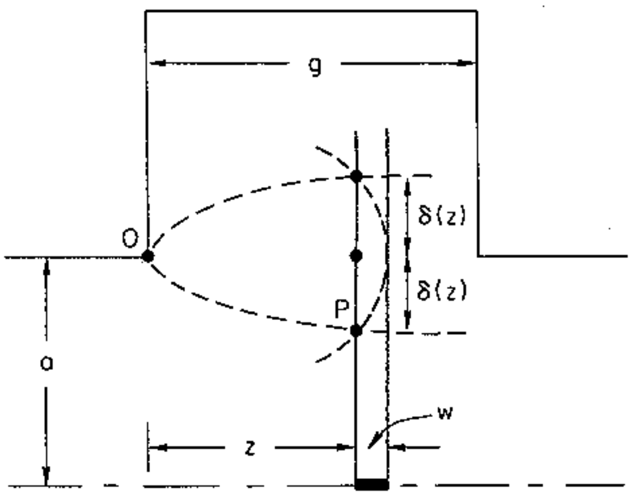
Extra slides

Short range wake critical ("catch-up") distance



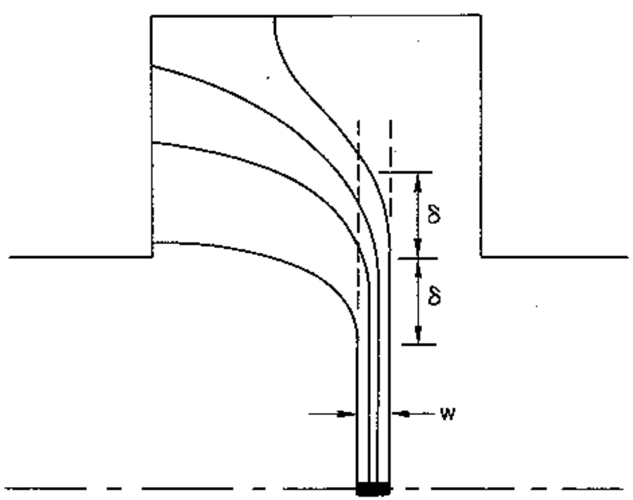
$$\sqrt{z^2+\delta^2}=\frac{z+w}{\beta}\quad \text{assuming}\quad w<<z;\quad \gamma^2>>\frac{z}{w}$$

then $\qquad \delta\approx\sqrt{2zw}$

In order to estimate critical number off cells:

assume $\quad \delta\approx a \quad$ then $\quad N_{crit}\approx\dfrac{a^2}{2Lw}$

To calculate the wake of infinitely long periodic structure at *w* we need more than N_{crit} cells



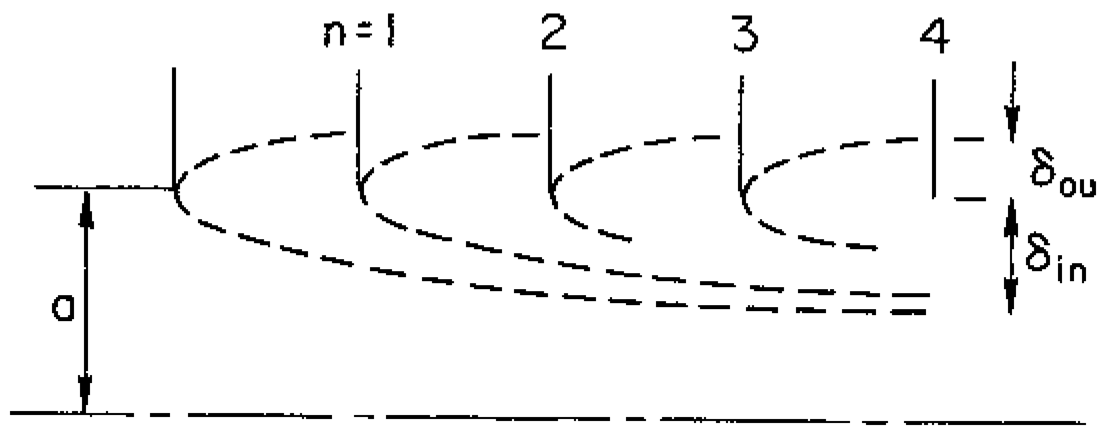


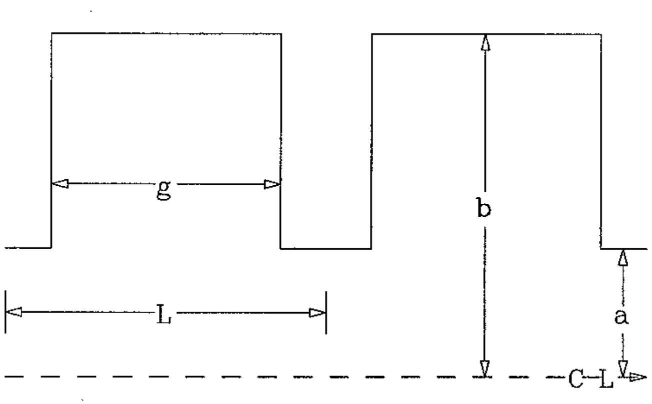
Image courtesy of R.B. Palmer, Particle Accelerators, vol.25, 1990

Validity of Karl Bane model: W_L

K.Bane et al., 'Calculation of the short-range longitudinal wakefields in the NLC linac', ICAP98, 1998

For const impedance periodic structure:

$$W_L = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_0}\right); \quad \text{where} \quad s_0 = 0.41 \frac{a^{1.8} g^{1.6}}{L^{2.4}} \qquad (1)$$



NLC structure at 1998 was tapered over 206 cells.
On the other hand, for this structure:
$N_{crit} = a_{max}^2/2L\sigma_z = 14 << N_c = 206$, what justifies averaging the wake of periodic structure over the 206 cells proposed by Karl Bane:

$$W_L^{TS} = \frac{1}{N_c}\sum_{n=1}^{N_c} W_L^{(n)}; \quad \text{where} \quad W_L^{(n)} = \frac{Z_0 c}{\pi a_n^2} \exp\left(-\sqrt{s/s_0^{(n)}}\right); \text{and} \quad s_0^{(n)} = 0.41 \frac{a_n^{1.8} g_n^{1.6}}{L_n^{2.4}} \qquad (2)$$

- However in CLIC for nominal σ_z = 44um , $N_{crit} = a_{max}^2/2L\sigma_z = 14 \approx N_c = 26$ in CLIC_G,
- Furthermore taking shorter bunch like for example in X-FELs: σ_z = 10 or 2 um N_{crit} => ~60 or ~300 which is bigger or much bigger than N_c. Eq. **(2) does not apply**

- In the extreme case, $s \to 0; \quad \gamma \to \infty; \quad W_L^{TS}(0) = \frac{Z_0 c}{\pi a_{min}^2} \neq \sum_{n=1}^{N_c} \frac{Z_0 c}{\pi a_n^2};$

Constant impedance travelling wave structure, length L

$$G(z) = G(0)e^{-\alpha z} - IR'(1 - e^{-\alpha z})$$

$$\alpha = \frac{\omega}{2Qv_g}$$

Note that: $G(\infty) = -IR'$ Ohms law!

$$\langle G \rangle = \frac{G(0)}{\alpha L}(1 - e^{-\alpha L}) + \frac{IR'}{\alpha L}(1 - \alpha L - e^{-\alpha L})$$

$$\eta = 2Y(1 - e^{-\alpha L}) + 2Y^2(1 - \alpha L - e^{-\alpha L})$$

Where: $Y = \frac{IR'}{G(0)}$