Problems Linac Basics

1) Calculate the relative longitudinal motion of two electrons with an energy of about $9\,\mathrm{GeV}$ and a difference of 3% over a distance of $21\,\mathrm{km}$.

2) A superconducting linac consists of cavities with a length of $L = 1.1\,\mathrm{m}$ and an external coupling of $Q_{ext} = 10^5$. It is operated in matched conditions (no reflected power) with a gradient $G_0 = 20\ \mathrm{MV/m}$ and a beam current of $I_0 = 10\,\mathrm{mA}$.
a) What is the input power per cavity P required?
b) The management wants to double the beam current but keep the gradient the same. In order to stay matched, which input power P_{new} does one need? Which other parameter needs to be changed and how?

3) A harmonic oscillator is a special case of Hill's equation: $K(s) = K_0 > 0$. Show that in this case $\beta(s) = \beta_0$ fullfils Hill's equation (using the differential equation for the evolution of the beta-function). Which value has β_0?

4) How much energy is roughly stored in one ILC cavity at nominal gradient?