Lecture B2: Superconductive RF fundamental

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Reference books

Hasan Padamsee, Jens Knobloch, and Tom Hays WILEY-VCH

RF Superconductivity for Accelerators

Second Edition







History of Superconductivity 1



Superconductive materials

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Ce	Pr	Nď	Pm	Sm	Eu	Gd	Тb	Dy	Но	Er	Tm	Yb	Lu
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr



SC materials in normal crystal condition



SC materials only in special conditions like high pressure, amorphous, etc.

Cu Materials for which SC phase has not been found yet.

Compound materials have also SC phase. → High-temperature superconductivity materials.



Non superconductive materials

- Even if they are metal, alkaline metals and good metals with high conductivity are non SC materials.
- Even if they are metal, transition elements and Rare Earth Elements (REE) with magnetization are non SC materials.

History of Superconductivity 2 (1/2)

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- 1908: H. K. Onnes and Van Der Waals liquefied He for the first time in Leiden.
- 1911: H. K. Onnes discovered superconductivity with Hg at 4.1 K in Leiden.
- 1914: H. K. Onnes discovered persistent current in loop superconductor.
- 1933: F. W. Meissner discovered Meissner effect.
- 1935: F. W. London and H. London established London equations which explained Meissner effect.
- 1935 37: L. V. Schvinikov, De Haas and Casimir-Jonker discovered two Hc's. Later this is called Hc1 and Hc2 of type-II superconductor.

History of Superconductivity 2 (2/2)

- 1950: V. L. Ginzburg and L. D. Landau established GL theory, introducing thermo-dynamical states in superconductor.
- 1952: A. Abrikosov predicted Type-II superconductor.
- 1953: Pippard introduced Pippard coherent length.
- 1953: SC phase of Nb3Sn was found with Tc = 17 K (at rather high temperature).
- 1957: J. Bardeen, L. N. Cooper, and J. R. Schrieffer established BCS theory (microscopic understanding of superconductivity).
- 1961: Nb3Sn was confirmed to be Type-II superconductor.
- 1980's: High-temperature SC materials were found.

Superconductivity in DC



Persistent DC current

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Decay time of DC superconductive current > 10⁵ years

The perfect conductivity is the first hallmark of superconductivity. But the superconductivity is not identical to the perfect conductivity.

Perfect conductivity

Tc = Critical temperature (a) H = 0H >0 H -> 0 T<T c T<T c T<T c (b) H > 0 H -> 0 H >0 T>T c T<T c T<T c

Figure 4.3: (a) Screening of external magnetic field by a perfect conductor. (b) Flux trapping in a perfect conductor.

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Superconductivity

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Tc = Critical temperature





Figure 4.4: (a) Screening of external magnetic field by a superconductor. (b) Meissner effect (flux expulsion) in a superconductor.

Superconductivity

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Superconductivity is the thermodynamic state.

If you give the thermodynamic parameters: T and H, the state is defined exactly.







Comparison of thermodynamic quantities in superconducting and normal states. $U_{en}(0)$ is chosen as the zero of ordinates in (c) and (d). Because the transition is of second order, the quantities S, U, and F are continuous at T_c . Moreover, the slope of F_{es} joins continuously to that of F_{en} at T_c , since $\partial F/\partial T = -S$.

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Type-I and Type-II superconductors

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H = External magnetic field

Empirical expression of Hc(T)



It was found empirically that Hc(T) is quite well approximated by a parabolic law.





Cryostat



V = V1 + V2

V1 = $-(d\phi / dt) = -(d NBS / dt) = -NS (dB / dt)$ = -NS (d µo H / dt) = -µo NS (dH / dt)



V2 = $(d\phi / dt) = (d NBS / dt) = NS (dB / dt)$ = NS { d {µo H + µo M } / dt } = µo NS (dH / dt) + µo NS (dM / dt)



V = V1 + V2 = - μο NS (dH / dt) + μο NS (dH / dt) + μο NS (dM / dt) = μο NS (dM / dt) External H is canceled out.

 $\int V dt = \mu o NS \int (dM / dt) dt = \mu o NS (M(t) - M(0))$ $= \mu o NS M(t) (because M(0) = 0)$

Pickup coil and sample x-sec parameters:

N = 250 turns S = 4.65 mm x 2.8 mm = 13.02 x 10**-6 m**2 NS = 250 x 13.02 x 10**-6 = 3.23 x 10**-3 m**2

External B parameters : External Coil Current 1 A ⇔ B = 653.7 gauss = 0.06537 Wb/m**2 Ramping rate of B : 0.1 A / sec ⇔ 6.537 x 10**-3 Wb/m**2 / sec dB / dt = 6.537 x 10**-3 Wb/m**2 / sec

V1 =- NS (dB / dt) =
$$3.23 \times 10^{**}-3 \times 6.537 \times 10^{**}-3$$

= $21 \times 10^{**}-6$ Volt

Output voltage from the pickup coil is about 20 micro-volt.

Hc(T) measurement if Type-I













Proposed a 2-fluid model with a normal fluid and superfluid components

 n_s : density of the superfluid component of velocity v_s n_n : density of the normal component of velocity v_n

$$m\frac{\partial \vec{\upsilon}}{\partial t} = -e\vec{E}$$
 superelectrons are accelerated by E
$$\vec{J}_s = -en_s \vec{\upsilon}$$

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E} \qquad \text{superelectrons}$$

 $\vec{J}_n = \sigma_n \vec{E}$ normal electrons

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$

Maxwell: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow \frac{\partial}{\partial t} \left(\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} \right) = 0 \qquad \Rightarrow \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = \text{Constant}$$

F&H London postulated:

$$\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = 0$$

combine with
$$\overline{\nabla} \times \overline{B} = \mu_0 \overline{J}_s$$

$$\overline{\nabla^2 \overline{B}} - \frac{\mu_0 n_s e^2}{m} \overline{B} = 0$$

$$B(x) = B_A \exp\left[-x/\lambda_L\right]$$

$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$
London penetration depth

The magnetic field, and the current, decay exponentially over a distance λ (a few 10s of nm)

Penetration depth in thin film



$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$

From Gorter and Casimir two-fluid model

$$n_s \propto \left[1 - \left(\frac{T}{T_C}\right)^4\right]$$





Fig. 21. Penetration depth as a function of temperature. (After Shoenberg, Nature, 43, 433, 1939.)

London Equation:
$$\lambda^2 \nabla \times \vec{J}_s = -\frac{\vec{B}}{\mu_0} = -\vec{H}$$

 $\nabla \times \vec{A} = \vec{H}$
choose $\nabla \cdot \vec{A} = 0$, $A_n = 0$ on sample surface (London gauge)

$$\vec{J}_s = -\frac{1}{\lambda^2} \vec{A}$$

Note: Local relationship between \vec{J}_s and \vec{A}

Pippard coherent length

Pippard introduced the concept of coherent length for the first time.

Analogy to the chambers's nonlocal generation of Ohm's law : $J(\mathbf{r}) = \sigma \mathbf{E}(\mathbf{r})$

$$\mathbf{J}(\mathbf{r}) = \frac{3\sigma}{4\pi\ell} \int \frac{\mathbf{R}[\mathbf{R} \cdot \mathbf{E}(\mathbf{r}')]e^{-R/\ell}}{R^4} d\mathbf{r}' \qquad \text{where } \mathbf{R} = \mathbf{r} - \mathbf{r}'$$

throughout a volume of radius I (mean free path) about r

Pippard extended
$$\mathbf{J}_s = n_s e \langle \mathbf{v}_s \rangle = \frac{-n_s e^2 \mathbf{A}}{mc} = \frac{-\mathbf{A}}{\Lambda c}$$

To
$$\mathbf{J}_{s}(\mathbf{r}) = -\frac{3}{4\pi\xi_{0}\Lambda c} \int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}(\mathbf{r}')]}{\mathbf{R}^{4}} e^{-R/\xi} d\mathbf{r}'$$
 where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$
Coherent length $\xi_{0} = a \frac{\hbar v_{F}}{kT_{c}}$ $\frac{1}{\xi} = \frac{1}{\xi_{0}} + \frac{1}{\ell}$

If choosing a = 0.15, Pippard could fit the experimental data on both tin and aluminum. This form was later confirmed by BCS theory.

- Ginzburg-Landau theory is a particular case of Landau's theory of second order phase transition
- Formulated in 1950, before BCS
- Masterpiece of physical intuition
- Grounded in thermodynamics
- Even after BCS it still is very fruitful in analyzing the behavior of superconductors and is still one of the most widely used theory of superconductivity

- Theory of second order phase transition is based on an order parameter which is zero above the transition temperature and non-zero below
- For superconductors, GL use a complex order parameter Ψ(r) such that |Ψ(r)|² represents the density of superelectrons
- The Ginzburg-Landau theory is valid close to T_c

- Assume that Ψ(r) is small and varies slowly in space
- Expand the free energy in powers of $\Psi(\textbf{r})$ and its derivative

$$f = f_{n0} + \alpha |\psi|^{2} + \frac{\beta}{2} |\psi|^{4} + \frac{1}{2m^{*}} \left| \left(\frac{\hbar}{i} \nabla - \frac{e^{*}}{c} \mathbf{A} \right) \psi \right|^{2} + \frac{\hbar^{2}}{8\pi}$$

$$\boxed{\mathbf{Derivative in}}$$

$$\frac{h = \text{external}}{\text{magnetic field}}$$



London penetration depth: length over which magnetic field decay

$$\lambda_{L}(T) = \left(\frac{m^{*}\beta}{2e^{2}\alpha'}\right)^{1/2} \sqrt{\frac{T_{c}}{T_{c}-T}}$$

Coherence length: scale of spatial variation of the order parameter (superconducting electron density)

$$\xi(T) = \left(\frac{\hbar^2}{4m^*\alpha'}\right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$

The critical field is directly related to those 2 parameters

$$H_c(T) = \frac{\phi_0}{2\sqrt{2}\,\xi(T)\,\lambda_L(T)}$$

$$\phi_0 \equiv \frac{h}{2e} = 2.07 \times 10^{-15} \text{Wb}$$

 $\kappa = \frac{\lambda}{\epsilon}$

The ratio of the two characteristic lengths defines the GL parameter:



$$\sigma \simeq \frac{1}{8\pi} \Big[H_c^2 \xi - H^2 \lambda \Big]$$

 $\frac{H^2\lambda}{8\pi}$ $\frac{H_c^2\xi}{8\pi}$

Energy that can be gained by letting the fields penetrate Energy lost by "damaging" superconductor

Positive surface energy for a Type I superconductor



Negative surface energy for a Type I superconductor



Interface is stable if σ >0

If
$$\xi >> \lambda$$
 $\sigma > 0$

Superconducting up to H_c where superconductivity is destroyed globally

If
$$\lambda >> \xi$$
 $\sigma < 0$ for $H^2 > H_c^2 \frac{\xi}{\lambda}$

Advantageous to create small areas of normal state with large area to volume ratio \rightarrow quantized fluxoids vortex

More exact calculation (from Ginzburg-Landau):

$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} \qquad : \text{Type I}$$
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} \qquad : \text{Type I}$$



FIGURE 5-2

Comparison of magnetization curves for three superconductors with the same value of thermodynamic critical field H_c , but different values of κ . For $\kappa < 1/\sqrt{2}$, the superconductor is of type I and exhibits a first-order transition at H_c . For $\kappa > 1/\sqrt{2}$, the superconductor is type II and shows second-order transitions at H_{c1} and H_{c2} (for clarity, marked only for the highest κ case). In all cases, the area under the curve is the condensation energy $H_c^2/8\pi$.

FIGURE 1-5

Comparison of flux penetration behavior of type I and type II superconductors with the same thermodynamic critical field H_c . $H_{c2} = \sqrt{2} \kappa H_c$. The ratio of B/H_{c2} from this plot also gives the approximate variation of R/R_n , where R is the electrical resistance for the case of negligible pinning, and R_n is the normal-state resistance.

 H_{c2}





Vortex lines in Pb_{.98}ln_{.02}

At the center of each vortex is a normal region of flux h/2e

The energy is low to make vortex lines which have the boundary of SC / NC.

Structure of a vortex



- (a) SC current surrounding quantized magnetic flux (= h/2e)
- (b) Distribution of rrder parameter, or the density of superconducting electrons, surrounding a vortex.
- (c) Distribution of magnetic field surrounding a vortex.

Even though it is more energetically favorable for a type I superconductor to revert to the normal state at H_c , the surface energy is still positive up to a superheating field H_{sh} > H_c \rightarrow metastable superheating region in which the material may remain superconducting for short times.

Type I H_c Thermodynamic critical field $H_{sh} \simeq -\frac{H_c}{\sqrt{\kappa}}$ Superheating critical field

Type II H_c Thermodynamic critical field $H_{c2} = \sqrt{2} \kappa H_c$ $H_{c1} \simeq \frac{H_c^2}{H_{c2}}$ $\simeq \frac{1}{2\kappa} (\ln \kappa + .008) H_c$ (for $\kappa \gg 1$)



Figure 3-1 Phase diagram for a long cylinder of a Type II superconductor.

2,5

Ginsburg-Landau: 2.0 $H_{sh} \sim \frac{0.9H_c}{\sqrt{\kappa}}$ for *k* << 1 1,5 1.2 H_c for $\kappa \sim 1$ H He $0.75 H_c$ for $\kappa >> 1$ 0,5 The exact nature of the rf critical field of superconductors is still

an open question

Niobium H_{sh} $\frac{H_{C2}}{H_c} = \sqrt{2} \pi$ NORMAL STATE Mixed State Data for / lead Superheated State Meissner State 2.0 1.2 (.6 0.8 0,4 [≡ <u>}</u>] GL Parameter K

Figure 5.9: Phase diagram showing the Meissner, normal, mixed, and superheated states. The normalized critical fields H_{c1} , H_{c} , H_{c2} and H_{sh} are shown as functions of the GL parameter. The range of data for the rf critical field of lead is also shown.

Material parameter for various superconductors

Superconductor	$\lambda_L(0)$ (nm)	ξ_{0} (nm)	κ	$2\Delta(0)/kT_c$	$T_c(\mathbf{K})$
Al	16	1500	0.011	3.40	1.18
In	25	400	0.062	3.50	3.3
Sn	28	300	0.093	3.55	3.7
Pb	28	110	0.255	4.10	7.2
Nb	32	39	0.82	3.5-3.85	8.95-9.2
Та	35	93	0.38	3.55	4.46
Nb ₃ Sn	50	6	8.3	4.4	18
NbN	50	6	8.3	4.3	≤ 17
Yba ₂ Cu ₃ o _x	140	1.5	93	4.5	90



- What needed to be explained and what were the clues?
 - Energy gap (exponential dependence of specific heat)
 - Isotope effect (the lattice is involved)
 - Meissner effect





Figure 26: The critical temperature of various tin isotopes.

Interaction of two electrons via phonon

Assumption: Phonon-mediated attraction between electron of equal and opposite momenta located within $\hbar\omega_{_{D}}$ of Fermi surface

Moving electron distorts lattice and leaves behind a trail of positive charge that attracts another electron

Fermi ground state is unstable

moving in opposite direction

Electron pairs can form bound states of lower energy

Bose condensation of overlapping Cooper pairs into a coherent Superconducting state













Figure 22: Cooper pairs and single electrons in the crystal lattice of a superconductor. (After Essmann and Träuble [12]).



Figure 23: Various Cooper pairs $(\vec{p}, -\vec{p}), (\vec{p}', -\vec{p}'), (\vec{p}'', -\vec{p}''), \dots$ in momentum space.

The size of the Cooper pairs is much larger than their spacing They form a coherent state





Figure 3.7: The change in the density of states that accompanies the superconducting state, showing the gap in the energy levels.



- The BCS model is an extremely simplified model of reality
 - The Coulomb interaction between single electrons is ignored
 - Only the term representing the scattering of pairs is retained
 - The interaction term is assumed to be constant over a thin layer at the Fermi surface and 0 everywhere else
 - The Fermi surface is assumed to be spherical
- Nevertheless, the BCS results (which include only a very few adjustable parameters) are amazingly close to the real world



T/Te

At finite temperature:

Implicit equation for the temperature dependence of the gap:

$$\frac{1}{V\rho(0)} = \int_0^{\hbar\omega_D} \frac{\tanh\left[\left(\varepsilon^2 + \Delta^2\right)^{1/2} / 2kT\right]}{\left(\varepsilon^2 + \Delta^2\right)^{1/2}} d\varepsilon$$



Variation of the order parameter Δ with temperature in the BCS approximation.



Specific heat



Fig. 22. Reduced electronic specific heat in superconducting vanadium and tin. [From Biondi et al., (150).]

$$H_0\phi + H_{ex} \phi = i\hbar \frac{\partial \phi}{\partial t}$$

$$H_{ex} = \frac{e}{mc} \sum A(r_i, t) p_i$$

 H_{ex} is treated as a small perturbation

$$H_{\it rf} << H_c$$

There is, at present, no model for superconducting surface resistance at high rf field

$$J \propto \int \frac{R[R \cdot A]I(\omega, R, T)e^{-\frac{R}{l}}}{R^4} dr$$
$$J(k) = -\frac{c}{4\pi}K(k)A(k)$$
$$K(0) \neq 0: \text{ Meissner effect}$$

similar to Pippard's model



Represented accurately by $\lambda \sim \frac{1}{\left(\frac{T_{c}}{2}\right)^{4}}$ near T_{c} 2000 1200 1000 800 $\frac{d\lambda}{dy}$ BCS THEORY 600 40.4 0 U o 400 200 0 2.0 2.5 3.0 3.5 1.0 1.5 4.0 4.5 5.0 $y = \frac{1}{\sqrt{1 + 4}}$

Fig. 30. Temperature dependence of $d\lambda/dy$ for tin obtained by Schawlow and Devlin (207) compared with the theoretical curve obtained from the BCS theory.





Fig. 1. Measured values of the surface resistance ratio r of superconducting aluminum as a function of the reduced temperature t at several representative wavelengths. The wavelengths and corresponding photon energies are indicated on the curves [After Biondi and Garfunkel (15).]



Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and Nb₃Sn as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

Temperature dependence

-close to T_c :

dominated by change in $\lambda(t)$

$$\frac{t^4}{\left(1-t^2\right)^{3/2}}$$

-for $T < \frac{T_c}{2}$:

dominated by density of excited states $\sim e^{-\frac{1}{kT}}$

$$R_s \sim \frac{A}{T} \omega^2 \exp\left(-\frac{\Delta}{\mathbf{kT}}\right)$$

Frequency dependence

 ω^2 is a good approximation



Fig. 5. The surface resistance of Nb at 4.2 K as a function of frequency [62,63]. Whereas the isotropic BCS surface resistance $(-\cdot - \cdot)$ resulted in $R \propto \omega^{1.8}$ around 1 GHz, the measurements fit better to $\omega^2 (---)$. The solid curve, which fits the data over the entire range, is a calculation based on the smearing of the BCS density-of-states singularity by the energy gap anisotropy in the presence of impurity scattering [61]. The authors thank G. Müller for providing this figure.

A reasonable formula for the BCS surface resistance of niobium is

$$R_{BCS} = 9 \times 10^{-5} \frac{f^2 (\text{GHz})}{T} \exp\left(-1.83 \frac{T_c}{T}\right)$$





Fig. 2. Temperature dependence of surface resistance of niobium at 3.7 GHz measured in the TE_{011} mode at $H_{rf} \simeq 10$ G. The values computed with the BCS theory used the following material parameters:

$$\begin{split} T_c &= 9.25 \text{ K}; \qquad \lambda_L(T=0, \, l=\infty) = 320 \text{ Å}; \\ \Delta(0)/k \, T &= 1.85; \quad \xi_F(T=0, \, l=\infty) = 620 \text{ Å}; \quad l=1\,000 \text{ Å or } 80 \text{ Å}. \end{split}$$

At around 4 – 3 K, the total resistance of SRF Nb cavity is dominated by BCS resistance. The fitting of these data gives the band-gap width of Nb material.

The total resistance of SRF Nb cavity at around 2 K is not dominated by the BCS resistance. It is dominated by the residual resistance which is caused by impurity of Nb material and magnetic flux trapping.

$$R_s \sim \frac{A}{T} \omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

In the dirty limit $l \ll \xi_0 \qquad R_{BCS} \propto l^{-1/2}$

In the clean limit $l \gg \xi_0$ $R_{BCS} \propto l$

R_{res}:

Residual surface resistance

No clear temperature dependence

No clear frequency dependence

Depends on trapped flux, impurities, grain boundaries, ...

Surface Resistance - Nb - 1500 MHz



BCS surface resistance [n Ohm]



- The surface resistance of superconductors depends on the frequency, the temperature, and a few material parameters
 - Transition temperature
 - Energy gap
 - Coherence length
 - Penetration depth
 - Mean free path
- A good approximation for T<T_c/2 and $\omega{<<}\Delta/h$ is

$$R_s \sim \frac{A}{T} \omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$



- Normal Conductors
 - Skin depth proportional to $\omega^{-1/2}$
 - Surface resistance proportional to $\omega^{1/2} \rightarrow ^{2/3}$
 - Surface resistance independent of temperature (at low T)
 - For Cu at 300K and 1 GHz, $R_s = 8.3 \ m\Omega$
- Superconductors
 - Penetration depth independent of ω
 - Surface resistance proportional to ω^2
 - Surface resistance strongly dependent of temperature
 - For Nb at 2 K and 1 GHz, R_s≈7 nΩ