Homework Set 1 (10/29/15), C1 class (X-Ray Free Electron Laser Theory), 9th ILC school, 2015

1 Energy conservation in 1D FEL theory

The purpose of this problem is to prove the FEL energy conservation relation that was mentioned in slide 19 of the 1D theory notes. Correcting the typo involving the factor of 2, this is

$$\frac{d}{dz}\left[\int d\theta(2\varepsilon_0|E(\theta,z)|^2)+\sum_j n_e\gamma_r(1+\eta_j)mc^2\right]=0$$

Hint: apart from the Maxwell relation mentioned in slide 19 and the energy exchange equation $d\eta_j/dz=\chi_1(E_je^{i\theta_j}+E_j^*e^{-i\theta_j})$, it is also useful to take into account the fact that $\int d\theta f(\theta)\left\langle e^{i\theta_j}\right\rangle_\Delta=\sum_j f_je^{i\theta_j}$, where f_j is the value of the function f at the location of the jth electron.

2 1D SASE parameters

Using the SASE quantitative analysis presented in slides 27-28 of the 1D theory notes, explicitly derive the results of slide 30 for the case of zero energy spread. In particular, find the dispersion relation and the expressions for g_A, g_S and σ_ν. Note that you can use the stated expansion of μ in Taylor series without actually proving it (including a proof will be considered extra credit).