Lecture B2: Superconductive RF

RF Test of Cavity Homework

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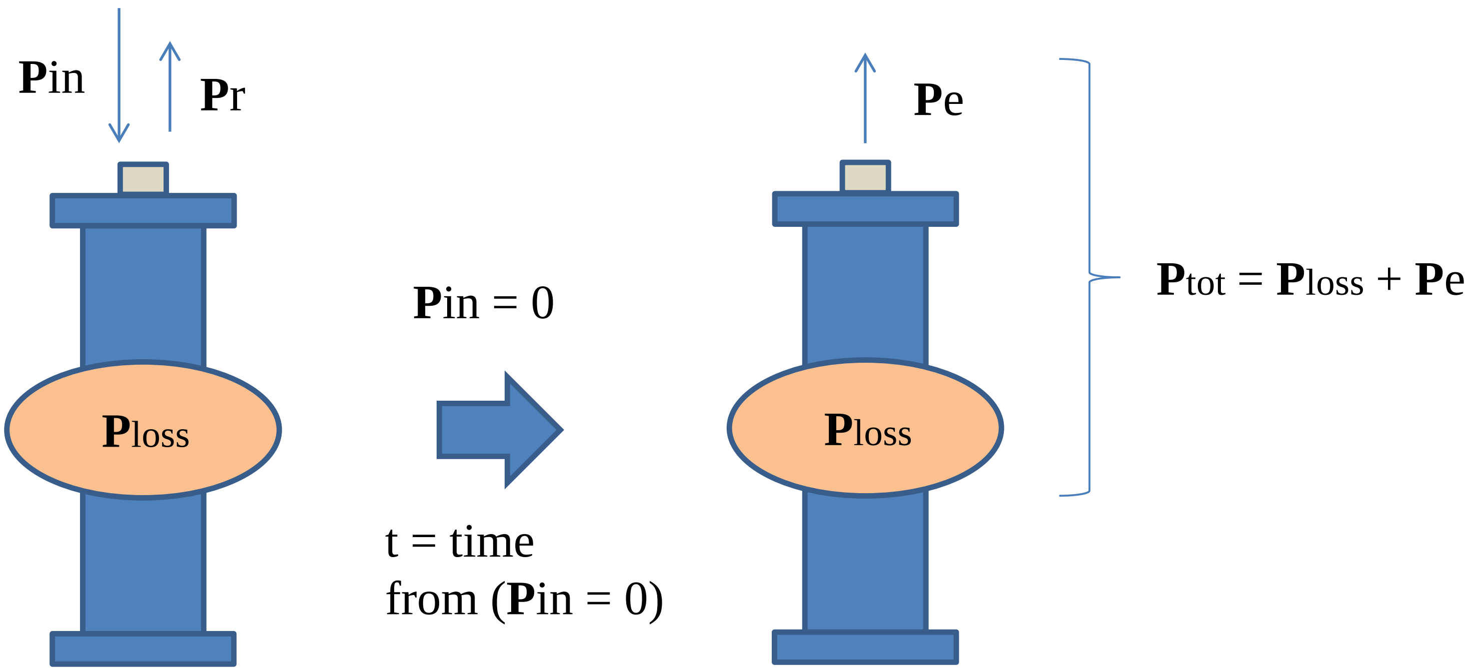
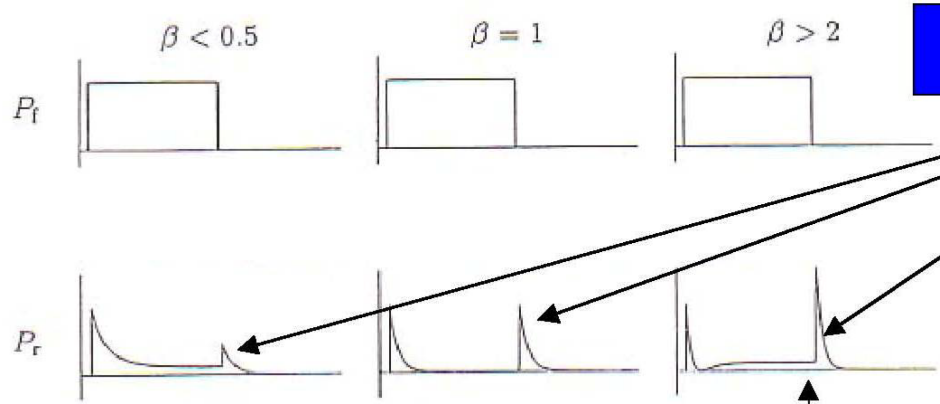
Solve this problem and confirm following pages by yourself.



U (t) = Stored energy in the cavity, $\mathbf{Q}_L$ = Loaded **Q**, **ω** = Frequency.
Think the meaning of following equation.
And find **U**(t) by resolving the equation with the initial condition: **U**(0) = $\mathbf{U}_0$.

$$\frac{d\mathbf{U}(t)}{dt} = -\,\mathbf{P}_{tot} = -\,\frac{\boldsymbol{\omega}\mathbf{U}(t)}{\mathbf{Q}_L}$$

Theory of Measurement



Pulse method

$$P_t(t) = P_o \exp\left(-\frac{\omega}{Q_L} t\right)$$

$$\omega = 2\pi f, \quad Q_L : \text{Loaded Q}$$

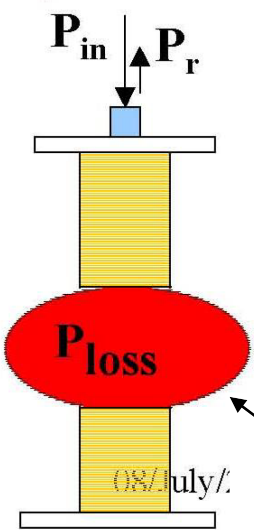
One-port

Measure the period during when P becomes half .

Decatime : $\tau_{1/2}$

$$P_t(\tau_{1/2}) = \frac{1}{2} P_o = P_o \exp\left(-\frac{\omega}{Q_L} \cdot \tau_{1/2}\right)$$

$$\ln(2) = \frac{2\pi f}{Q_L}$$



$$Q_L = 2\pi f \cdot \frac{\tau_{1/2}}{\ln(2)}$$

Ploss = heat caused by RF field in the cavity.

One-Port Cavity

$$Q_O \equiv \frac{\omega U}{P_{loss}},$$

Ploss = consumption on cavity wall

Pe = Power that goes out from port when Pin=0

$$Q_L \equiv \frac{\omega U}{P_{loss} + P_e} = \frac{\omega U}{P_{loss}\left(1 + \frac{P_e}{P_{loss}}\right)}$$

(for one port)

(When **P**in = 0)

$$= \frac{Q_O}{(1+\beta_{in})}$$

$$Q_O = (1+\beta_{in}) \cdot Q_L$$

$\tau_{1/2}$, P_{in}, P_r — Measured values

Calculation Q_L, β_{in}

Calculation of Equivalent Circuit

The wave shape of Pr

$$\beta_{in} \equiv \frac{1 \pm \sqrt{P_r / P_{in}}}{1 \mp \sqrt{P_r / P_{in}}}$$

(over > 1 / under < 1)

(When **P**in > 0)

$$\beta_{in} \equiv \frac{P_e}{P_{loss}}$$

(When **P**in = 0)

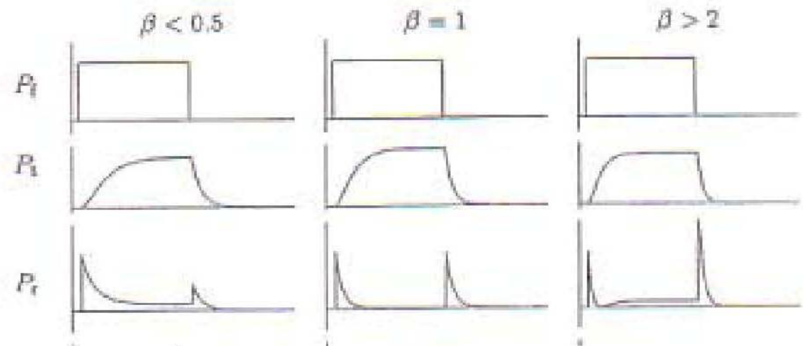
Calculation Q_O

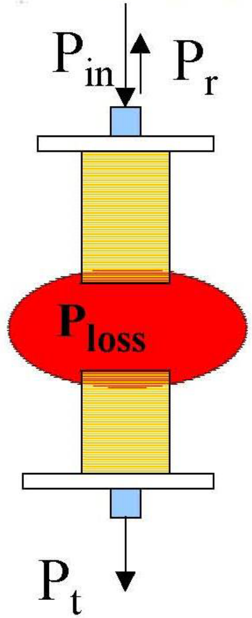
$$R_s = \frac{\Gamma}{Q_o}$$

Geom. factor

Surface resistance

Two-Port Cavity





$$P_{loss}^{*} = P_{loss} + P_t$$ Ploss of one-port

$$Q_o^{*} = \frac{\omega U}{P_{loss}^{*}} = \frac{\omega U}{P_{loss} + P_t}$$

$$= \frac{\omega U}{P_{loss}\left(1 + \frac{P_t}{P_{loss}}\right)}$$

$$= \frac{Q_o}{(1+\beta_t)} \quad \because \ \beta_t \equiv \frac{P_t}{P_{loss}}$$

$$= (1+\beta_{in}^{*})Q_L$$

Measured value

Measured value

When **P**in = any.

$$\beta_{in}^{*} = \frac{1 \pm \sqrt{P_r / P_{in}}}{1 \mp \sqrt{P_r / P_{in}}}$$ (over > 1 / under < 1)

(When **P**in > 0)

$$\beta_{in}^{*} \equiv \frac{P_e}{P_{loss}^{*}}$$ (When **P**in = 0)

$Q_L = \omega U / (P_{loss} + Pe + Pt)$
$= Q_0 / (1+\beta in + \beta t)$ when $Pin = 0$

$$Q_o^* = \frac{Q_o}{(1+\beta_t)} = (1+\beta_{in}^*)\cdot Q_L$$

$$Q_o = (1+\beta_{in}^*)\cdot(1+\beta_t)\cdot Q_L$$
$$= \left[1+(1+\beta_t)\cdot\beta_{in}^* + \beta_t\right]\cdot Q_L$$

$$Q_0 = (1+\beta_{in}+\beta_t)\cdot Q_L \Leftrightarrow \beta_{in} = (1+\beta_t)\cdot\beta_{in}^*$$

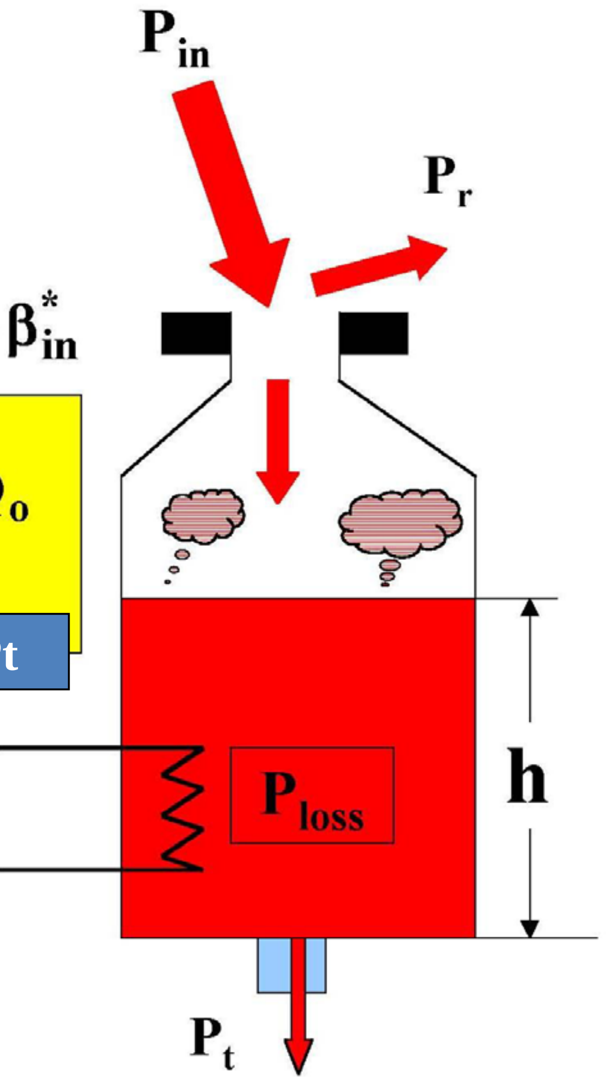
$$Q_o \equiv \frac{\omega U}{P_{loss}},\ Q_t \equiv \frac{\omega U}{P_t} = \frac{\omega U / P_{loss}}{P_t / P_{loss}} = \beta_t^{-1}\cdot Q_o$$
$$\omega U = Q_o \cdot P_{loss} = Q_t \cdot P_t$$

$Qt = Q_0 \cdot P_{loss} / Pt$

$$P_{loss} = P_{in} - P_r - P_t$$

Pin, Pr, Pt : Measured values

Stable Status : h=const $\Leftarrow$ U const



Calculation of Acceleration Gradient

$$R_{sh} = \frac{V^2}{P_{loss}} \qquad \because\ V = E_{acc} \cdot d_{eff}$$

Rsh = Shunt impedance

deff = Effective length of cavity along the beam axis.

$$= \left(E_{acc} \cdot d_{eff}\right)^2 \ / \ P_{loss}$$

$$E_{acc} = \frac{1}{d_{eff}} \cdot \sqrt{R_{sh} \cdot P_{loss}} = \frac{1}{d_{eff}} \cdot \sqrt{\left(R_{sh}/Q_O\right) \cdot \left(Q_O \cdot P_{loss}\right)}$$

$$= Z\sqrt{Q_O \cdot P_{loss}} \quad \text{(One-port)}$$

$$= Z\sqrt{Q_t \cdot P_t} \quad \text{(Two-port)}$$

Rsh /Qo = $V^2/\omega U$

Not dependent on material

$$\because Q_t = \frac{\omega U}{P_t} = \frac{Q_o \cdot P_{loss}}{P_t}, \quad Q_o \cdot P_{loss} = Q_t \cdot Pt$$

Set Qt >> Q_0 (Pt << Ploss).

Qt is constant during the measurement if using a fixed antenna.

Z is independent of surface resistance (material of cavity).

Summary:

Measured parameters:

$f, \tau_{1/2} \rightarrow$

$P_{in}, P_r, P_t \rightarrow$

Stable state (CW)

$$Q_L = 2\pi f \tau_{1/2} / \ln(2)$$

$$\beta_{in}^* = \frac{1 \pm \sqrt{\frac{P_R}{P_{IN}}}}{1 \mp \sqrt{\frac{P_R}{P_{IN}}}}$$

$$P_{loss} = P_{in} - P_r - P_t$$

$$\beta_t = P_t / P_{loss}$$

$$Q_t = Q_O / \beta_t$$

$$\beta_{in} = (1+\beta_t)\beta_{in}^*$$

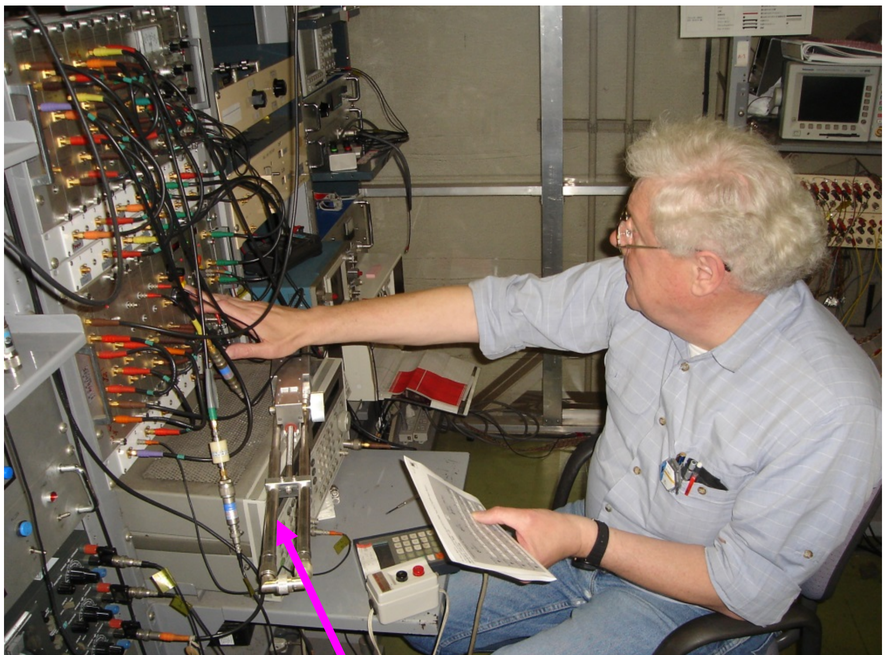
$$Q_O = (1+\beta_{in}+\beta_t)\, Q_L$$

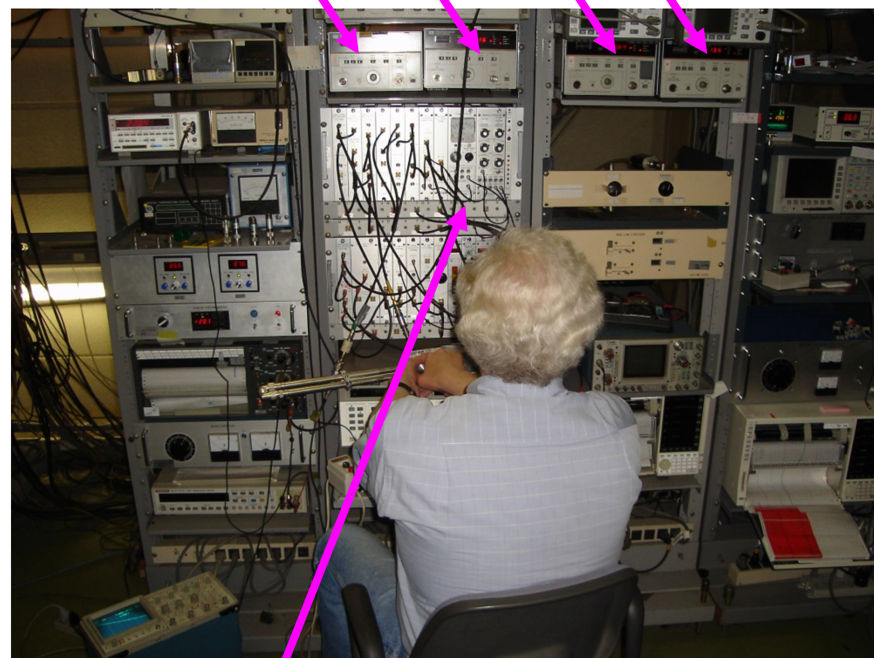
$$R_s = \frac{\Gamma}{Q_o}$$

$$Eacc = Z \cdot \sqrt{P_t \cdot Q_t}, \qquad Q_o = Q_t \beta_t$$

Control Room of Vertical Test (VT)

Power-meter (Pin, Pr, Pt)





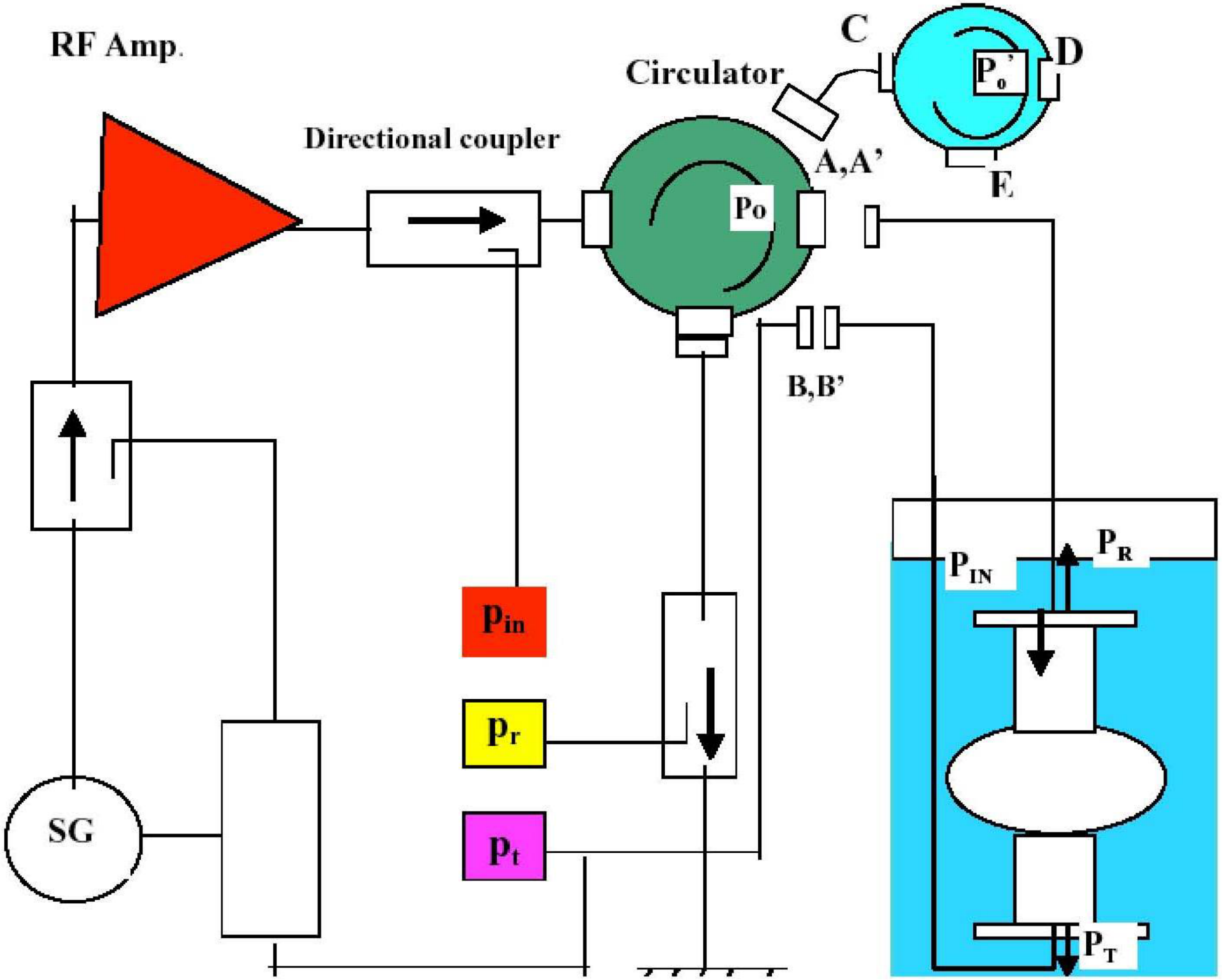
Signal Generator (SG)

Feed-back system (PLL)

Cable Correction

p_{in}, p_r, p_t : measured in the measurement room

P_{IN}, P_R, P_T : Power at the cavity (cooled), $P_{IN} = c_{in} \cdot p_{in}$, $P_R = c_r \cdot p_r$, $P_T = c_t \cdot p_t$



p_{in}, P_O at A,

p_r: short A

p_t : connect B to A

P_O/p_{in}, P_O/p_r, P_O/p_t

P_O' at E : connect A and C , and short D

p_{in}' at E : connect D to A'

p_t' at E : connect D to B'

$C_{in}=(P_O/p_{in}) \cdot (p_{in}'/P_O')^{1/2}$

$C_r=(P_O/p_r) \cdot (P_O'/p_{in}')^{1/2}$

$C_t=(P_O/p_t) \cdot (P_O'/p_t')^{1/2}$

$P_{IN} = c_{in} \cdot p_{in}$

$P_R = c_r \cdot p_r$

$P_T = c_t \cdot p_t$

Feed Back System