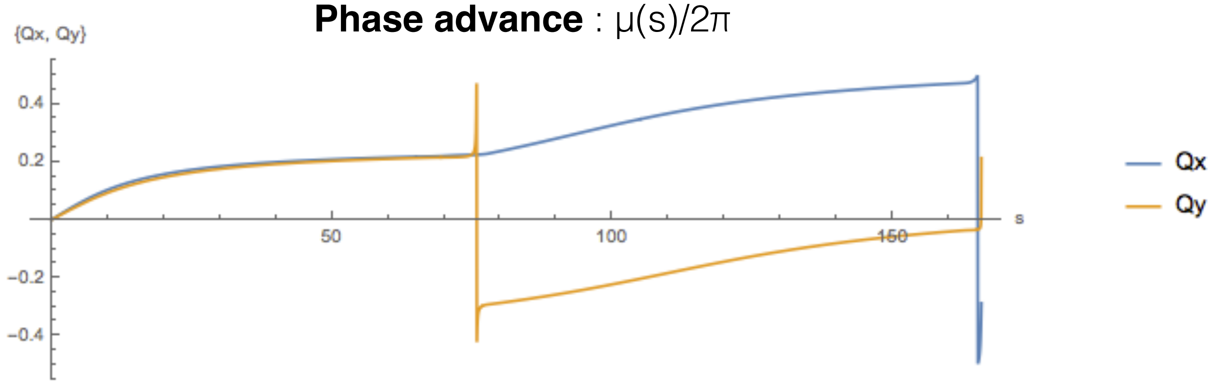
Chromatic amplitude : W(s)



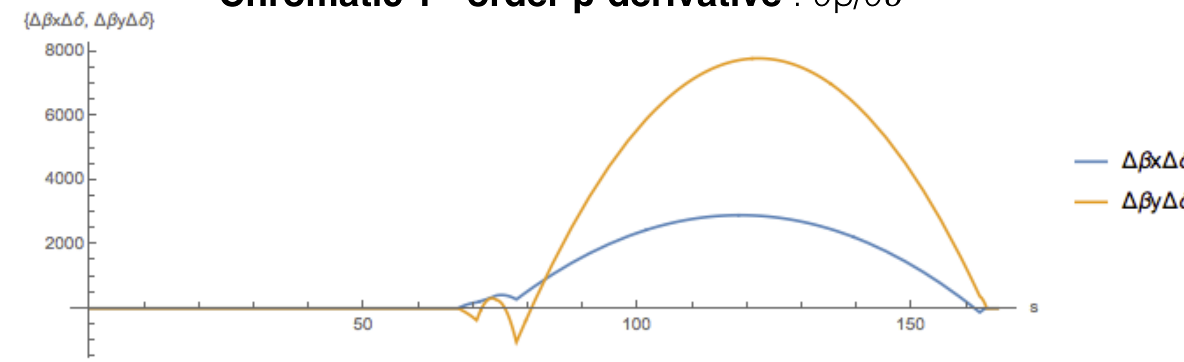
Chromatic phase : Φ(s)



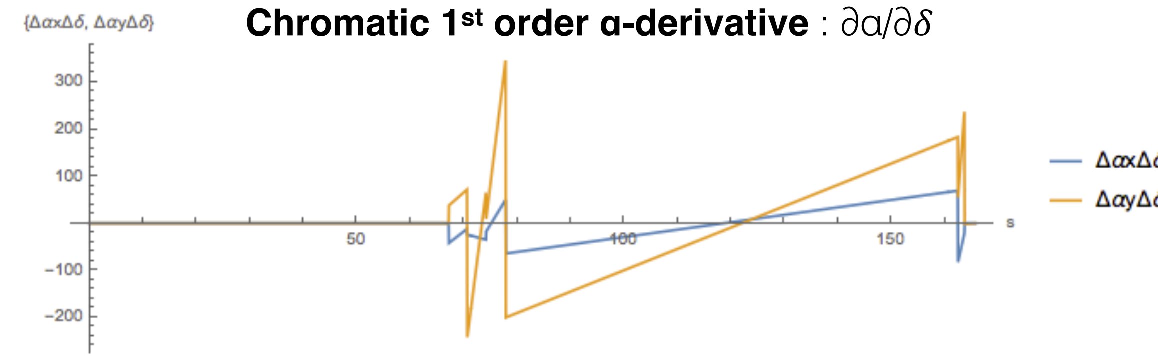
Phase advance : $\mu(s)/2\pi$



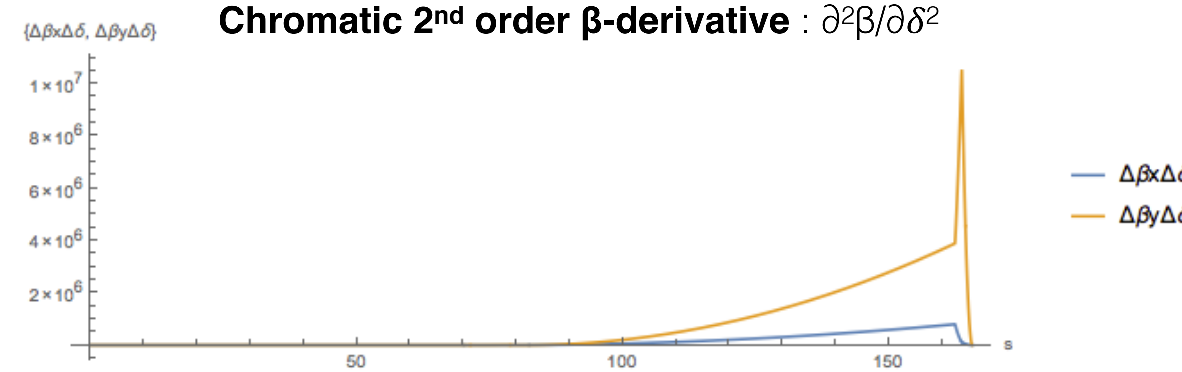
Chromatic 1st order β-derivative : $\partial\beta/\partial\delta$



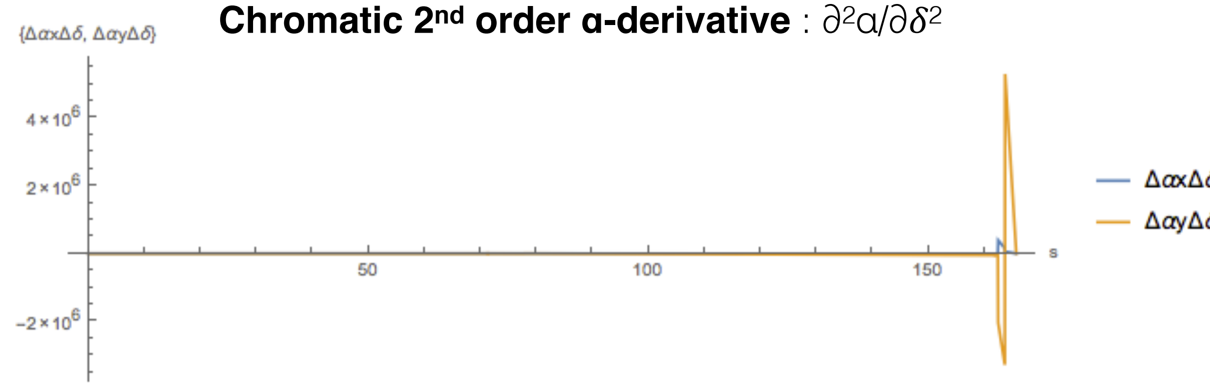
Chromatic 1st order α-derivative : $\partial\alpha/\partial\delta$



Chromatic 2nd order β-derivative : $\partial^2\beta/\partial\delta^2$



Chromatic 2nd order α-derivative : $\partial^2\alpha/\partial\delta^2$



Open questions / Work in progress

- Can a linear lattice achieve **same demagnification / energy acceptance / length** as a non-linear lattice with sextupoles?

- Sextupoles cancel W, as can linear lattices.
  Emittance growth is then due to **2nd order chromatic terms**.
  Why would this **scale better in a non-linear lattice than in a linear lattice**?

- Currently solving on local (laptop) computer (up to 2nd order), in timeframe of minutes. Reduced to a computational one (finding a global minimum). **Applying more computing power (and experts), what would be the limitations?**

- Can the FF be **made in separate stages?** (beta matcher, chromaticity compensator, 90° phase advance, FD)

Summary

- A sextupole-free final focus in possible in principle.
- An attempt was made, with moderate success (not as good as Seryi/Raimondi FFS).
- Possibilities for improvement, with potentially great payoff.

Plans of further work

- Document principle and methodology in an article.
- We are happy to discuss/get input on current work.

Thank you for your attention!