Undulators for FELs

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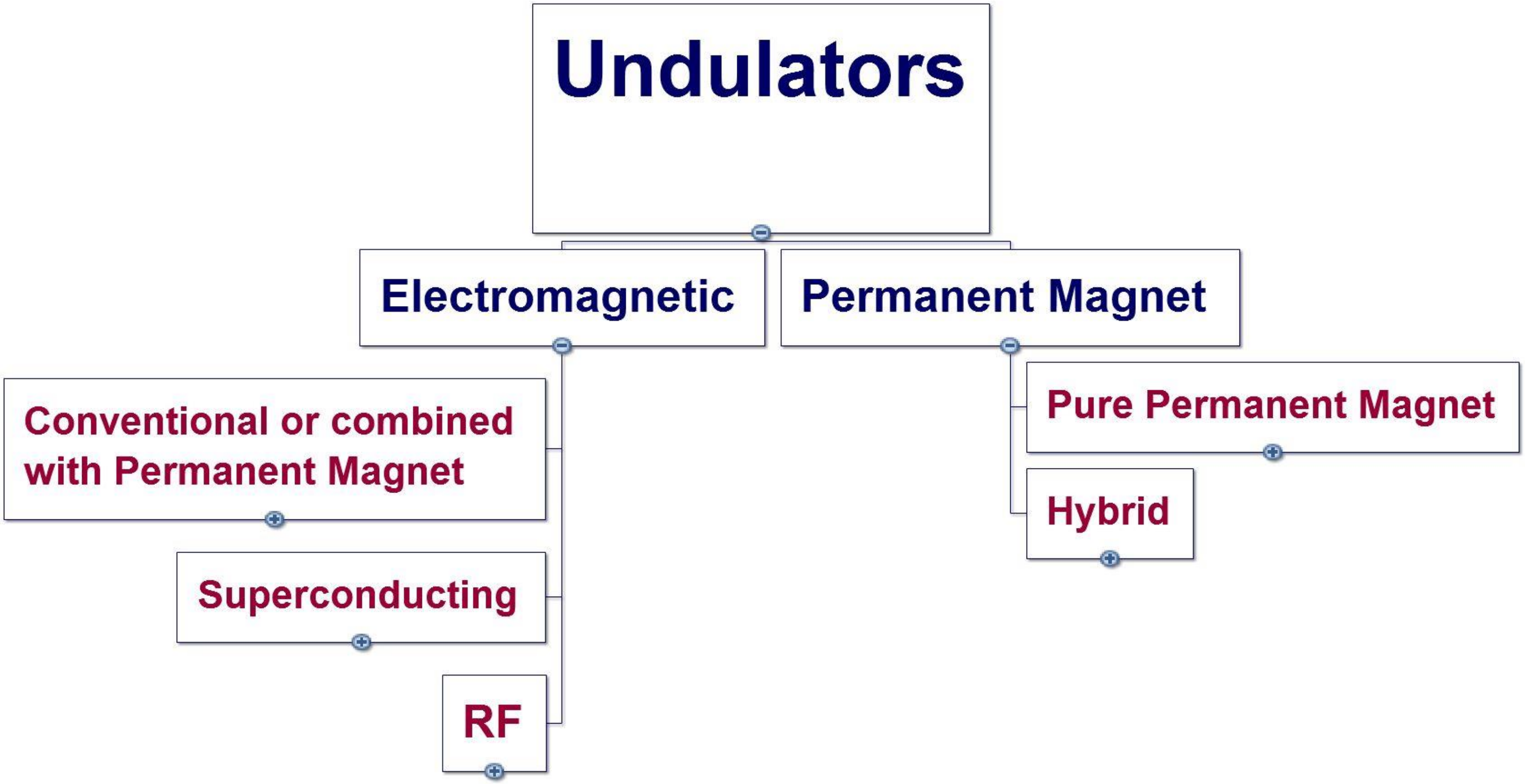
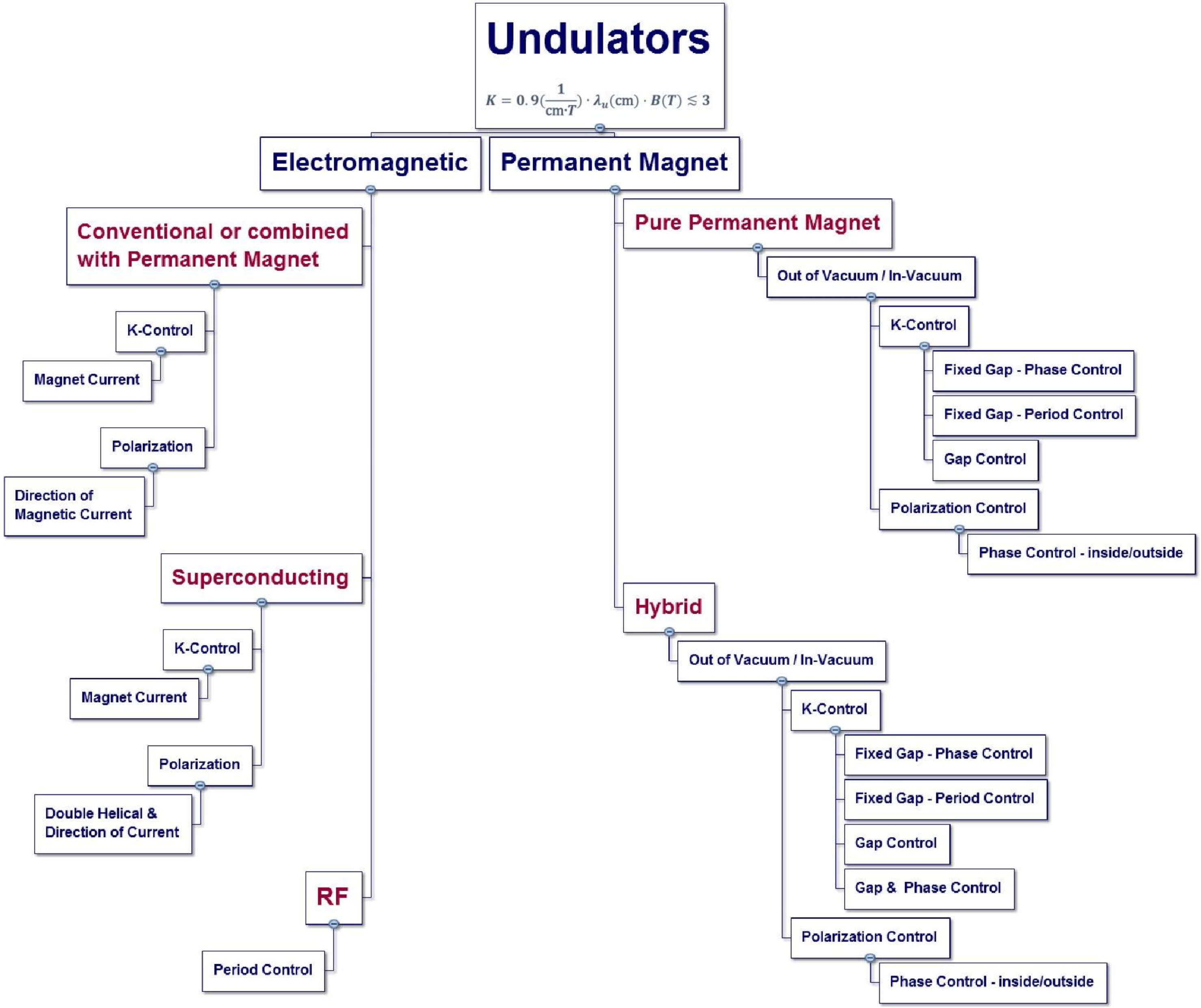
Topics

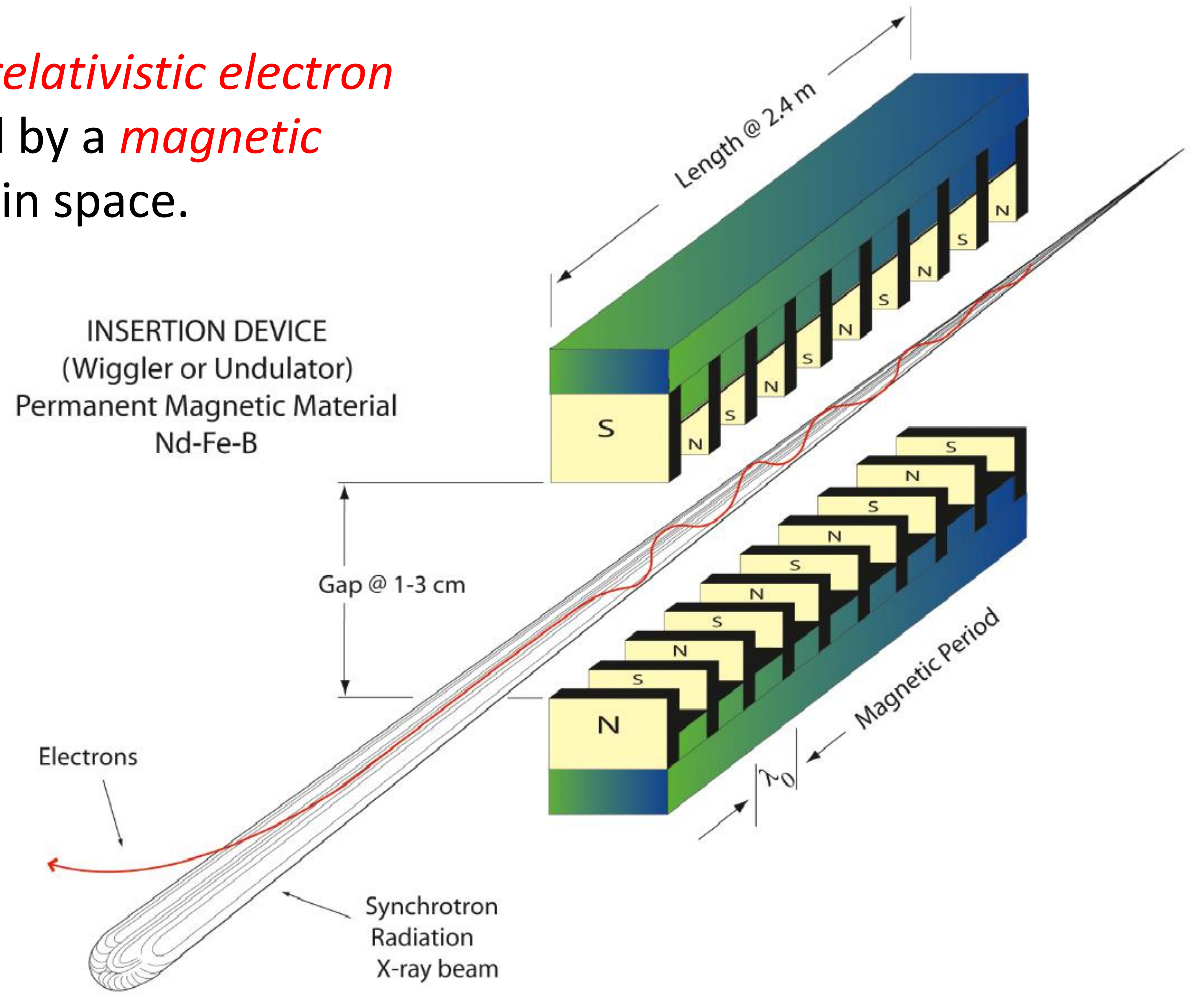
- Motion of the relativistic electron in alternating magnetic field and some properties of its radiation
- Different types of undulators
  - magnetic field generation
  - undulator mechanical systems
- Magnetic performance of undulators
- Undulator line: an FEL amplifier
- Undulators for future FELs


Undulators
Electromagnetic
Permanent Magnet
Conventional or combined with Permanent Magnet
Superconducting
RF
Pure Permanent Magnet
Hybrid


Undulators
$K = 0.9(\frac{1}{\text{cm}\cdot T}) \cdot \lambda_u(\text{cm}) \cdot B(T) \lesssim 3$
Electromagnetic
Permanent Magnet
Conventional or combined with Permanent Magnet
K-Control
Magnet Current
Polarization
Direction of Magnetic Current
Superconducting
K-Control
Magnet Current
Polarization
Double Helical & Direction of Current
RF
Period Control
Pure Permanent Magnet
Out of Vacuum / In-Vacuum
K-Control
Fixed Gap - Phase Control
Fixed Gap - Period Control
Gap Control
Polarization Control
Phase Control - inside/outside
Hybrid
Out of Vacuum / In-Vacuum
K-Control
Fixed Gap - Phase Control
Fixed Gap - Period Control
Gap Control
Gap & Phase Control
Polarization Control
Phase Control - inside/outside

Permanent magnet undulator schematic

The wiggles in the *relativistic electron* beam are produced by a *magnetic* field that oscillates in space.



Several simple formulas for the relativistic motion

- Electron speed u and $b = \dfrac{u}{c} \to 1$
- Momentum $\vec{p} = \dfrac{m\vec{\upsilon}}{\sqrt{1-\beta^2}} = \gamma m\vec{\upsilon}$

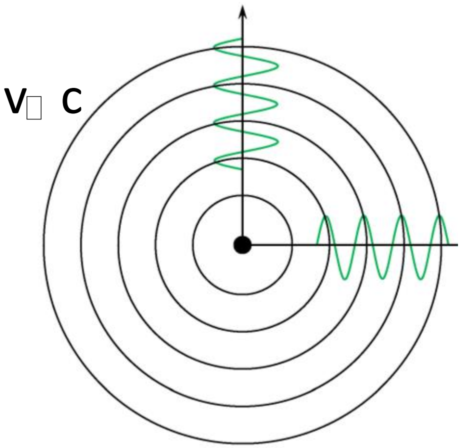
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} \simeq 1-\frac{1}{2\gamma^2} \Rightarrow 1+\beta^2\gamma^2 = \gamma^2$$

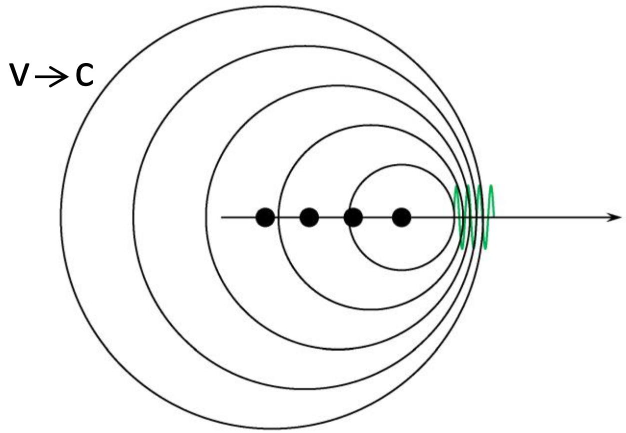
- Energy $E = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2} = gmc^2$

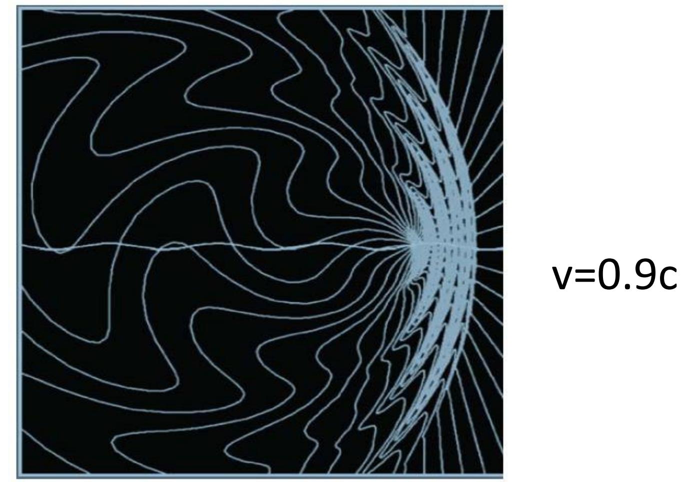
For 1GeV electron $g \Box 2000$ and $\dfrac{1}{2g^2} \Box 10^{-7}$ $\left(mc^2_{e^-} \Box 0.51 MeV\right)$

Except of energy eV units, Gaussian units are used through the lecture

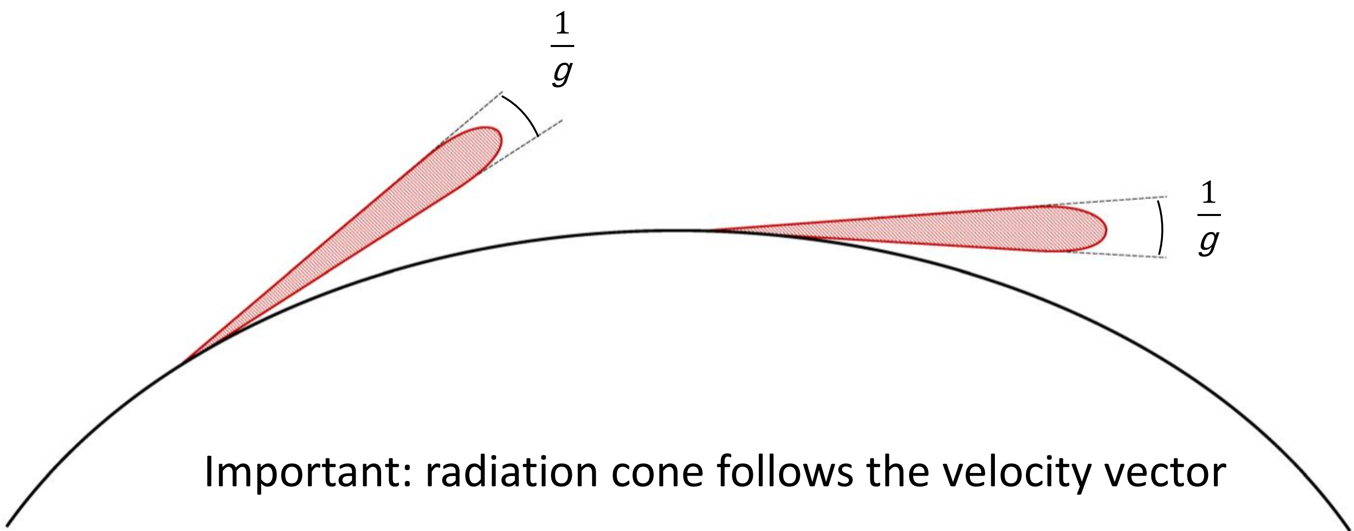
“Sweeping” radiation cone





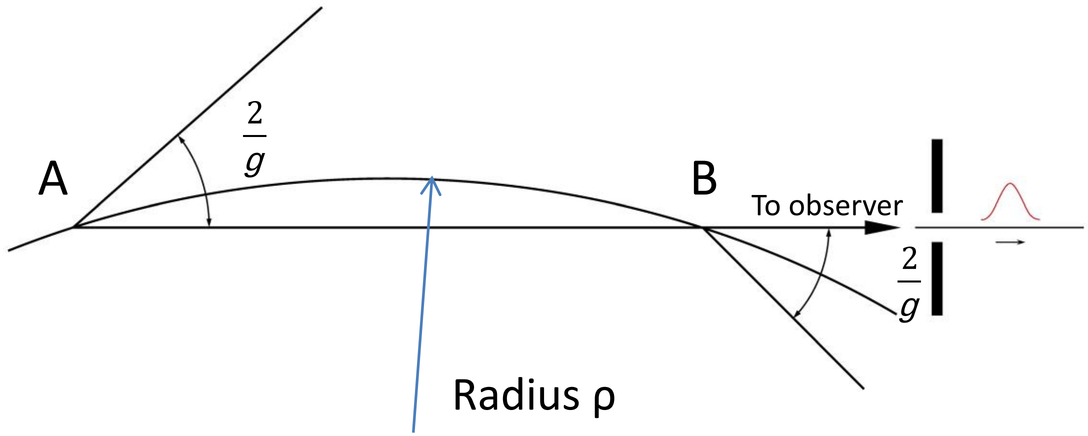


Radiation field of relativistic particle in an undulator



Important: radiation cone follows the velocity vector

Radiation pulse length and bandwidth



Electron emits at the point A, and it takes t_g for radiation to propagate from A to B. An observer will see the first radiation front at the point B at:

$$t_g = \frac{2r\sin\frac{1}{g}}{c}$$

Radiation pulse length and bandwidth (cont.)

It takes electron t_e time interval to travel from A to B where the last radiation seen by an observer is emitted at:

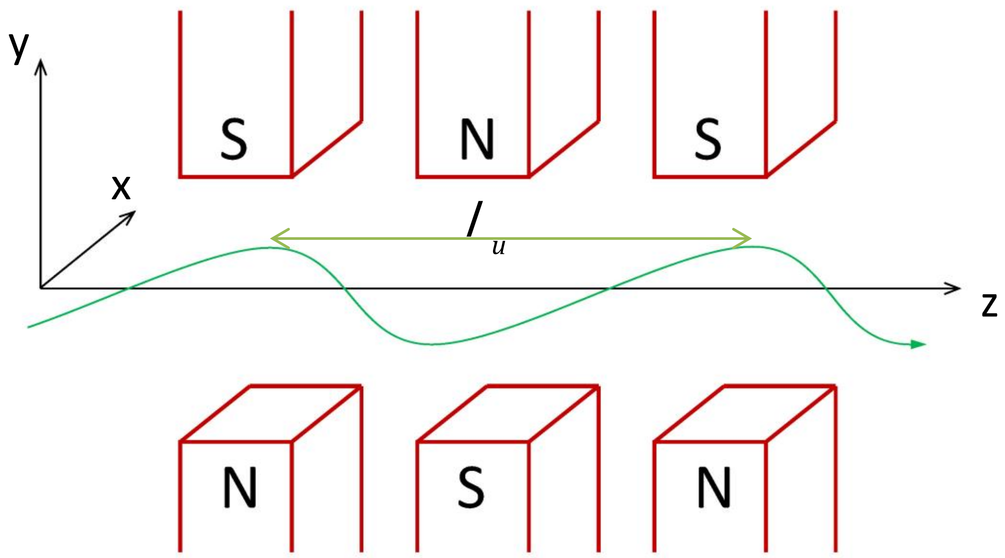
$$t_e = \frac{2r}{bcg}$$

The total time an observer was able to detect radiation:

$$\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta c\gamma} - \frac{2\rho \sin\frac{1}{\gamma}}{c} \cong \frac{2\rho}{c}\left[\frac{1}{\gamma\left(1-\frac{1}{2\gamma^2}\right)} - \frac{1}{\gamma} + \frac{1}{6\gamma^3}\right] \simeq \frac{2\rho}{c}\left[\frac{1}{\gamma}\left(1+\frac{1}{2\gamma^2}\right) - \frac{1}{\gamma} + \frac{1}{6\gamma^3}\right] = \frac{4\rho}{3c\gamma^3}$$

$$\mathrm{D}w \cdot \mathrm{D}t \,\square\, 1 \Rightarrow \mathrm{D}w \,\square\, \frac{3cg^3}{4r}$$

Motion equations in a planar undulator



$$B_y = B_0 \sin\left(k_u z\right) \qquad k_u = \frac{2\pi}{\lambda_u}$$

λ_u - Undulator period

Motion equations in a planar undulator (cont.)

Motion in XZ plane:

$$\frac{d^2x}{dt^2} = \frac{e}{gmc}\left(-u_z B_y\right) = -\frac{eB_0}{gmc}\sin\left(k_u z\right) \times \frac{dz}{dt}$$

$$\frac{d^2z}{dt^2} = \frac{e}{gmc}\left(u_x B_y\right)$$

$$\frac{dx}{dt} = u_x = \frac{eB_0}{gmck_u}\cos\left(k_u z\right)$$

$$b_x = \frac{u_x}{c} = \frac{K}{g} \times \cos\left(k_u z\right)$$ Where $K = \frac{eB_0 l_u}{2pmc^2} = 0.934 \cdot B_0\left[T\right] \cdot l_u\left[cm\right]$

Electron maximum deflection angle $\frac{K}{g} \square\ 1$

Motion equations in a planar undulator (cont.)

$$b_x^2 + b_z^2 = b^2 \left(= const\right) \qquad b_z = \frac{u_z}{c}$$

$$b_z^2 = b^2 - b_x^2 \Rightarrow b_z = b\sqrt{1-\frac{b_x^2}{b^2}} @ b\left[1-\frac{K^2\cdot\cos^2\left(k_u z\right)}{2g^2}\right] =$$

$$= b\left(1-\frac{K^2}{4g^2}\right) - \frac{K^2}{4g^2}\cdot b\cos\left(2k_u z\right)$$

$$\overline{b_z} = b\left(1-\frac{K^2}{4g^2}\right) @ \left(1-\frac{1}{2g^2}\right)\left(1-\frac{K^2}{4g^2}\right) @ 1 - \underline{\underline{\frac{1}{2g^2}}} - \underline{\underline{\frac{K^2}{4g^2}}}$$

Motion equations in a planar undulator (cont.)

$$\frac{dz}{dt} = \beta_z c \cong \overline{\beta_z} - \frac{K^2}{4\gamma^2} c \cdot \cos\left(2k_u z\right)$$

$$\frac{dx}{dt} = \frac{K}{\gamma} c \cdot \cos\left(k_u z\right)$$

$$k_u z = \Omega t \qquad z = \overline{\beta_z} c \cdot t \qquad \Omega = \frac{2\pi \overline{\beta_z} c}{\lambda_u}$$

$$\frac{dz}{dt} = \overline{\beta_z} \cdot c - \frac{K^2}{4\gamma^2} c \cdot \cos\left(2\Omega t\right)$$

$$\frac{dx}{dt} = \frac{K}{\gamma} c \cdot \cos\left(\Omega t\right)$$

Motion equations in a planar undulator (cont.)

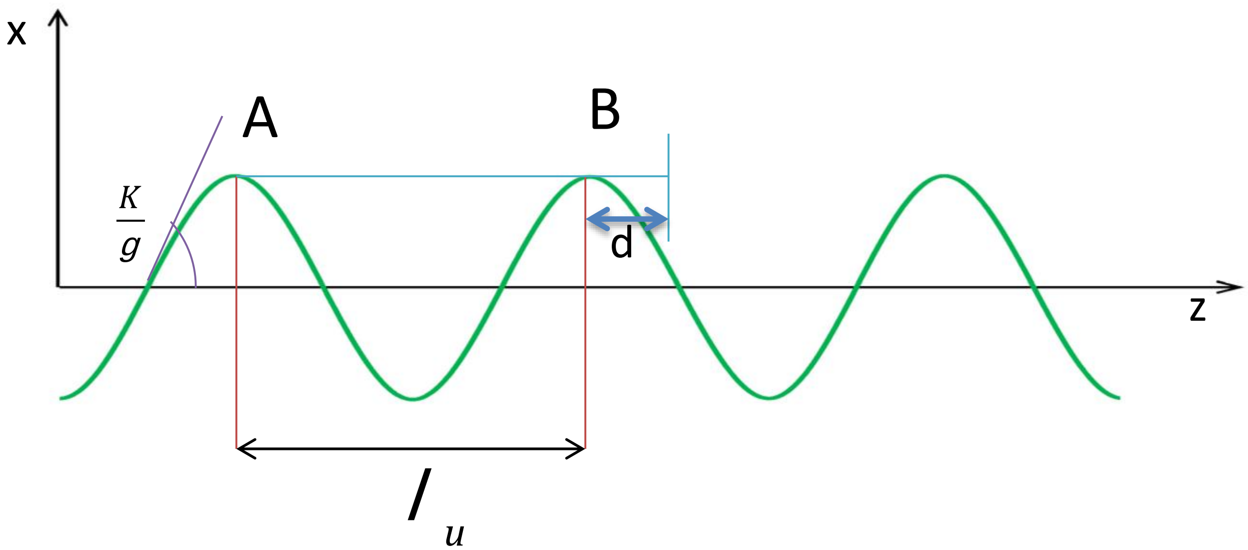
$$z = \overline{\beta_z} c \cdot t - \frac{K^2 c}{8\gamma^2 \Omega} \sin(2\Omega t)$$

$$x = \frac{K}{\gamma} \cdot \frac{c}{\Omega} \sin(\Omega t)$$

$$x \approx \frac{K}{\gamma} \cdot \frac{c}{\Omega} = \frac{K \cdot \lambda_u}{\gamma \cdot 2\pi \cdot \overline{\beta_z}} \simeq \frac{K \cdot \lambda_u}{2\pi\gamma} \sim \frac{2 \cdot 3cm}{10^4 \cdot 6} \sim 1\mu k$$

5 GeV electron wiggles away from the straight line in the 3-cm period undulator with K=2 about 1 micron

Undulator wavelength equation



Time for the electron to travel from A to B is: $\tau_e = \dfrac{\lambda_u}{\beta_z \cdot c}$. During this time the radiation

wavefront travels distance: $z_{rad} = c \cdot \tau_e = \dfrac{\lambda_u}{\beta_z}$ and will be ahead of electron by d.

Undulator wavelength equation

$$\delta = \frac{\lambda_u}{\bar{\beta}_z} - \lambda_u$$

δ is called "slippage"

When the distance δ is equal to integer number n – harmonic number - of radiation wavelengths, there is a constructive interference for that wavelength:

$$\delta = n\lambda = \frac{\lambda_u}{\bar{\beta}_z} - \lambda_u = \lambda_u \frac{1 - \bar{\beta}_z}{\bar{\beta}_z} = \lambda_u \frac{1 - 1 + \frac{1}{2\gamma^2} + \frac{K^2}{4\gamma^2}}{1} =$$

$$= \underline{\underline{\frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}}$$

$$\boxed{\lambda = \frac{\lambda_u}{n \cdot 2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

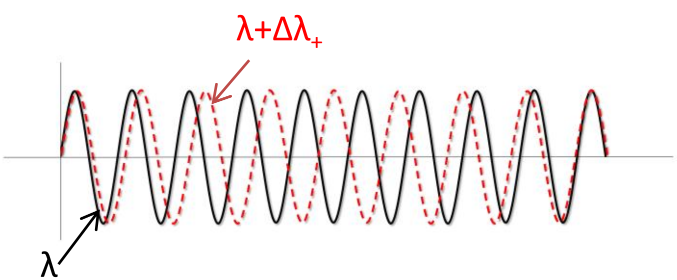
Undulator harmonic spectral width

The constructive interference condition for an undulator with ***N*** periods and total length ***L***= λ_u***N*** at zero angle:

$$\frac{L}{\beta} - L = n\lambda \cdot N$$

Let's ask a question: at what $\lambda_+ = \lambda + \Delta\lambda_+$ the interference becomes destructive on the undulator length?

$$\frac{L}{\beta} - L = n\lambda_+ N + \frac{\lambda_+}{2} = nN\left(\lambda + \Delta\lambda_+\right) + \frac{\lambda + \Delta\lambda_+}{2}$$



Subtracting two above equations from each other $\Rightarrow$

Undulator harmonic spectral width (cont.)

$$0 = nN\lambda - nN\left(\lambda + \Delta\lambda_+\right) - \frac{\lambda + \Delta\lambda_+}{2}$$

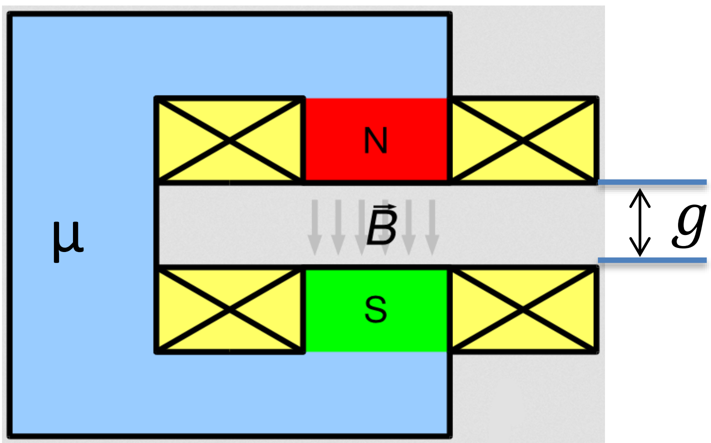
$$\frac{\Delta\lambda_+}{\lambda} = \frac{1}{2nN - 1} \approx \frac{1}{2nN}$$

The same derivation for $\lambda_- = \lambda - \lambda_-$

Therefore: $\dfrac{\Delta\lambda}{\lambda} \approx \dfrac{1}{nN}$

Electromagnet

Simple relation between current I, gap g and magnetic field B.



C-shaped dipole

$$\mu \cdot (N \cdot I) \approx B \cdot \frac{g}{2}$$

N - number of coil turns

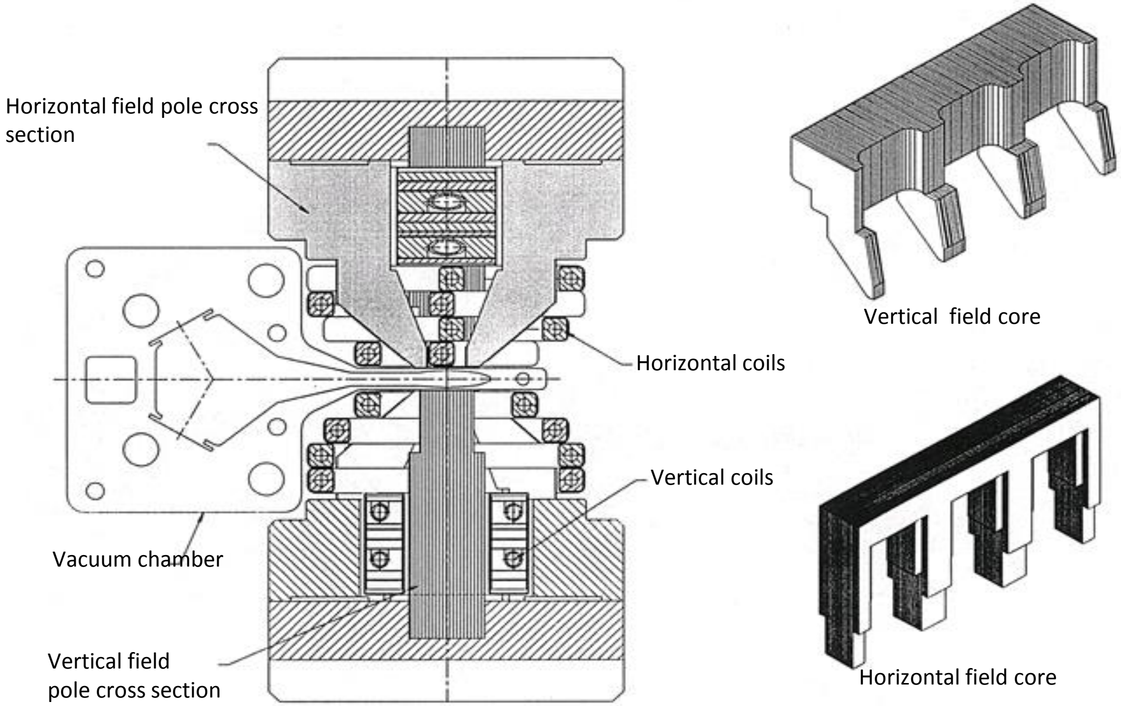
μ - permeability

Then $K \approx \dfrac{\lambda_u \cdot \mu \cdot \{N(\lambda_u) \cdot I\}}{g}$

Performance, i.e. max B per coil volume, is limited by the heat transfer in the coil. Current density is limited by the water cooling capacity: $j \le 1\frac{kA}{cm^2}$. And that defines practically attainable undulator periods: $\lambda_u \ge 10cm$.

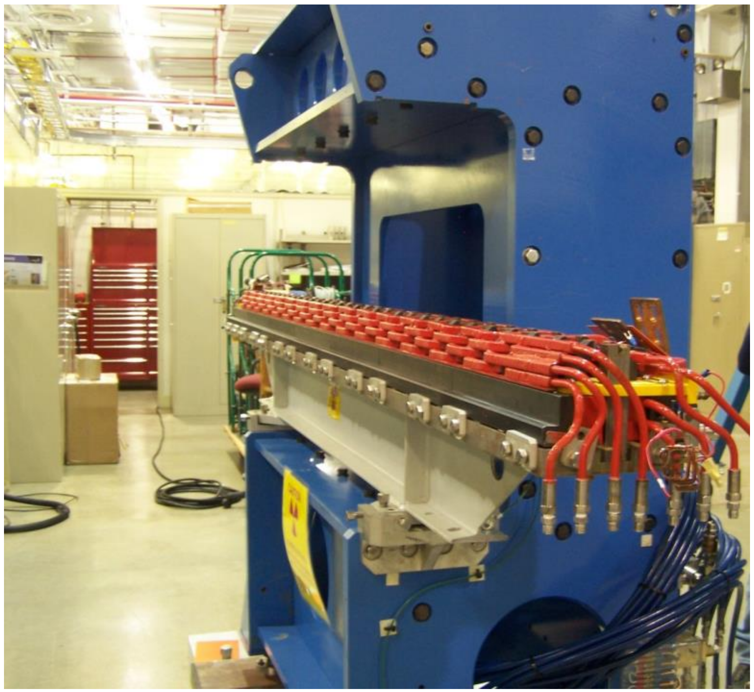
Superconducting coil brings more than two orders of magnitude in the performance enhancement!

Example of electromagnetic IDs

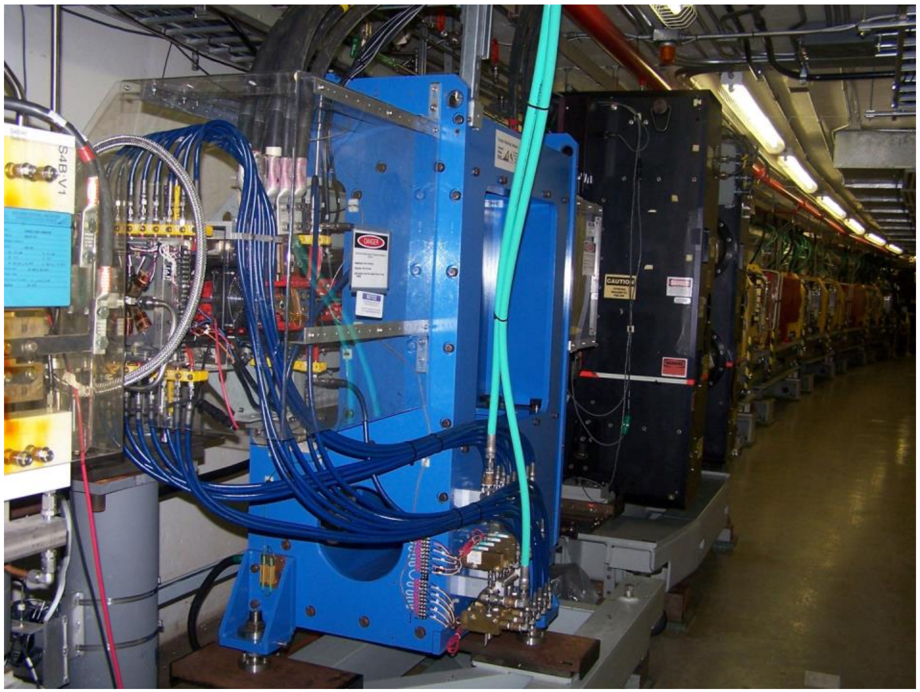


Electromagnetic undulator - APS

Electromagnetic IDs - continued



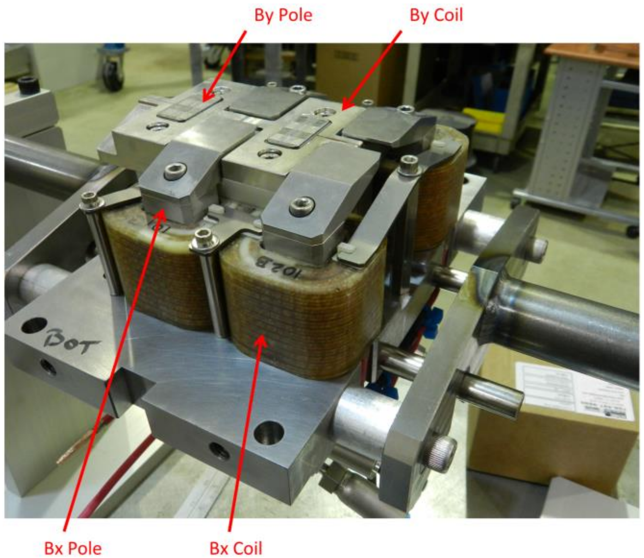
Undulator in the assembly process

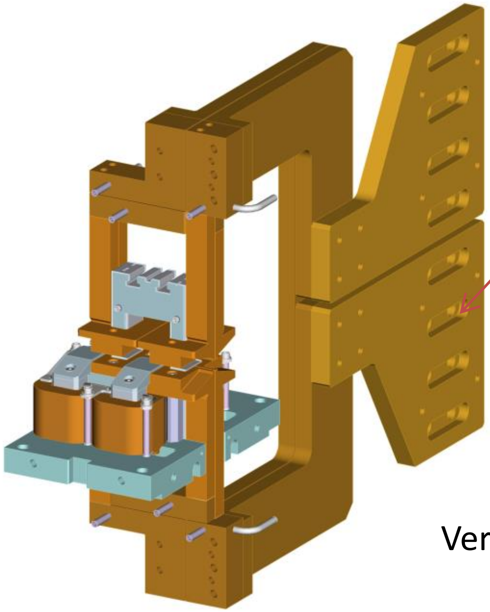


Undulator in the storage ring tunnel

There is also a large power supply to feed the coils

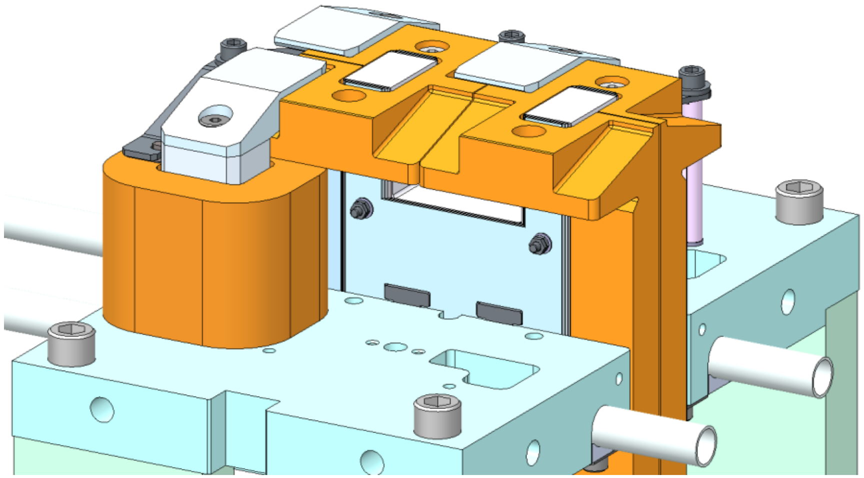
APS Electromagnetic VPU w/fast switching

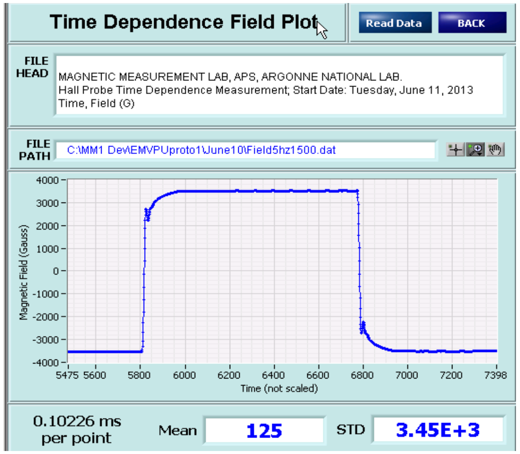




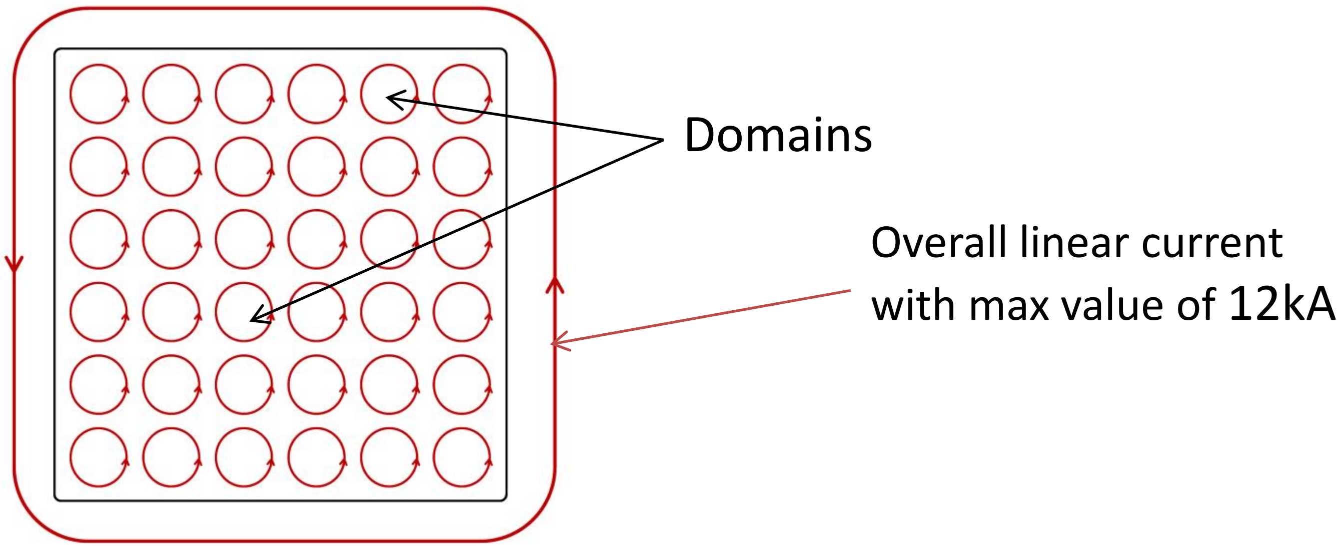
High current bus bar

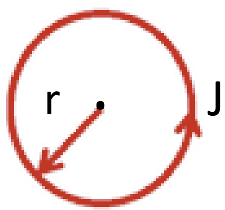
Very low L and R





Permanent magnet





$$B_{center} = \frac{1}{c}\oint \frac{\left[\vec{dl}\times\vec{r}\right]\cdot J}{r^3} = \frac{J}{c}\int_0^{2\pi}\frac{dl\cdot r}{r^3} = \int_0^{2\pi}\frac{d\theta}{r} = \frac{2\pi J}{c\cdot r}$$

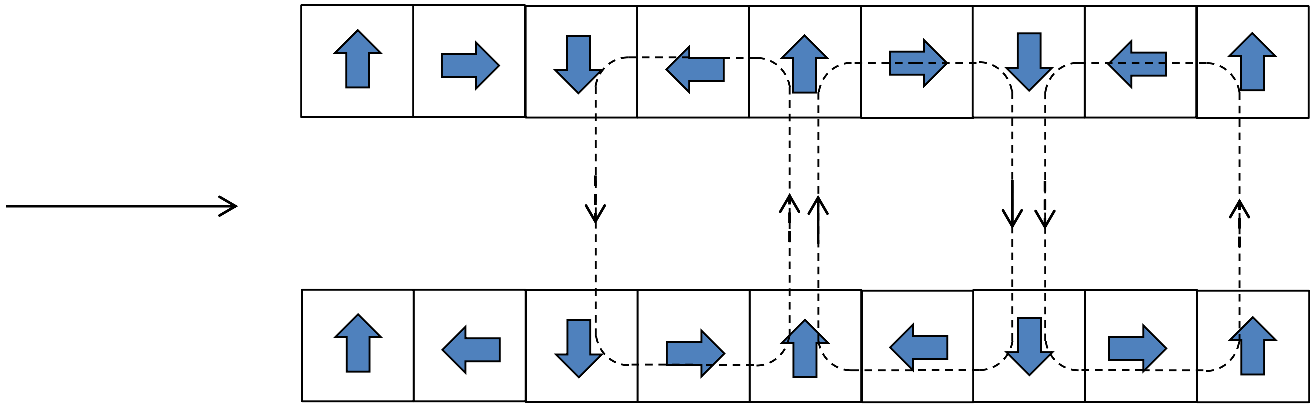
For r=1cm and J=12kA

$$B_{center} = \frac{24\pi\cdot 10^3\, {}^{q}\!/_{\sec\cdot cm}}{3\cdot 10^{10}\, {}^{cm}\!/_{\sec}} = 24\pi\cdot 10^2 Gauss \simeq 7500 Gauss$$

Pure permanent magnet design

The undulator is assembled from many individual magnet blocks, with their magnetic moments oriented in different directions.

Electron beam

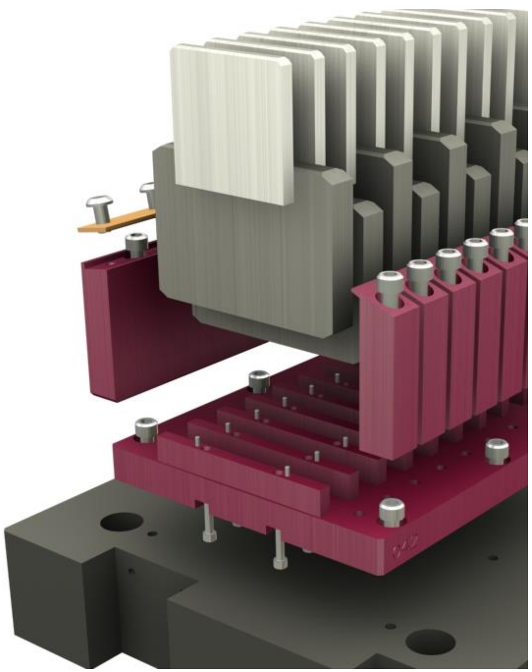


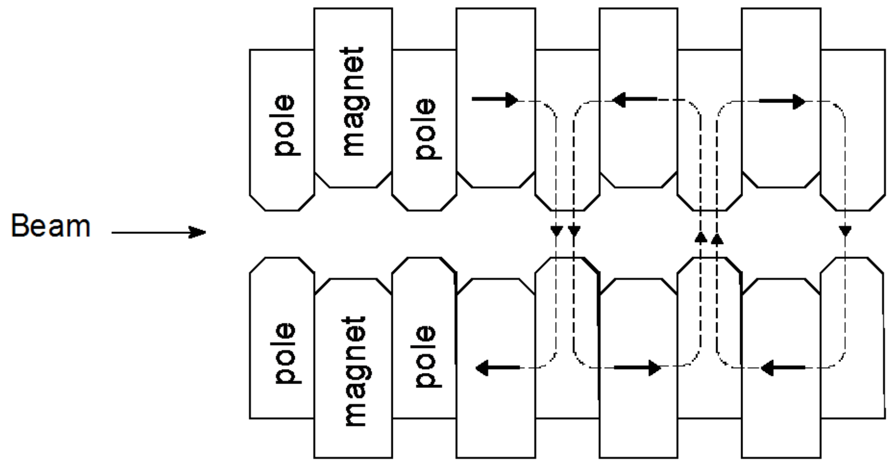
The magnet blocks are typically NdFeB-type magnets. Occasionally SmCo magnets are used.

Design of a hybrid magnetic structure

The sizes, shapes, chamfers, and relative positions of the magnets and poles in a hybrid magnetic structure are carefully optimized in order to produce a strong undulator field on axis.

The magnets must be taller than the poles on the side away from the beam. Otherwise as much flux would be lost out the back of the pole as would go across the gap.
Similarly, the magnets are wider than the poles to keep flux from going directly to the next pole around the side of the magnet.





$$B_u = a \cdot e^{-b \cdot \frac{g}{\lambda_u} + c \cdot \frac{g^2}{\lambda_u^2}}$$

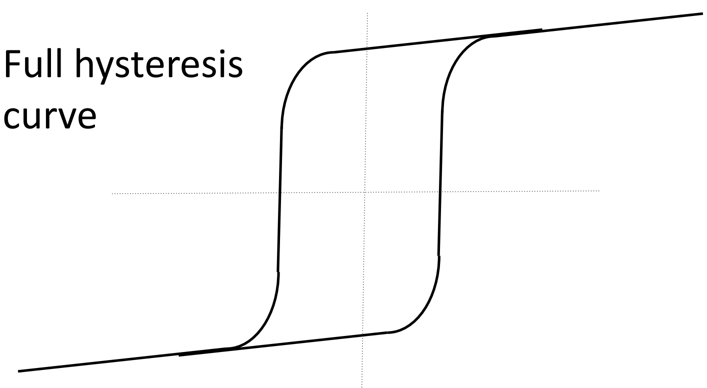
Magnetic field on undulator axis as a function of gap g

B-H curve for a grade of magnet

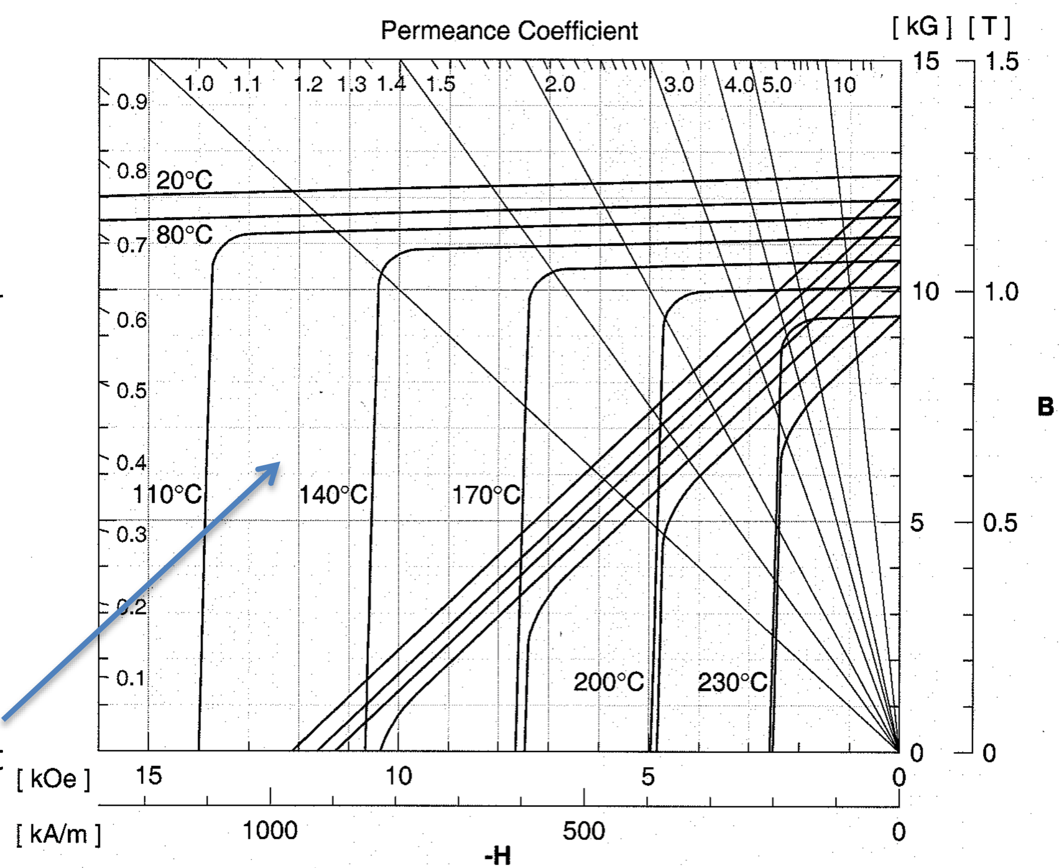
Important characteristics:

- ✓ Remanence at room temperature (i.e., magnet strength)
- ✓ Field where permanent demagnetization starts – note that it is temperature dependent.
- ✓ The remanent strength is also temperature dependent, typically -0.1%/K.
- ✓ Two coercivities: one is where the B-H curve crosses B=0; the other, sometimes called 'intrinsic coercivity,' is where the magnet demagnetizes.

Full hysteresis curve



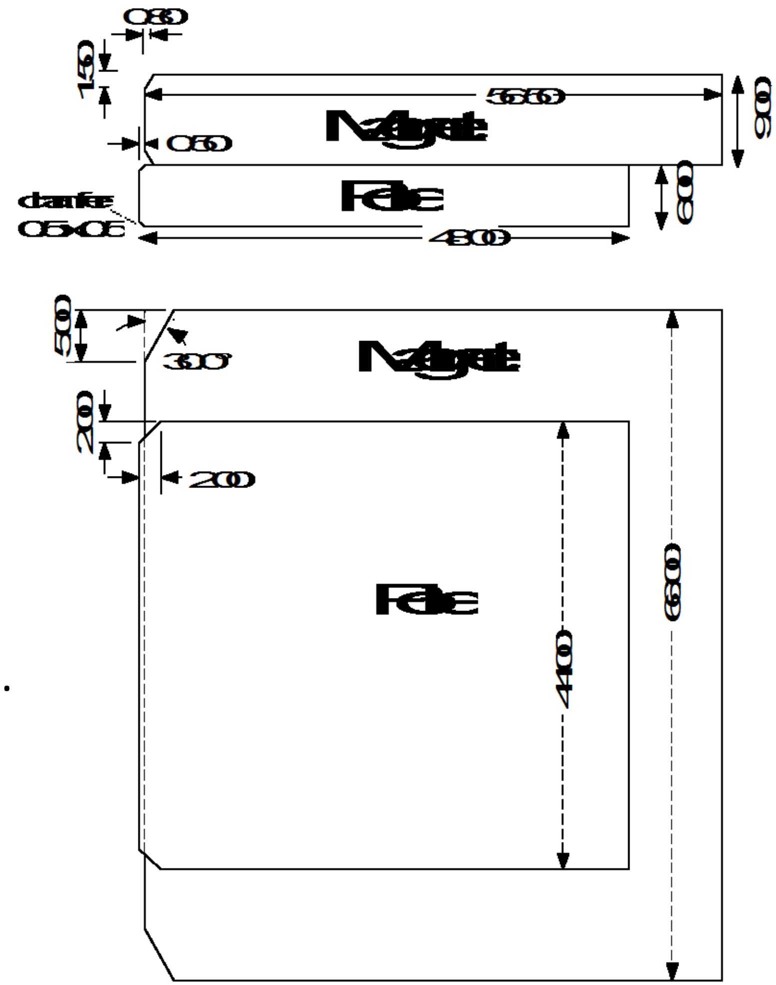
2nd quadrant



Example: the LCLS undulator magnetic design

Period: 30 mm

Optimized at 6 mm gap.



The process of optimization includes the variation of the magnet thickness, width and height, as well as "chamfers" dimensions. Poles dimensions are also part of the optimization.

Magnetic structure design considerations

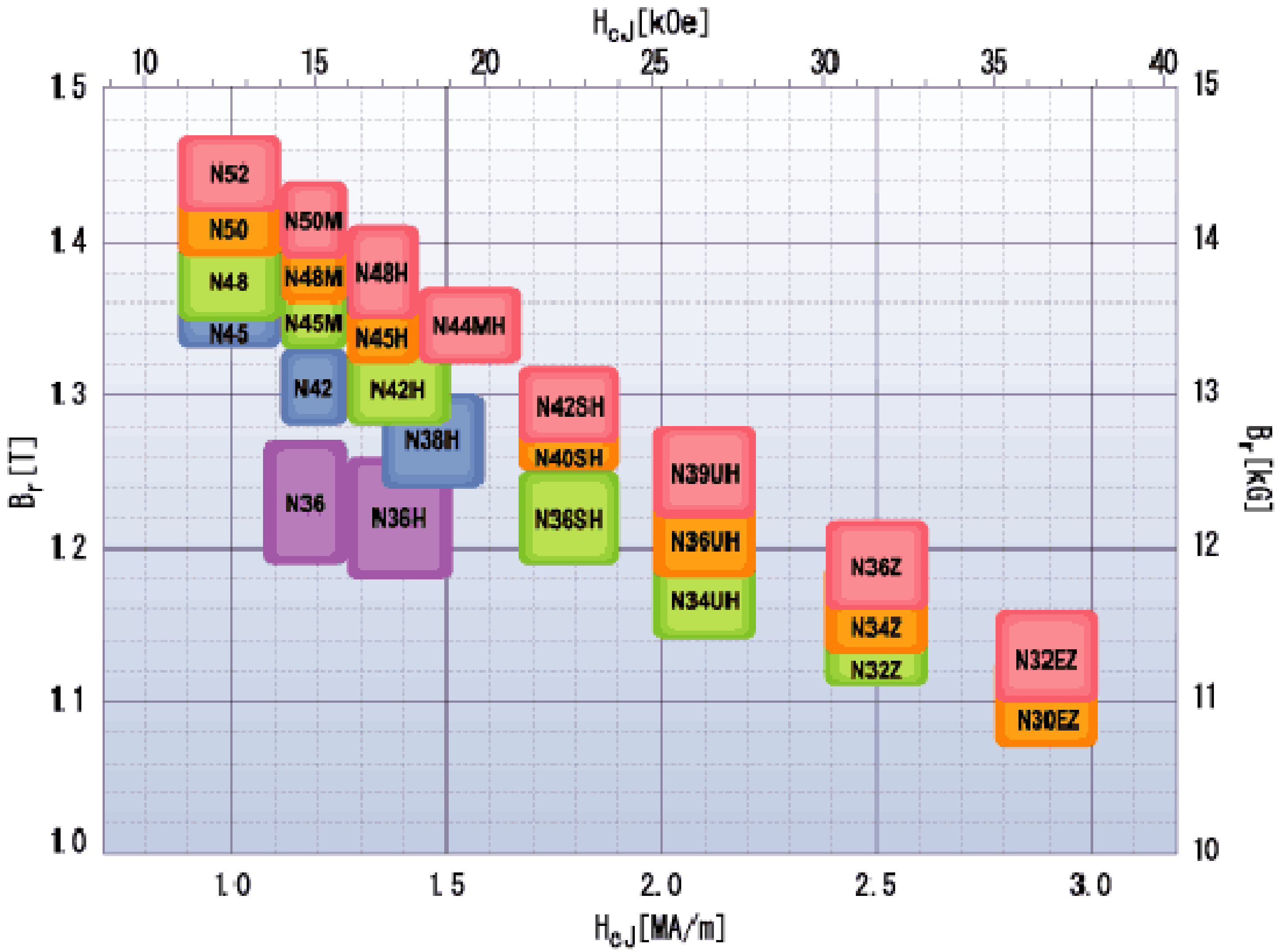
The magnetic structure is designed to produce the largest field on axis at the smallest anticipated gap.

Constraints:

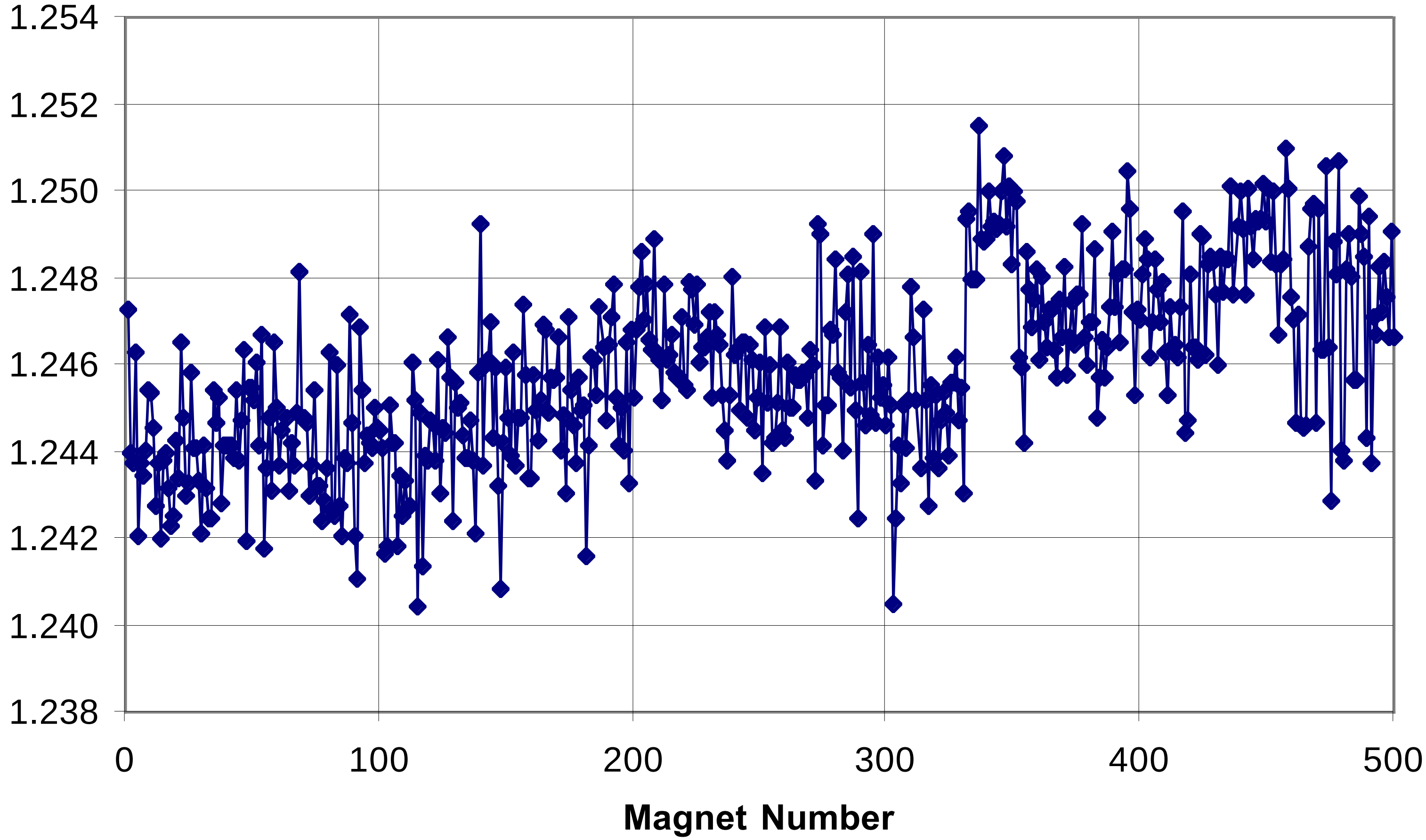
- Don't demagnetize the magnets
- Don't oversaturate the poles (less important – but nonuniformities in the pole material may show up close to saturation.)
- If the undulator gets warm (e.g., the air conditioning goes out, or it is shipped by truck in the summer), the magnets still shouldn't demagnetize
- The magnets shouldn't demagnetize during assembly

Radiation damage is also a consideration. If the magnets are close to demagnetizing, radiation is more likely to result in damage.

Different grades of magnets (Shin-Etsu as example)

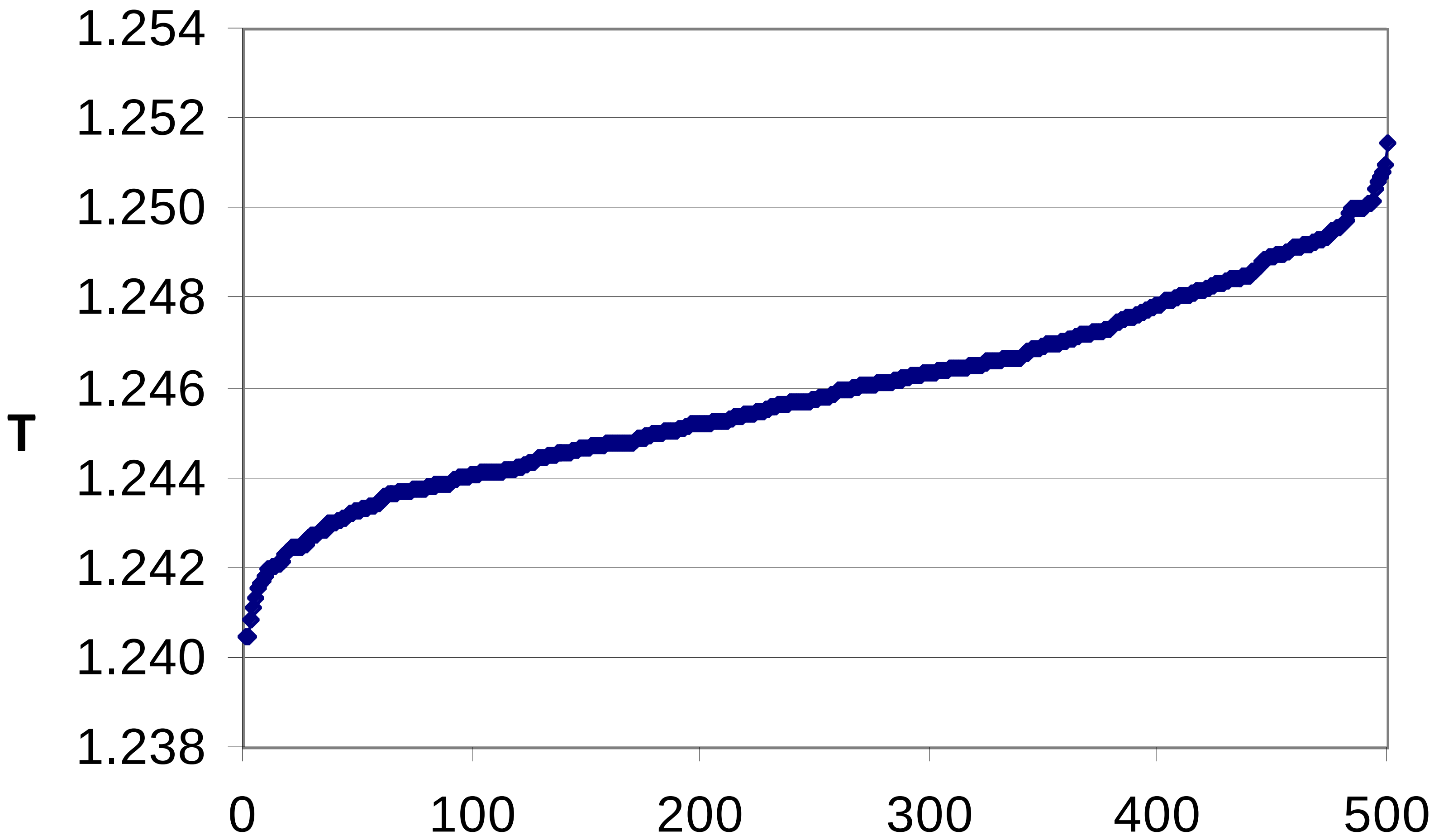


Field strength distribution of set of magnets



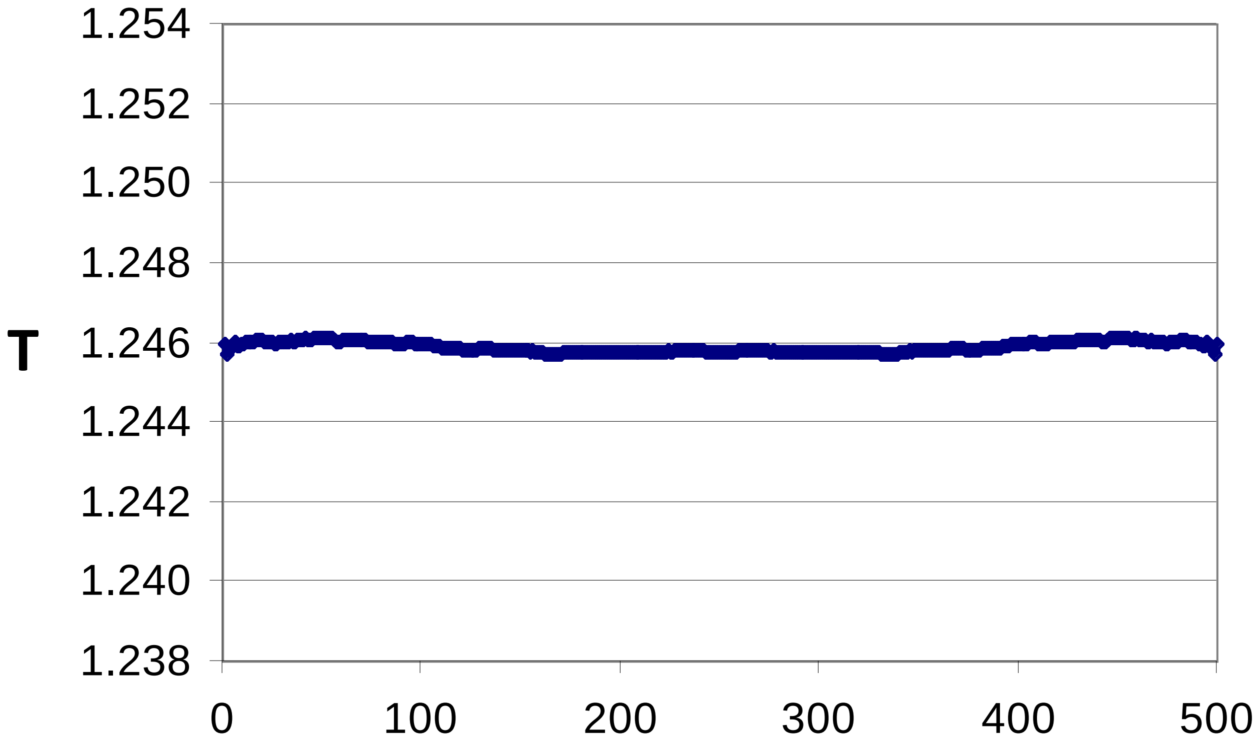
Total magnetic moment of the magnet blocks for one of the LCLS undulator segment ordered by magnet serial number. Overall range of graph is ±0.6%.

Magnet field strengths, sorted by strength



Magnet block strength with the magnets in order of increasing magnetic moment.

After pairing the magnets by strength



After pairing the magnets by matching strong to weak to even out the distribution.

APS hybrid magnet undulator

Gap drive mechanism

Magnets and poles arrays

Strongback

C-Frame

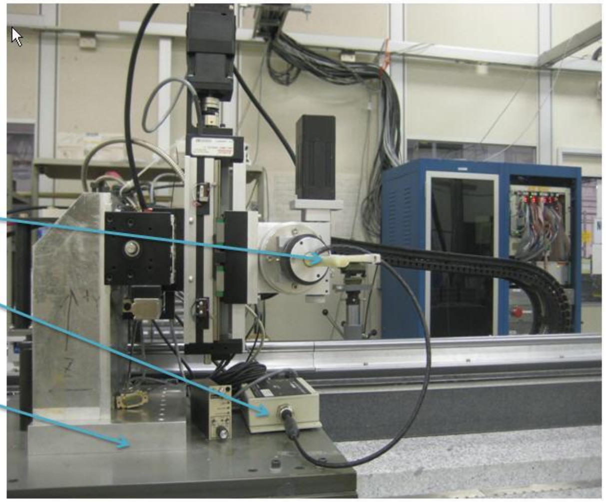
Undulator magnetic measurement system

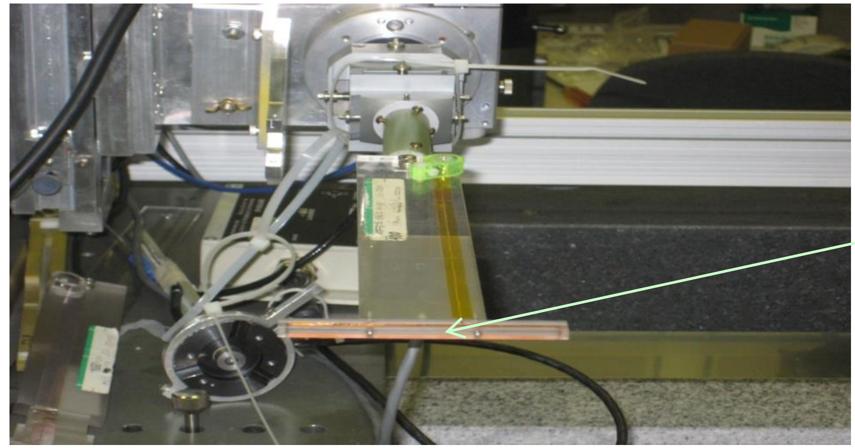
Precise, down to 1 micron, highly reproducible motion of magnetic sensors. Multitude of these sensors. Temperature-controlled to fraction of a degree environment.

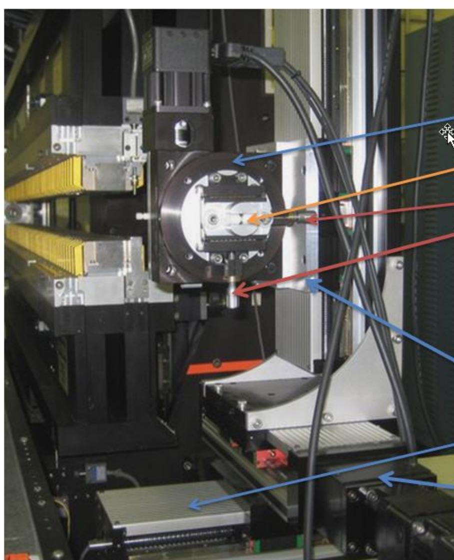
Magnetic sensors

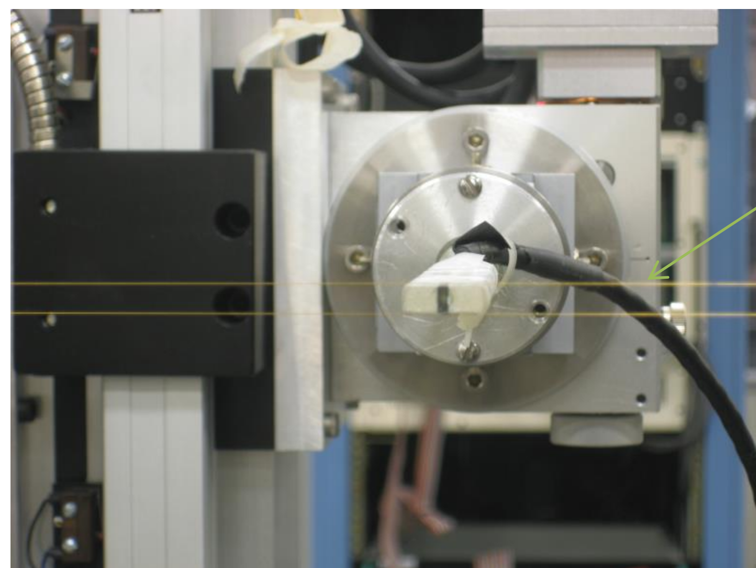
- Hall probes
  - accurate trajectory measurements
  - temperature sensitive
  - requires precise calibration
- Rotating/flipping coil
  - most reliable field integral measurements
- Moving coils
  - trajectory and field integral measurements
- Stretched pulsed wire
  - trajectory measurements for very small apertures

Magnetic sensors









Field integrals

Electron motion equations for all three planes

$$\frac{d^2x}{dt^2} = \frac{e}{\gamma mc}\left(-v_z B_y\right)$$

$$\frac{d^2y}{dt^2} = \frac{e}{\gamma mc} v_z B_x$$

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt} \ll v_z$$

$$dt = \frac{c}{\bar{\beta}_z} dz$$

$$\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz} = \bar{\beta}_z c \frac{d}{dz}$$

$$\frac{d^2}{dt^2} = \bar{\beta}_z^2 c^2 \frac{d^2}{dz^2}$$

$$\bar{\beta}_z \to 1 \qquad v_z \to c$$

Field integrals (cont.)

$$x'' = -\frac{e}{\gamma m \bar{\beta}_z^2 c^3} \upsilon_z B_y \cong -\frac{e}{\gamma m c^2} B_y$$

$$y'' = \frac{e}{\gamma m \bar{\beta}_z^2 c^3} \upsilon_z B_x \cong \frac{e}{\gamma m c^2} B_x$$

$$x'(z) = \int_{z_0}^{z} x''(z_1)\,dz_1 = -\frac{e}{\gamma m c^2} \int_{z_0}^{z} B_y(z_1)\,dz_1$$

$$y'(z) = \int_{z_0}^{z} y''(z_1)\,dz_1 = \frac{e}{\gamma m c^2} \int_{z_0}^{z} B_x(z_1)\,dz_1$$

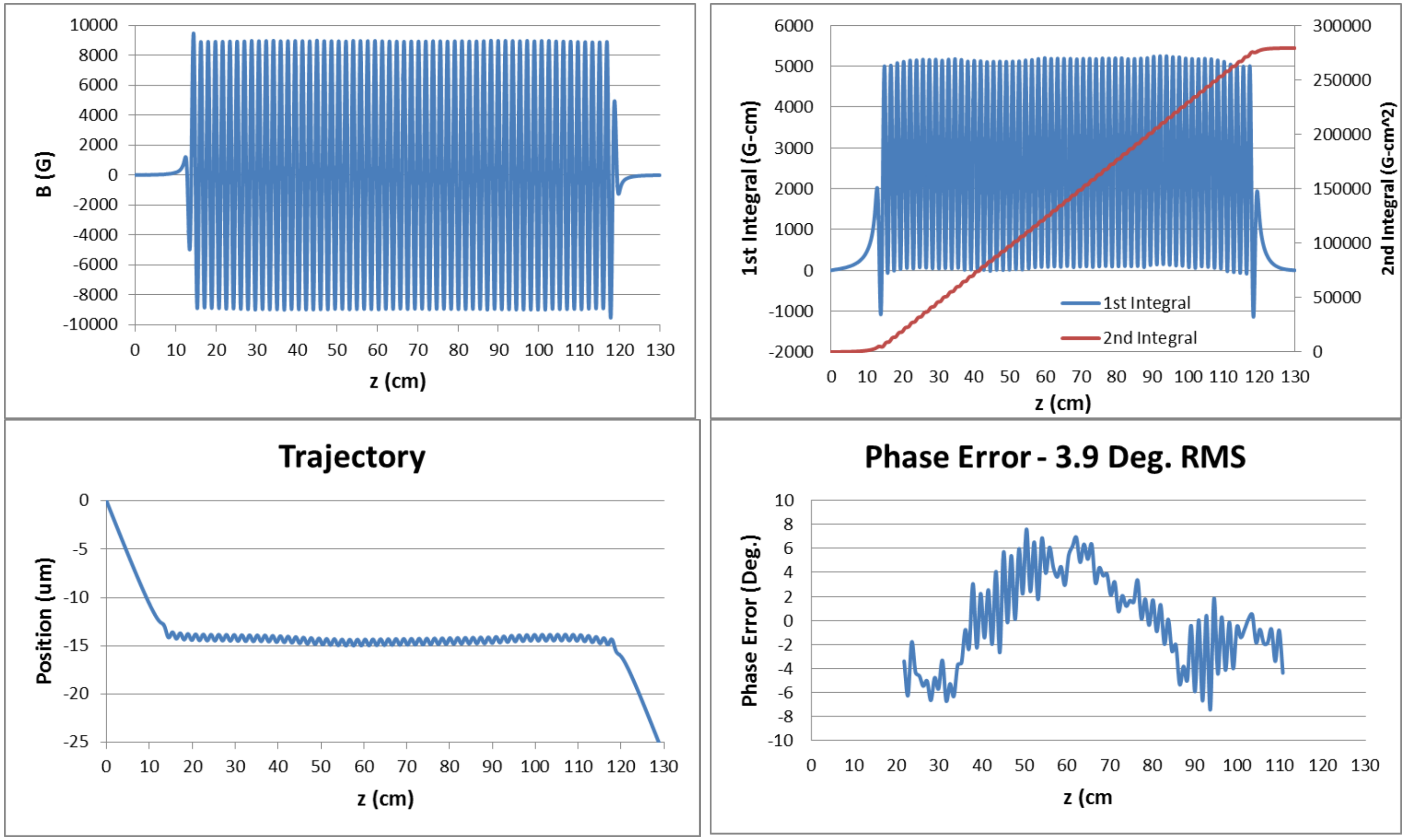
First integrals: **angles**

$$x(z) = \int_{z_0}^{z} x'(z_2)\,dz_2 = -\frac{e}{\gamma m c^2} \int_{z_0}^{z} dz_2 \int_{z_0}^{z_2} B_y(z_1)\,dz_1$$

$$y(z) = \int_{z_0}^{z} y'(z_2)\,dz_2 = \frac{e}{\gamma m c^2} \int_{z_0}^{z} dz_2 \int_{z_0}^{z_2} B_x(z_1)\,dz_1$$

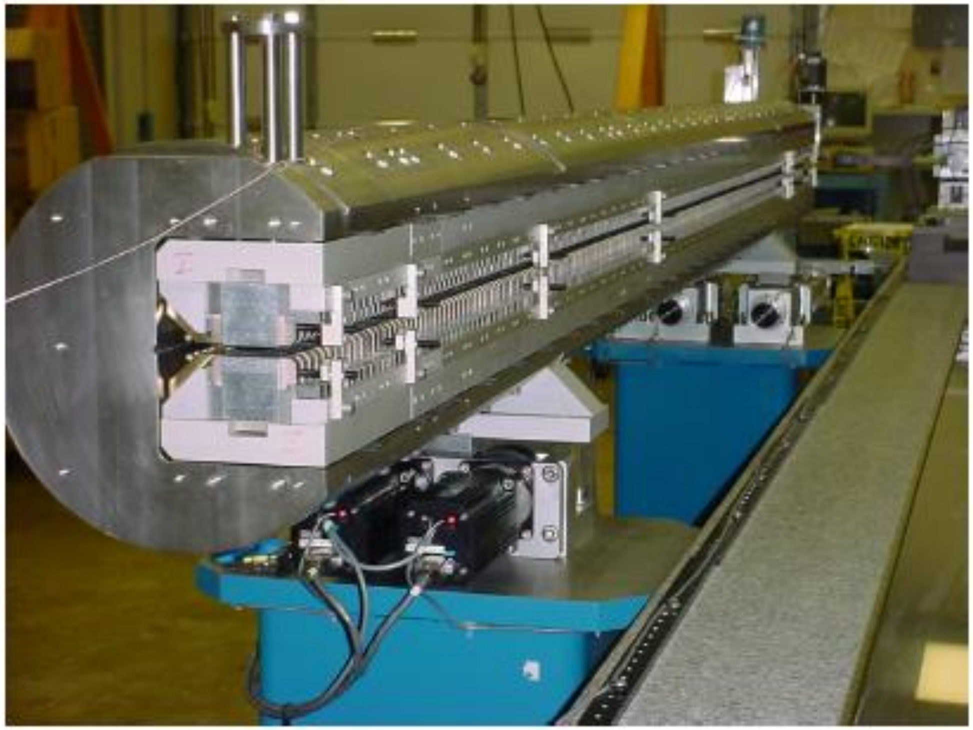
Second integrals: **trajectories**

Example of undulator magnet measurements results



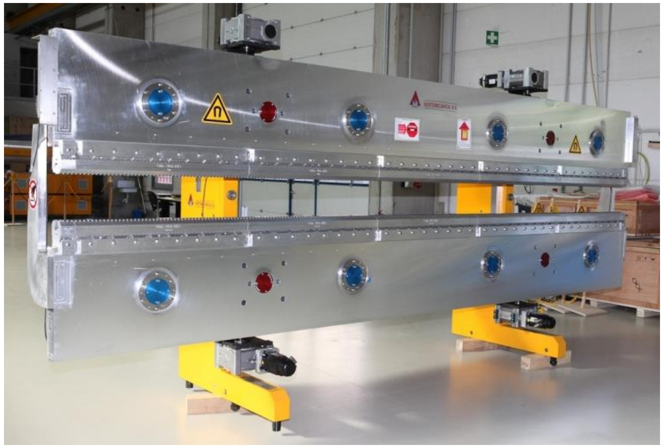
Undulator performance depends on how accurately one could maintain K - value along the device. Few slides down it will be shown how this requirement relates to undulator phase errors and undulator performance.

LCLS-I fixed gap undulator



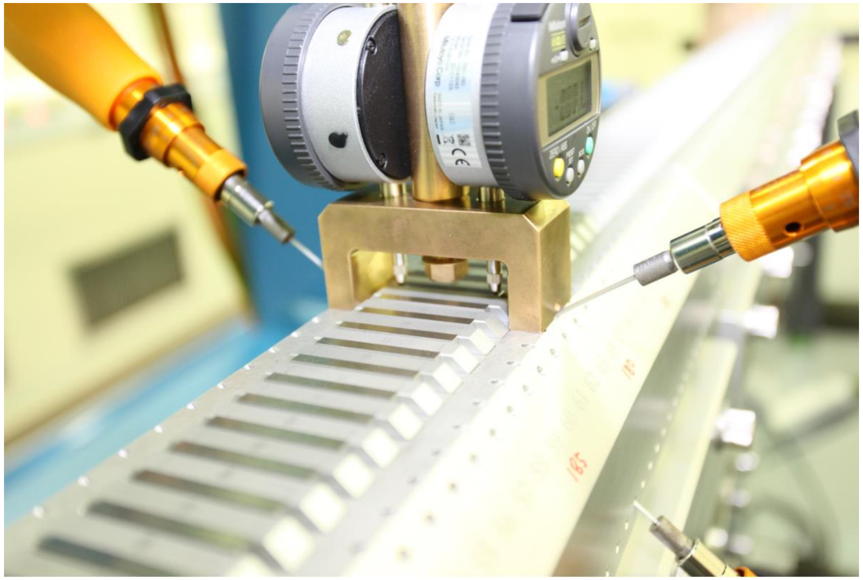
Since the gap of the device is fixed the performance is defined only by its magnetic quality. Although to achieve that one have to pay attention to the accuracy of the positioning of poles and magnets in their holders.

European XFEL undulator



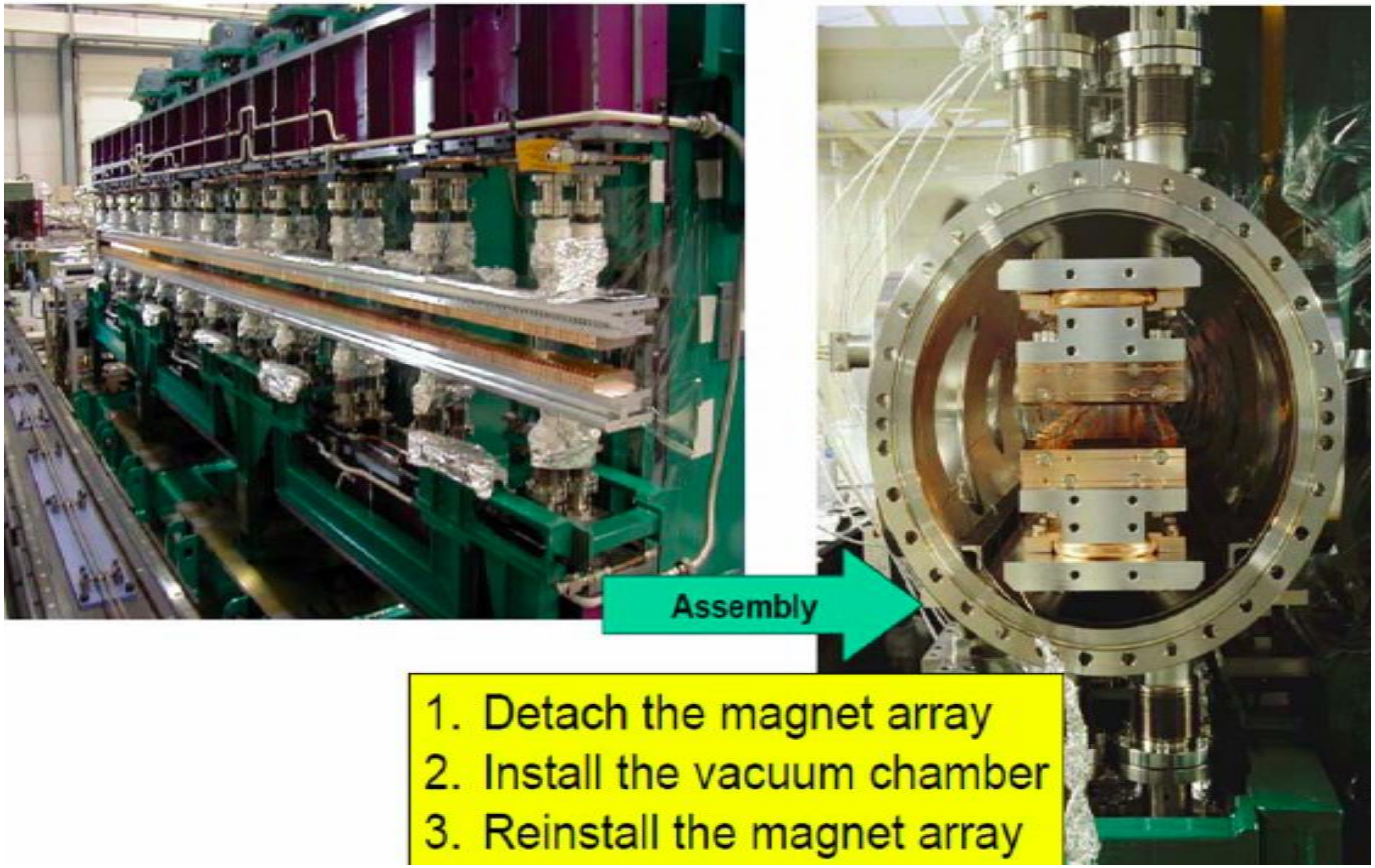
Support Mechanics	
Length [m]	5
Gap Range [mm]	10-200
Gap Accuracy [µm]	±1
4 Servo Axes, No Gears	
Weight [to]	8
Standardized Interface for Magnet Structures	
Requested Number	91

Magnet Structures	U40	U68
Length [m]	5	5
Period Length [mm]	40	68
Number of Poles	248	146
Operational Gap Range [mm]	10-20	10-20
B_0 @ 10mm Gap [T]	1.14	1.66
K @ 10mm Gap	3.9	9.3
Requested Number	70	21

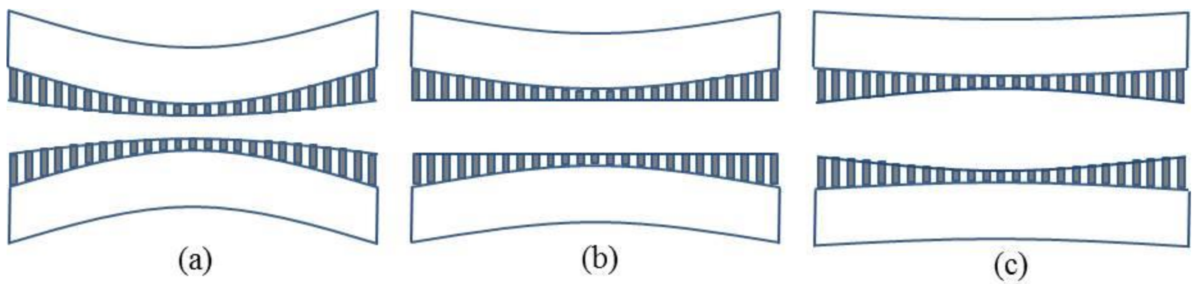


This 5-meter long undulator represents a significant challenge in the construction of highly precise device in large quantity

In-vacuum IDs



Strongbacks/magnetic arrays bending



Force/pole= $\dfrac{H^2}{8p}\times S$ where H-magnetic field and S-pole surface size

Typical H=10^4 Gauss, and S≈2.4 cm^2. Number of poles≈200

$$F_{total} = \frac{H^2 s \cdot N}{8\pi} \simeq \frac{10^8 Gauss^2 \cdot 2.4cm^2 \cdot 200}{24} = 2\cdot 10^9 dyn \approx 2000kg$$

Phase errors

Phase evolution for electron moving in real – not ideal - undulator field

Non ideal magnetic field $\Rightarrow$ variations of K along undulator

$$\Phi = k\left(\frac{l}{\beta} - z\right) \text{ where } k = \frac{2\pi}{\lambda} \text{ and } l - \text{ electron path length}$$

$$dl^2 = dx^2 + dz^2 \qquad dl = dz\sqrt{1+\left(\frac{dx}{dz}\right)^2} \simeq dz\left[1+\frac{1}{2}\left(\frac{dx}{dz}\right)^2\right]$$

From motion equations – slide 13: $$\frac{dx}{dz} = \frac{K}{\gamma}\cdot\cos\left(k_u z\right)\cdot\frac{1}{\beta_z} \cong \frac{K}{\gamma}\cdot\cos\left(k_u z\right)$$

$$dl \cong dz\left[1+\frac{K^2}{2\gamma^2}\cdot\cos^2\left(k_u z\right)\right]$$

Phase errors (cont.)

$$l_{pole} = \int_0^{\frac{l_u}{2}} dl = \frac{l_u}{2} + \frac{K^2}{2g^2} \cdot \frac{1}{2} \cdot \frac{l_u}{2} = \frac{l_u}{2}\left(1 + \frac{K^2}{4g^2}\right)$$

$$dl_{pole} = \frac{l_u \cdot 2K \cdot dK}{8g^2} = \frac{K^2 l}{2\left(1 + \frac{K^2}{2}\right)} \cdot \frac{dK}{K} = \frac{pK^2}{k\left(1 + \frac{K^2}{2}\right)} \cdot s$$

$$\delta\Phi_{pole} = k\frac{\delta l}{\beta} \cong \frac{\pi K^2}{1 + \frac{K^2}{2}} \cdot \sigma = \tilde{K}\sigma$$

where $s = \frac{dK}{K}$

where $\tilde{K} = \frac{\pi K^2}{1 + \frac{K^2}{2}}$

Phase errors (cont.)

Radiation field along the undulator poles evolves with the phase

$$E,H \sim \prod_{2N} e^{i\delta\Phi} \simeq \left\langle e^{i\delta\Phi} \right\rangle^{2N}$$

To simplify phase averaging assume that the distribution of errors in undulator parameter K is gaussian:

$$\mathrm{R}(s) = \frac{1}{\sqrt{2p} \times s_K} \times e^{-\frac{s^2}{2s_K^2}} \quad \text{where } s_K = \sqrt{\overline{s^2}}$$

Phase errors (cont.)

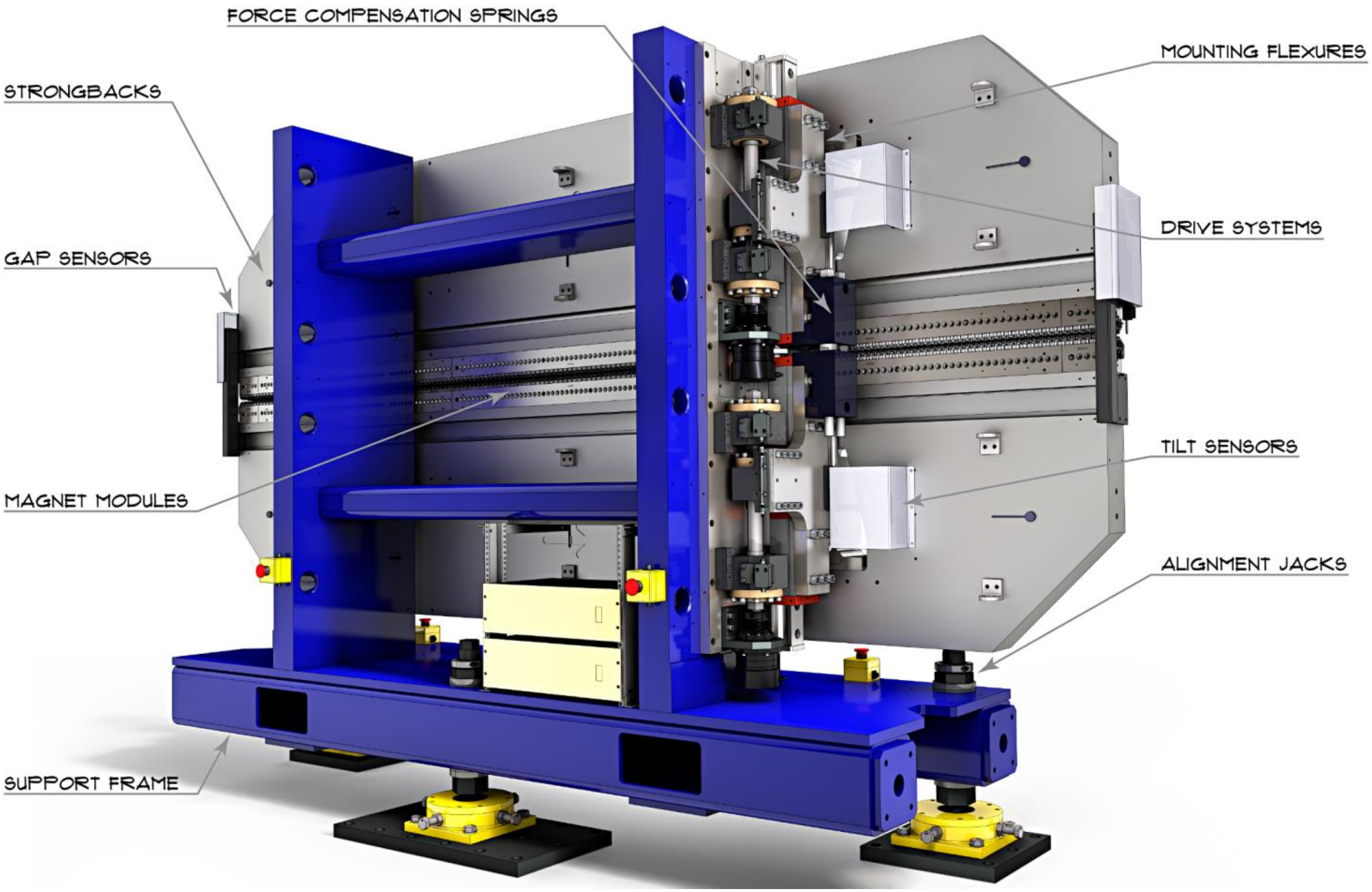
$$\left\langle e^{i\delta\Phi}\right\rangle=\int e^{i\delta\Phi}\cdot\frac{1}{\sqrt{2\pi}\cdot\sigma_K}\cdot e^{-\frac{\sigma^2}{2\sigma_K^2}}=\frac{1}{\sqrt{2\pi}\cdot\sigma_K}\int e^{\frac{2i\widetilde{K}\sigma\sigma_K^2-\sigma^2}{2\sigma_K^2}}d\sigma=$$

$$=\frac{1}{\sqrt{2\pi\sigma_K}}\int e^{-\frac{\left(\sigma-i\widetilde{K}\sigma_K\right)^2}{2\sigma_K^2}}\cdot e^{-\frac{\widetilde{K}^2\sigma_K^2}{2}}d\sigma=C\cdot e^{-\frac{\pi^2K^4}{2\cdot\left(1+\frac{K^2}{2}\right)^2}}$$

$$E,H\propto\left\langle e^{i\delta\Phi}\right\rangle^{2N}=e^{-\frac{N\pi^2K^4}{\left(1+\frac{K^2}{2}\right)^2}\cdot\sigma_K^2}$$

For N~100 and K~2 to keep field losses below 1% σ_K should less than 10^{-3}

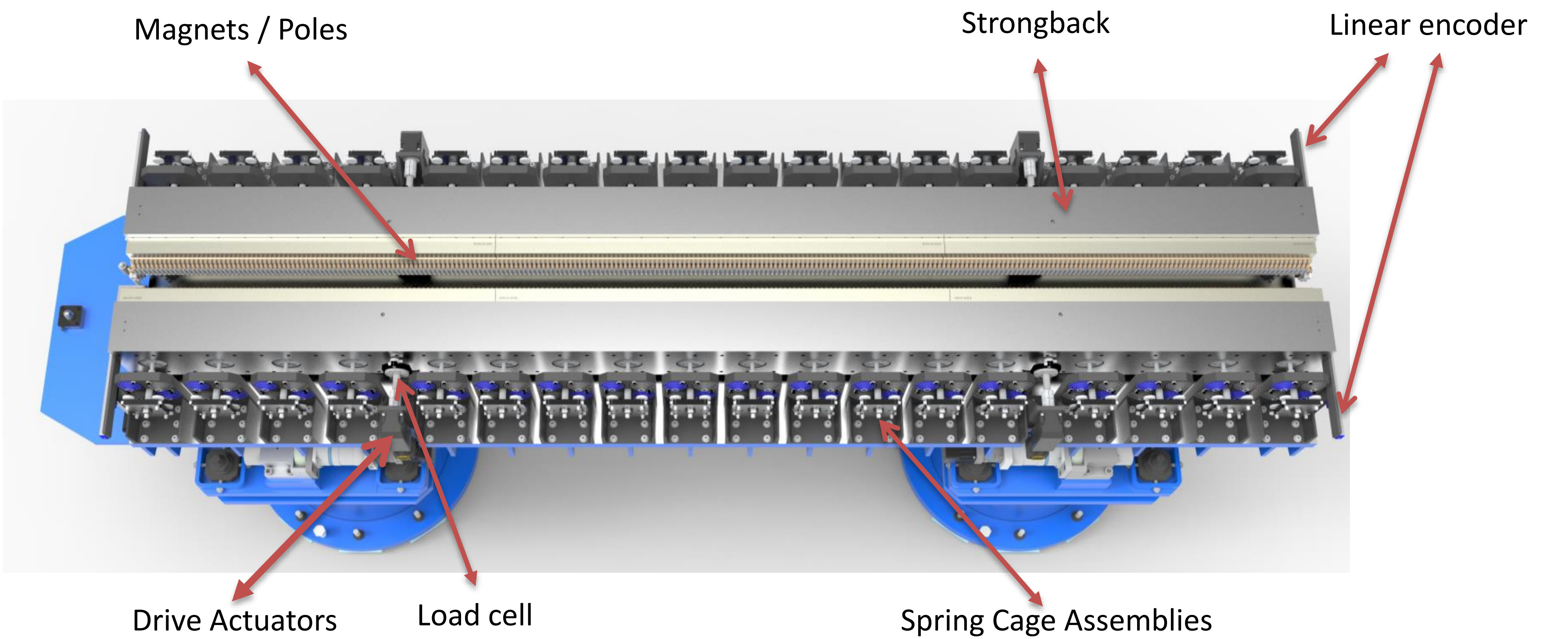
LBL/SLAC undulator



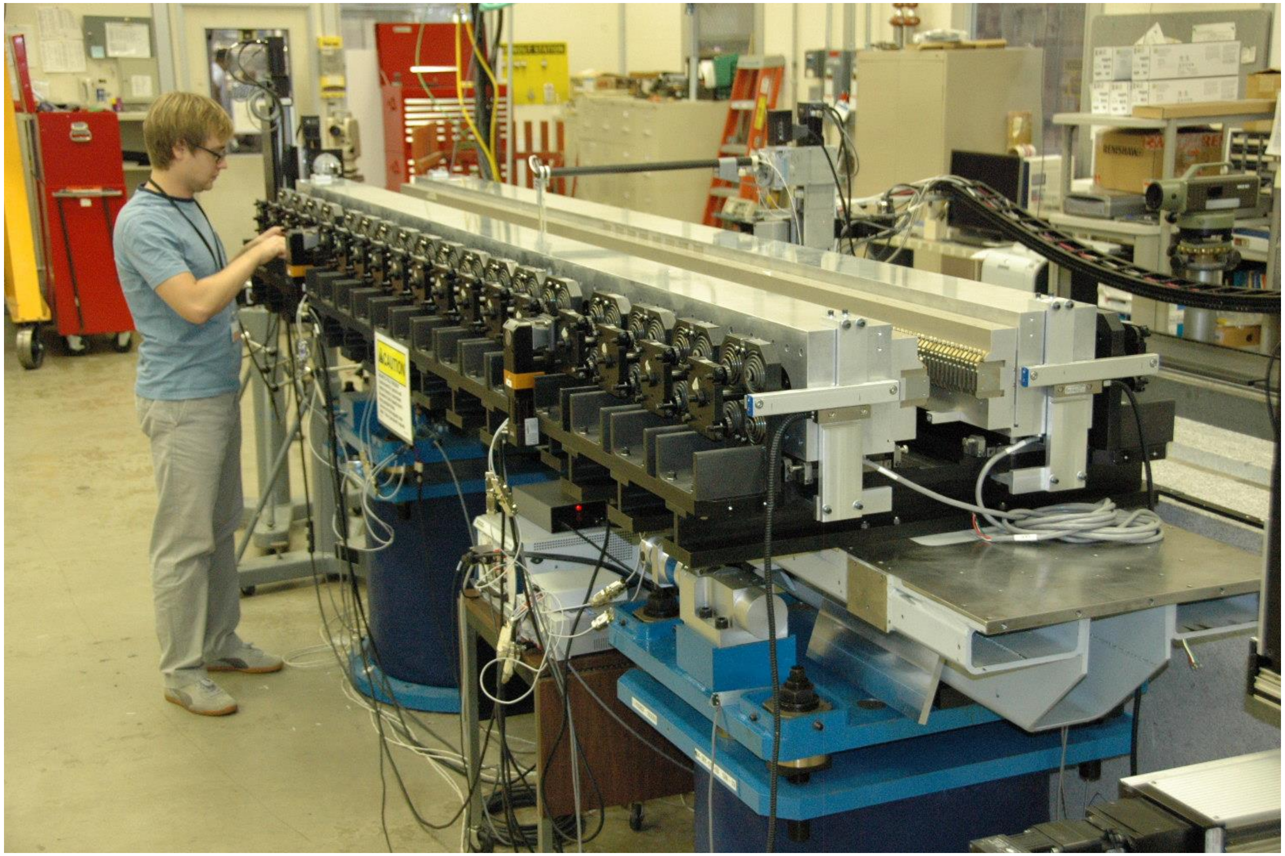
This design exploits different compare with EXFEL approach in supporting heavy moving structures

General Design Concept of Undulator with Horizontal gap

Attractive magnetic forces, that are perpendicular to the gravity force, are compensated by an array of conical springs. These springs are designed to exhibit non-linear force characteristics that can be closely tuned to match the force curve exerted by the magnetic field.

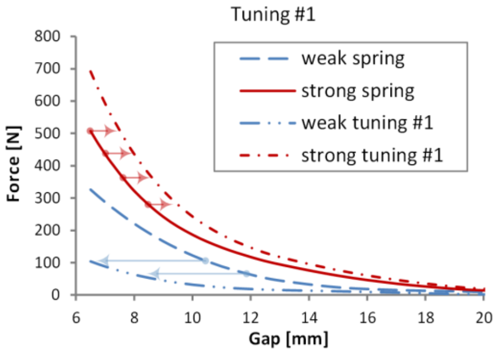


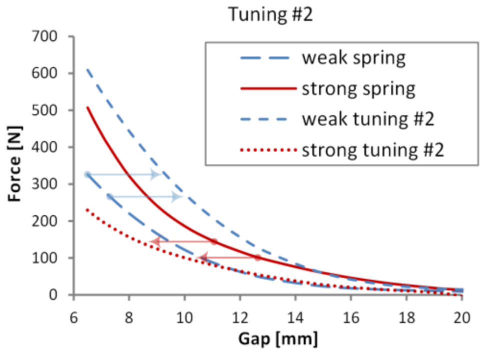
HGVPU at the APS magnet measurement facility

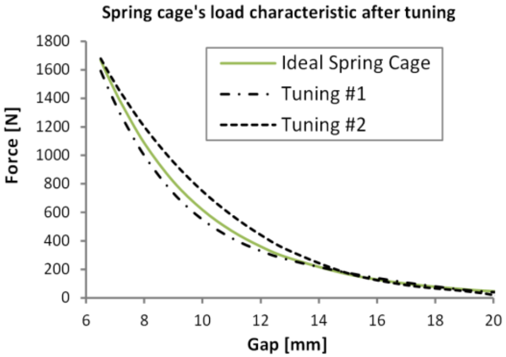


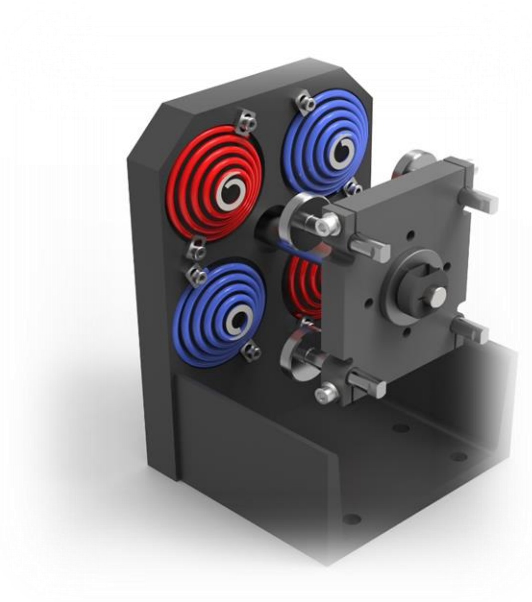
This device has demonstrated the remarkable performance: phase errors are below 3^0 for all operational gaps

Spring system for undulator with horizontal gap









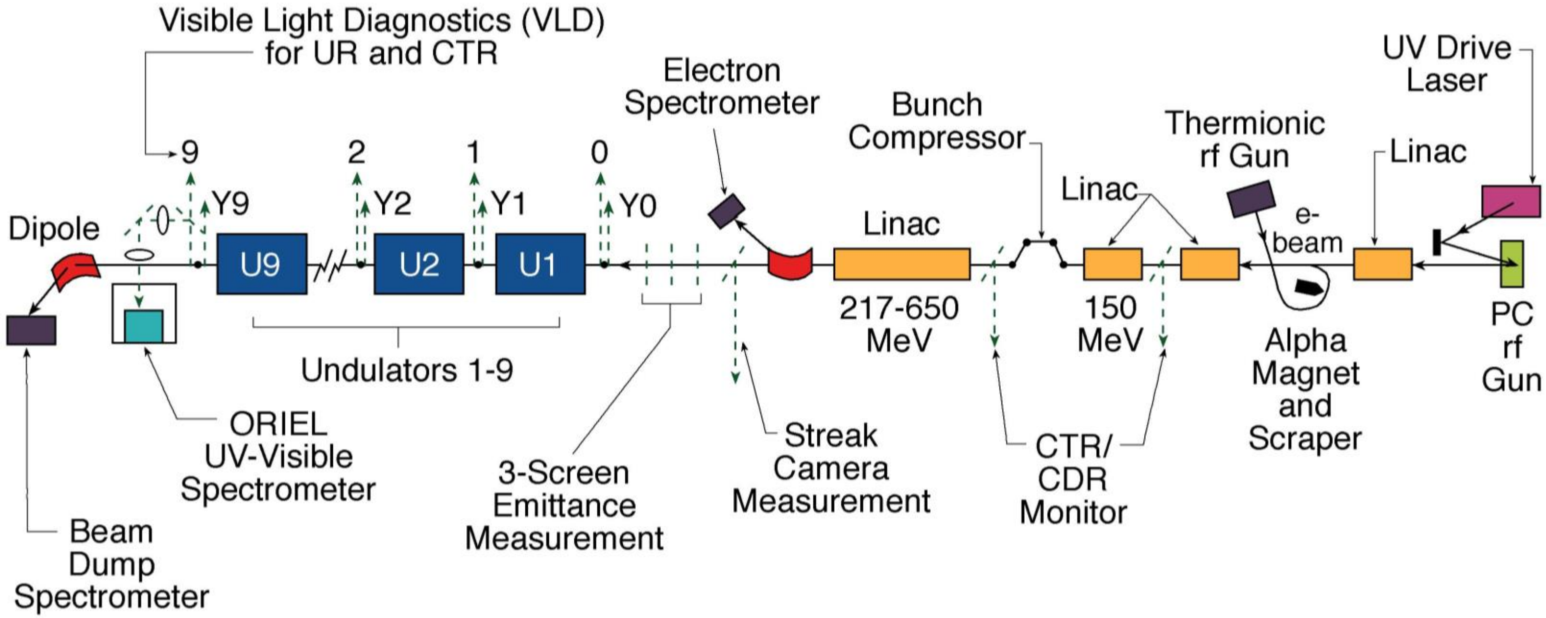
Conical springs permit to closely match magnetic and mechanical forces

LEUTL Undulator Line

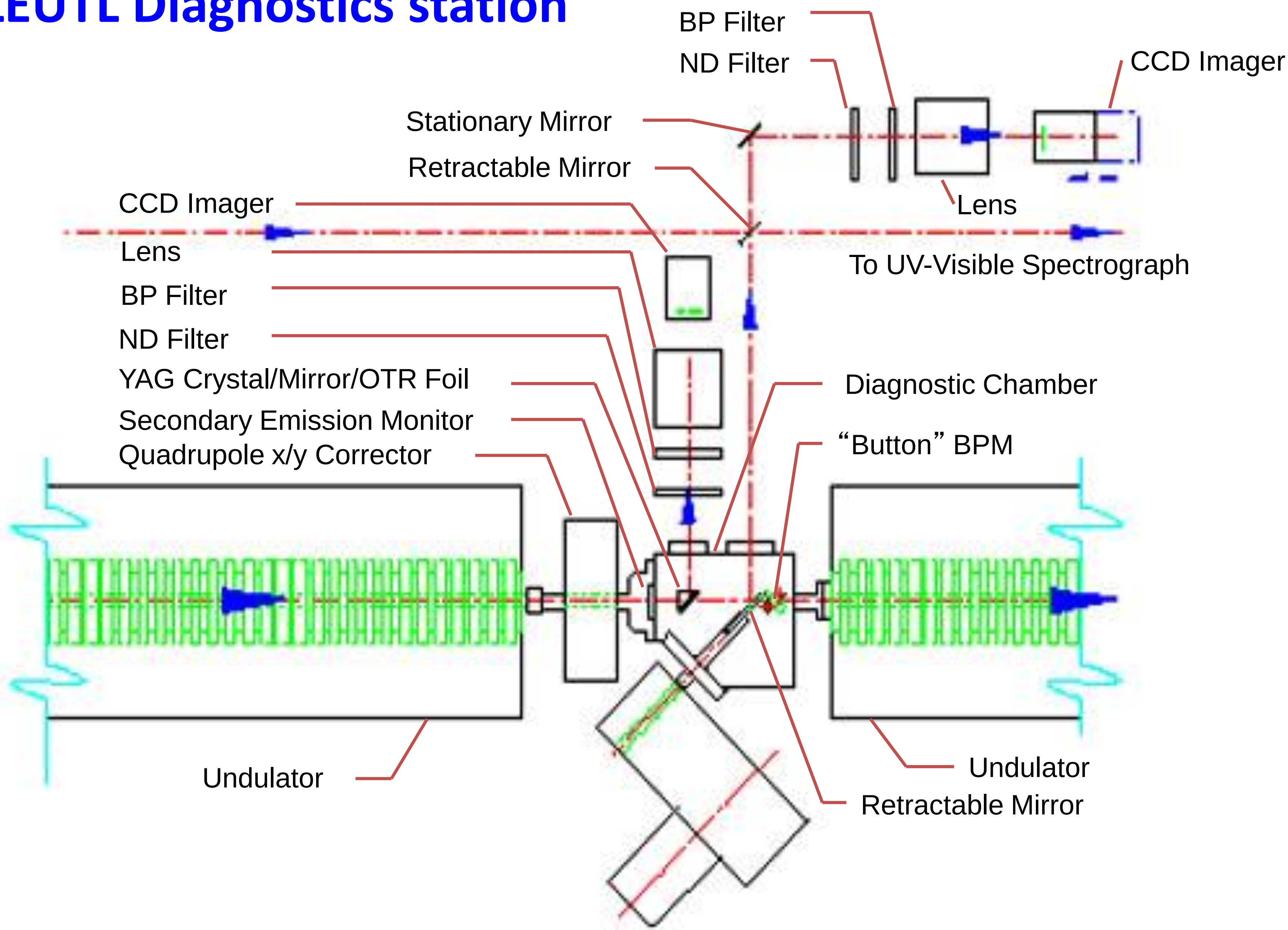
Length	9 x 2.4 m
Period	3.3 cm
Gap	9.4 mm
Field	1 T
K	3.1
Intermodule gap	33 cm

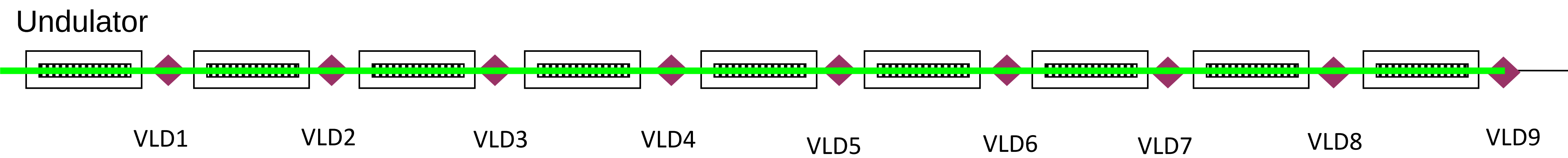
Advanced Photon Source, 2000

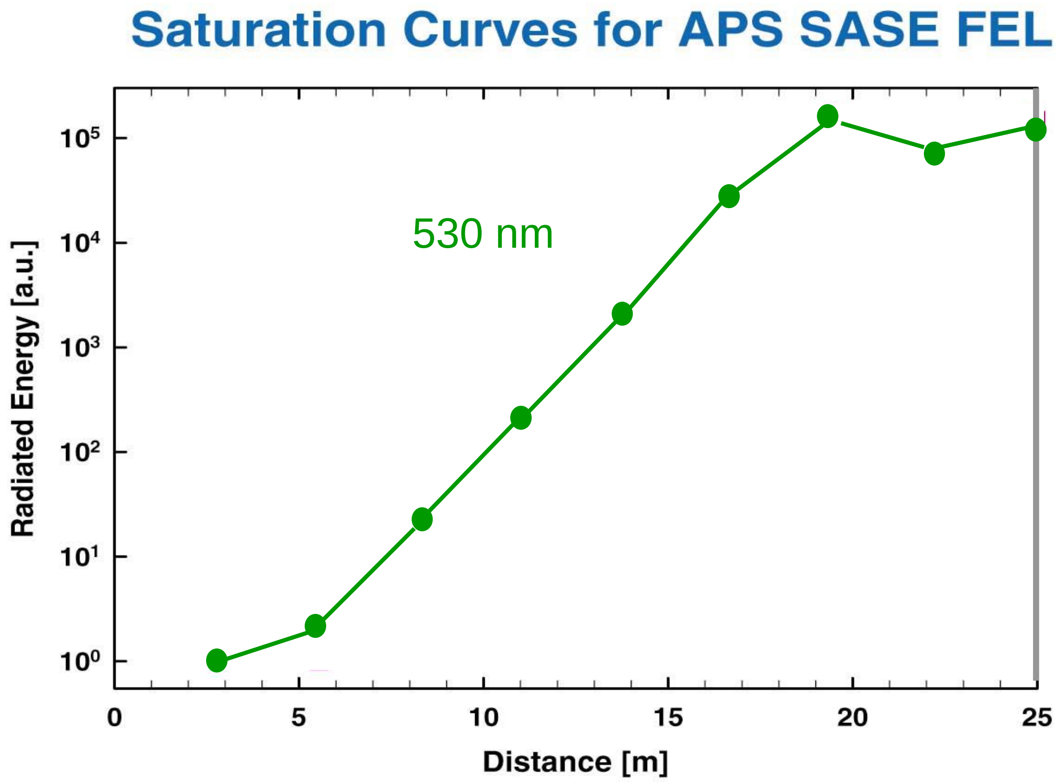
Schematic of LEUTL SASE FEL experiment

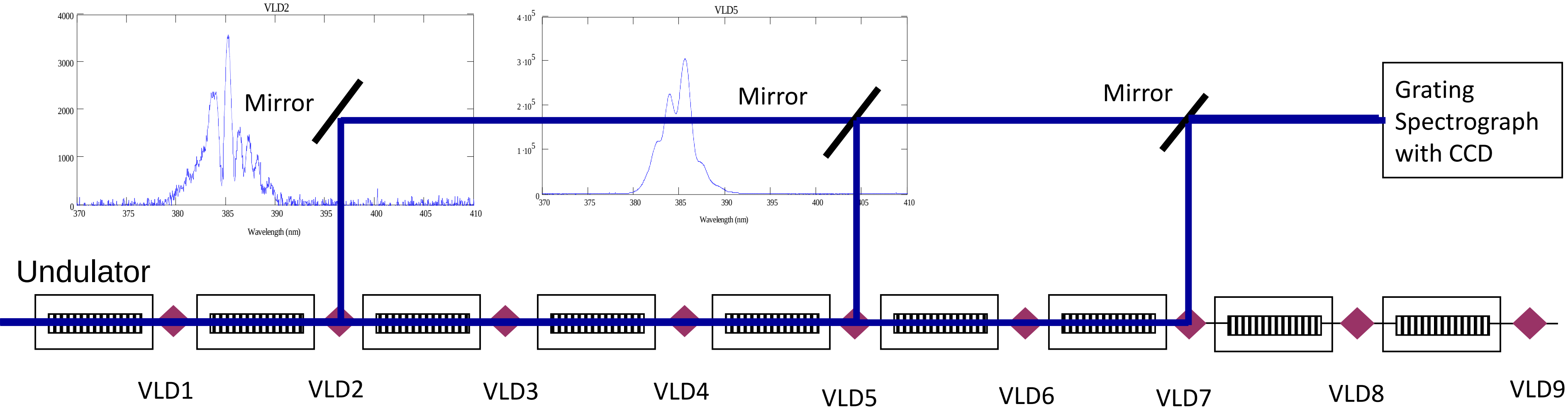


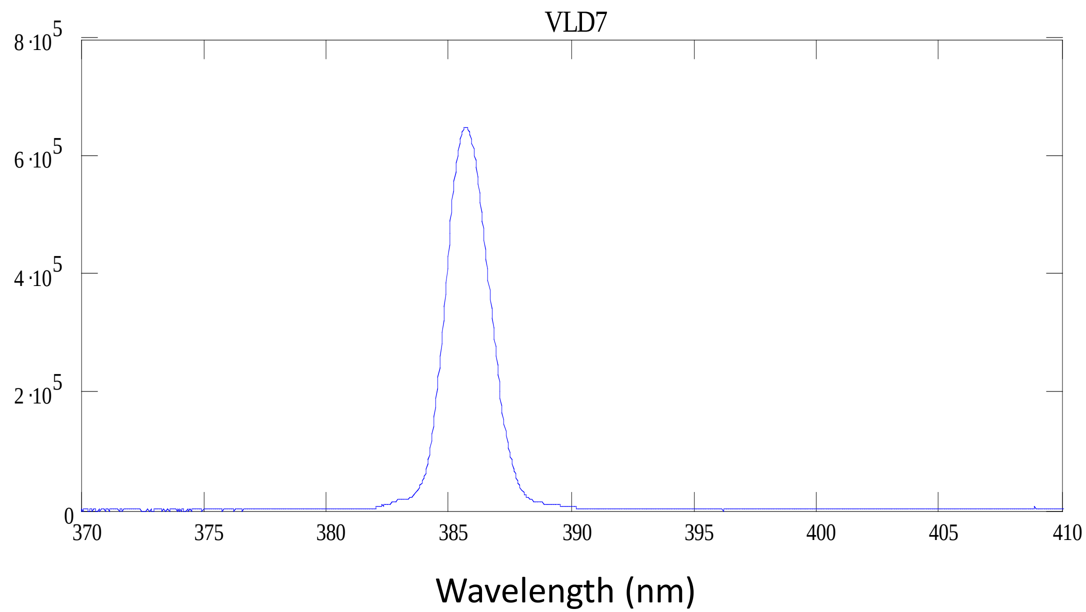
LEUTL Diagnostics station




Undulator
VLD1
VLD2
VLD3
VLD4
VLD5
VLD6
VLD7
VLD8
VLD9


Saturation Curves for APS SASE FEL
530 nm
Radiated Energy [a.u.]
Distance [m]
10⁰
10¹
10²
10³
10⁴
10⁵
0
5
10
15
20
25


VLD2
VLD5
Wavelength (nm)
Mirror
Mirror
Mirror
Grating Spectrograph with CCD
Undulator
VLD1
VLD2
VLD3
VLD4
VLD5
VLD6
VLD7
VLD8
VLD9

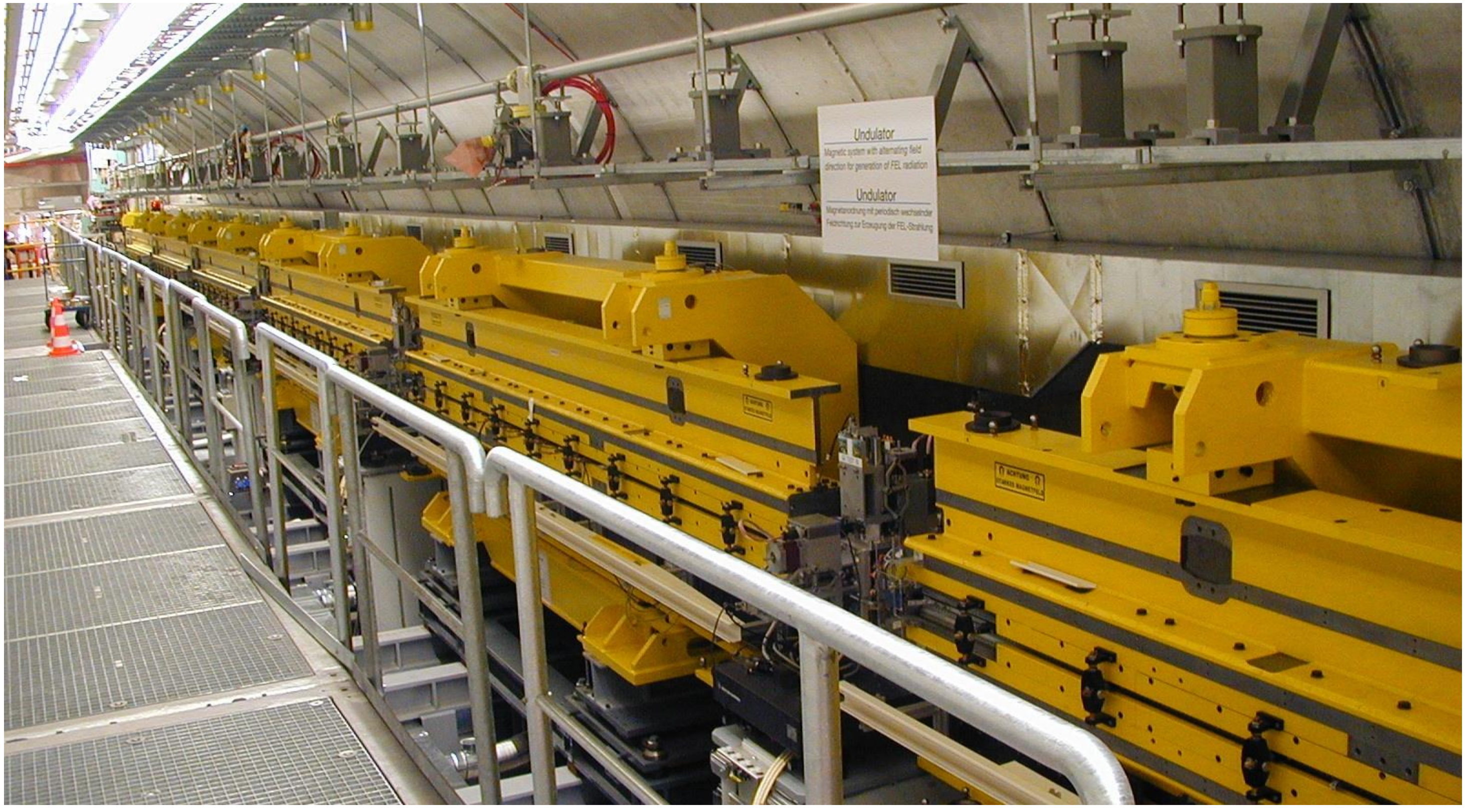

VLD7
8 ·10^5
6 ·10^5
4 ·10^5
2 ·10^5
0
370
375
380
385
390
395
400
405
410
Wavelength (nm)

84 meters of FEL undulator installed at LCLS-I

SACLA FEL Undulator line

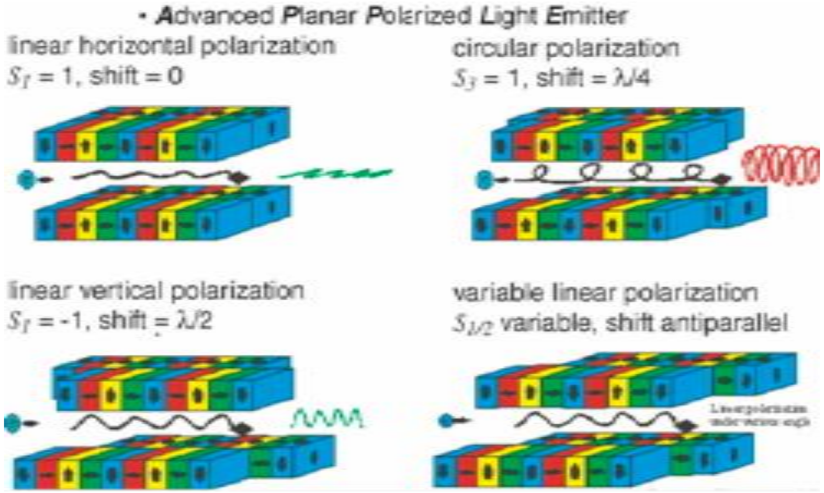
Largest density of in-vacuum undulators one would ever see

FLASH undulator line



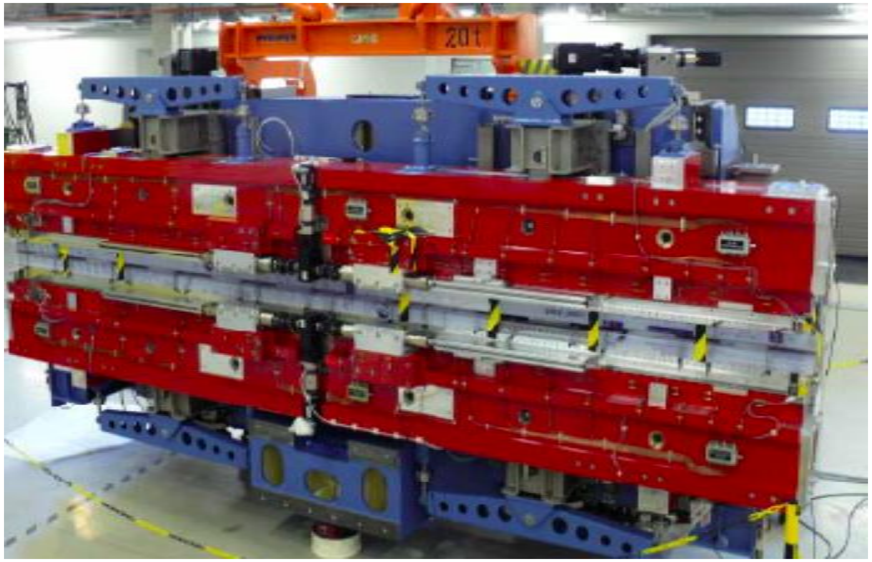
The first SASE FEL dedicated for user operations

APPLE-type ID

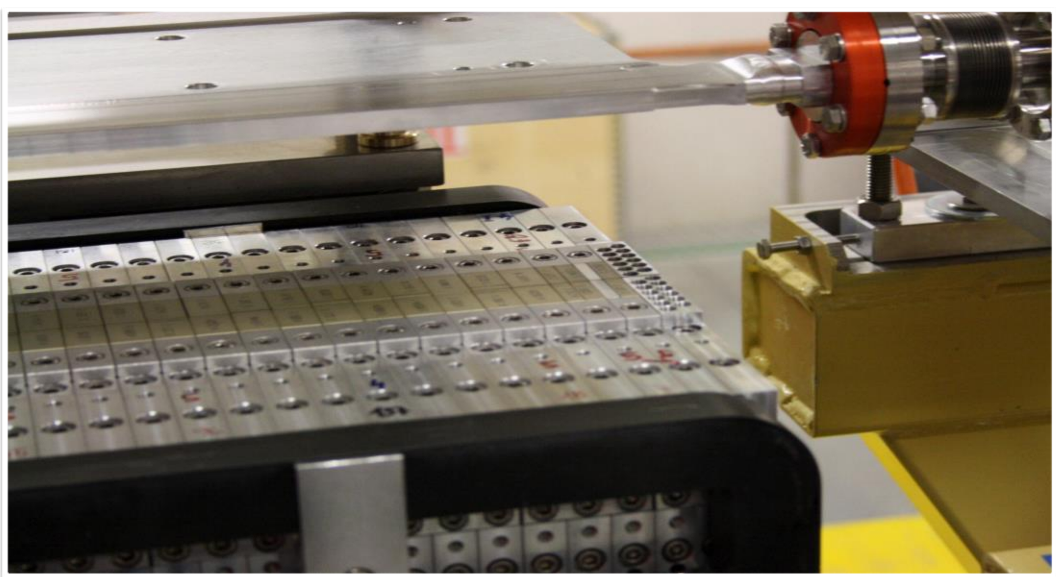


A lot of flexibility to control radiation polarization

Extremely complex mechanically, relatively weak fields

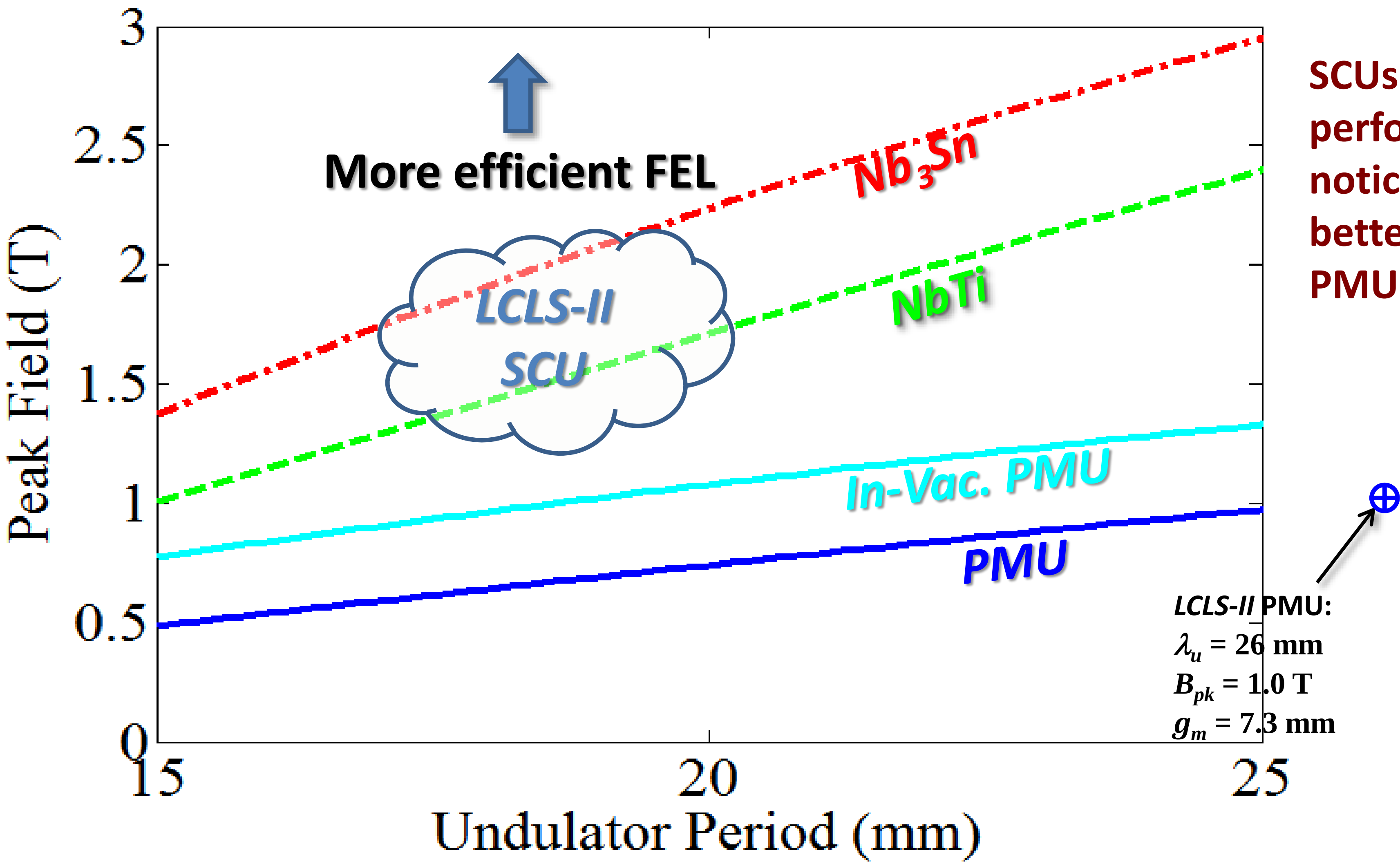


FERMI FEL Undulator line



Soft X-Ray FEL undulator lines are relatively short, but require/prefer control of radiation polarization

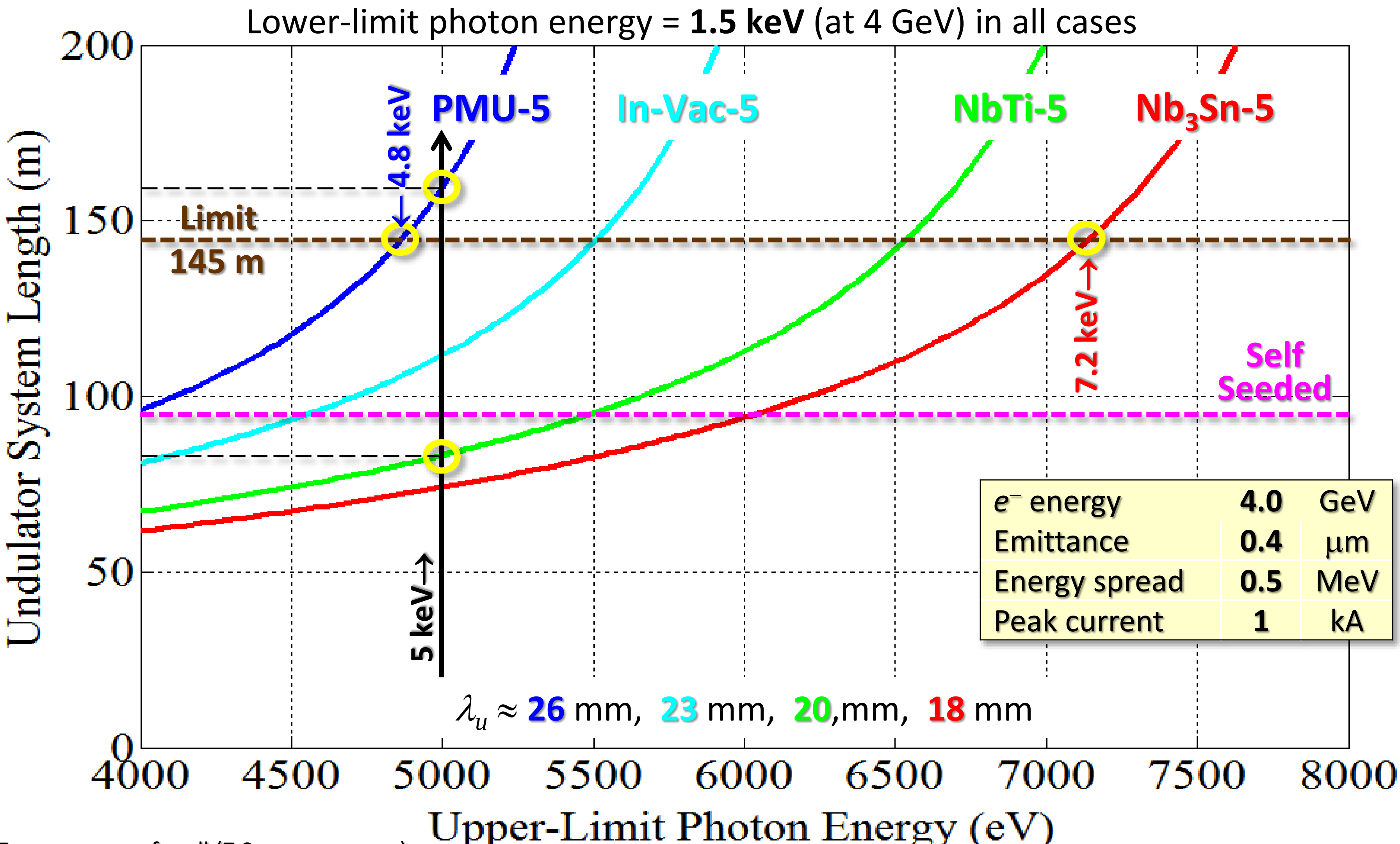
Peak field at SCUs and PMUs



SCUs perform noticeably better than PMUs

- 5-mm vac. gap for all (7.3-mm mag. gap)
- *In-Vac* → same vac. gap (5.3-mm mag. gap)

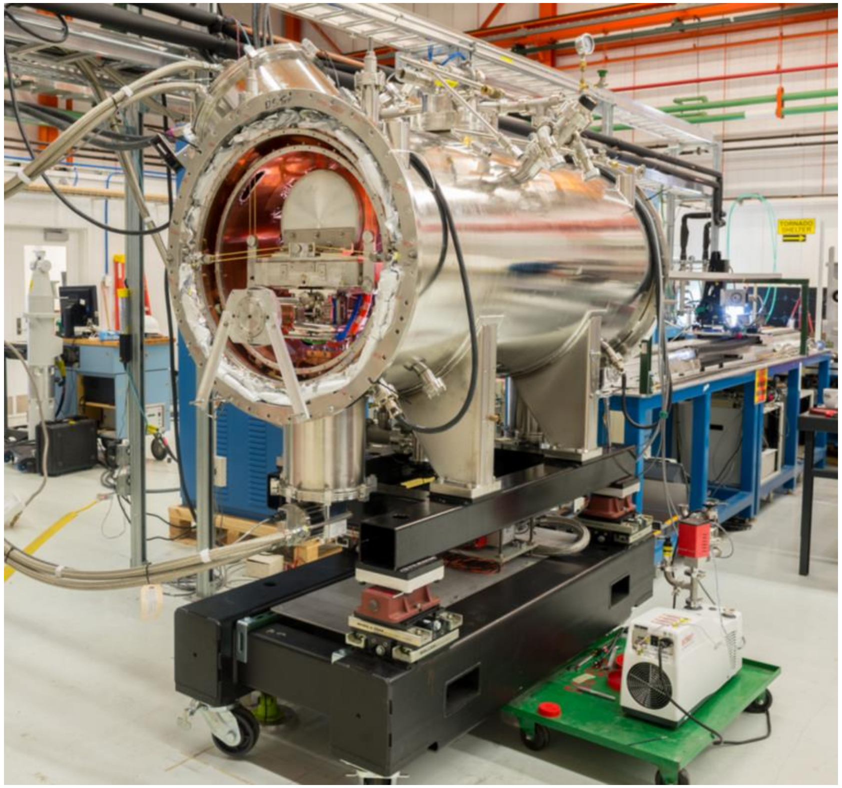
Advantage of SCU versus IVU, PMU for LCLS FEL



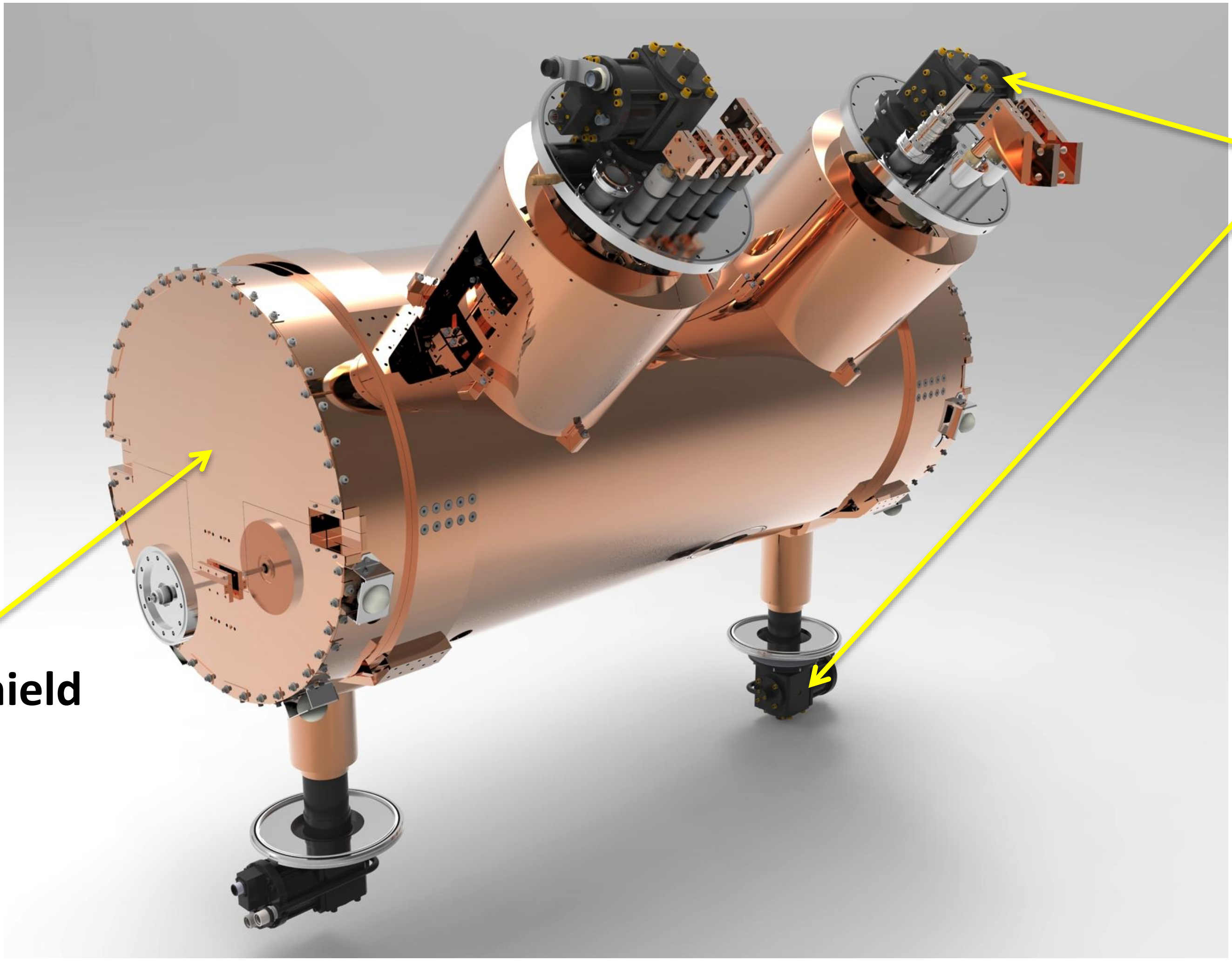
e^- energy	**4.0**	GeV
Emittance	**0.4**	μm
Energy spread	**0.5**	MeV
Peak current	**1**	kA

- 5-mm vac. gap for all (7.3-mm mag. gap)
- *In-Vac* → same vac. gap (5.3-mm mag. gap)

SCU: assembled cryostat



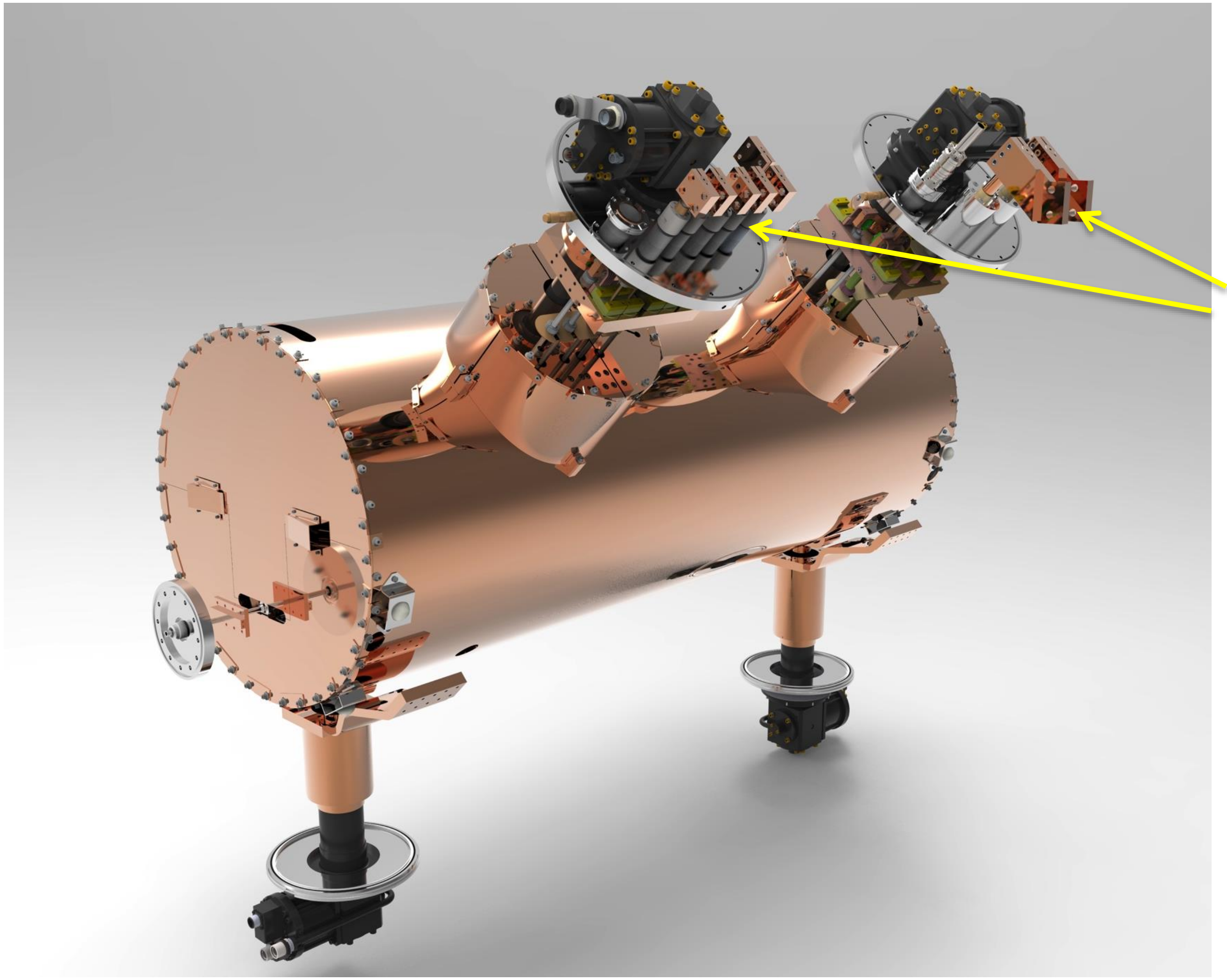
SCU: inside the vacuum vessel



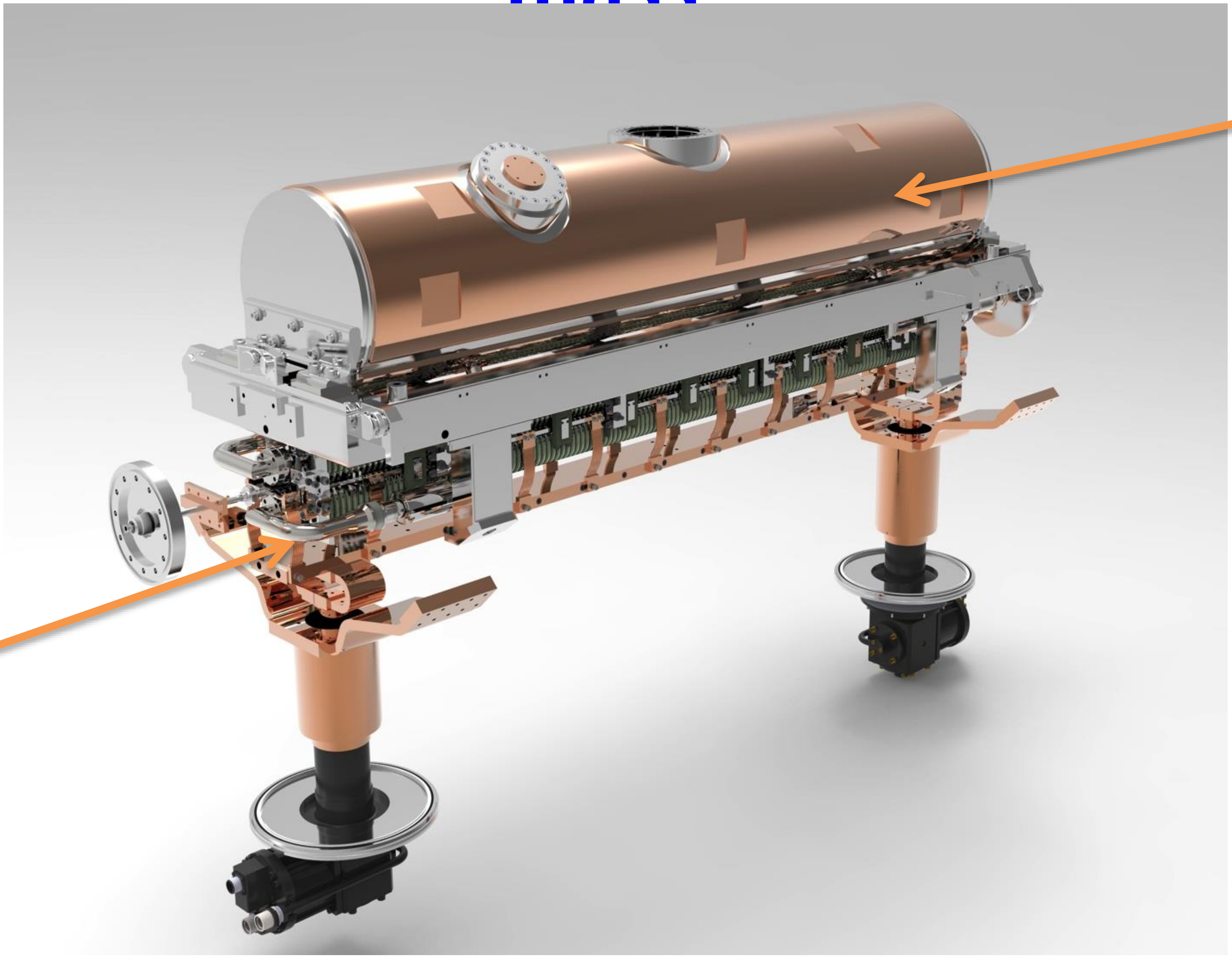
Cryocoolers

Thermoshield

SCU: under 1st thermoshield



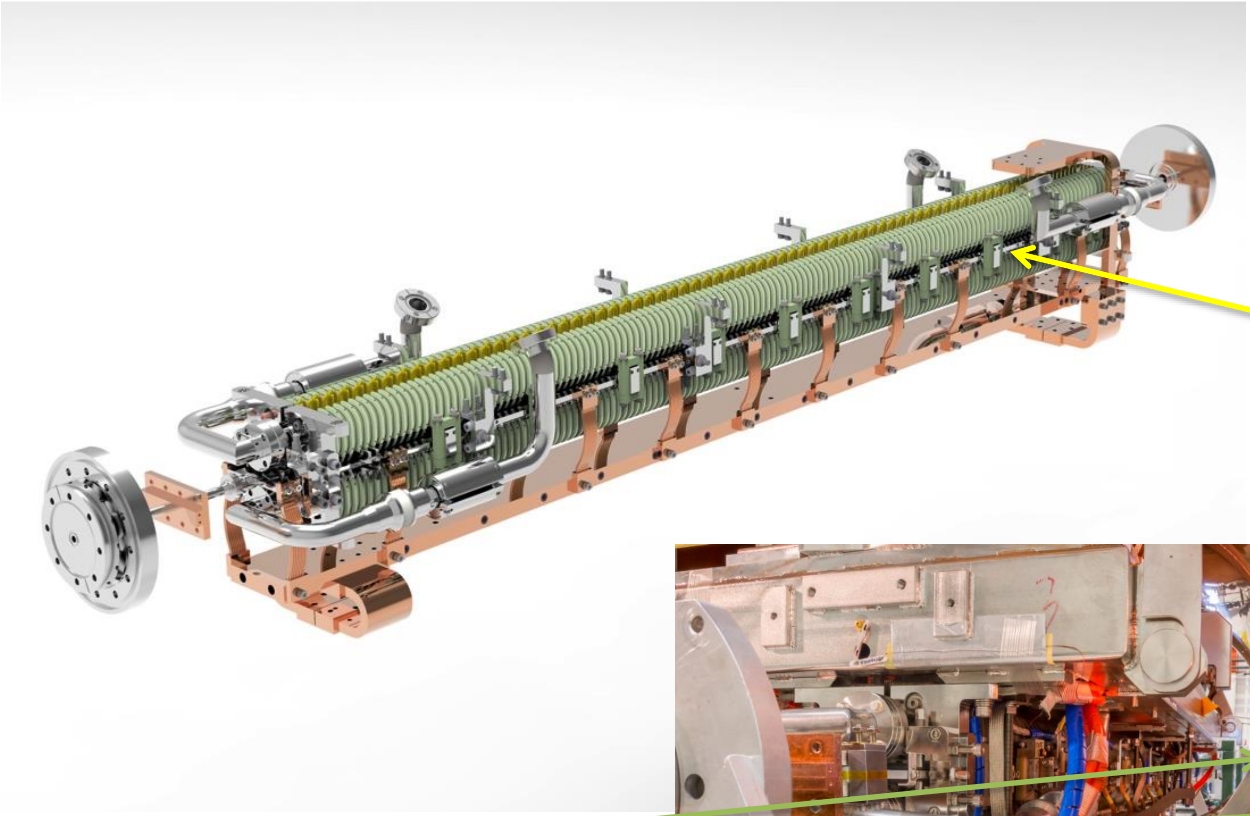
SCU: under 2nd thermoshield - cold mass

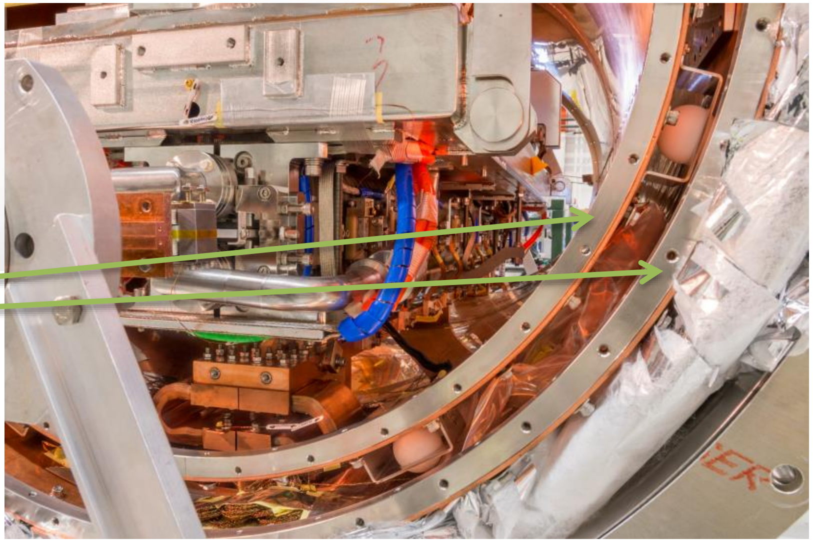


Liquid He vessel

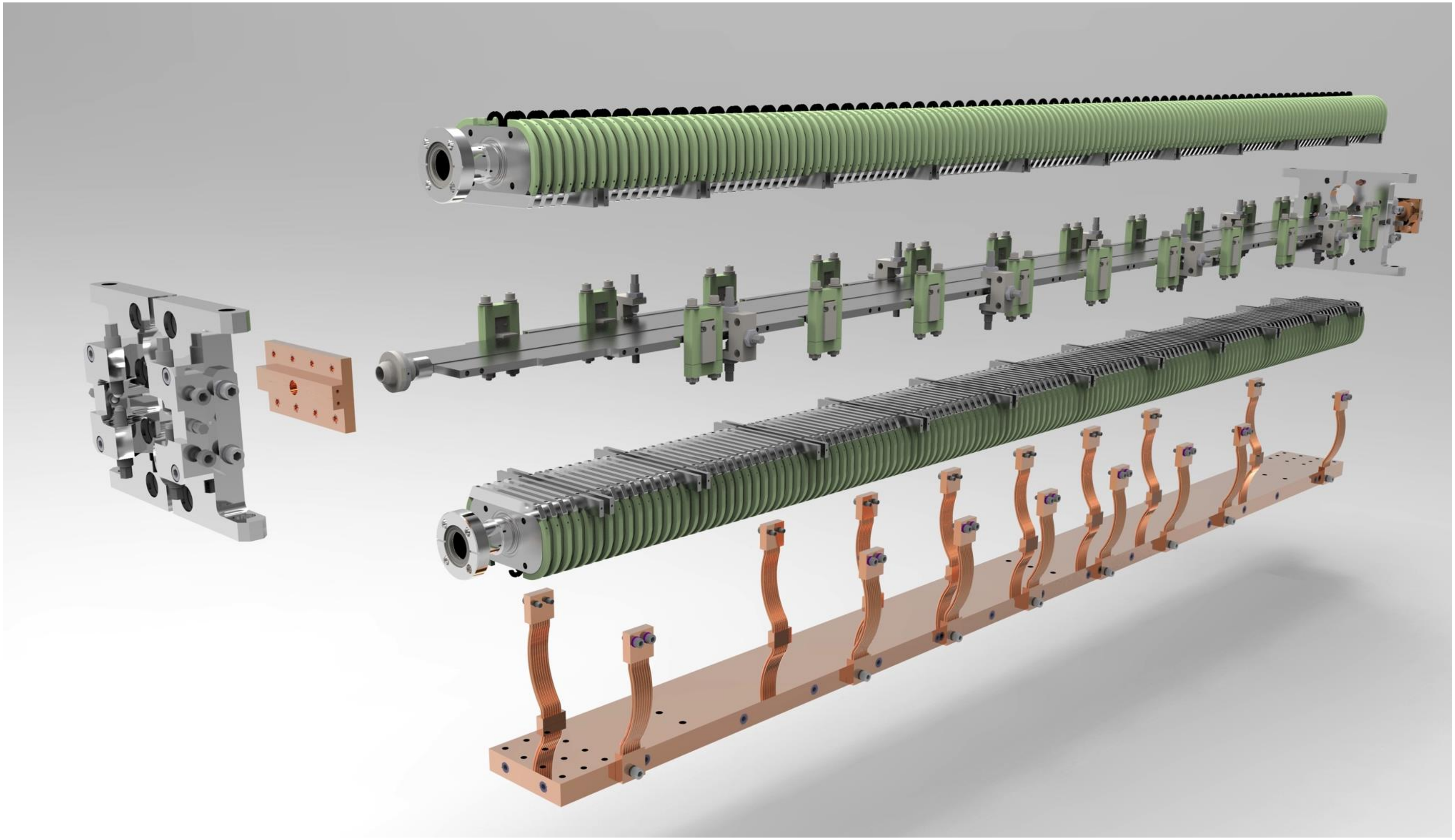
Liquid He plumbing

SCU: undulator and vacuum chamber

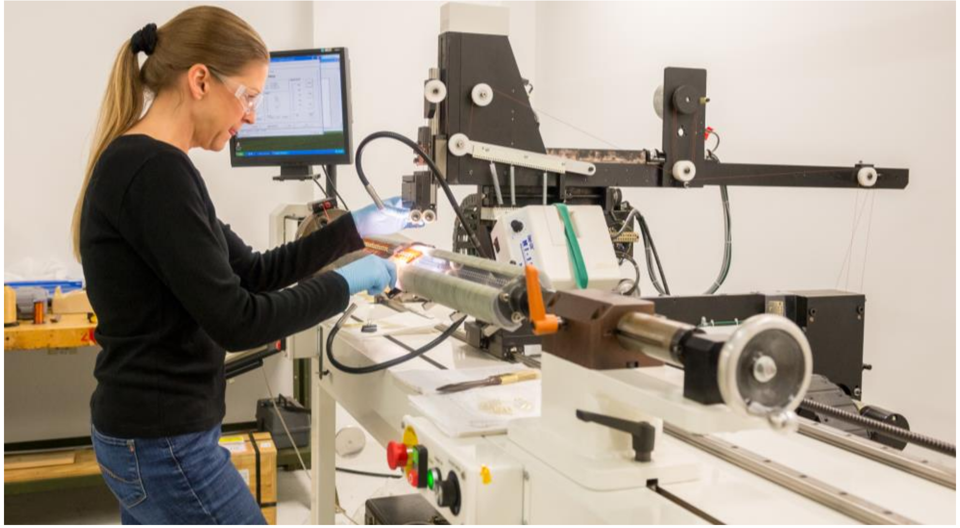




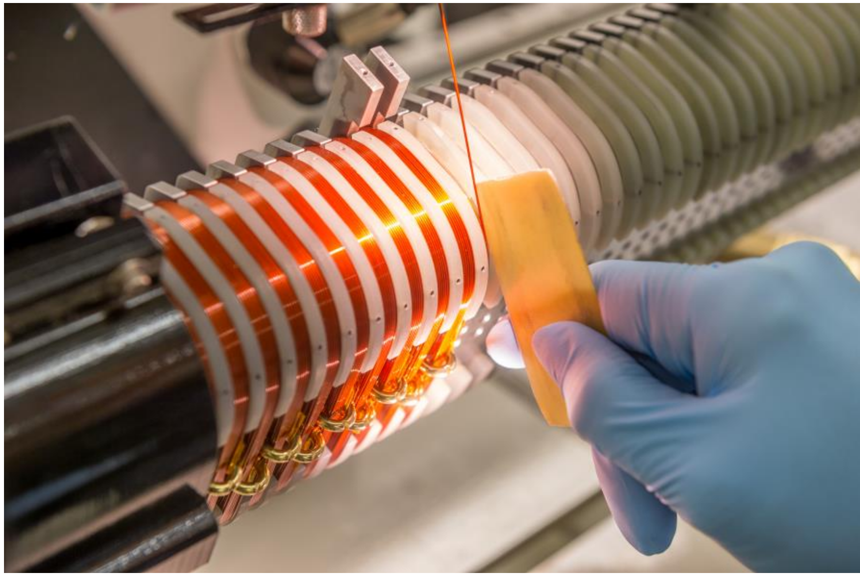
Undulator coils and vacuum chamber in parts



SCU magnet winding



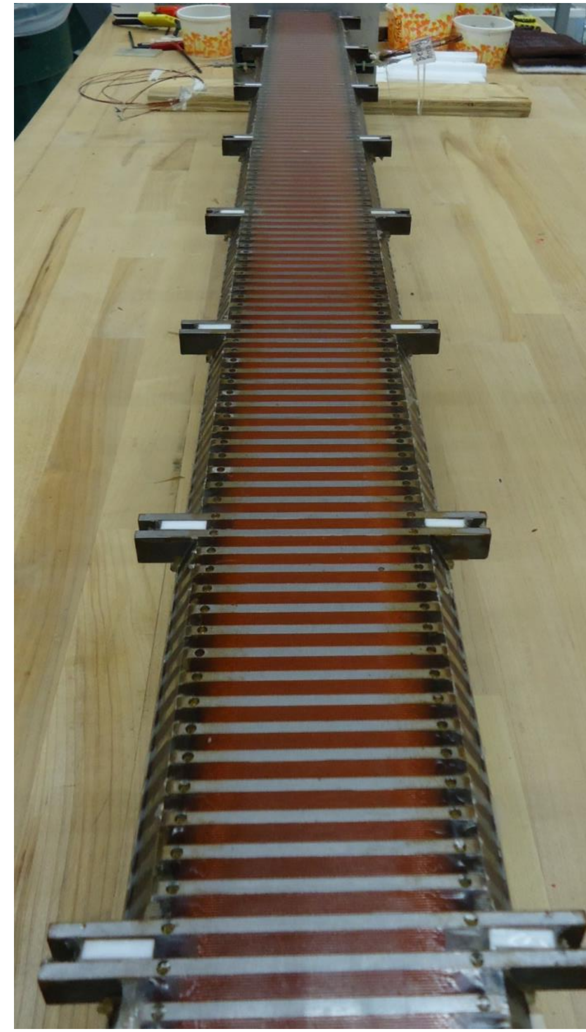
“Automated” winding machine

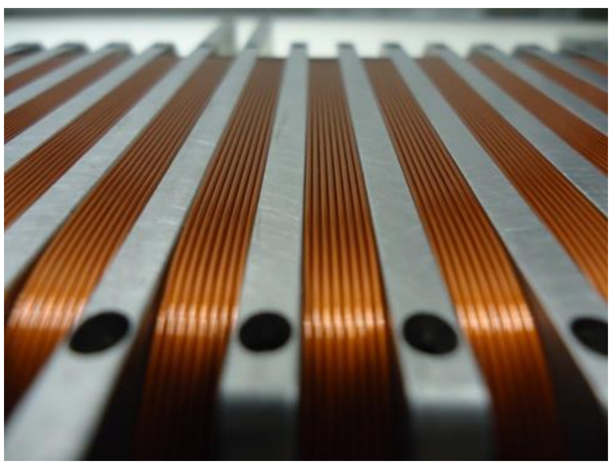


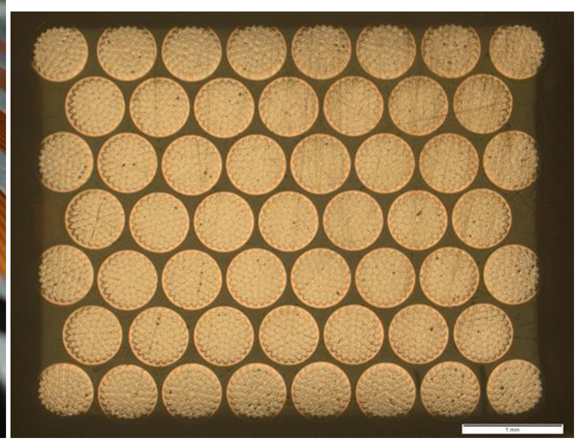
Careful persuasion helps

SCU1 Magnet

1.08-m long, 1.8-cm period

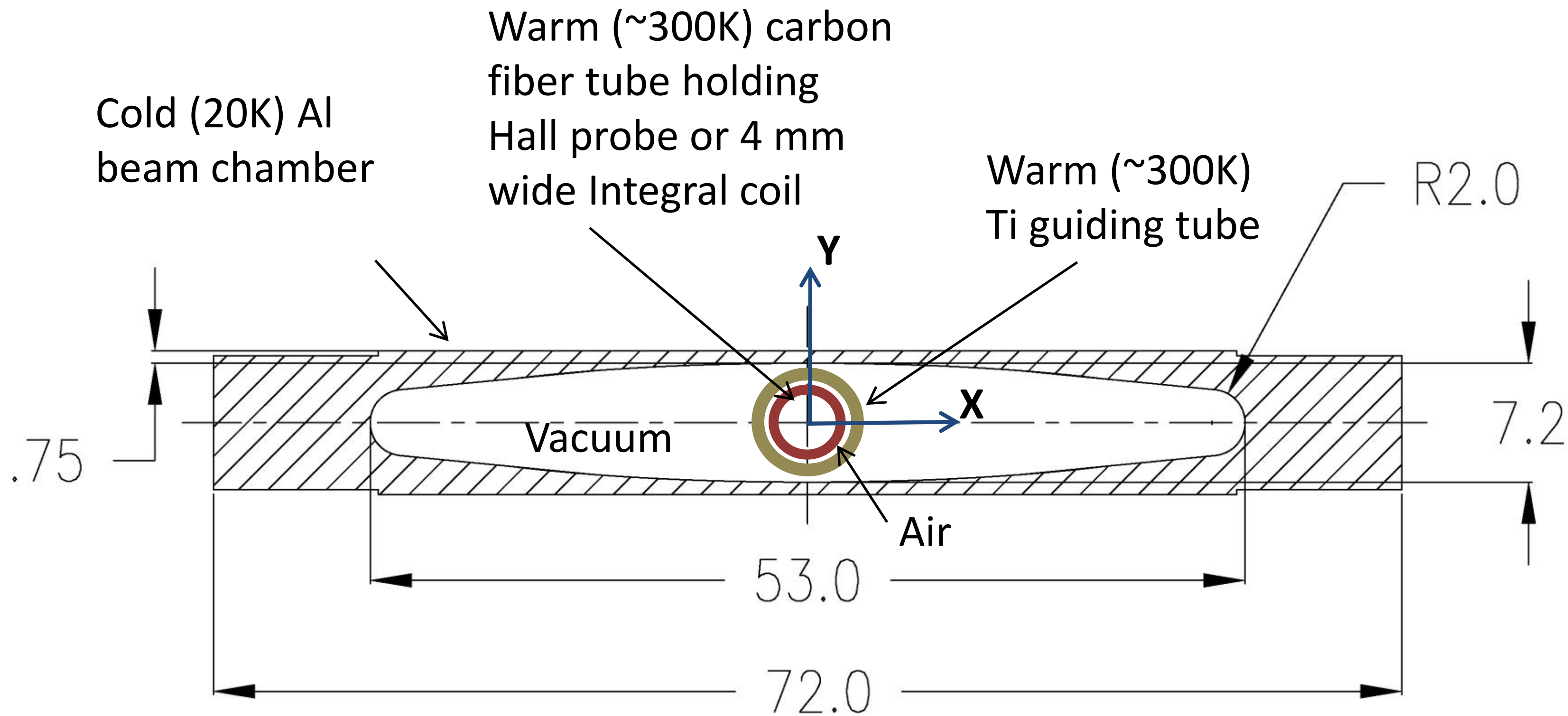




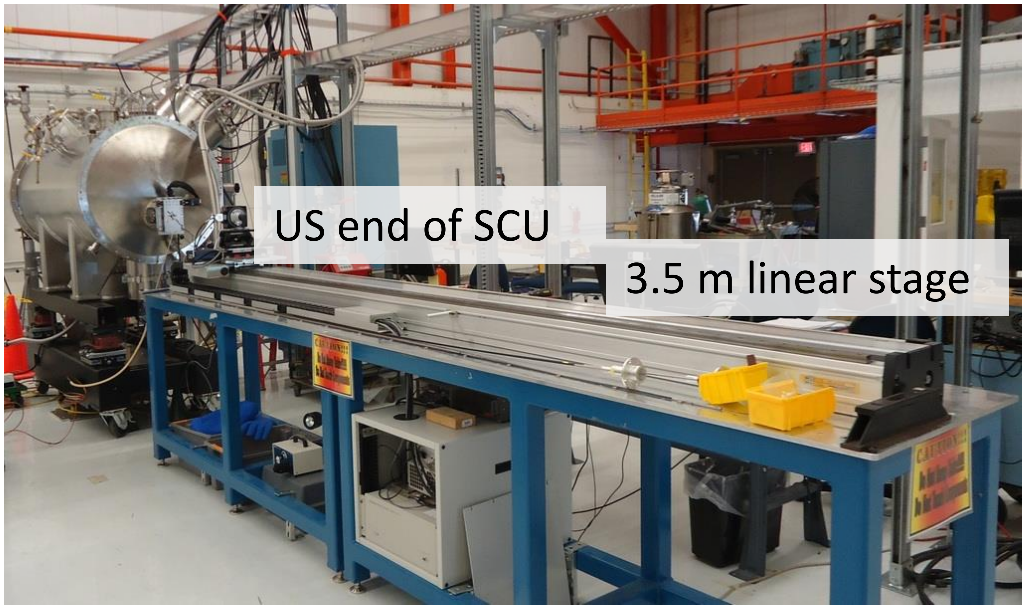


Exceptional quality of winding

SCU magnetic measurement system design: Beam chamber and guide tube cross section

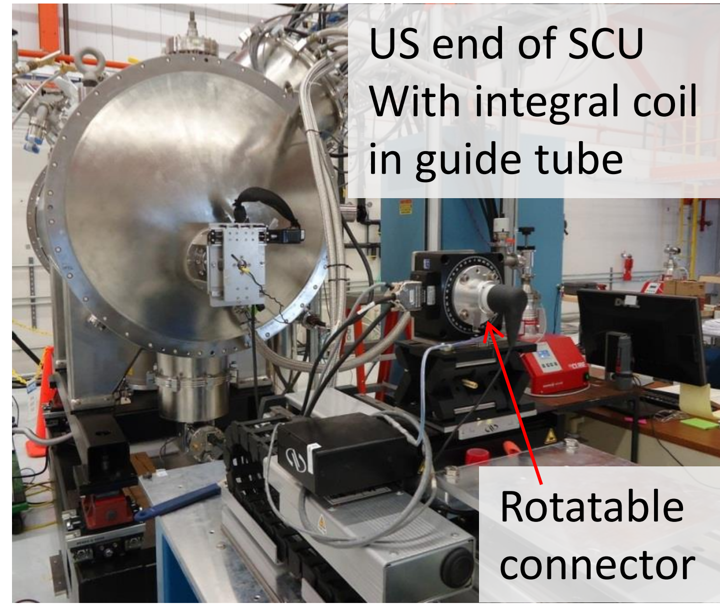


SCU magnetic measurement system



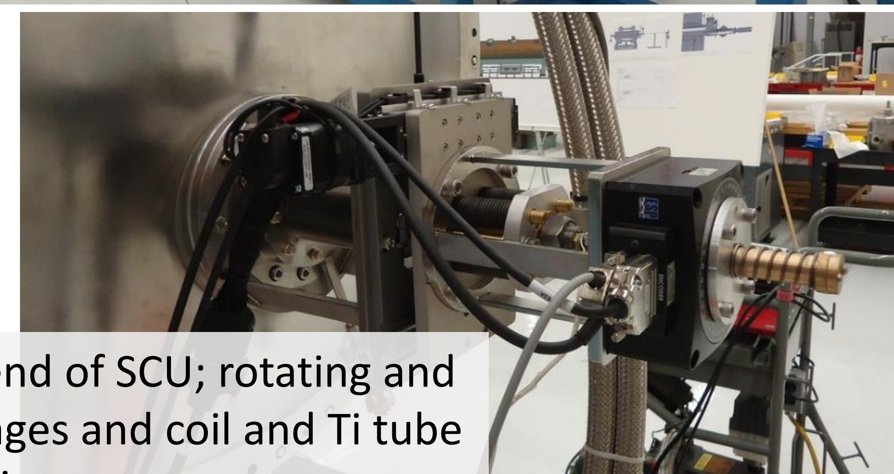
US end of SCU

3.5 m linear stage

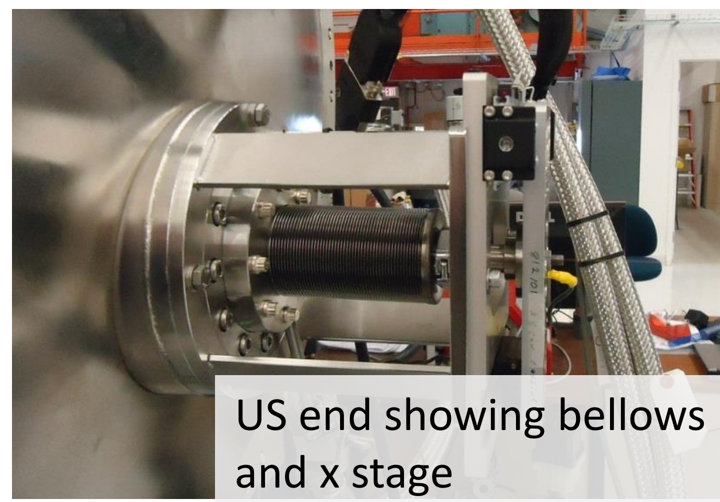


US end of SCU
With integral coil
in guide tube

Rotatable
connector

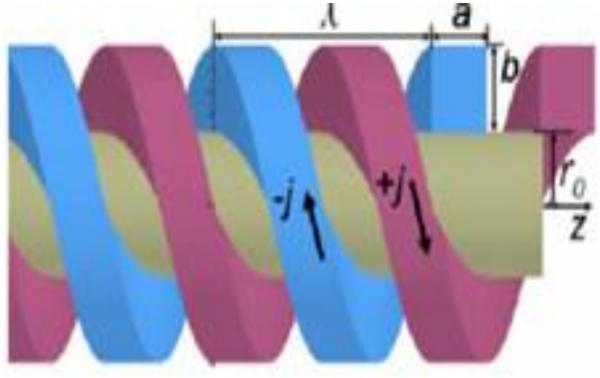


DS end of SCU; rotating and
x stages and coil and Ti tube
tensioners

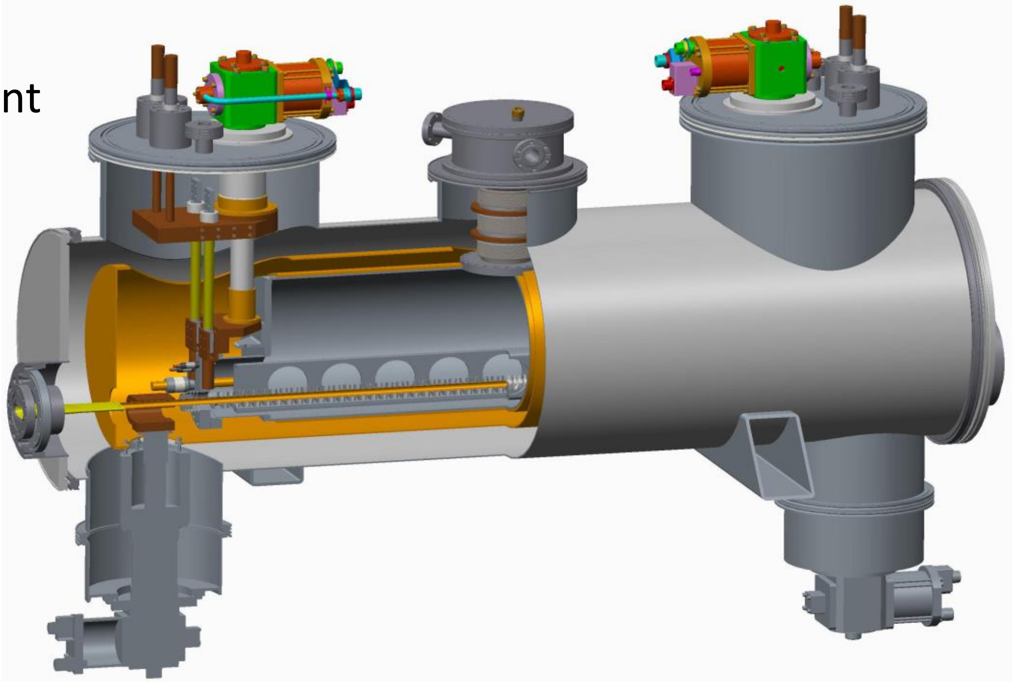


US end showing bellows
and x stage

Helical SCU



Helic bifilar to eliminate longitudinal field component



Motion equations in helical undulator

For left hand helicity

$$\frac{d^2x}{dt^2} = \frac{e}{\gamma mc}\left(-\upsilon_y B_z + \upsilon_z B_y\right)$$

$$\frac{d^2y}{dt^2} = \frac{e}{\gamma mc}\left(\upsilon_x B_z - \upsilon_z B_x\right)$$

$$\frac{d^2z}{dt^2} = \frac{e}{\gamma mc}\left(-\upsilon_x B_y + \upsilon_y B_x\right)$$

$$B_x = B_0 \cos\left(k_u z\right)$$

$$B_y = B_0 \sin\left(k_u z\right)$$

$$B_z = 0$$

$$k_u = \frac{2\pi}{\lambda_u}$$

$$\frac{d\upsilon_x}{dt}\upsilon_z = \frac{e}{\gamma mc}\upsilon_z B_y = \frac{eB_0\upsilon_z}{\gamma mc}\cos\left(k_u z\right) \Rightarrow \upsilon_x = \frac{eB_0}{\gamma mck_u}\sin\left(k_u z\right)$$

$$\frac{d\upsilon_y}{dt}\upsilon_z = -\frac{e}{\gamma mc}\upsilon_z B_x = -\frac{eB_0\upsilon_z}{\gamma mc}\sin\left(k_u z\right) \Rightarrow \upsilon_y = \frac{eB_0}{\gamma mck_u}\cos\left(k_u z\right)$$

Motion equations in helical undulator (cont.)

$$\beta_x = \frac{\upsilon_x}{c} = \frac{K}{\gamma}\sin(k_u z) \qquad K = \frac{e\lambda_u B_0}{2\pi mc^2}$$

$$\beta_y = \frac{\upsilon_y}{c} = \frac{K}{\gamma}\cos(k_u z)$$

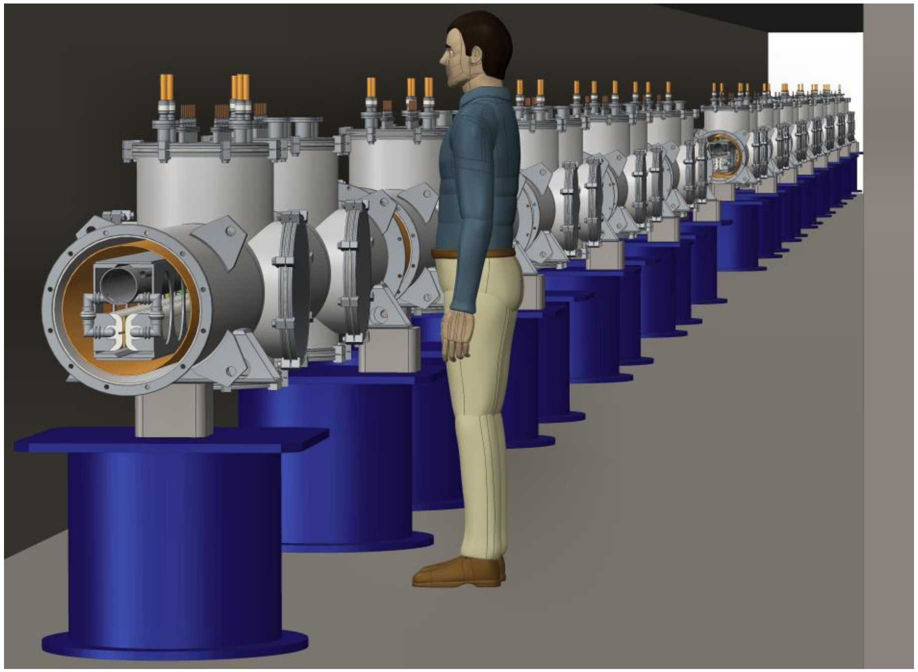
$$\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2 = const \qquad \beta_x^2 + \beta_y^2 = \frac{K^2}{\gamma^2} = const \Rightarrow \beta_z = const$$

$$\beta_z = \sqrt{\beta^2 - \left(\beta_x^2 + \beta_y^2\right)} = \sqrt{\beta^2 - \frac{K^2}{\gamma^2}} = \sqrt{\left(1 - \frac{1}{2\gamma^2}\right)^2 - \frac{K^2}{\gamma^2}} \approx \sqrt{1 - \frac{1}{\gamma^2} - \frac{K^2}{\gamma^2}} \approx 1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \frac{K^2}{\gamma^2}\right) =$$

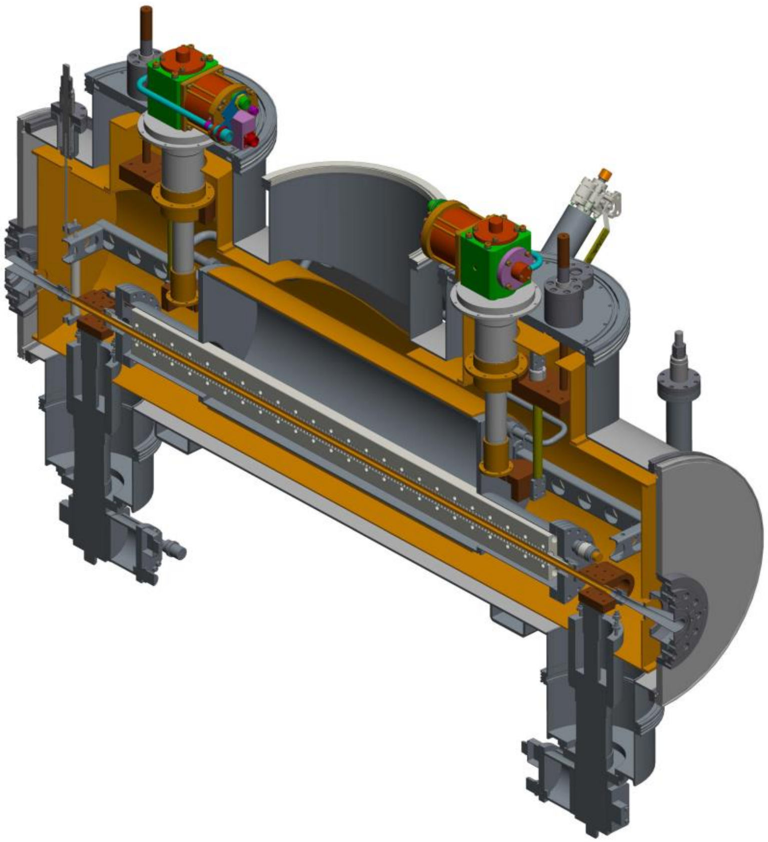
$$= 1 - \frac{1}{2\gamma^2}\left(1 + K^2\right)$$

$$\lambda = \frac{\lambda_{u_h}}{2\gamma^2}\left(1 + K^2\right)$$

FUTURE SCU undulator line



String of cryostats with helical undulators



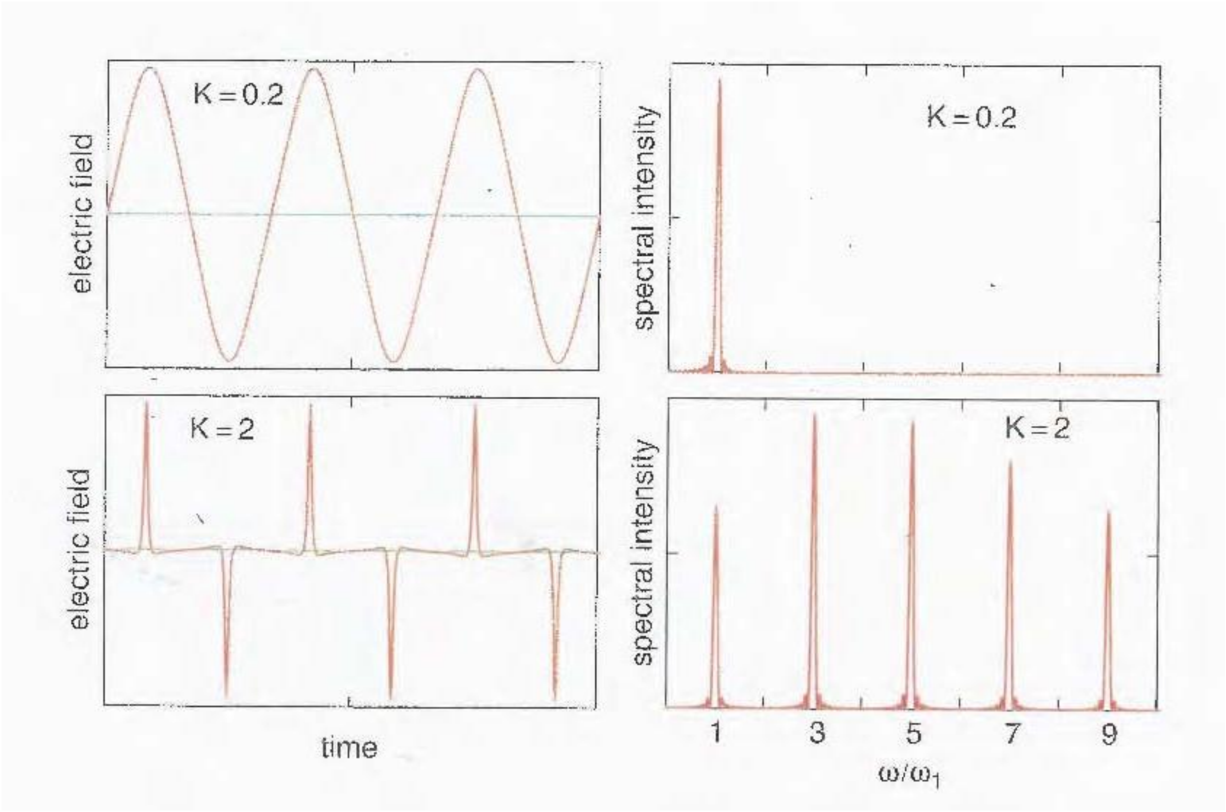
Cross-section of one cryostat

Extras

What to read:

1. R.Walker, CERN Accelerator School on Synchrotron Radiation and Free Electron lasers, CERN 98-04, 3 August 1998
2. J.Clarke, The Science and Technology of Undulators and Wigglers, Oxford University Press, Oxford, 2004
3. P.Schmüster, M.Dohlus, J.Rossbach, Ultraviolet and Soft X-Ray FELs, Springer Tracts in Modern Physics, V.229, 2008
4. P.Luchini and H.Motz, Undulators and Free Electron Lasers, Oxgrod University Press, Oxford, 1990

Undulator harmonics



Schematic view of electric field of undulator emitted radiation: top plots for a small undulator parametr; bottom plots for K=2 when harmonics are clearly pronaunced