

# Constant Beam Size and Transverse Beam Phase Space Tomography

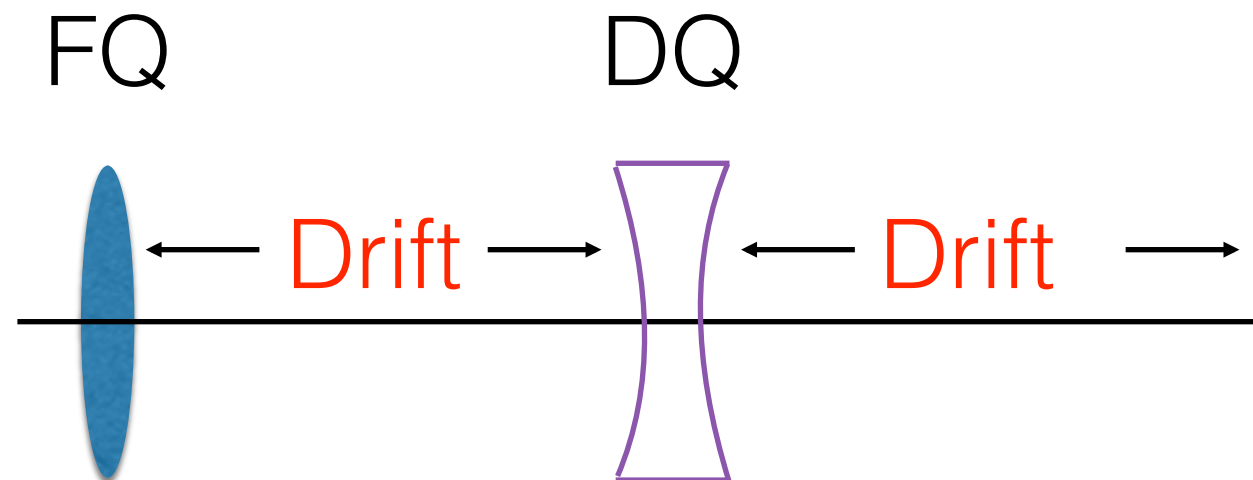
Lucy Martin

# Structure of programs

- Currently two separate programs in Matlab
- The first produces a beam of constant size and then conducts a quadrupole scan to find the Twiss parameters and emittance of the beam
- The second uses tomography to reconstruct the phase space at the start of a line and produce an image of this phase space
- The tomography reconstruction was then tested on old CTF3 quad scan data

# Constant beam size

- Aim is to create constant beam size to reduce effect of any aberrations in the optical lines used to acquire the image from the screen in the beam line.
- Beam should be large enough to be sensitive to changes in beam size, but small enough to avoid nonlinear aberrations
- A FODO transfer line was considered, using two quadrupoles



# Background theory

- The transfer matrix describing the transformation of the position and momentum phase space down a line can be constructed when details of the lattice are known

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M \begin{pmatrix} x \\ x' \end{pmatrix} \text{ where } M = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \quad M_{foc} = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}s) \\ -\sqrt{K} \sin(\sqrt{K}s) & \cos(\sqrt{K}s) \end{pmatrix}$$

$$M_{Total} = M_{Drift} \cdot M_{QD} \cdot M_{Drift} \cdot M_{QF}$$

- The equation of motion along the line is not constant, it changes with position. The general solution of Hill's equation for particle position:

$$x(s) = \sqrt{\epsilon} \sqrt{\beta(s)} \cos(\psi(s) + \phi)$$

- Definition of the Twiss parameters:

$$\sigma_x = \sqrt{\epsilon\beta_x}$$

$$\alpha(s) = -(1/2)\beta'(s) \quad \gamma(s) = \frac{1 + \alpha(s)^2}{\beta(s)}$$

- The transformation of the Twiss parameters down the transfer line can be described by:

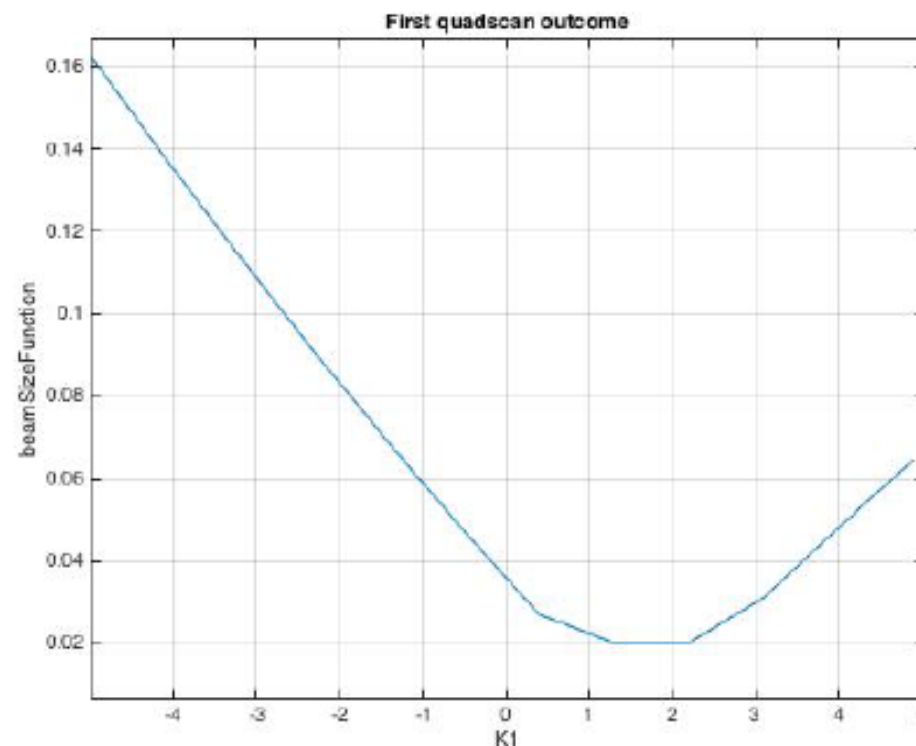
$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{s_1} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21}m_{12} & m_{21} + m_{22}m_{11} & -m_{12}m_{22} \\ m_{12}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{s_0}$$

- Using the first line of this transformation of beta down the same line with many different quadrupole current values can be described by:

$$\begin{pmatrix} \beta(0) \\ \beta(1) \\ \vdots \end{pmatrix}_{s_1} = \begin{pmatrix} m(0)_{11}^2 & -2m(0)_{11}m(0)_{12} & m(0)_{12}^2 \\ m(1)_{11}^2 & -2m(1)_{11}m(1)_{12} & m(1)_{12}^2 \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{s_0}$$

# Process

- 1, The varying current values to be set to the first quadrupole are given to the program, along with the desired beam size.
- 2, Initially a quadrupole scan is conducted, changing the currents through one of the quadrupoles and keeping the current to the second quadrupole constant. A parabola is produced, like the one below.
- 3, The size of the beam at the end of the line is found and recorded.





4, Transfer matrix for the lattice is constructed for each set of quadrupole values.

5, The matrix containing the first line of the transfer matrix for each set of quadrupole values is then constructed.

6, The program multiplies through by the pseudo inverse to give an estimate of the Twiss parameters and the emittance of the beam.

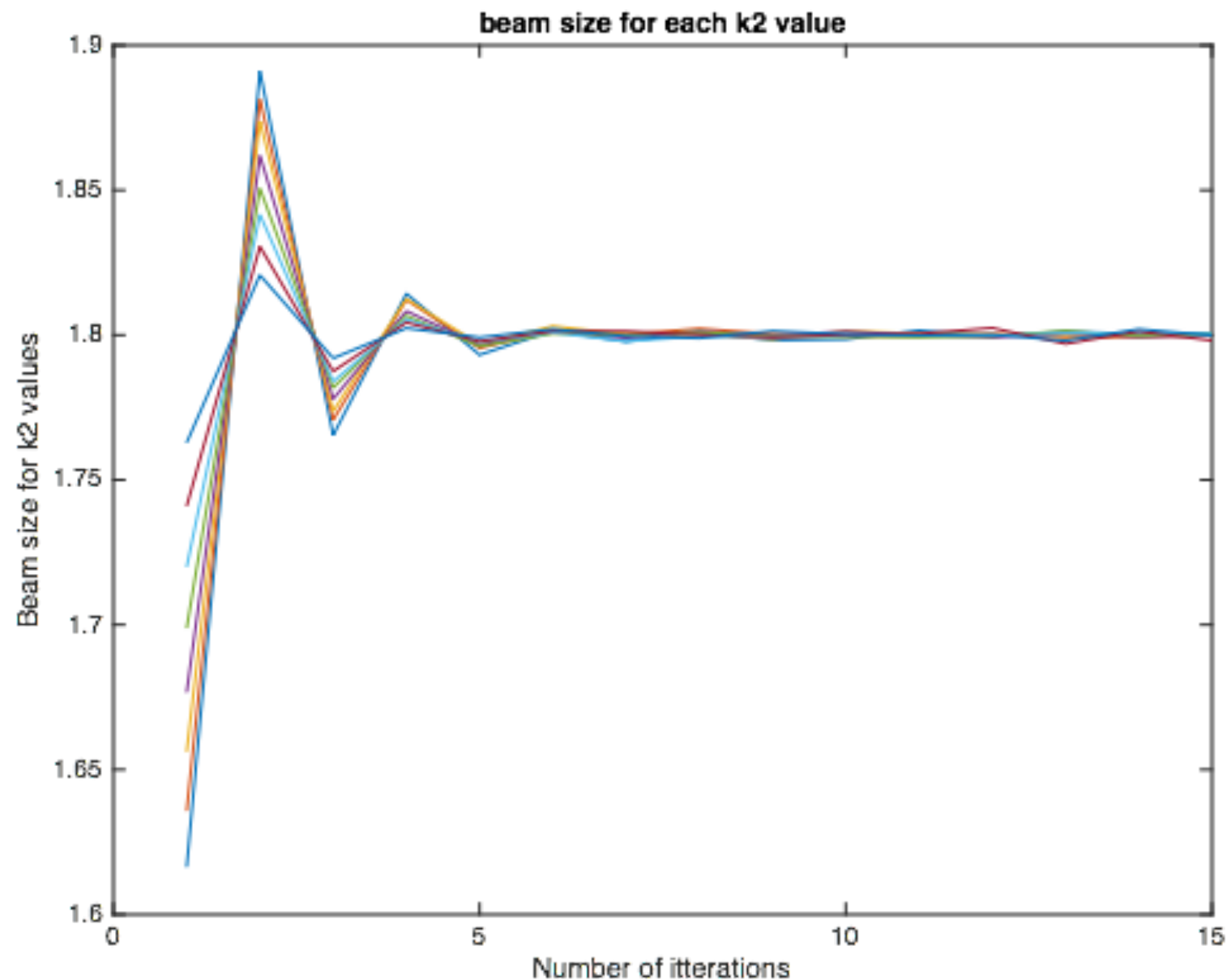
7, The desired beam size, the quadrupole currents for the first quadrupole and the current estimate of the Twiss parameters are inputted into the transfer equation which is solved numerically to give the current through the second quadrupoles needed to give this beam size.

8, The calculated values are then set to the second quadrupoles.

9, The size of the beam at the end of the line is found and recorded.

10, The beam size, quadrupole values and Twiss parameters are outputted.

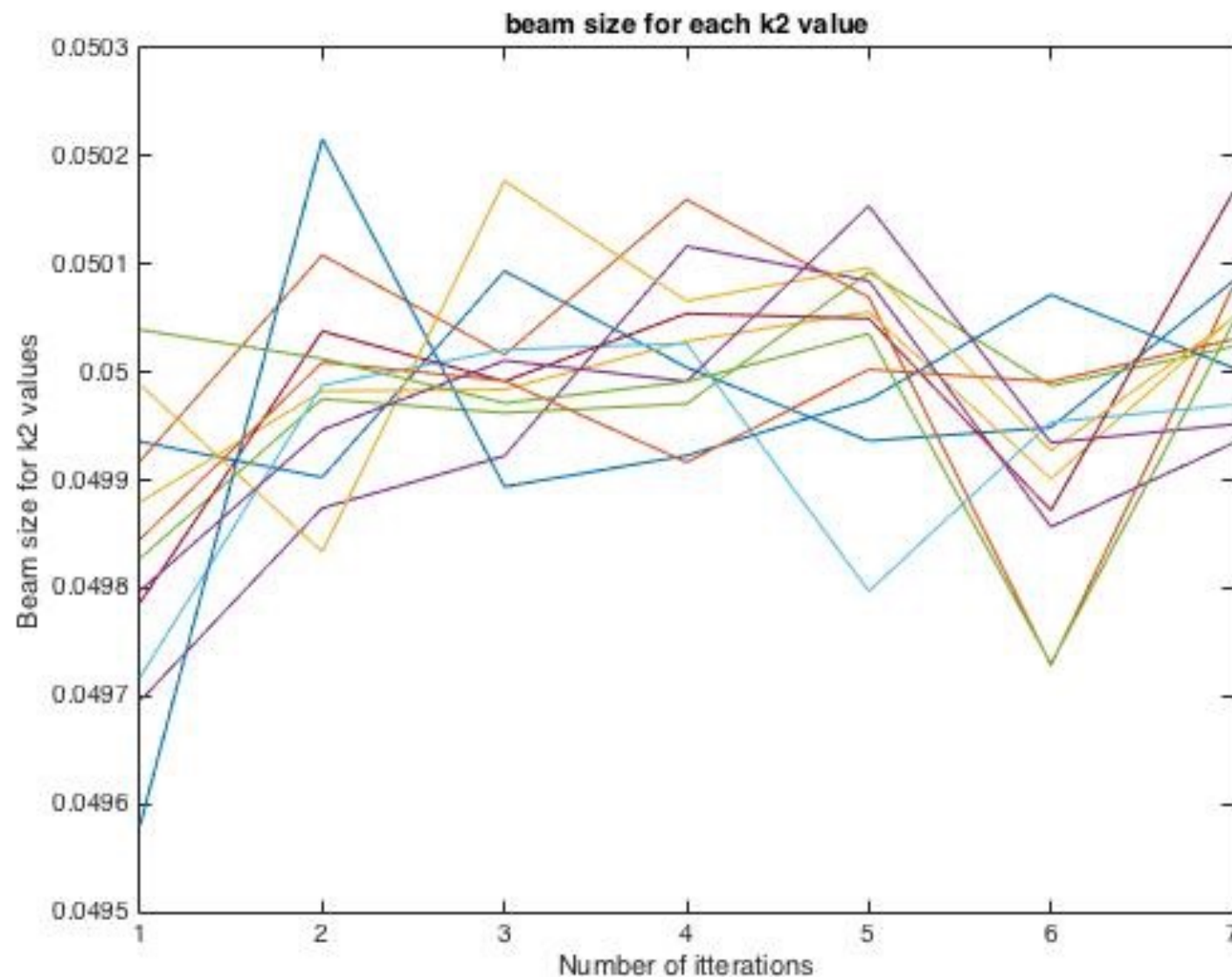
Dramatic example, including error and with and average taken



```
startk1Value = 35;  
endk1Value = 50;  
step1 = 2;  
desiredBeamSize = 1.8;  
k2Value = -2;  
driftL = 1;  
orientation = 1
```

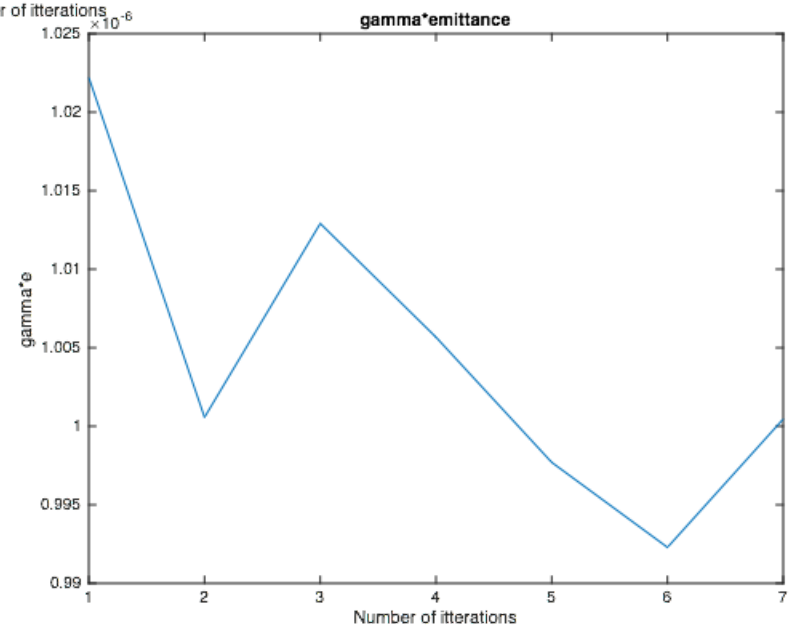
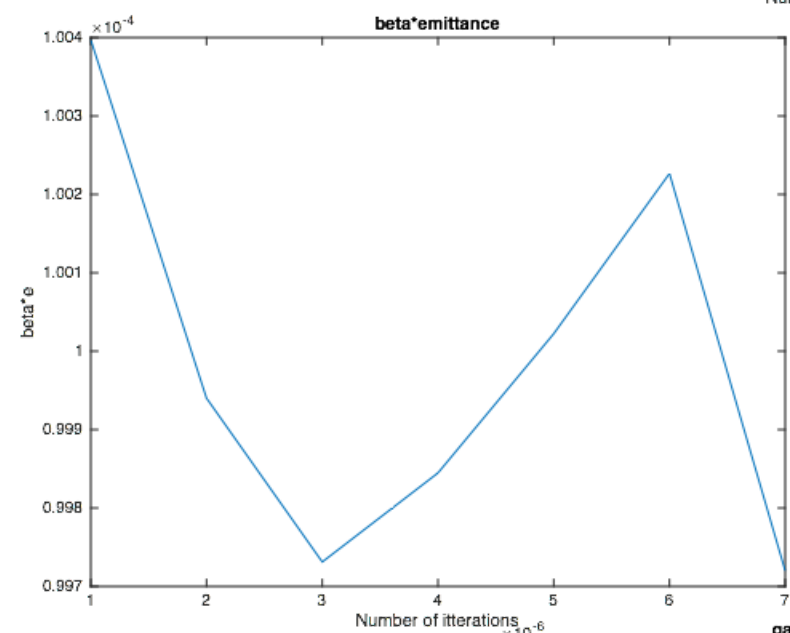
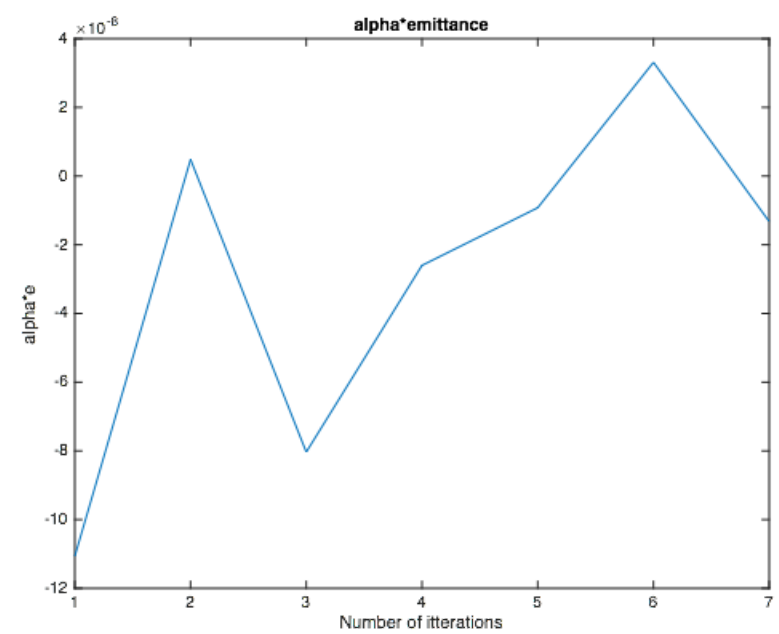
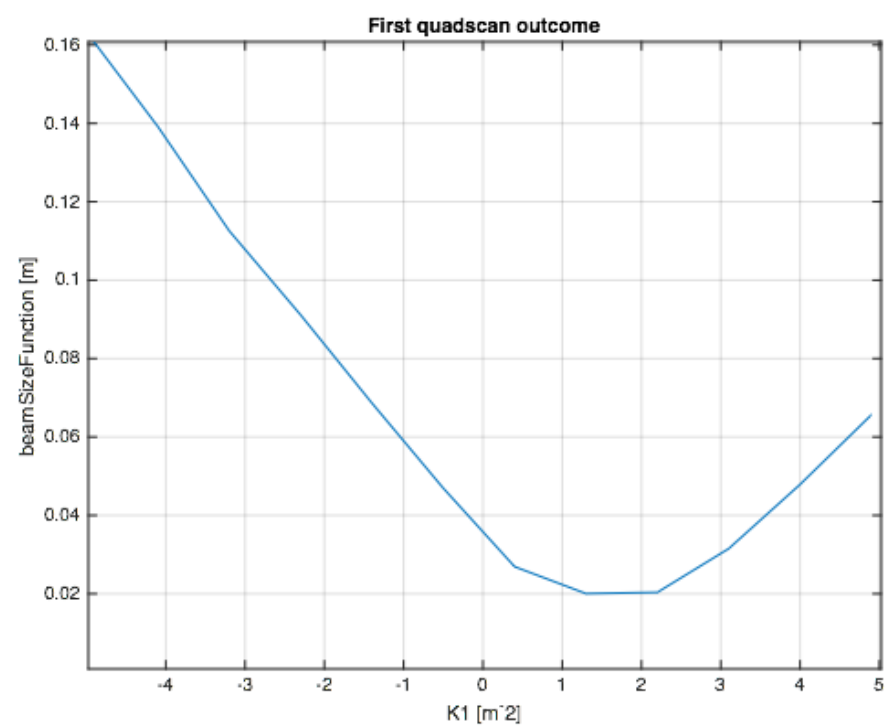
beta = 16  
alpha = 0.8  
emittance = 1





```
realBeta0=10;  
realAlpha0=0;  
realEmittance=0.00001;
```

```
startk1Value = -5;  
endk1Value = 5;  
step1 = 0.9;  
desiredBeamSize = 0.05;  
k2Value = -5;  
driftL = 1;  
orientation = 1;  
quadL = 0.38;  
numberOfIterations = 8;
```



# Tomography

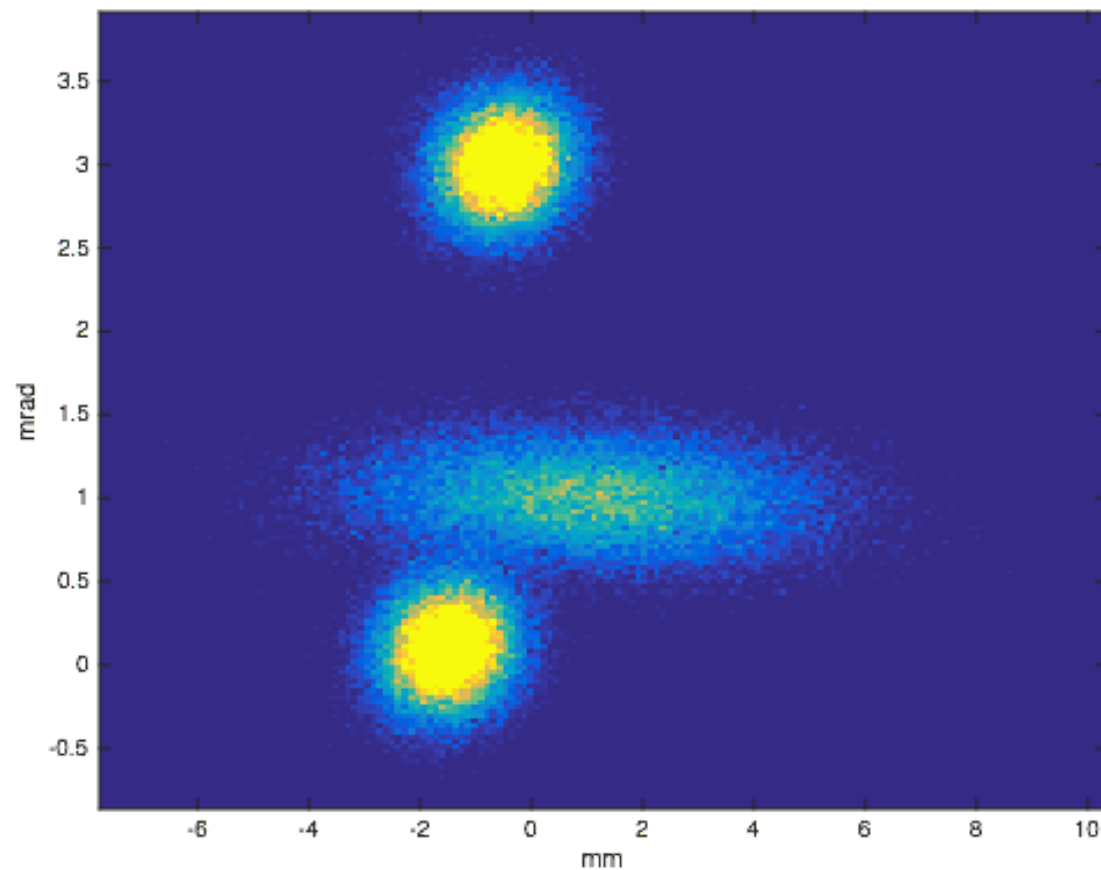
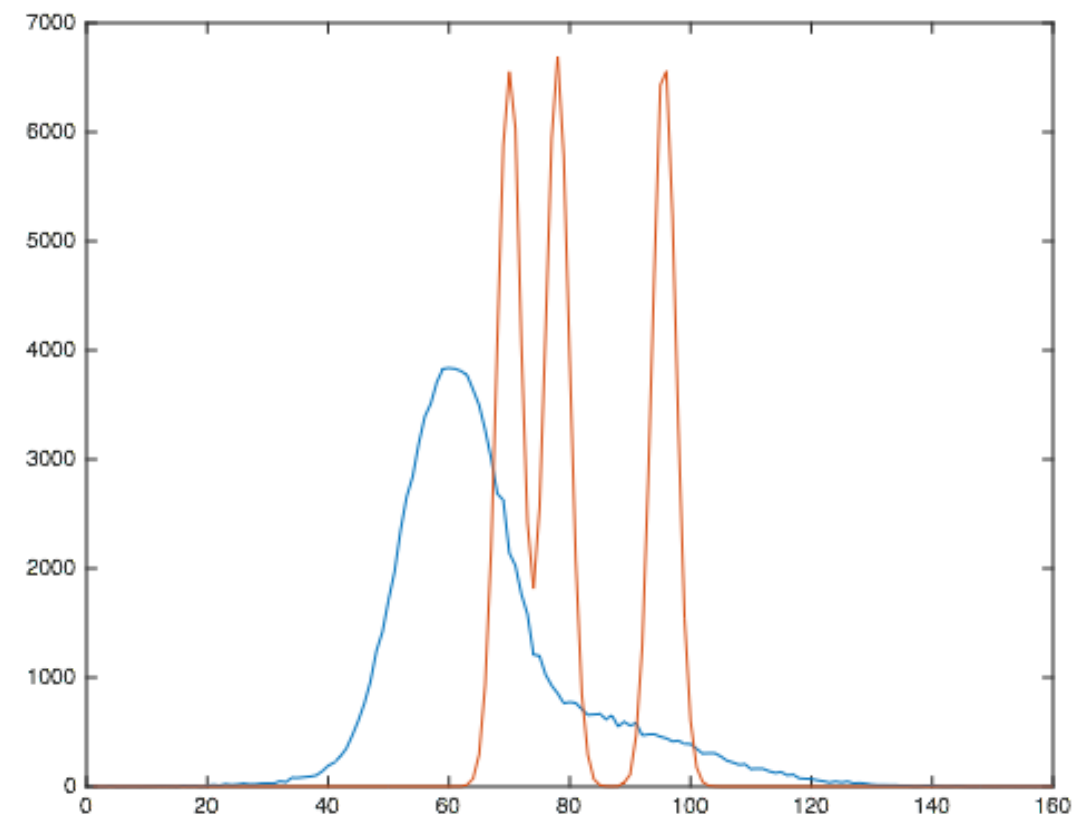
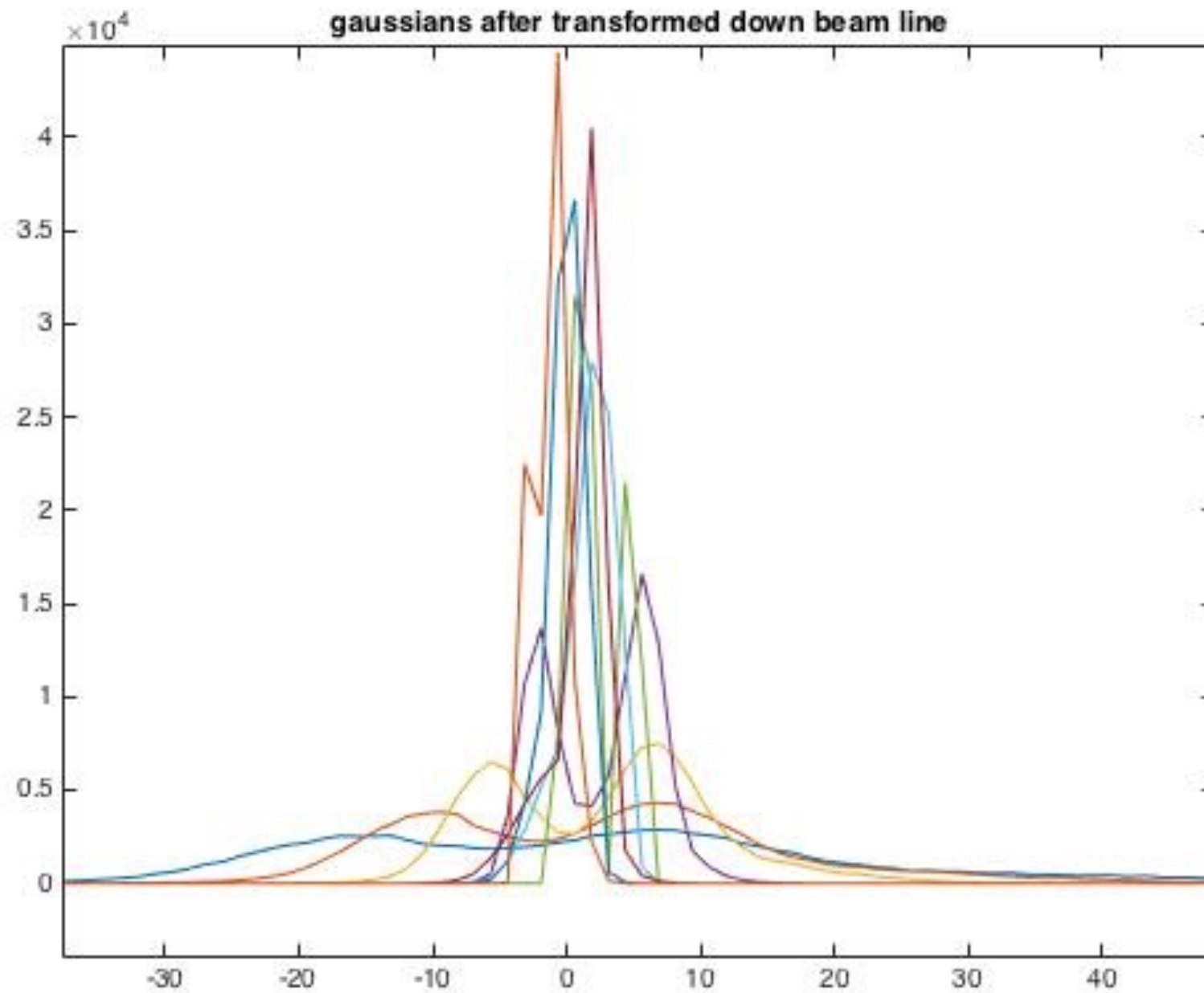


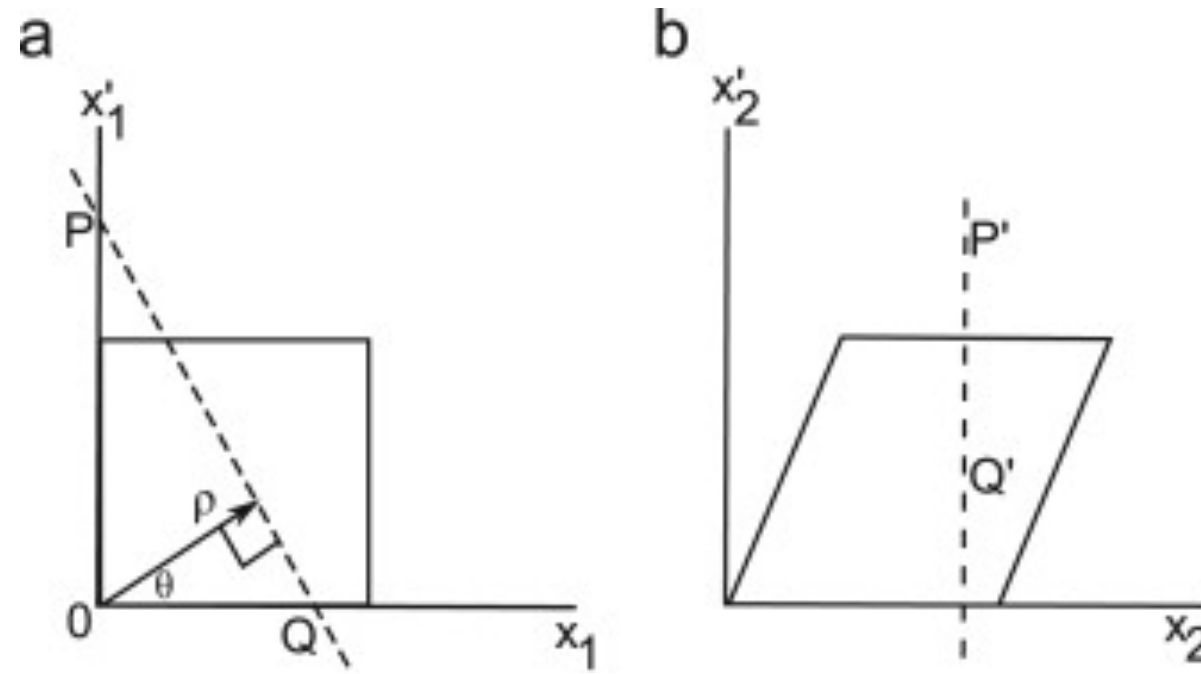
Image of the initial phase space created.  
Complex to test program capabilities.

Projection of this phase space onto both the position and momentum axis.



The program uses the projection of the beam onto a spacial axis for different values of the current in each quadrupole





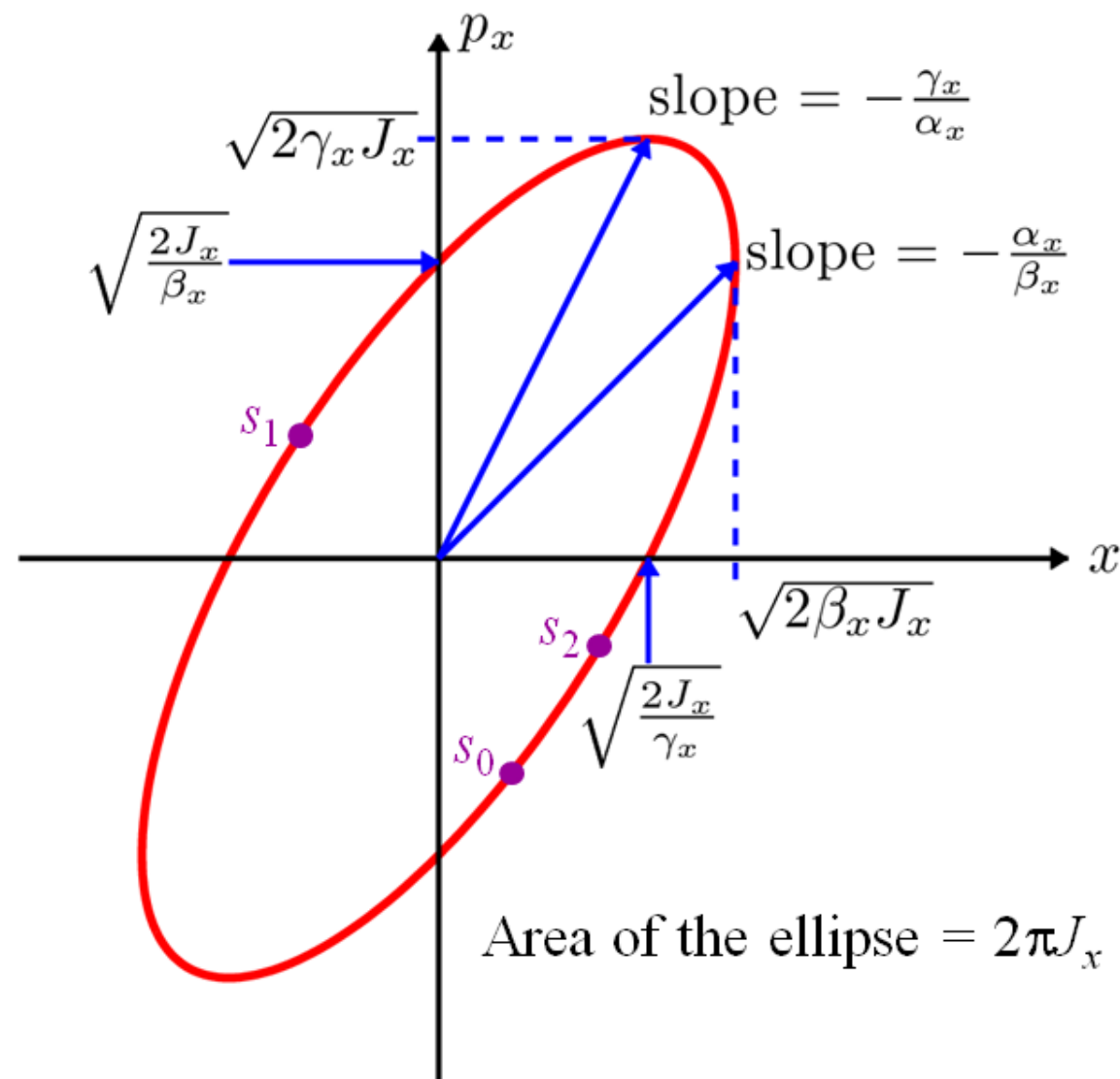
- As the beam moves down the lattice the phase space rotates. To transform the phase space projection back to the start of the line the projection must be rescaled and angle of rotation found.
- Projection is equivalent to a series of integrals parallel to  $PQ$
- Using the transfer matrix :

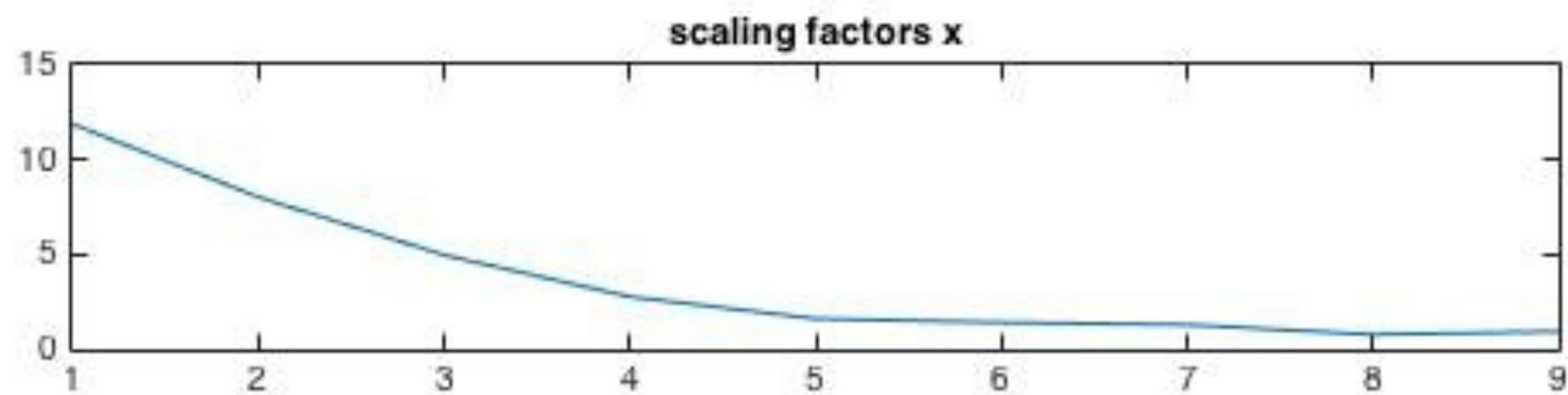
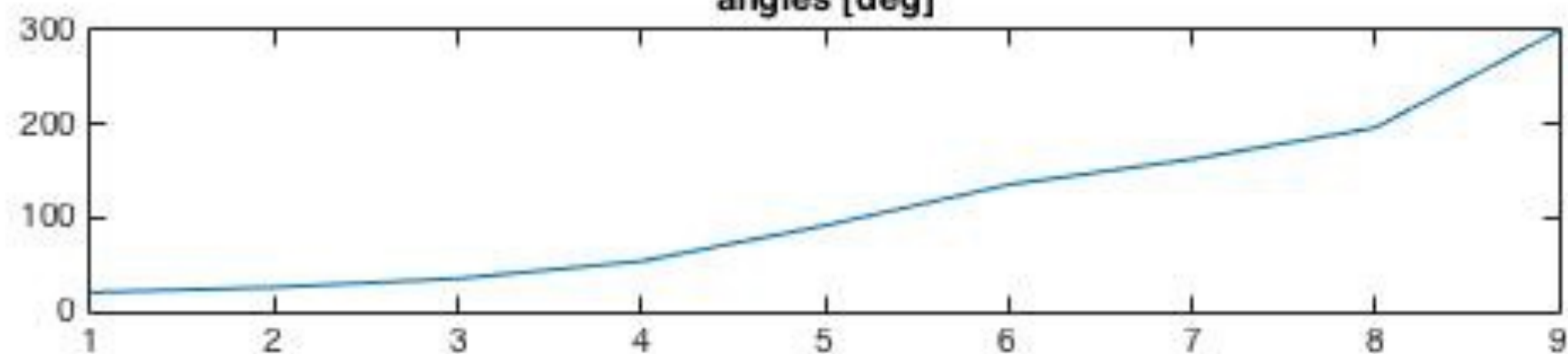
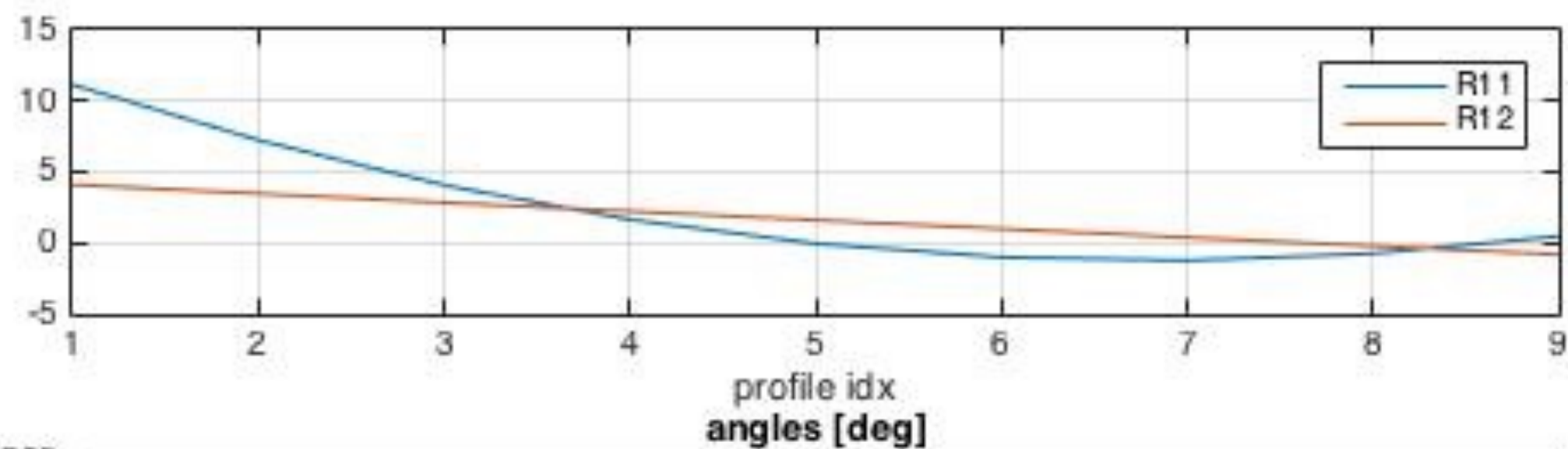
$$\tan \theta = \frac{R_{12}}{R_{11}}$$

$$\rho = \frac{x_2}{s} \quad \text{where : } s = \sqrt{R_{12}^2 + R_{11}^2}$$

The Standard form of a matrix that rotates a vector counterclockwise by theta:

$$M = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$





# Process

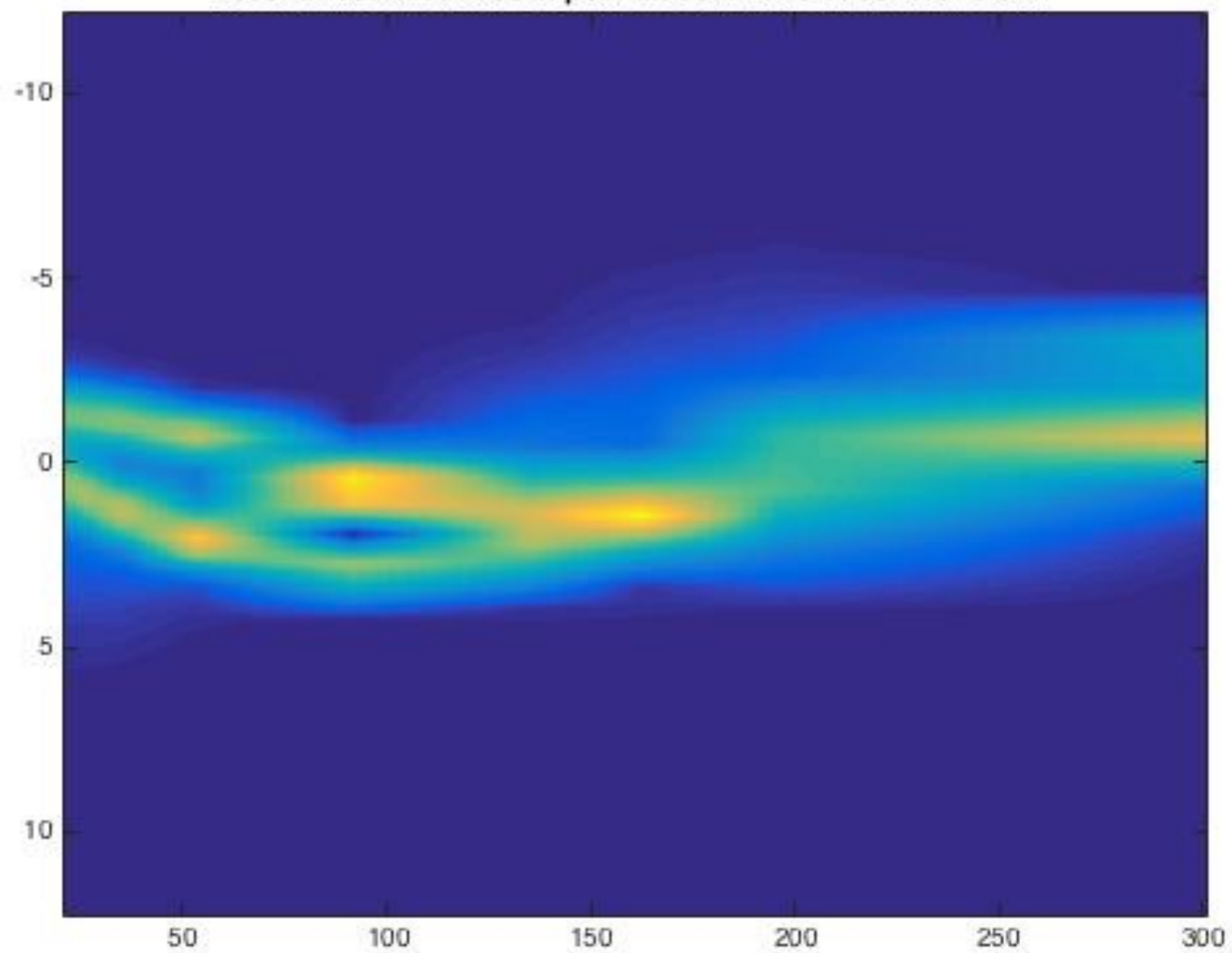
1. The transfer matrix of the line is constructed and an initial test phase space is generated.
2. The  $x$  and  $x'$  components of each particle are transformed down the line, for a range of different quadrupoles.
3. The rotation angle and  $s$  are computed for each set of quadrupole current values.
4. Histograms of the projection of the momenta onto the position axis are created, these histograms are produced so that all the bin centres are in the same place, so that the distribution isn't skewed.
5. The distribution at the start of the line and the angle that this is "seen from" is then calculated

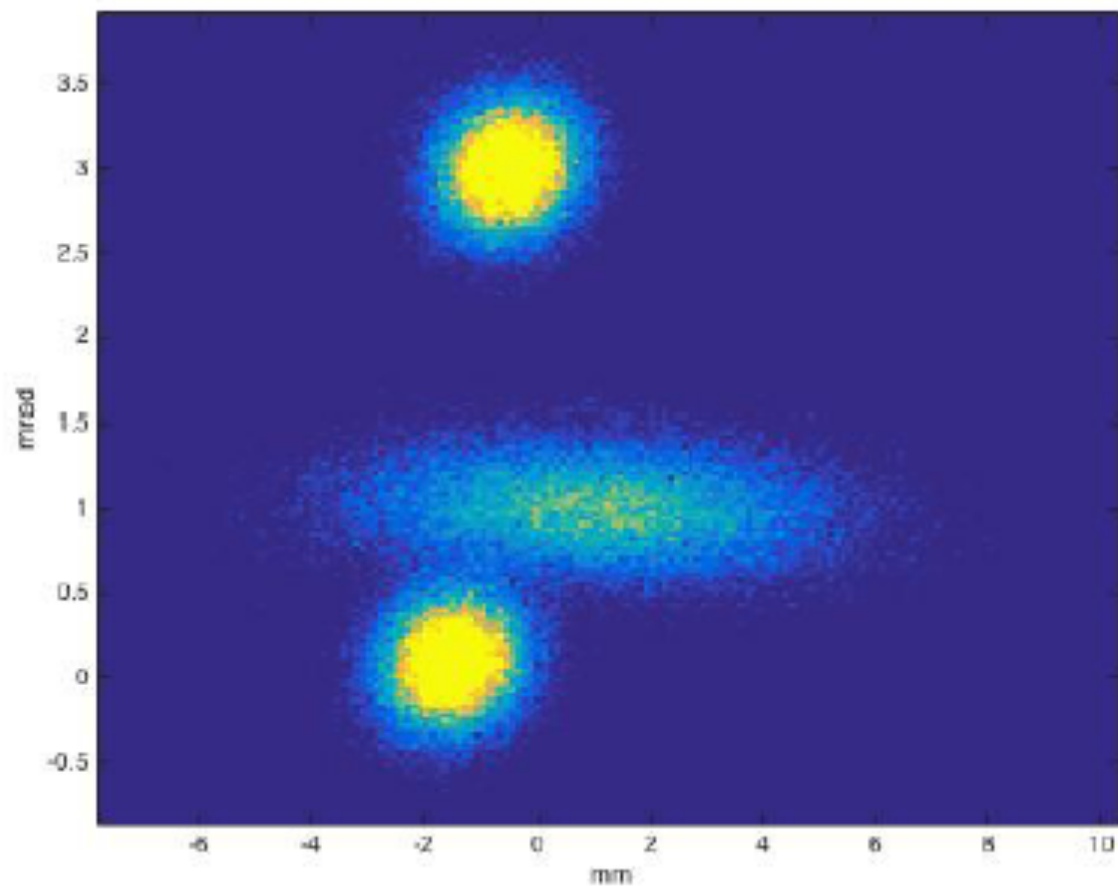


The Iradon Transform function in Matlab only takes equally spaced projection angles to reconstruct the image.

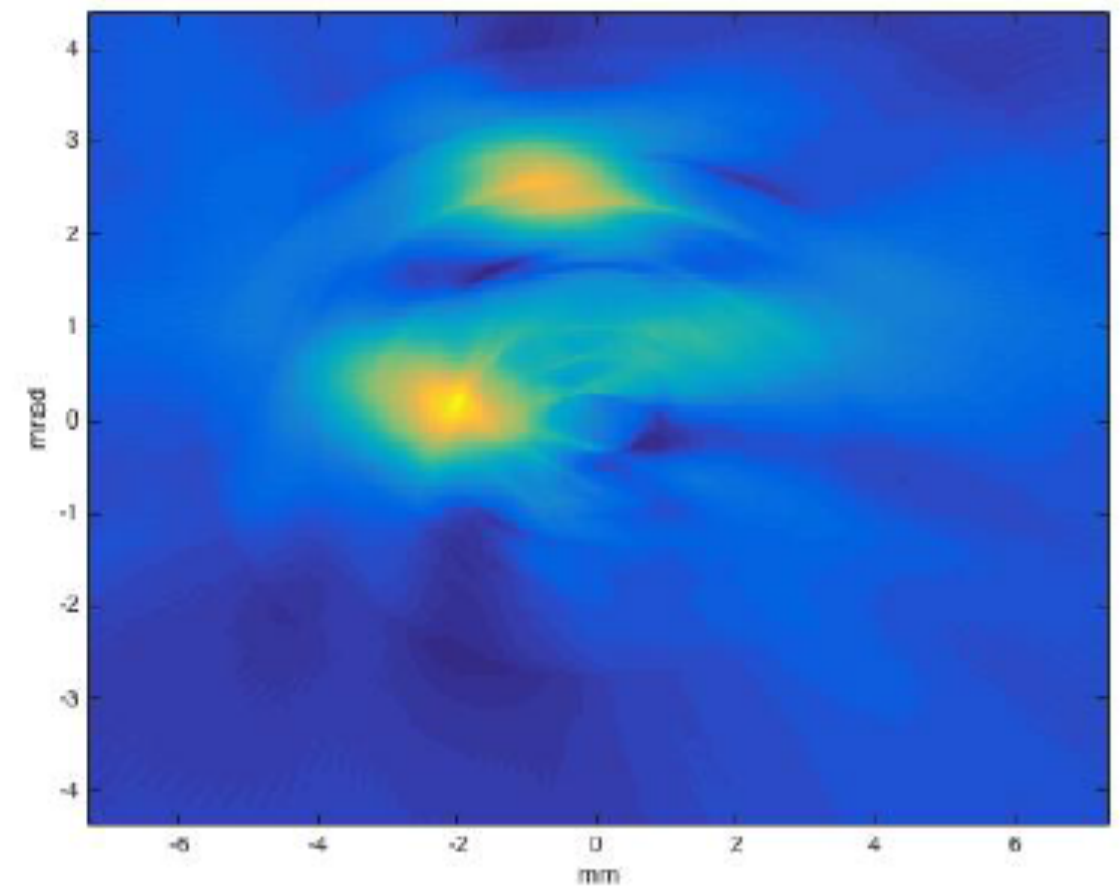
4. A 3d surface is constructed allowing interpolation between the angles. This also means that a larger number of angles can be used in the reconstruction.
5. Each histogram is rescaled differently when transformed back to the start of the line. To reconstruct the surface they must be equally spaced, the bin centres are interpolated at predefined points.
6. The distribution can then be read off for a given number of angles and inputted into the radon transform to give the image of the phase space as the start of the line.

reconstructed surface at points used in the iradon transform





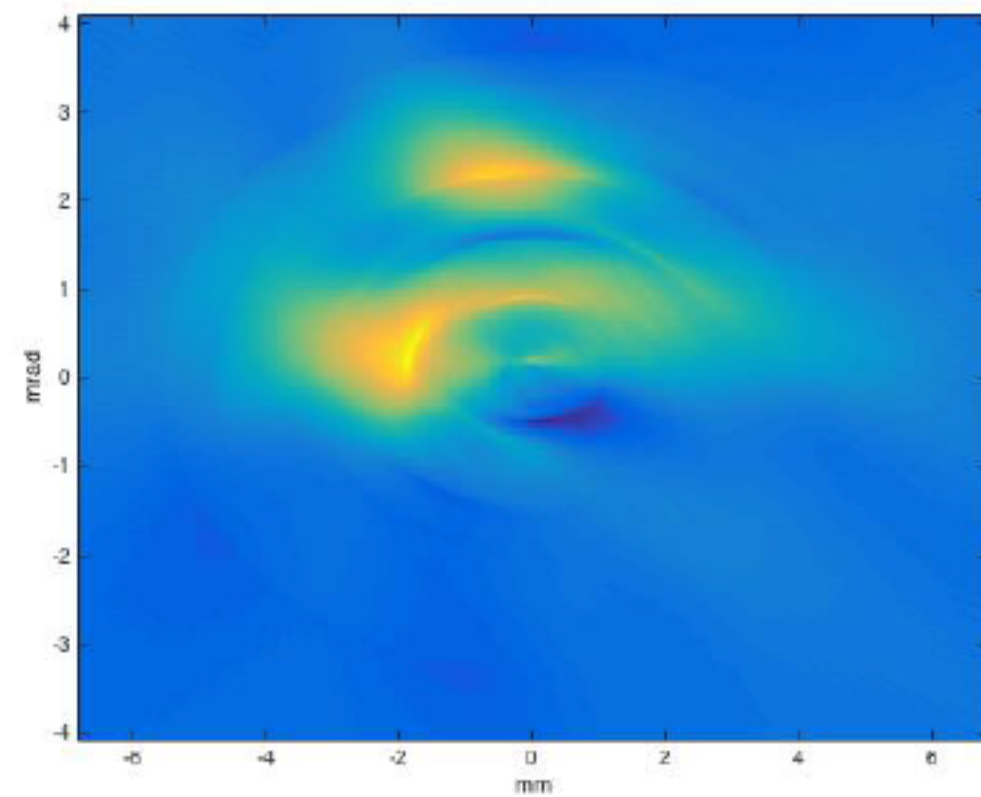
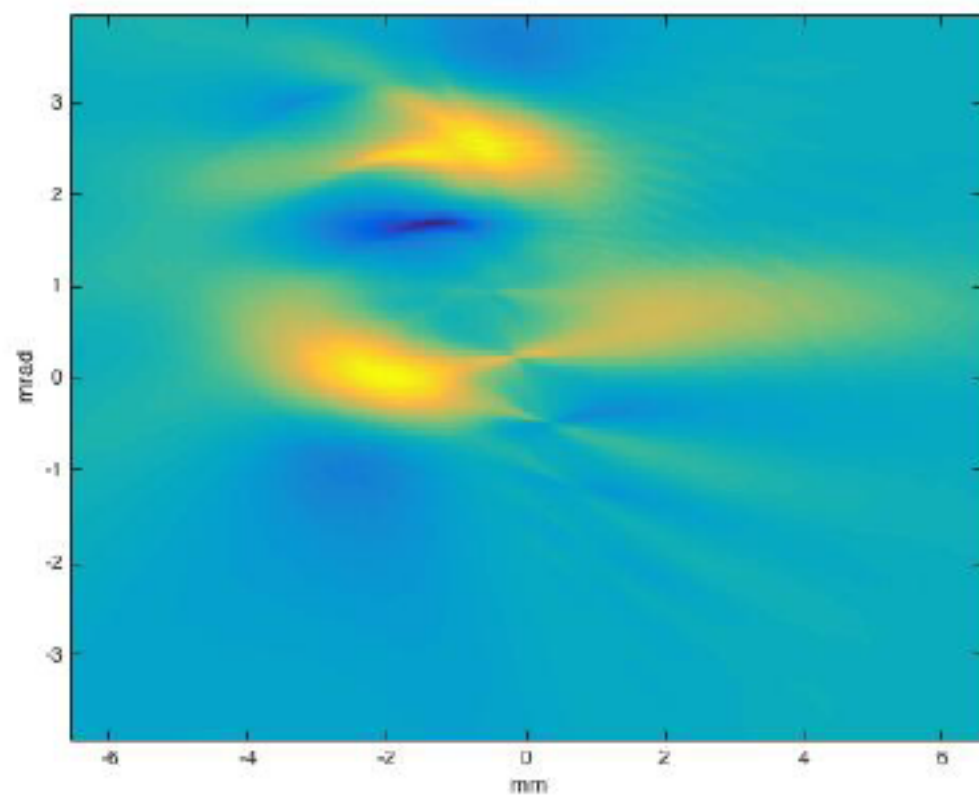
(a) Phase space inputted into the simulation.



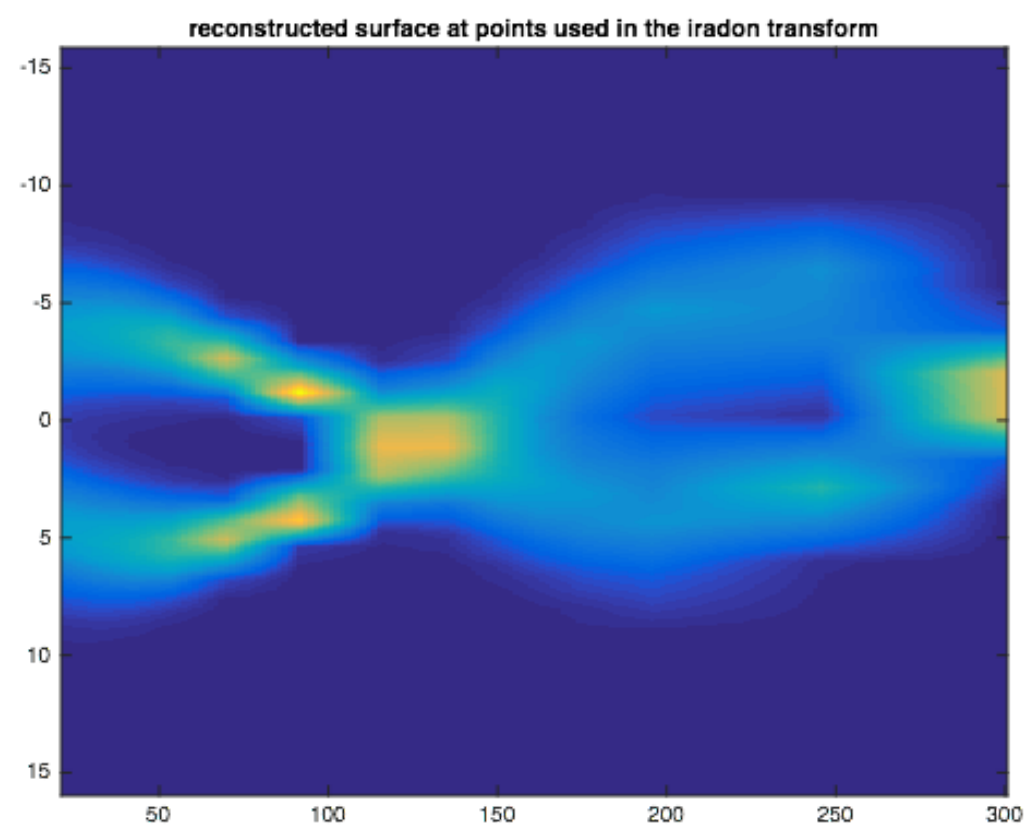
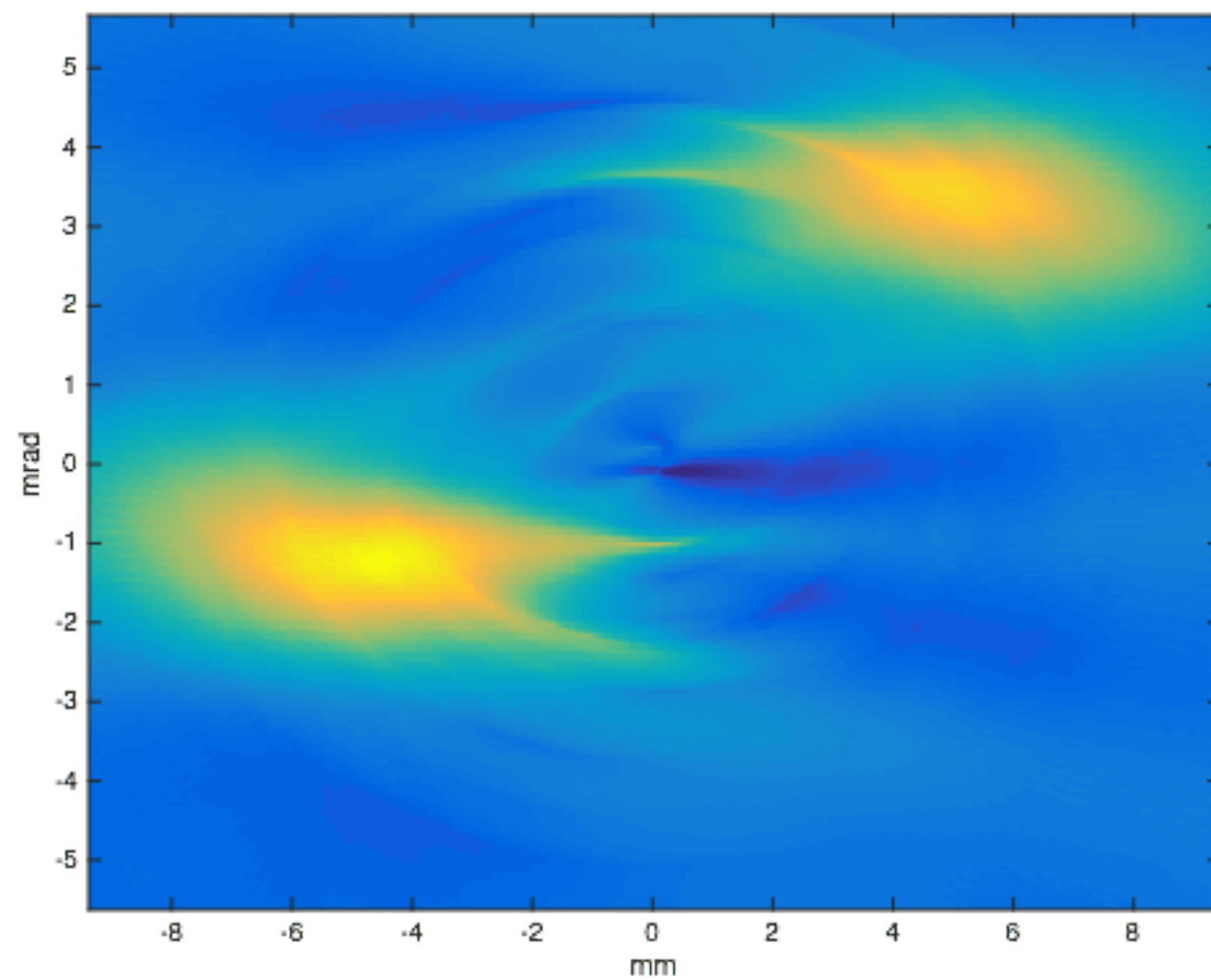
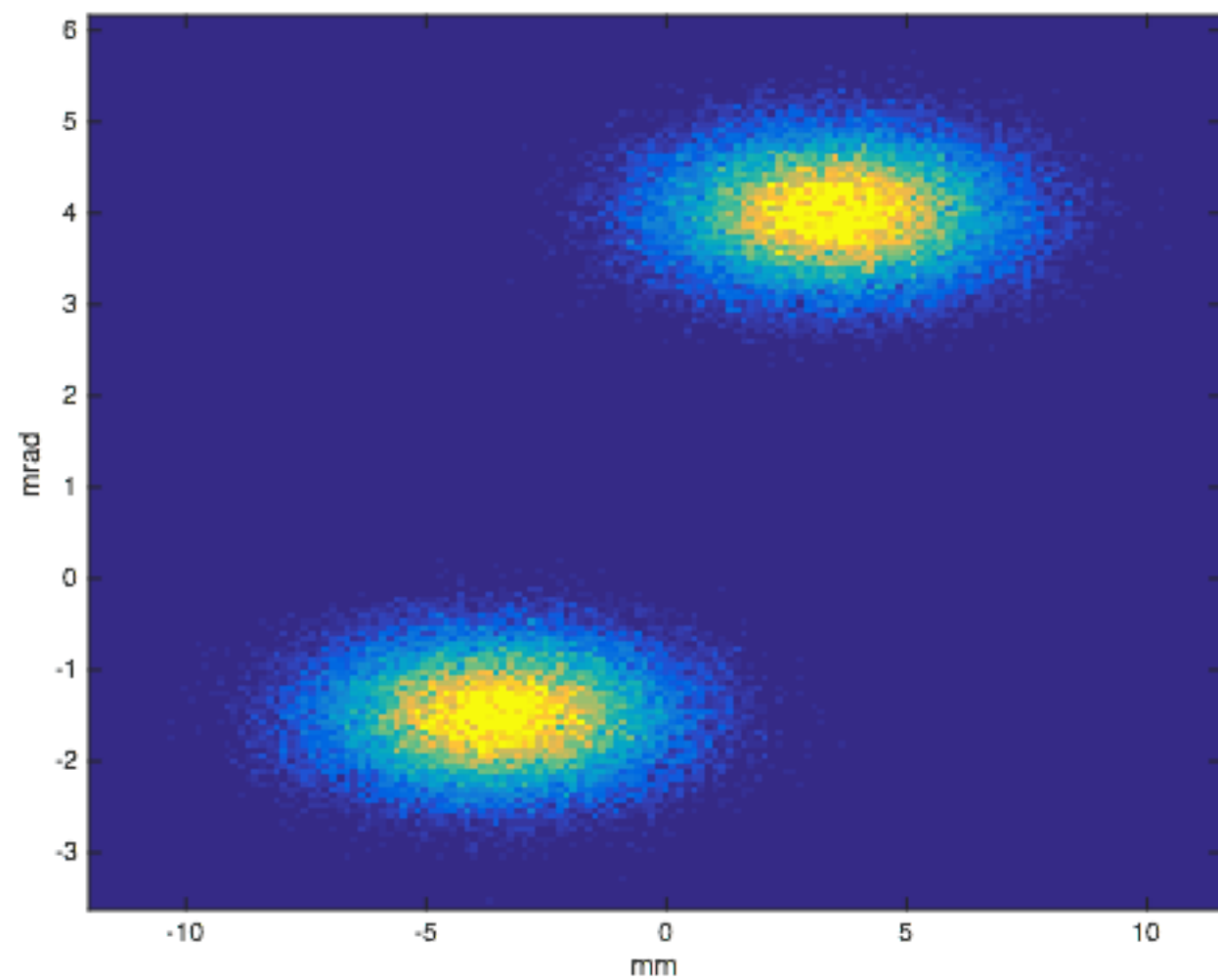
(b) Reconstructed image of the phase space.

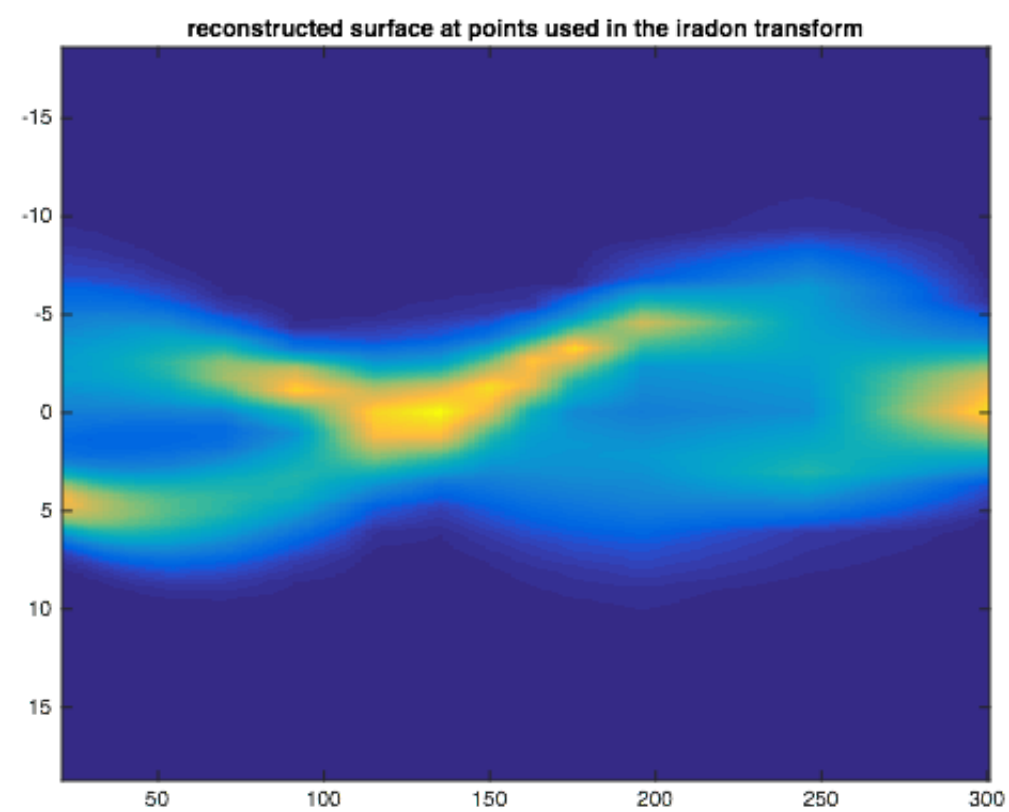
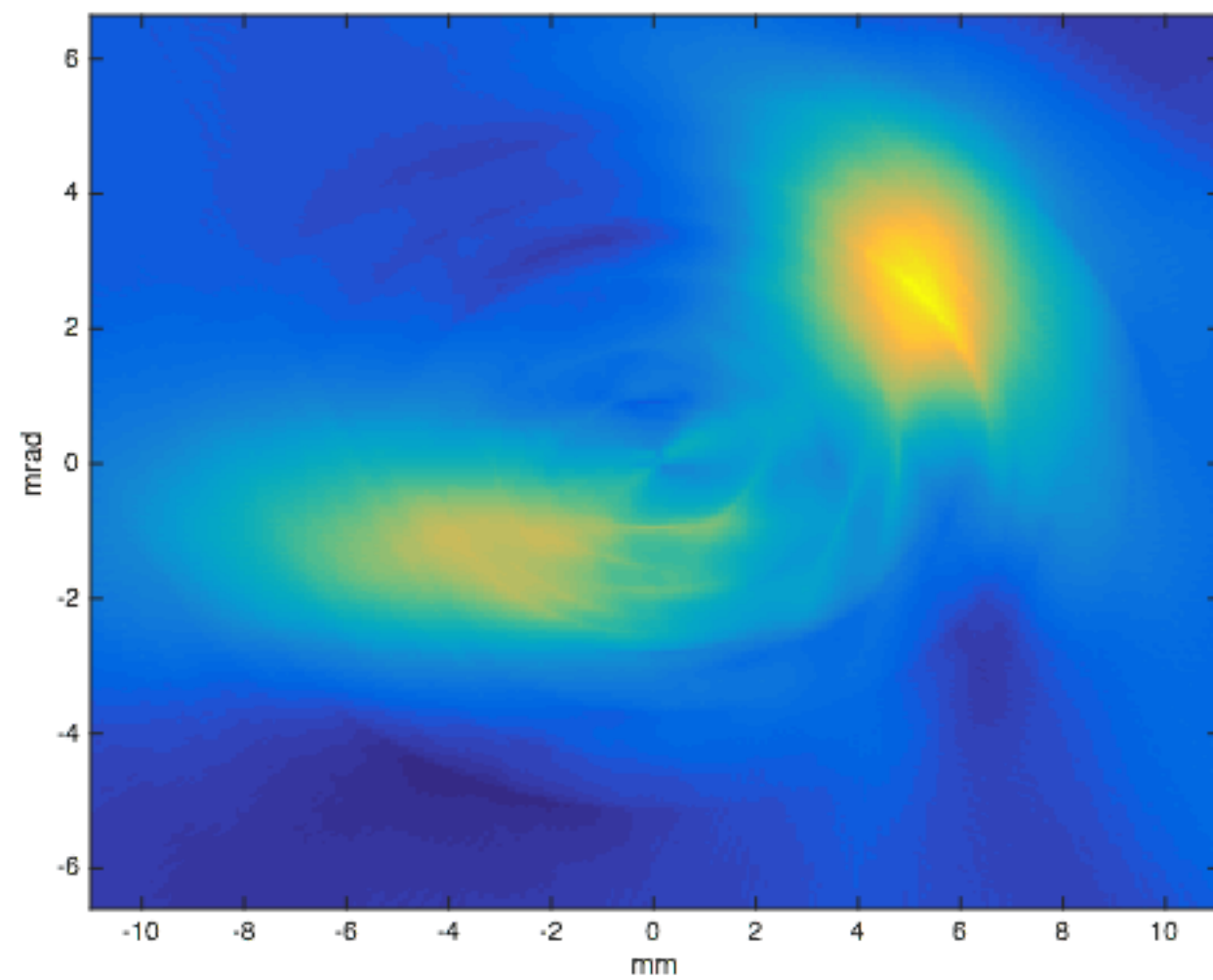
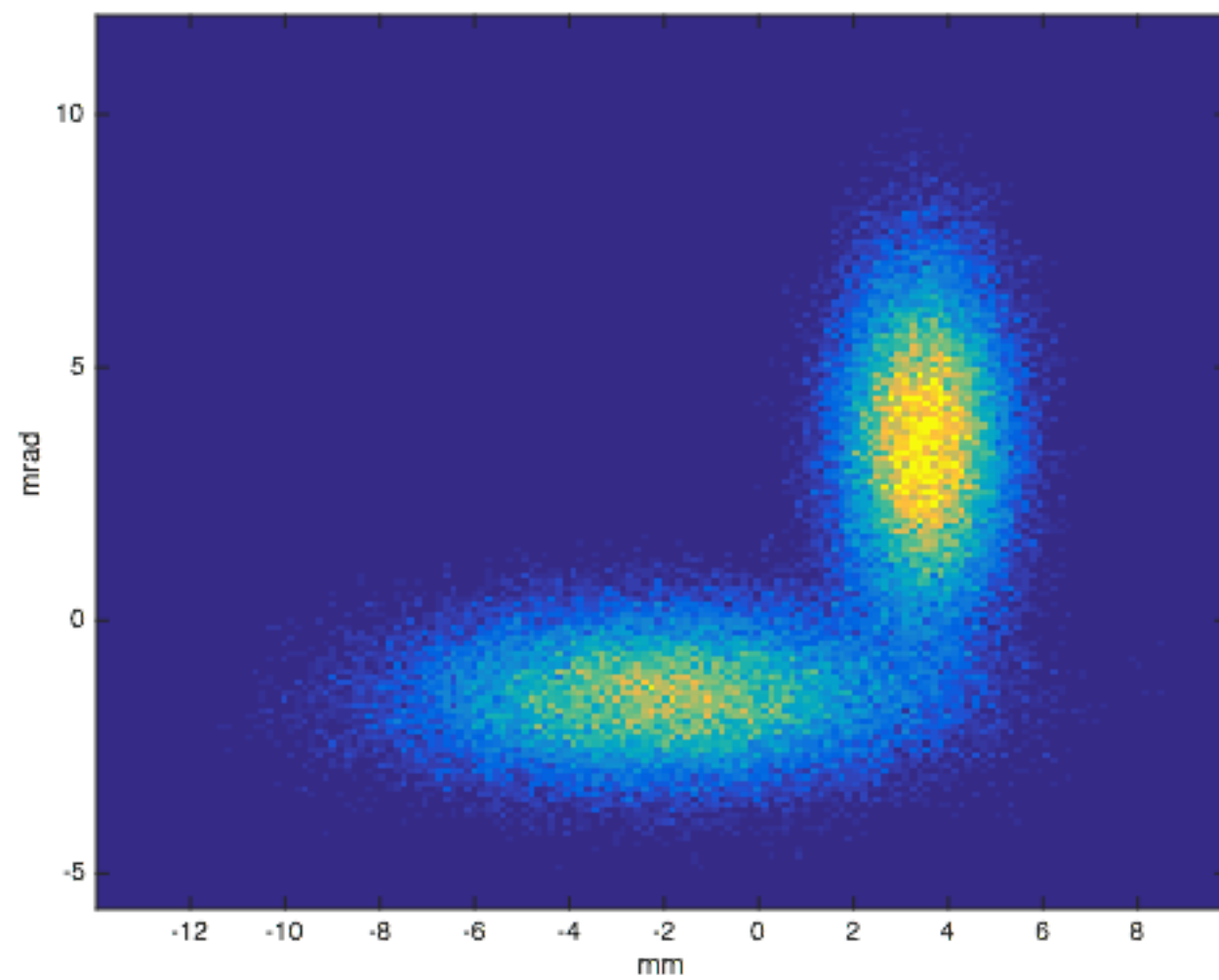
Images comparing the reconstructed image to the initially inputted image for 16 theta value over a 20 to 300 degree range.

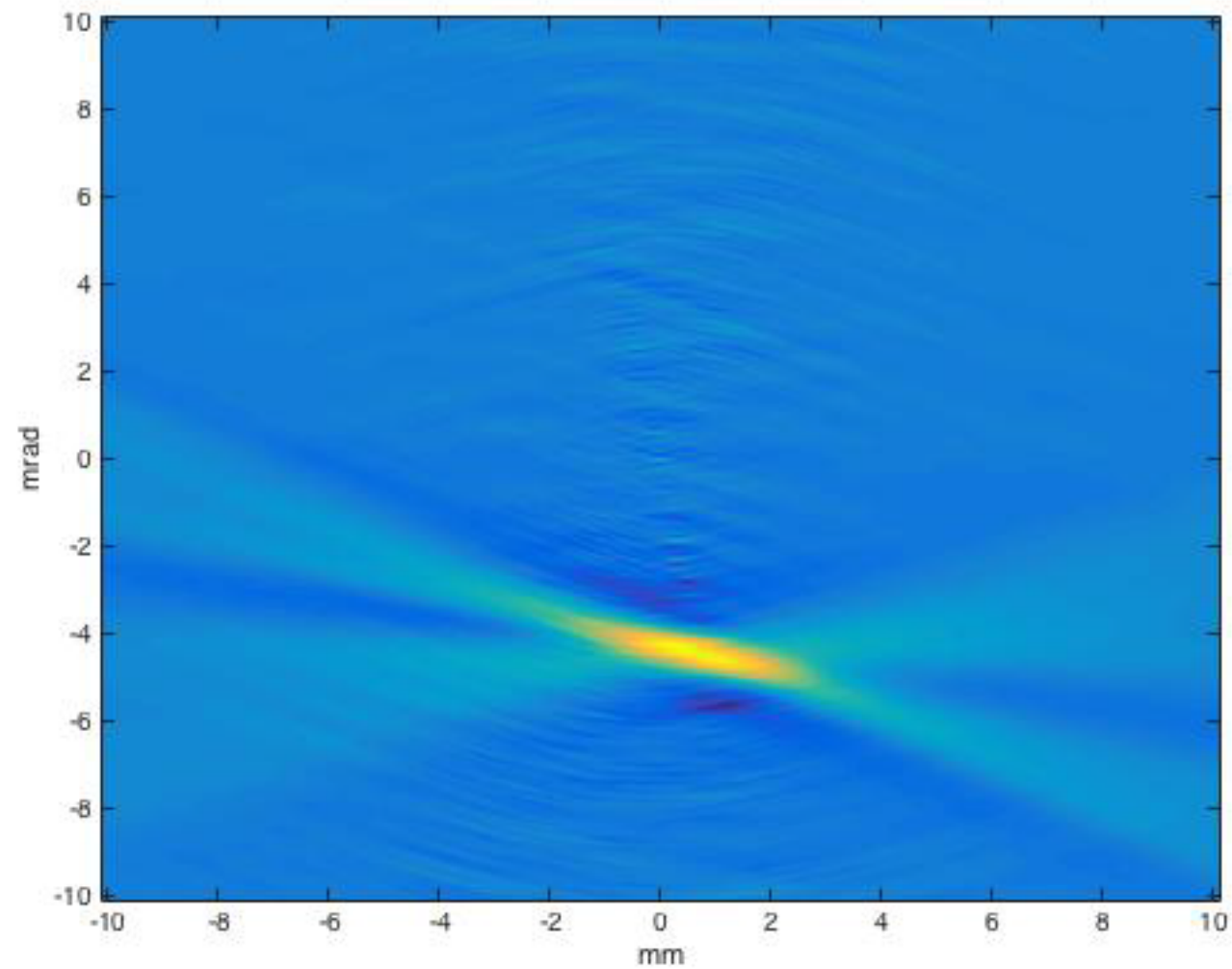
- Linear Interpolation
- 100000 particles



(a) Reconstruction with 16 rotation angles covering approximately  $100^\circ$ . (b) Reconstruction with 8 rotation angles covering approximately  $300^\circ$ .







Reconstruction from old CTF3 data taken on 01/07/15 from girder 15.  
Angle of reconstruction between 60 and 111 degrees.



# Conclusions

- The program successfully keeps the beam size at the end of the lattice constant when currents through one quadrupole are predefined.
- The correct value of the twiss parameters are given unless a beam size smaller than pixel size is asked for.
- In a FODO line similar to CFT 3 the twiss parameters and current values are found quickly.
- Creating a beam of constant size reduces the effect of aberrations.
- Tomography was successfully (to a certain extent) used to reconstruct complex phase spaces.
- A large range of rotation angles and a large number of angles (approximately 20) is needed for complex phase spaces, whereas for a simple ellipse the range of rotation angle is the more important factor.



These two techniques are complementary, with the quadrupole scan giving the Twiss parameters and the emittance of the beam, assuming that the beam is gaussian. The tomography reveals the true phase distribution of the beam, which may be non gaussian. If the quadrupole scan can rotate the phase space through a large enough angle, then both a quadrupole scan and the tomography can be used.

# Acknowledgements

Professor Philip Burrows

Davide Gambe

Jack Roberts

# References

- [1] <https://indico.cern.ch/category/345/>  
The CERN summer student lectures "Introduction to Accelerator Physics",  
Verena Kain.
- [2] A.T.Green et al, Implementation of Quadrupole-Scan Emittance Measurement at Fermilab's Advanced Superconducting Test Accelerator
- [3] <http://www.sciencedirect.com/science/article/pii/S0168900211007108>  
Beam tomography in transverse normalised phase space
- [4] <http://www.slaney.org/pct/pct-toc.html>  
Principles of Computerised Tomographic Imaging
- [5] <http://uk.mathworks.com/help/images/ref/iradon.html>  
iradon transform, Mathworks
- [6] <http://uk.mathworks.com/help/matlab/ref/scatteredinterpolant-class.html>  
Scattered Interpolant, Mathworks
- [7] <http://uk.mathworks.com/help/matlab/ref/interp1.html>  
Interp1, Mathworks
- [8] <http://accelconf.web.cern.ch/accelconf/IPAC2013/papers/mopme070.pdf>  
Emittance and Beta Function Measurement for the Max IV Linac
- [9] <http://madx.web.cern.ch/madx/>  
MadX Documentation