Track fitting in the ILD non-uniform magnetic field

Bo Li[1] Keisuke Fujii[2]

[1] Laboratoire Leprince-Ringuet, École polytechnique

[2]Institute of Particle and Nuclear Studies
High Energy Accelerator Research Organization (KEK)

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Outline

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Introduction

- High performance of tracking is essential to the physics program on the future linear collider experiment:
  - **Tracking detectors** should have excellent spatial resolution and minimized track distortion;
  - **Tracking algorithm** need to has the ability to get momentum with high accuracy in the real non-uniform magnetic field.
- The Kalman filter tracking software package, **KalTest**:
  - It has been worked successfully in the physics studies based on ILD uniform magnetic field and LCTPC large prototype beam test;
  - Since ILCSoft v01-17-07, KalTest can deal with the track fitting for both uniform and non-uniform magnetic field;
  - To use track fitting for non-uniform magnetic field, you should provide a field map by TEveMagField class to KalTest.
    (see KalTest/src/bfield/TBField.h)

Outline

Equation of motion for a charged particle

- The equation of motion of a charged particle in a magnetic field is

$$m\frac{d^2\boldsymbol{x}}{dt^2} = Q\frac{d\boldsymbol{x}}{dt} \times \boldsymbol{B}(\boldsymbol{x}), \tag{1}$$

  where m is the relativistic mass, and Q is the charge of particle.

- If the magnetic is uniform, and we assume its direction is parallel with z axis of coordinate system, then the trajectory of charge particle can be solved analytically,

$$\left\{ \begin{array}{lllllll} x & = & x_0 & + & d_\rho \cos\phi_0 & + & \frac{\alpha}{\kappa}\left[\cos\phi_0 - \cos(\phi_0+\phi)\right] \\ y & = & y_0 & + & d_\rho \sin\phi_0 & + & \frac{\alpha}{\kappa}\left[\sin\phi_0 - \sin(\phi_0+\phi)\right] \\ z & = & z_0 & + & d_z & - & \frac{\alpha}{\kappa}\tan\lambda\cdot\phi \end{array} \right. , \tag{2}$$

  which is a helix equation.

Helical track model

- According to the paramtrized track equation, helix in xy plane is plotted as:

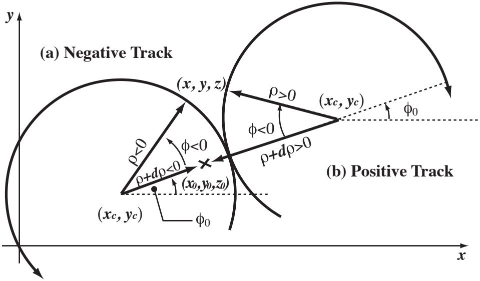


Figure: Helical track model.

- The **state vector** of a track is defined as

$$\boldsymbol{a}_k = \left(\; d_\rho, \phi_0, \kappa, d_z, \tan\lambda \; \right)^{\mathrm{T}}. \tag{3}$$

Kalman Filter

- For each site, Kalman filter algorithm has two steps:
  - Prediction:
$$\boldsymbol{a}_k^{k-1} = \boldsymbol{f}_{k-1}(\boldsymbol{a}_{k-1}), \tag{4}$$
    in which, $\boldsymbol{f}_k$ is **propagation function**. And the corresponding **propagation matrix** is defined by
$$\boldsymbol{F}_{k-1} = \frac{\partial \boldsymbol{f}_{k-1}}{\partial \boldsymbol{a}_{k-1}}. \tag{5}$$
  - Filtering:
$$\boldsymbol{a}_k = \boldsymbol{a}_k^{k-1} + \boldsymbol{K}_k \left(\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^{k-1})\right), \tag{6}$$
    where $\boldsymbol{K}_k$ is the gain matrix which is related to the propagation matrix, $\boldsymbol{h}_k$ is the measurement function.
- Kalman filter is implemented in KalTest[1], together with track models and basic detector geometries.

[1] KalTest manual is at http://www-jlc.kek.jp/jlc/en/subg/soft/tracking/kaltest.

Basic idea

To use the helical track model of KalTest in the non-uniform magnetic field, we can

- assume the magnetic field between two nearby layers is uniform;
- transform the frame to make the z axis point to the direction of magnetic field.

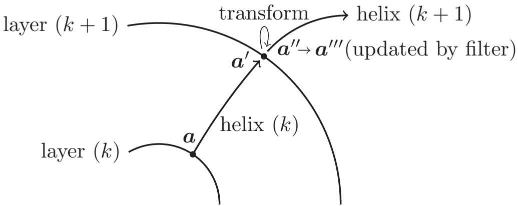


Figure: The updated track propagation procedure.

Therefore we now have a **segment-wise helical track model**, and we should recalculate the propagation function and propagation matrix.

How to transform the frame

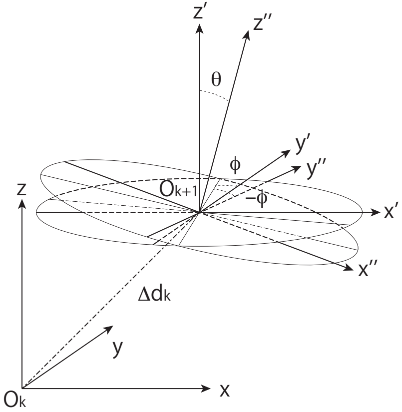


Figure: Transformation: translation and rotation

For details of the algorithm, see "Kalman-filter-based track fitting in non-uniform magnetic field with segment-wise helical track model. Computer Physics Communications 185 (2014) 754-761".

Implementation

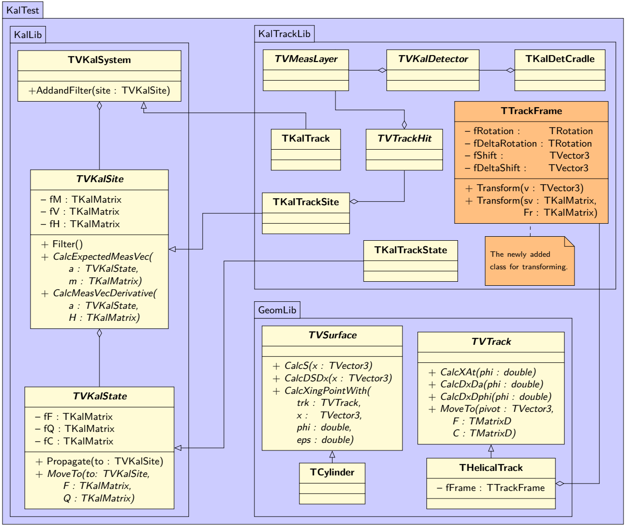


Figure: Class diagram of KalTest

Outline

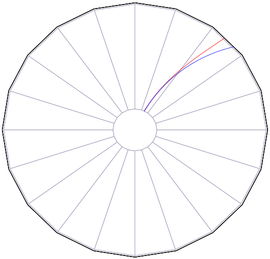
Simulation conditions

- Suppose the non-uniform magnetic field has a form of

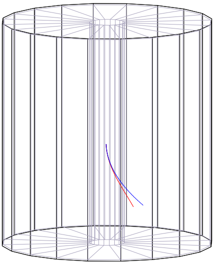
$$\left\{ \begin{array}{lll} B_x & = & B_0 k x z \\ B_y & = & B_0 k y z \\ B_z & = & B_0(1 - k z^2) \end{array} \right. ,$$

  in which, $k = \frac{k_0}{z_m r_m}$, $B_0 = 3$ T, $z_m = r_m = 3000$ mm;
- Runge-Kutta track generator: `TEveTrackPropagator` in ROOT and bisection method are used;
- Track parameters: dip angle $\lambda \in [0, 0.5]$, azimuth angle $\phi \in [0, 2\pi]$;
- Detector:
  - 251 layers;
  - distance between two nearby layers is 6 mm;
  - $R_{\text{in}} = 300$ mm;
  - Point resolution $\sigma_{r\phi} = 100$ µm.
- To see the effect of the non-uniform magnetic field, the track with the same initial parameters are also simulated in uniform magnetic field.

Event display



(a) xy view



(b) 3D view

Figure: Event display. $2\ \mathrm{GeV}$ tracks generated in uniform magnetic field (blue curve), and non-uniform magnetic field (red curve, $k_0 = 5$).

Momentum resolution

- $k_0 = 1$, $p = 10$ GeV;
- Tracks are reconstructed in **uniform** magnetic field.

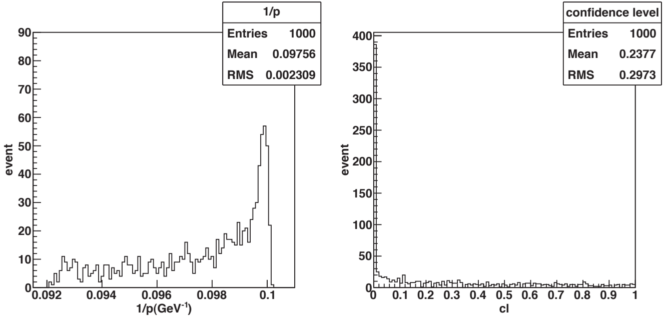


Figure: Momentum and confidence level by the original algorithm.

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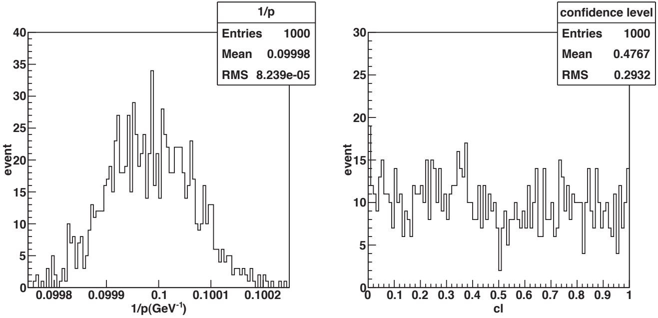


Figure: Momentum and confidence level by the updated algorithm.

Results with different non-uniformity and tracking step size

Table: Mean and RMS of $\frac{1}{p}$ (in units of $10^{-1} \cdot (\mathrm{GeV}/c)^{-1}$ and $10^{-5} \cdot (\mathrm{GeV}/c)^{-1}$ respectively) at $10\,\mathrm{GeV}/c$.

(a) Step size 6 mm

k_0	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
1	1.0000/8.03	0.9998/7.89	0.9995/7.65
2	1.0000/8.05	0.9997/8.09	0.9990/8.36
3	0.9999/8.07	0.9995/8.31	0.9984/9.20

(b) Step size 1 mm

k_0	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
1	1.0000/8.03	1.0000/7.89	0.9999/7.65
2	1.0000/8.05	0.9999/8.10	0.9998/8.36
3	1.0000/8.07	0.9999/8.32	0.9997/9.21

CPU expense

- MacBook Pro, OS X 10.6; 2.4 GHz Intel Core 2 Duo; 4 G Memory.
- 1,000 tracks

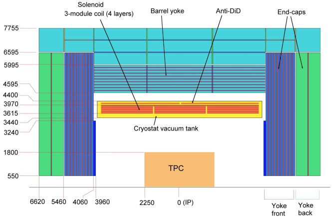
Table: Time consumption of functions (sec.)

Function	Time expense
Total	18.82
TVKalState::Propagate	11.53
TVKalSite::Filter	7.27
TTrackFrame::TTrackFrame	0.87
TTrackFrame::TransformVector	6.59
TTrackFrame::TransformSv	2.58
TVSurface::CalcXingPointWith	5.90

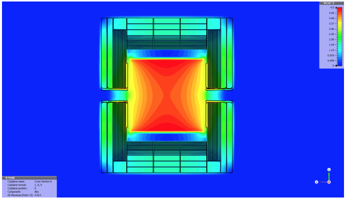
- The total CPU time is only about twice of the original code.

Test in ILD magnetic field

- The Anti-DID field, which was implemented in Mokka for pair background study, is used in this study
- The non-uniform field in Mokka database is dumped to a local file as a field map for KalTest.



(a) The magnet of ILD

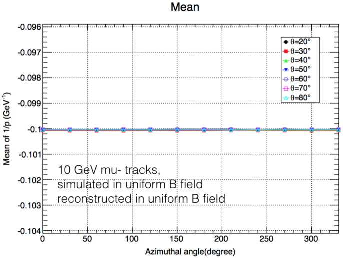


(b) The B field map of ILD

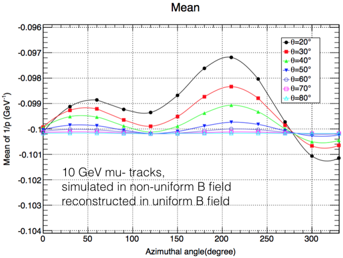
Figure: ILD magnetic field (Karsten Buesser@ALCW15)

Mean

- Mean of 1/p at different track angle (simulation: 10 GeV track).
- The mean of momentum is shifted by the anisotropic magnetic field.

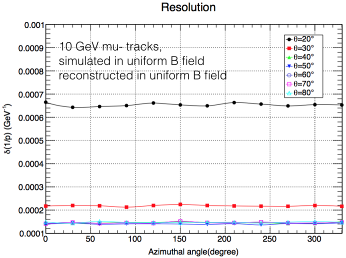


(a)

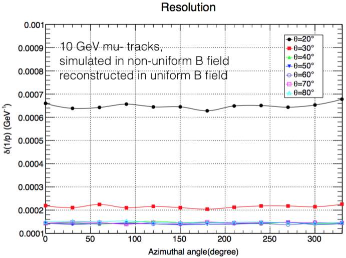


(b)

- Momentum resolution at different track angle.
- In this case the momentum resolution is not affected although the non-uniformity is not taken into account in track fitting.



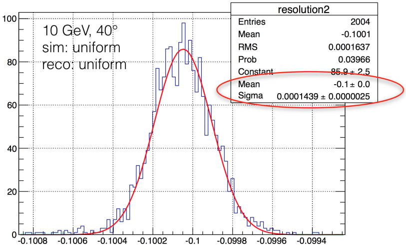
(a)



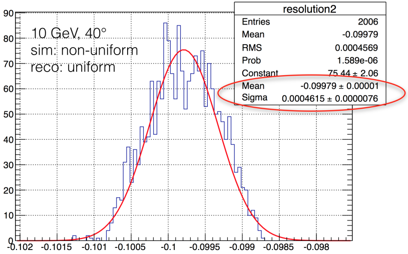
(b)

The effective momentum resolution

- If only fixing polar angle, the shift (or bias) of mean contributes to the momentum resolution, so we obtain a bigger effective momentum resolution in the non-uniform magnetic field.



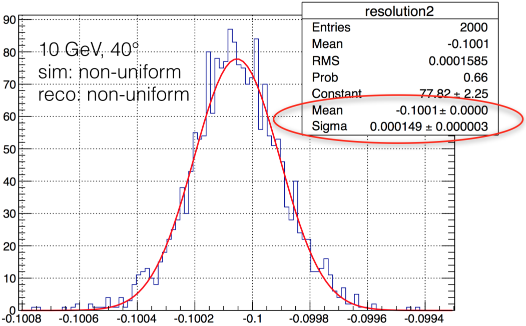
(a)



(b)

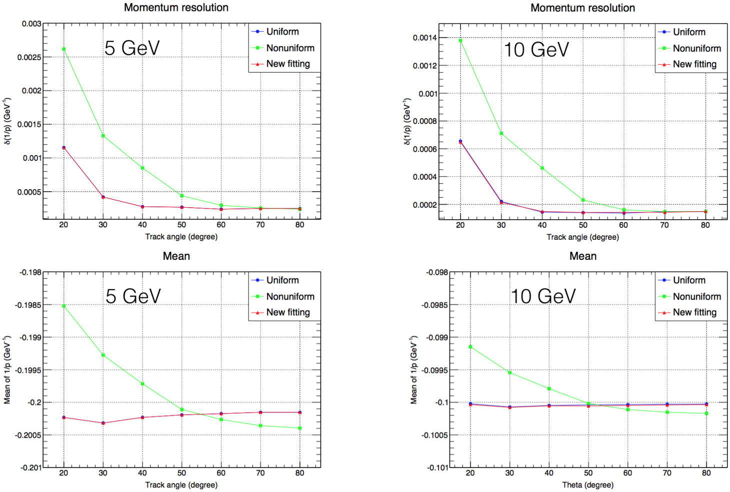
The effective momentum resolution

- The track fitting by the new KalTest can recover the momentum distribution in non-uniform magnetic field



(c)

Comparison of track fitting results



By taking field non-uniformity into account, the new KalTest gets consistent track fitting results with the original one for uniform case.

Outline

Summary

- Algorithm:
  - The algorithm for non-uniform magnetic field is based on Kalman filter;
  - By transforming the track frame, the segment-wise helical track model in KalTest is used and the non-uniformity of magnetic field is taken into account.
- Test results:
  - The updated algorithm can get correct momentum results at the ILD magnetic field;
  - The algorithm has good performance on CPU expense for non-uniform magnetic field.