Homework Set 2 (10/31/15), C1 class (X-Ray Free Electron Laser Theory), 9th ILC school, 2015

1 Parabolic model

In this problem, you will work out some of the details of the exact solutions for the parabolic model. In particular, using the mode equation given in slide 24 of the 3D theory notes, prove the dispersion relation for μ_l in the case of the fundamental mode. The final expression is a special case of the one shown in slide 25 for $m = l = 0$. To arrive at this, you should assume a solution of the form $A_l(\hat{\mathbf{x}}) = \exp(-a\hat{r}^2)$ and substitute back into the mode equation. Eliminating a from the resulting two equations would yield the desired result.

2 LCLS 3D optimization

Using Ming Xie's fitting formulas (see slides 33 and 34), plot the optimized gain length and saturation power as functions of the average beta function $\bar{\beta} = 1/k_\beta$ for the LCLS parameters. What is the approximate value of the optimum beta (i.e. what beta value results in a) minimum gain length and b) maximum saturation power)? A possible set of LCLS parameters is: radiation wavelength $\lambda_1 = 0.15$ nm, undulator period $\lambda_u = 3$ cm, $K = 3.7$, beam energy = 14.31 GeV, peak current $I = 3$ kA, normalized emittance $\epsilon_n = \gamma\epsilon = 0.5$ μm, relative energy spread $\sigma_\eta = 10^{-4}$. Recall that the beam size is given by $\sigma_x = (\epsilon\bar{\beta})^{1/2} = ((\epsilon_n/\gamma)\bar{\beta})^{1/2}$.