Exercises

1. We saw that the LLRF system reduces the station impedance experienced by the beam. Let's estimate the instability growth rates in the absence of feedback ("open loop") and in the presence of feedback ("closed loop").

   Two files have been uploaded. They include the LHC open and closed loop impedance evaluated at the positive ("imp") and negative ('impm") synchrotron sidebands, for modes -40 to 40, corresponding to frequencies "freq" and "freqm" respectively. Using the formulas from the lecture, estimate the most unstable mode for the LHC in open and closed loop. The constant depending on accelerator parameters scaling the impedance is equal to 9,300 for the LHC. $f_{RF} = 400.8$ MHz and $f_{rev} = 11245$.

2. What gain would get you from the open to the closed loop case? Remember, the closed loop transfer function is $GH/(1-GH)$ where G is the feedback gain and H the open loop response. You will not be able to get the closed loop exactly since the actual feedback is much more complicated that just a numerical gain, but you should be able to get an approximate result.

3. The bunch length growth rate due to RF noise is approximately equal to

$$\frac{d\sigma_\phi}{dt} = \frac{\omega_s^2}{4\pi\sigma_\phi} S_\phi(f_s)$$

   where $\sigma_\phi \approx 0.6$ rad is the bunch length, $\omega_s/(2\pi) \approx 25$ Hz the synchrotron frequency and $S_\phi(f)$ the noise power spectral density.

   - What is the maximum power spectral density (PSD) to achieve a bunch length growth of 5% per hour?
   - What total noise power does this PSD corresponds to, if the noise extends from 0 to 11245 Hz?

4. Assume we have one first order system which is controlled by a proportional controller (feedback gain is K), the system open loop transfer function is

$$G(s) = \frac{1360}{s + 1360}$$

   - What are the conditions for K to guarantee stability of the closed loop system.
   - If the loop delay is 1 μs, estimate the maximum gain the system can achieve.
   - If we want to reduce the disturbance of the system by 100 times, estimate the limit to the loop delay.

5. Consider only the π mode of the cavity, assume the cavity loaded quality factor is $Q_L = 3\times10^6$, the resonance frequency is $f_0 = 1.3$ GHz, and the shunt impedance is $r/Q = 1036\Omega$. The driving RF power is 200 kW.

    - Calculate the half-bandwidth $\omega_{1/2}$ and loaded resistance R_L of the cavity.
    - What is the maximum cavity voltage in steady state?
    - If the cavity is detuned by 200 Hz, what is the cavity voltage in steady state? What is the phase difference between the cavity input and output signal?
    - Given a beam current of 8mA (average DC current) with beam phase 0 (on-crest), determine the beam injection time into the cavity (detuning = 0) to achieve a flattop.
    - Calculate the beam induced voltage in the case above.
    - If there is no detuning, and we want to fill the cavity voltage to 25 MV in 500 μs, how much RF power is required? At 500 μs, we inject a beam of 5mA (DC current), beam phase is 0, how much RF power is needed to maintain this 25 MV flattop. How much power is reflected?

6. If we want the cavity flattop voltage to be 25MV, beam current 8mA, beam phase 30°, find out a group of optimized parameters to minimize the klystron power needed.

    - Find an optimized detuning.
    - Find an optimized loaded quality factor.
    - Find the minimum klystron power needed? How much reflection power now?