Precision QCD calculation for top-pair production at lepton collider in the continuum

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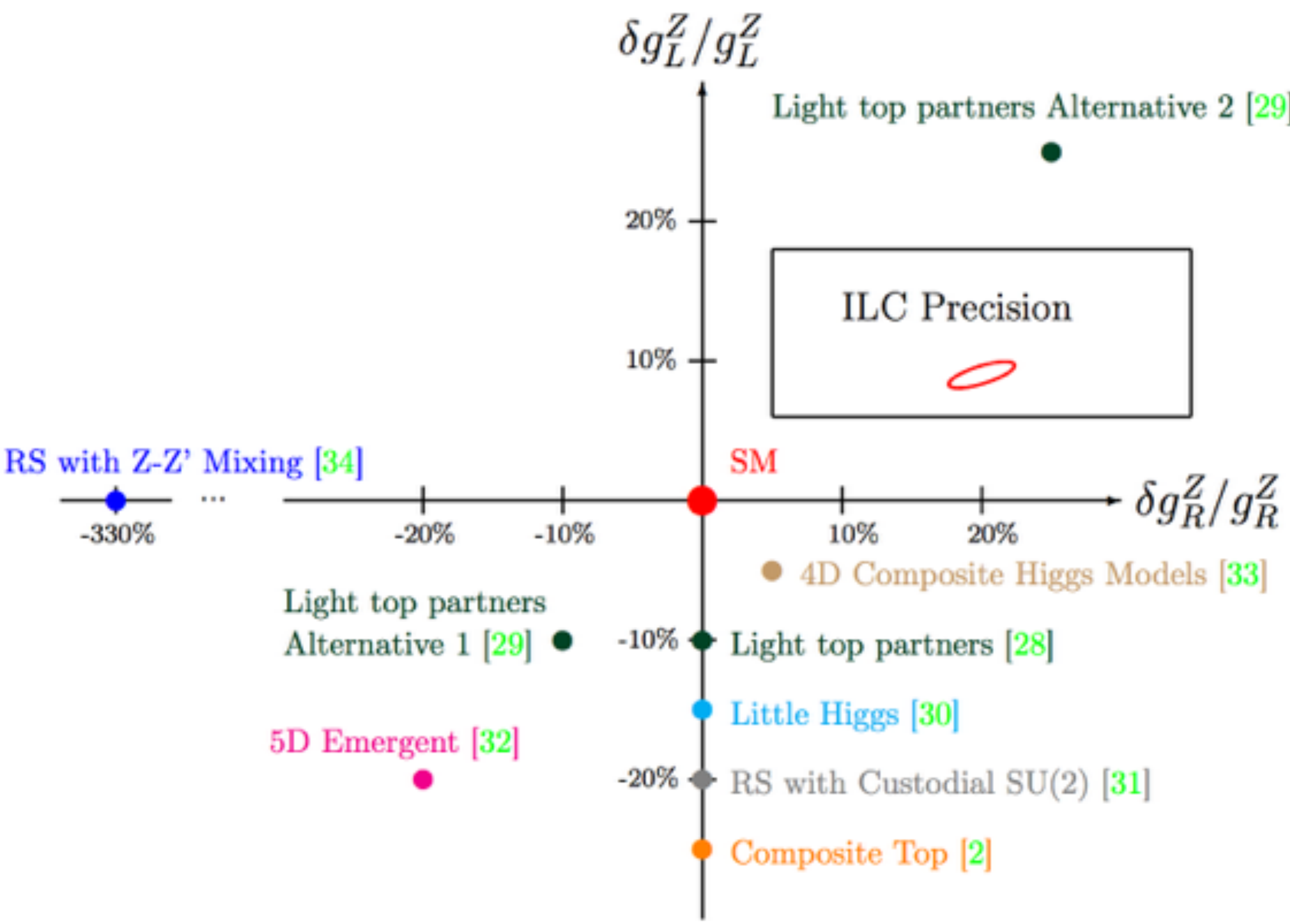
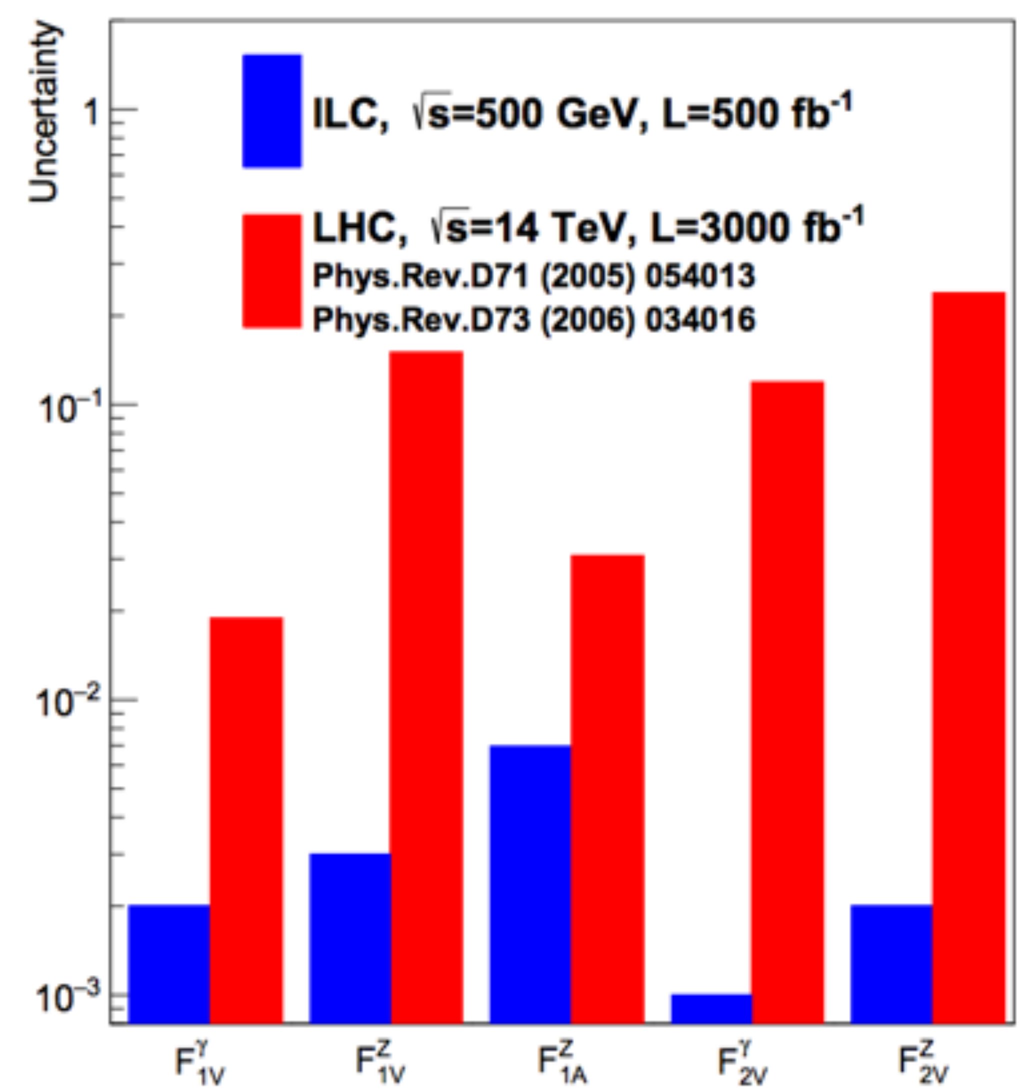
Perspective precision on top EW coupling from ILC

Amjad et al., EPCJ75(2015),10, 512

$$\Gamma_\mu^{t\bar{t}X}(k^2,q,\bar{q}) = ie\left\{\gamma_\mu\left(F_{1V}^X(k^2)+\gamma_5 F_{1A}^X(k^2)\right) - \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2)+\gamma_5 F_{2A}^X(k^2)\right)\right\}$$

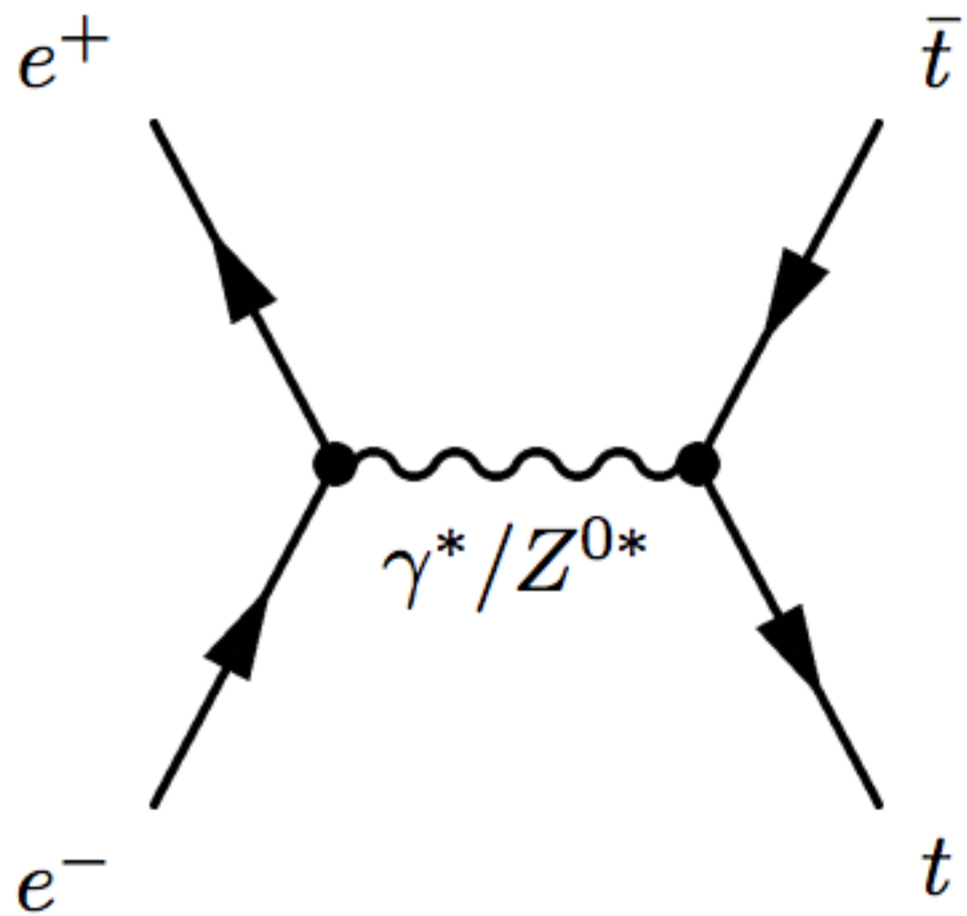




Theoretical effort on precision

- **QCD N3LO tT production at threshold [Beneke, et al., PRL115, 192001 (2015)]**
- **QCD N3LO inclusive cross section in high energy expansion up to $(m^2/s)^6$ [Chetyrkin et al., NPB503, 339 (1997)]**
- **Boosted top jet production at NLL [Fleming et al., PRD77(2008) 114003]**
- **One-loop EW corrections [Fleischer et al., EPJC31 (2003) 37]**
- **QCD NLO event generator including parton shower in WHIZARD**
- **One-loop EW corrections in GRACE**
- **…**

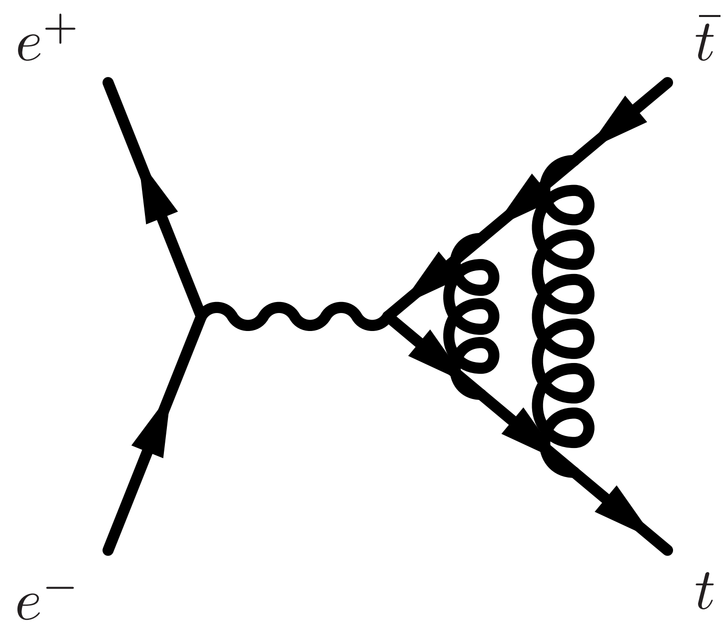
This talk: tT production in the continuum at QCD NNLO

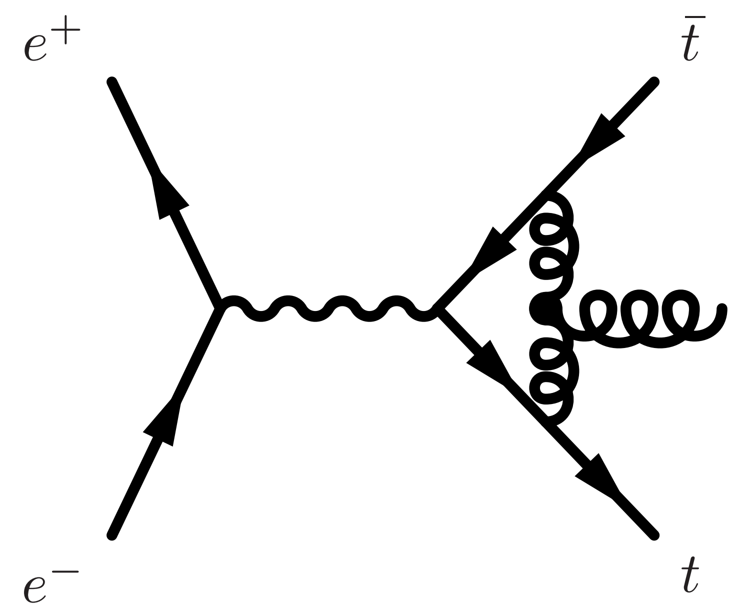


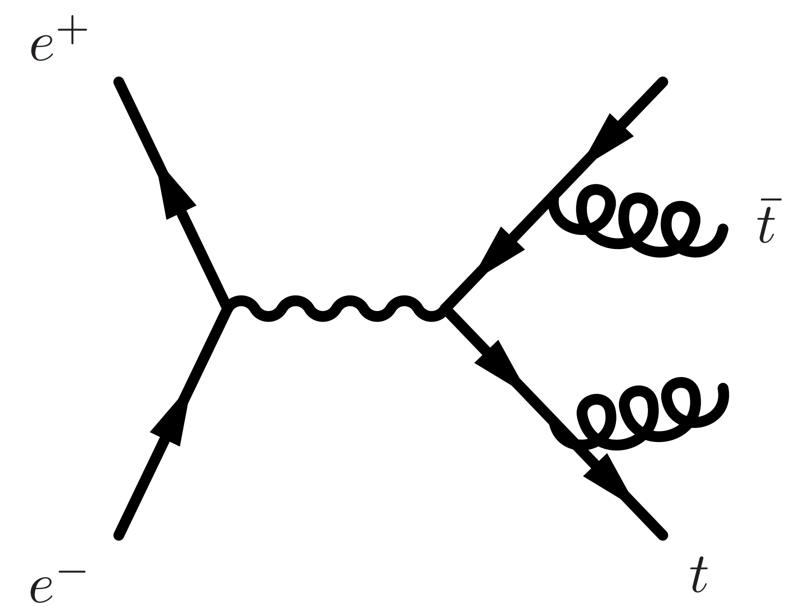
$$\sqrt{S} = 380, 500, 750, 1000? \text{ GeV}$$

Fully differential in top quark kinematics

tT production in the continuum at QCD NNLO







double virtual

two-loop heavy quark form factor

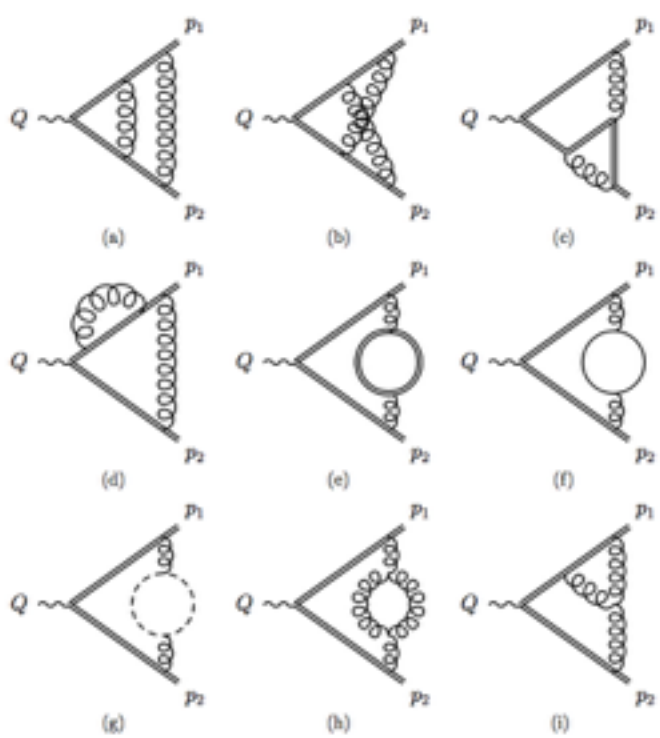
real-virtual **double real**

NLO QCD corrections to tT + jet production

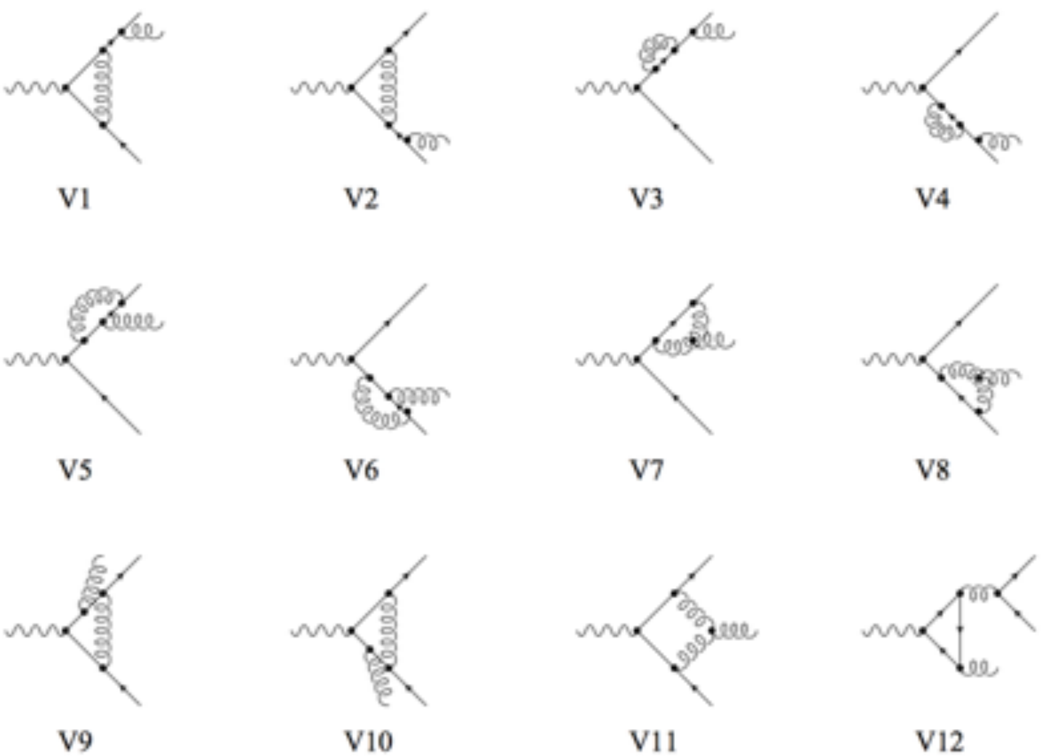
Two-loop heavy quark form factor

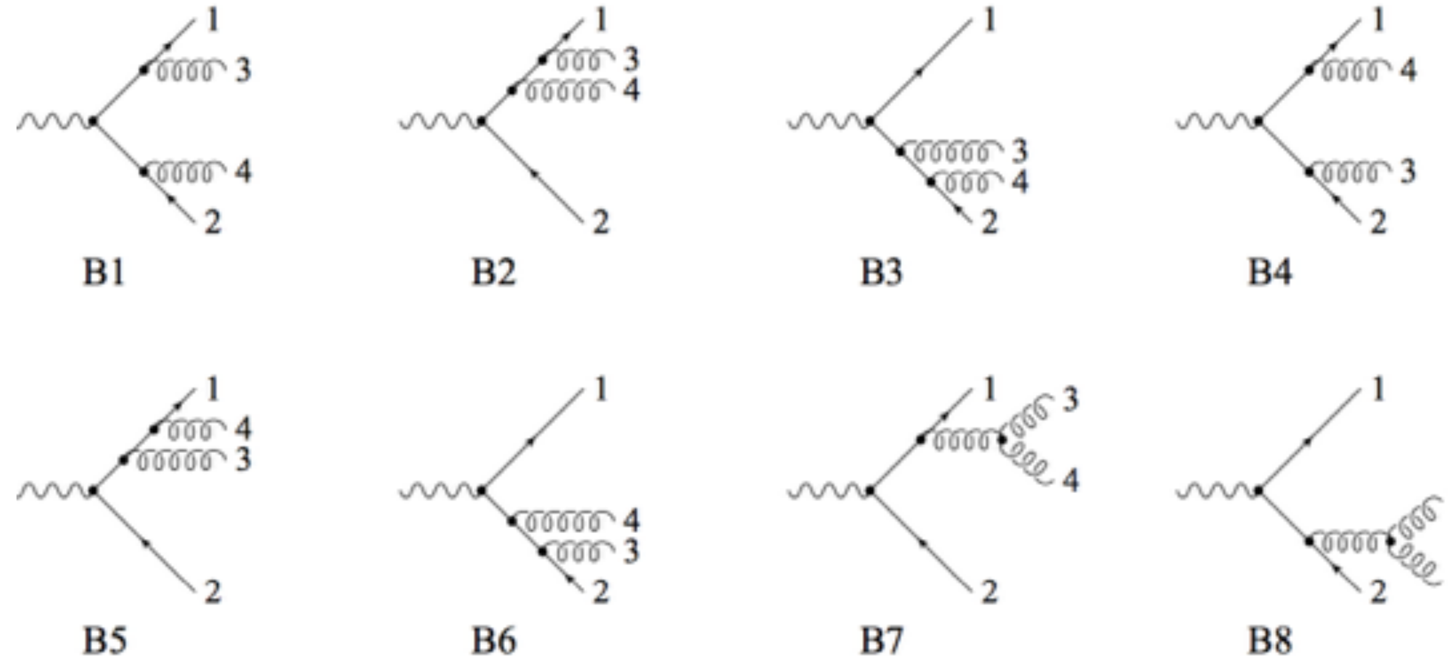
$$\Gamma_\mu^{t\bar{t}X}(k^2,q,\bar{q}) = ie\left\{\gamma_\mu\left(F_{1V}^X(k^2)+\gamma_5 F_{1A}^X(k^2)\right) - \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2)+\gamma_5 F_{2A}^X(k^2)\right)\right\}$$

- **The calculation of heavy quark form factor has been a subject of strong theoretical interest for a long time**
- **Two-loop contribution with closed fermion loops [Hoang et al., PLB338 (1994) 330]**
- **Full two-loop results available 10 years later [Bernreuther et al., NPB706, 245(2005); NPB712, 229(2005)]**
- **Application of many cutting-edge techniques at the time: integration-by-parts identities, Lorentz invariance, Laporta algorithm, method of differential equation**
- **Results written with 50 pages of harmonic polylogarithms.**



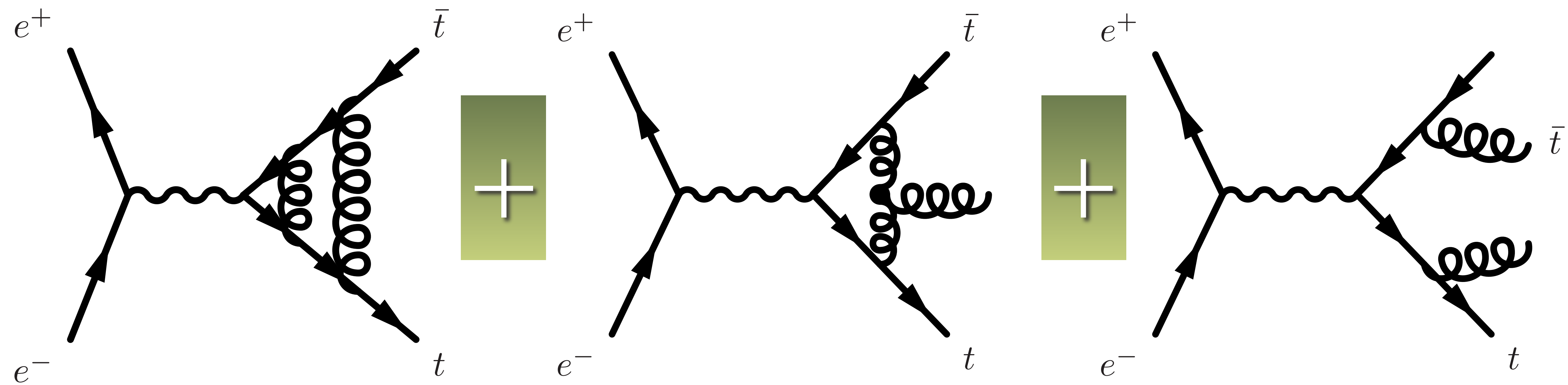
heavy quark pair + jet at NLO





- **NLO calculation available for about twenty years [Brandenburg, Uwer, NPB515(1998)279; Nason, Oleari, NPB521(1998)237; Rodrigo, Bilenky, Santamaria, NPB554(1999)257]**
- **These used to be difficult calculation. The remarkable progress in calculation technique for one-loop amplitude, and NLO subtraction of infrared/collinear singularity make such calculation "almost" trivial nowadays. For example can be automated using tools like Gosam.**

tT+X at QCD NNLO



double virtual

$$\frac{A_1}{\epsilon_{\mathrm{IR}}^2} + \frac{B_1}{\epsilon_{\mathrm{IR}}}$$

real-virtual

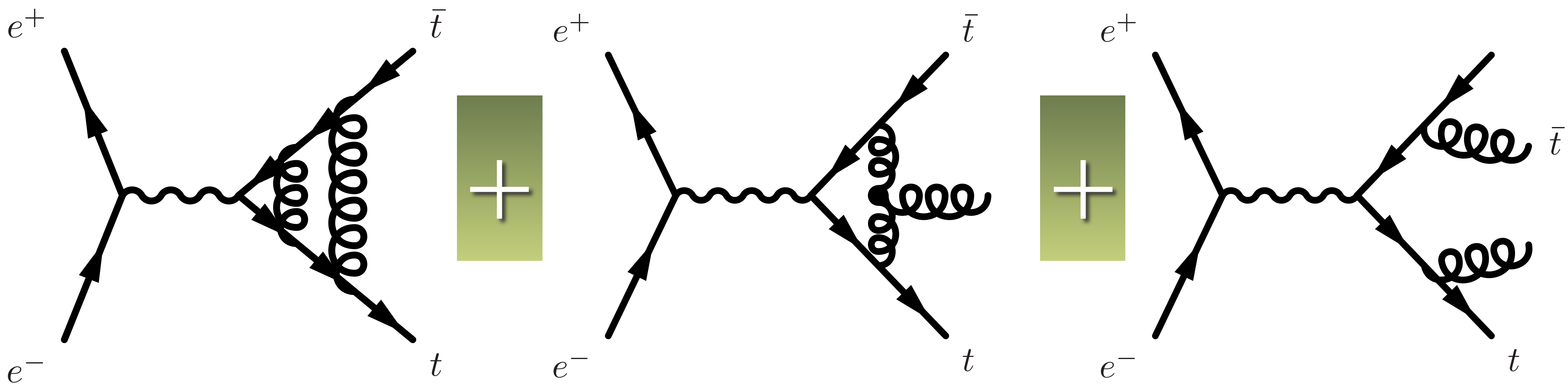
$$\frac{A_2}{\epsilon_{\mathrm{IR}}^2} + \frac{B_2}{\epsilon_{\mathrm{IR}}}$$

double real

$$\frac{A_3}{\epsilon_{\mathrm{IR}}^2} + \frac{B_3}{\epsilon_{\mathrm{IR}}}$$

For IR-safe cross section $\sum_i A_i = 0$ $\sum_i B_i = 0$

tT+X at QCD NNLO



double virtual **real-virtual** **double real**

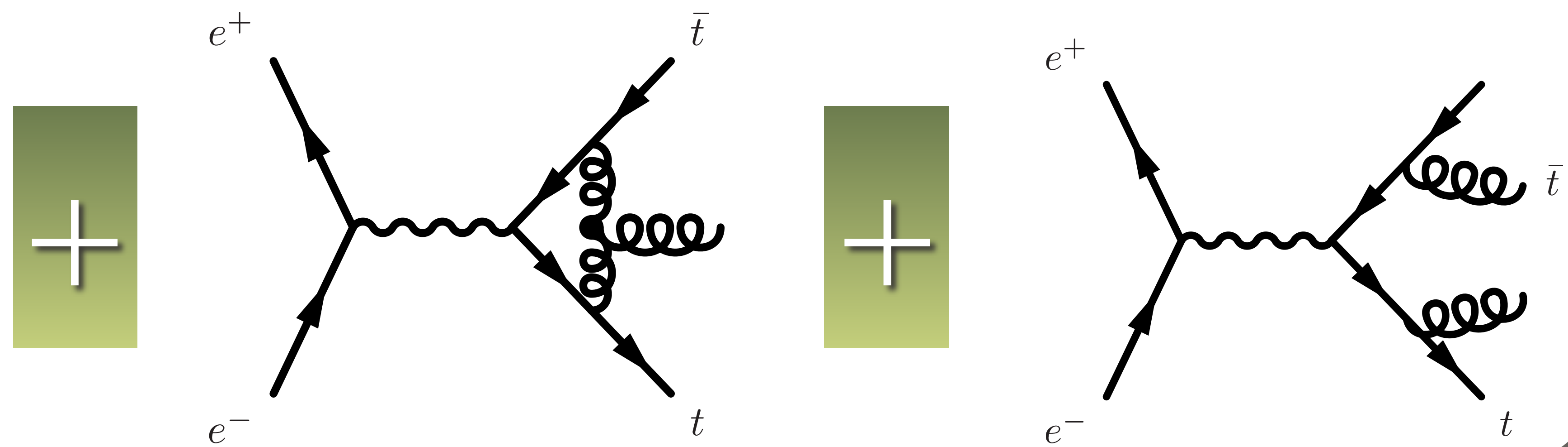
- **The infrared divergences origin from the real or virtual gluons become soft**
- **In the double virtual corrections, the IR poles are manifest, while in the real-virtual and double real they come from phase space integration**
- **The most successful method to deal with these poles at NLO is the subtraction, but at NNLO becomes too tedious**
- **Instead we generalize the more phase space slicing method to NNLO to overcome this problem**

Phase space slicing using radiation energy

von Manteuffel, Schabinger, H.X.Z, PRD92(2015)no.4,045034; J. Gao, H.X.Z, PRD90.114022; J. Gao, H.X.Z, PRL113(2014)262001

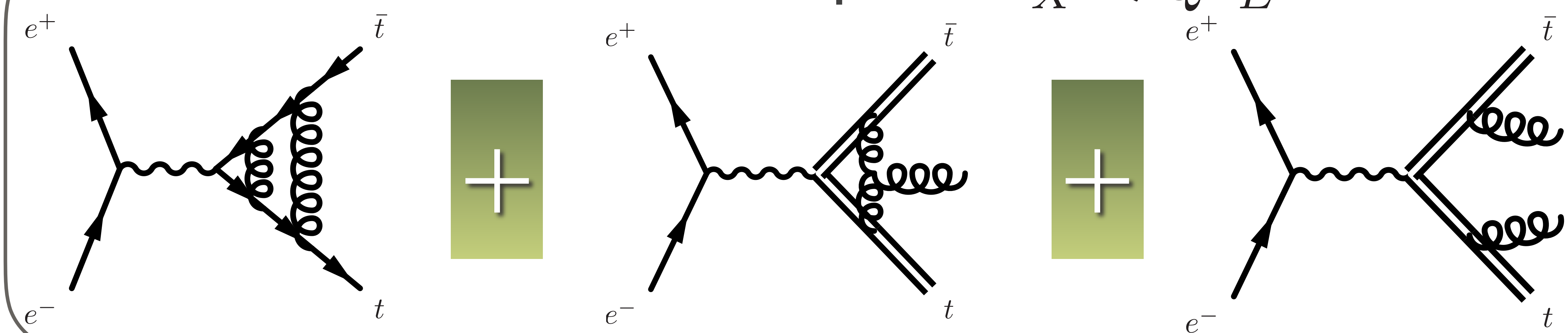
E_x: energy of QCD radiations from heavy quark pair

resolved part $E_X > Q\delta_E$



$\widetilde{C}_2 \ln^2 \delta_E + \widetilde{D}_2 \ln \delta_E$ $\qquad$ $\widetilde{C}_3 \ln^2 \delta_E + \widetilde{D}_3 \ln \delta_E$

unresolved part $E_X < Q\delta_E$



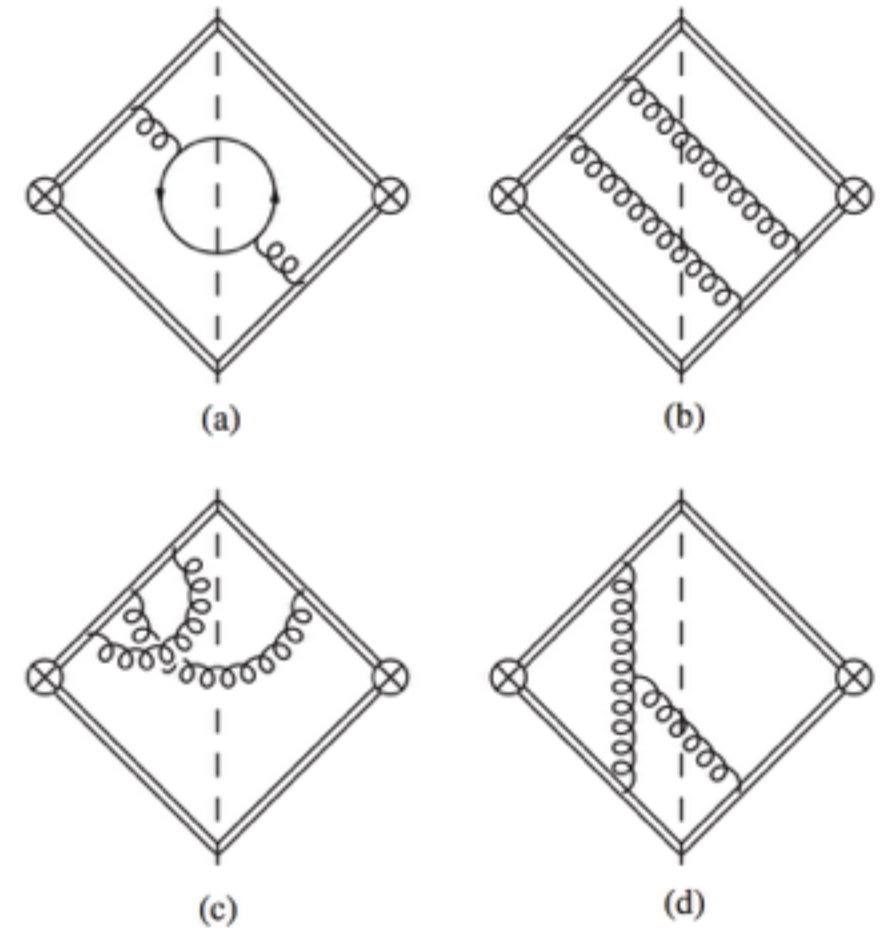
Classical color source moving at velocity v

$\frac{A_1}{\epsilon_{\rm IR}^2} + \frac{B_1}{\epsilon_{\rm IR}}$ $\qquad$ $\frac{A_2}{\epsilon_{\rm IR}^2} + \frac{B_2}{\epsilon_{\rm IR}} + \widetilde{A}_2 \ln^2 \delta_E + \widetilde{B}_2 \ln \delta_E$ $\qquad$ $\frac{A_3}{\epsilon_{\rm IR}^2} + \frac{B_3}{\epsilon_{\rm IR}} + \widetilde{A}_3 \ln^2 \delta_E + \widetilde{B}_3 \ln \delta_E$

Analytic calculation of the unresolved part

von Manteuffel, Schabinger, H.X.Z, PRD92(2015)no.4,045034

- The difficult phase space integral reduces to calculation of matrix element of time-like Wilson loop
- Can be treated analytically
- Results written in about two pages of harmonic polylogarithms

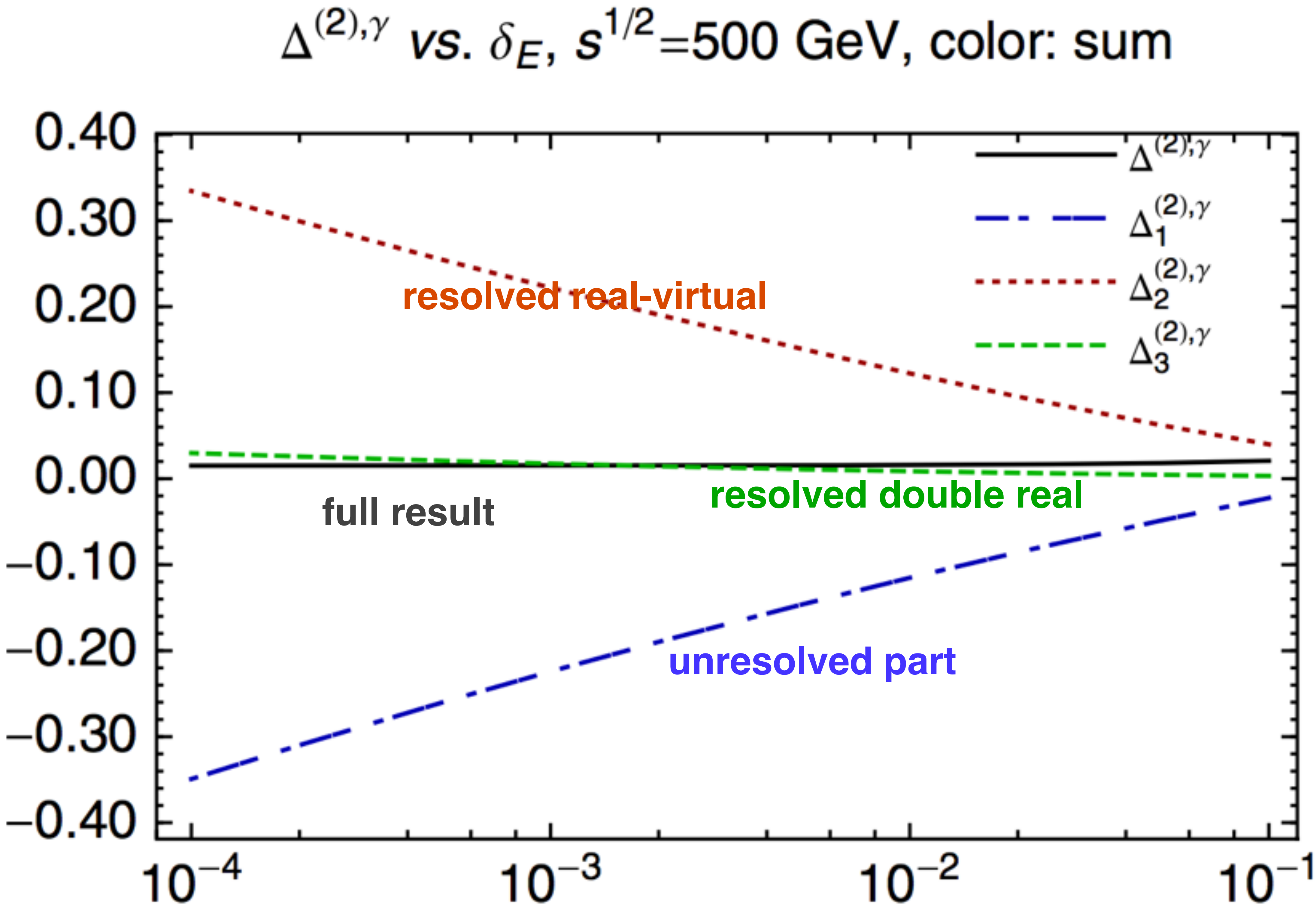


$$
\begin{aligned}
c_2(x) = C_F^2\Bigg[&2\frac{(1+x^2)^2}{(1-x^2)^2}\Bigg(G^4(0;x) - 8G(1;x)G^3(0;x) + 8\left(2G^2(1;x) + G(0,1;x) + \frac{\pi^2}{6}\right)G^2(0;x)\\
&-32\left(G(0,1;x)+\frac{\pi^2}{6}\right)G(1;x)G(0;x) + 16G^2(0,1;x) + \frac{16\pi^2}{3}G(0,1;x) + \frac{4\pi^4}{9}\Bigg)\\
&+8\frac{(1+x^2)}{(1-x)^2}\left(G^2(0;x) - 4G(1;x)G(0;x) + 4G(0,1;x) + \frac{2\pi^2}{3}\right)G(0;x) + 8\frac{(1+x)^2}{(1-x)^2}G^2(0;x)\Bigg]\\
&+C_F n_f T_F\Bigg[-\frac{224}{27} + \frac{16(1-3x+x^2)}{9(1-x^2)}\left((G(0;x) - 4G(1;x))G(0;x) + 4G(0,1;x) + \frac{2\pi^2}{3}\right)\\
&+\frac{8(1+x^2)}{3(1-x^2)}\Bigg(-\frac{1}{3}G^3(0;x) + 2G(1;x)G^2(0;x) - 4\left(G^2(1;x) + \frac{\pi^2}{6}\right)G(0;x) + \frac{4\pi^2}{3}G(1;x)\\
&+8G(1;x)G(0,1;x) - 4G(0,0,1;x) - 8G(0,1,1;x) + 4\zeta(3)\Bigg) + \frac{16(1+30x+x^2)}{27(1-x^2)}G(0;x)\Bigg]\\
&+C_A C_F\Bigg[-\frac{2(11x^4-84x^3+24x^2+12x-11)}{9(1-x^2)^2}G^3(0;x) + \frac{4(55x^2-294x+55)}{27(1-x^2)}G(0;x)\\
&+\frac{4(17x^3-32x^2+68x-17)}{9(1-x^2)(1-x)}G^2(0;x) + \frac{8\pi^2(5x^4+27x^3+6x^2+3x+7)}{9(1-x^2)^2}G(0;x)\\
&-4\zeta(3)\frac{(1+x^2)(9+x^2)}{(1-x^2)^2}G(0;x) + \frac{4\pi^2(11x^2+66x-25)}{27(1-x^2)} + \frac{\pi^4(1+x^2)(77+221x^2)}{90(1-x^2)^2}\\
&+\frac{16(26x^2-33x+26)}{9(1-x^2)}(G(0;x)G(1;x) - G(0,1;x)) + \frac{8\pi^2(1+x^2)(3+7x^2)}{3(1-x^2)^2}(G(0,1;x)\\
&-G(0;x)G(1;x)) + 8\frac{3x^3-7x^2-5x+1}{(1-x^2)(1-x)}(G(-1;x)G^2(0;x) - 2G(0,-1;x)G(0;x)\\
&+2G(0,0,-1;x)) + 16\frac{x^3-x^2+3x+1}{(1-x^2)(1+x)}(2G(0,0,-1;x) - G(0;x)G(0,-1;x))\\
&+\frac{8(13x^4-72x^2+11)}{3(1-x^2)^2}(2G(0,0,1;x) - G(0;x)G(0,1;x)) - 8\frac{(1+x^2)(3x^2-1)}{(1-x^2)^2}\\
&\times(6G(0,0,0,-1;x) + G(0,-1;x)G^2(0;x) - 4G(0,0,-1;x)G(0;x))\\
&+\frac{1+x^2}{1-x^2}\Bigg(\frac{8}{3}G(-1;x)G^3(0;x) + \left(\frac{4}{3}G(1;x) - 8G(0,-1;x)\right)G^2(0;x) + (24G^2(-1;x)\\
&-16G(1;x)G(-1;x) - \frac{16\pi^2}{3}G(-1;x) - 16G(-1;x) + \frac{16}{3}G^2(1;x) - \frac{8}{3}G(0,1;x)\Bigg)G(0;x)\\
&-\frac{4\pi^2}{3}G(-1;x) - \frac{100\pi^2}{9}G(1;x) - 48G(-1;x)G(0,-1;x) + 16G(1;x)G(0,-1;x)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{16\pi^2}{3}G(0,-1;x) + 16G(0,-1;x) + 16G(-1;x)G(0,1;x) - \frac{32}{3}G(1;x)G(0,1;x)\\
&+48G(0,-1,-1;x) - 16G(0,-1,1;x) + \frac{8}{3}G(0,0,1;x) - 16G(0,1,-1;x) + \frac{32}{3}G(0,1,1;x)\\
&+32G(0,0,0,-1;x)) + \frac{(1+x^2)(1+3x^2)}{(1-x^2)^2}\Bigg(\frac{8}{3}G(1;x)G^3(0;x) + 16G(0,0,1;x)G(0;x)\\
&+\left(\frac{2\pi^2}{3} - 8G(0,1;x)\right)G^2(0;x) - 16G(0,0,0,1;x)\Bigg) + \frac{x^2(1+x^2)}{(1-x^2)^2}\Bigg(-\frac{4}{3}G^4(0;x)\\
&+96G(0,0,1;x)G(0;x) - 288G(0,0,0,1;x)\Bigg) + \frac{(1+x^2)^2}{(1-x^2)^2}(16G(0,1;x)G^2(0;x)\\
&+(16G(0,1,-1;x) - 64G(0,0,1;x) - 32G(1;x)G(0,1;x) + 48G(0,-1,-1;x)\\
&-32G(1;x)G(0,-1;x) + 16G(0,-1,1;x) + 16G(0,1,1;x))G(0;x) - 24G^2(0,-1;x)\\
&+8G^2(0,1;x) - \frac{4\pi^2}{3}G(0,-1;x) - \frac{4\pi^2}{3}G(0,1;x) + 16G(0,-1;x)G(0,1;x)\\
&+64G(1;x)G(0,0,-1;x) + 64G(1;x)G(0,0,1;x) - 64G(0,0,-1,1;x) + 96G(0,0,0,1;x)\\
&-64G(0,0,1,-1;x) + 16G(1;x)\zeta(3) - 64G(0,0,1,1;x) - 32G(0,1,0,-1;x))\\
&+\frac{4\zeta(3)(13x^4-12x^2-49)}{3(1-x^2)^2} + \frac{592}{27}\Bigg].
\end{aligned}
$$

Validating the calculation: cancellation of slicing parameter δ_E

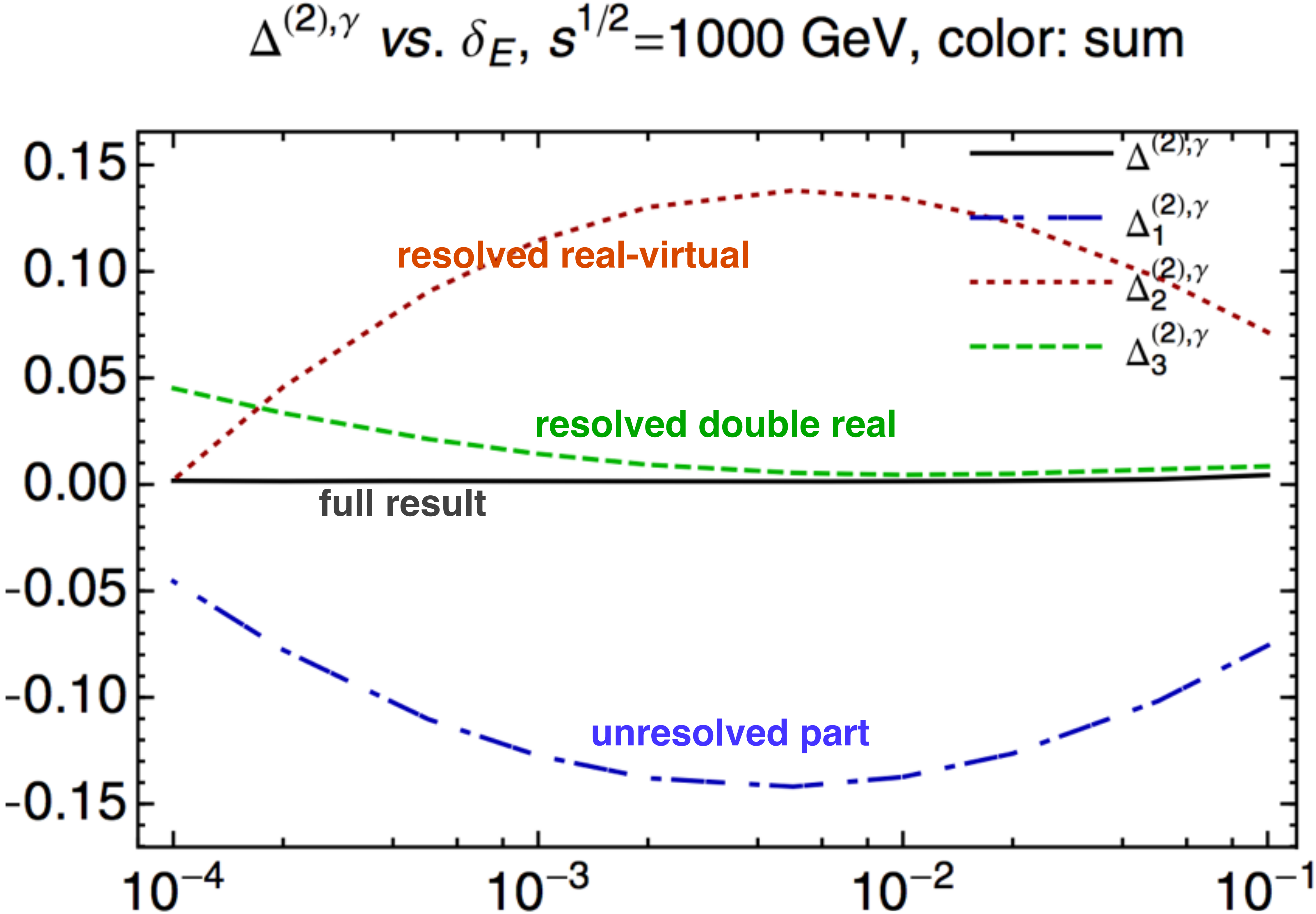
J. Gao, H.X.Z, PRD90.114022



$$\sigma_{\mathrm{NNLO},\gamma} = \sigma_{\mathrm{LO},\gamma}(1 + \Delta^{(1),\gamma} + \Delta^{(2),\gamma})$$

Validating the calculation: cancellation of slicing parameter δ_E

J. Gao, H.X.Z, PRD90.114022

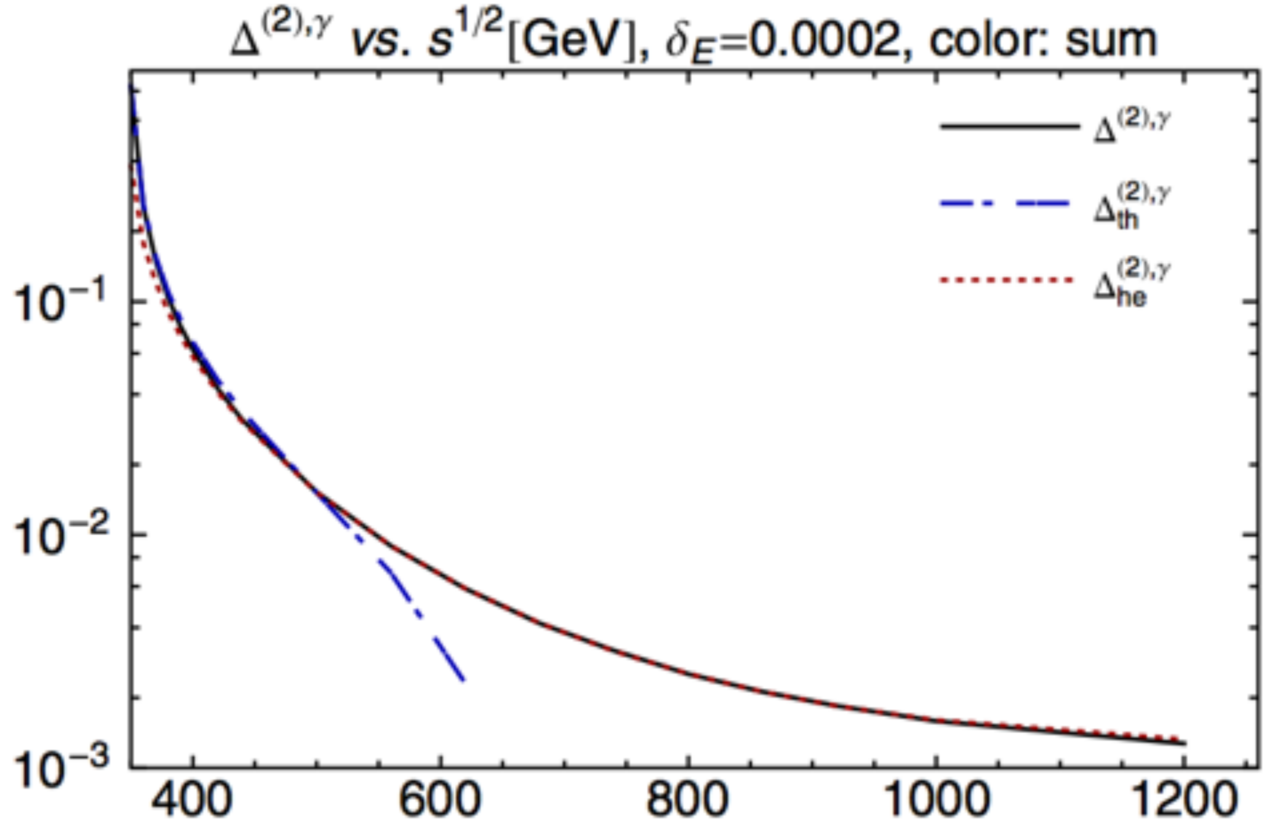


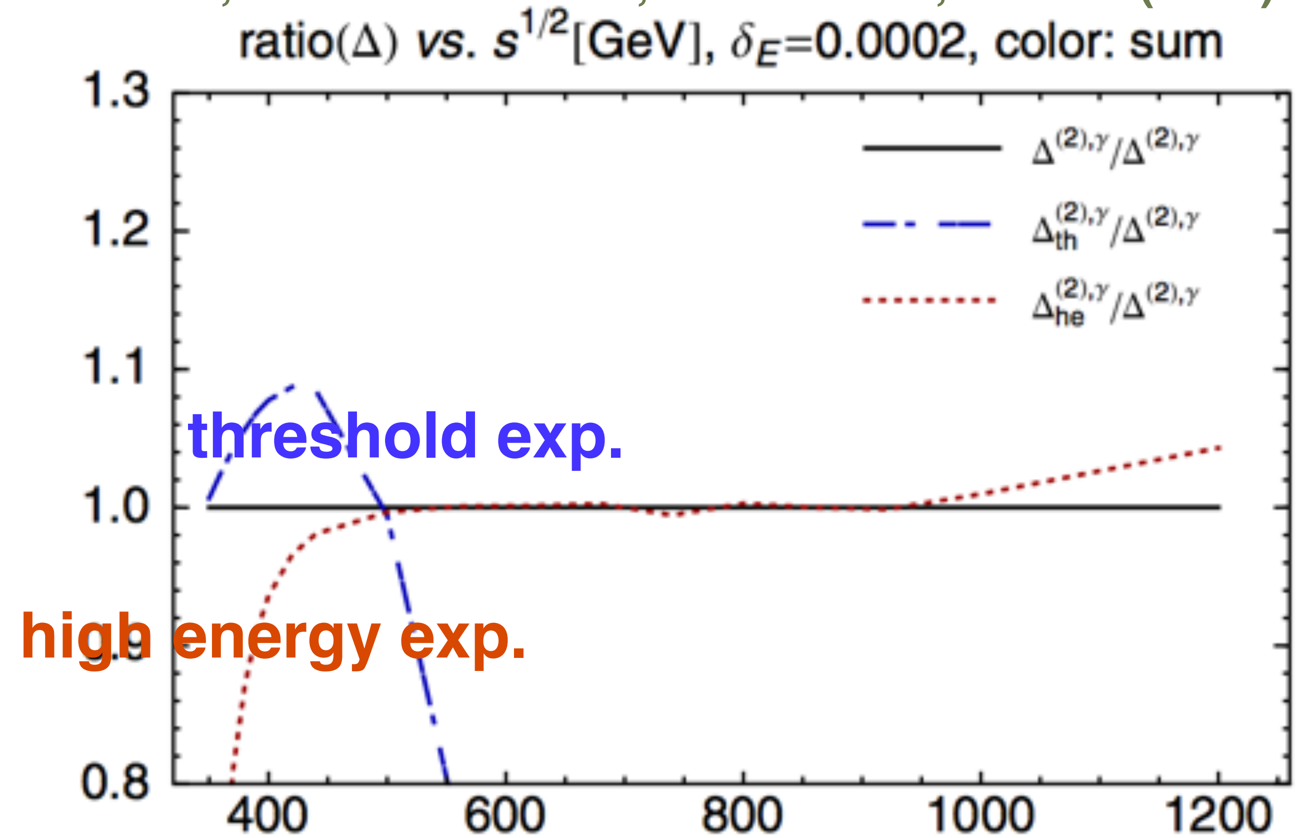
$$\sigma_{\mathrm{NNLO},\gamma} = \sigma_{\mathrm{LO},\gamma}(1 + \Delta^{(1),\gamma} + \Delta^{(2),\gamma})$$

Inclusive Xsec: compare with threshold and high energy expansion

$$\sigma_{\mathrm{NNLO},\gamma} = \sigma_{\mathrm{LO},\gamma}(1 + \Delta^{(1),\gamma} + \Delta^{(2),\gamma})$$

J. Gao, H.X.Z, PRD90.114022; see also Dekkers, Bernreuther, PLB738(2014)325



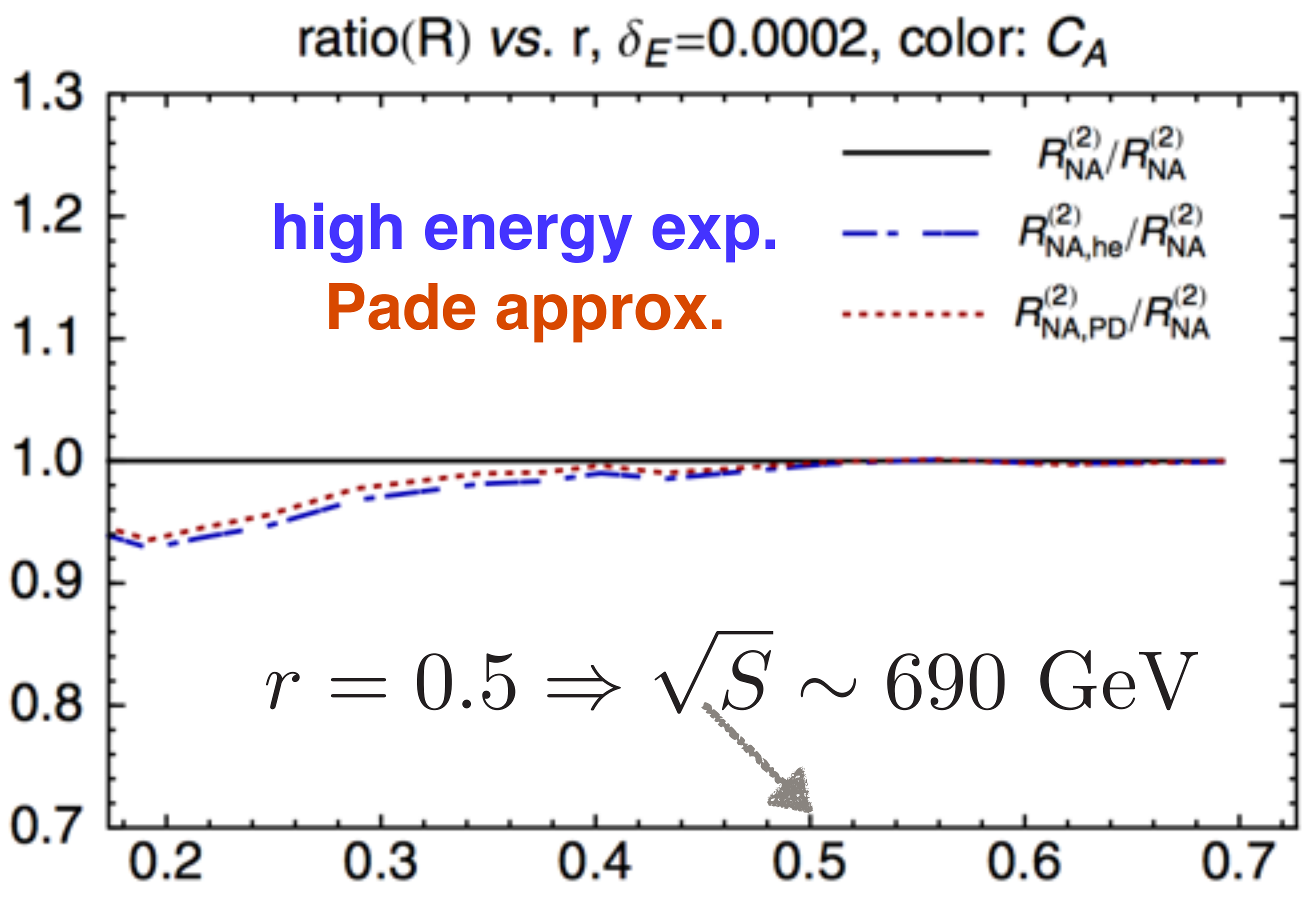


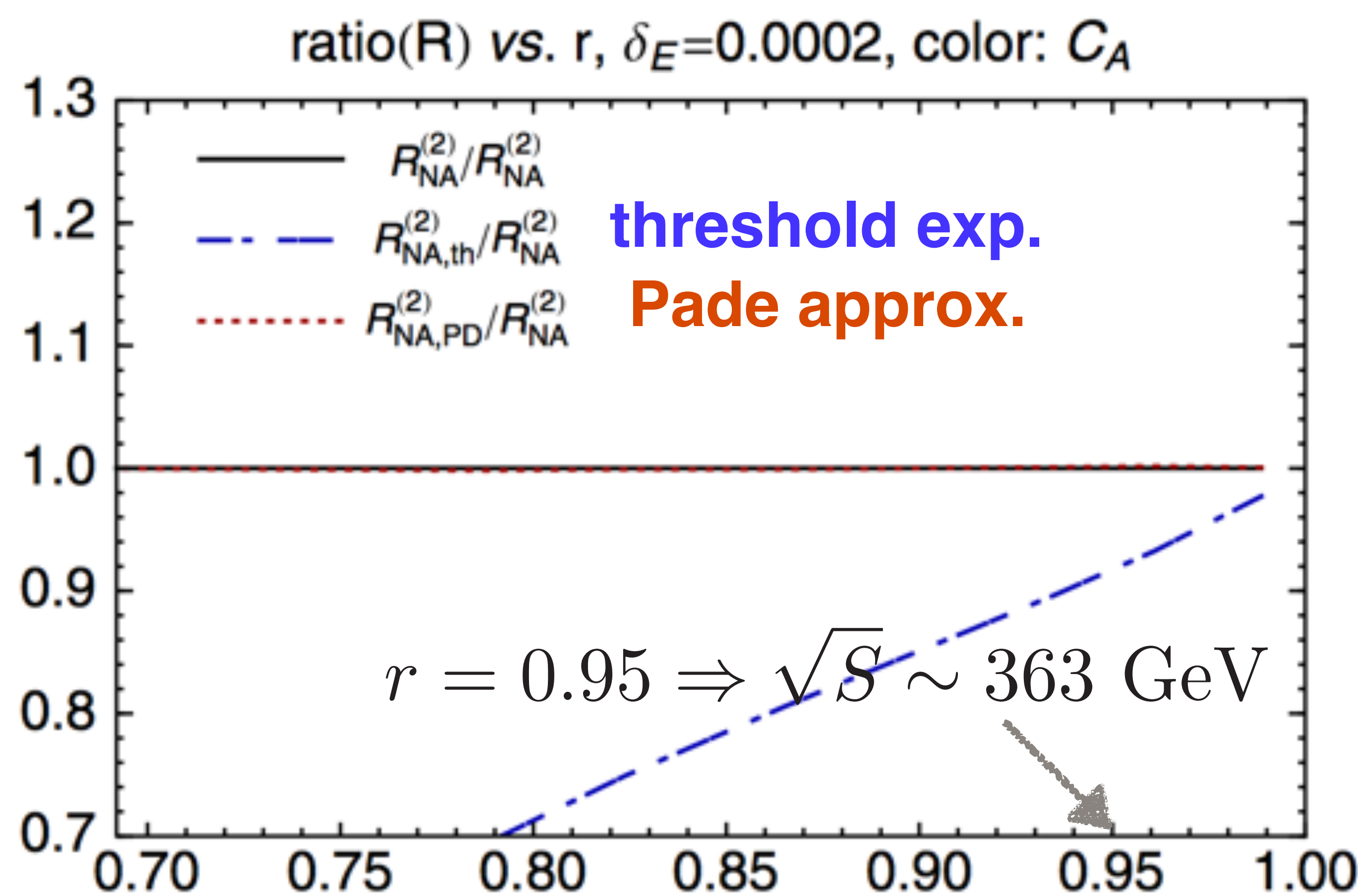
- **Two-loop threshold expansion:** [Czarnecki, Melnikov, 1998; Beneke, Signer, Smirnov, 1998; Hoang, Teubner, 1998…]
- **Two-loop high energy expansion:** [Chetyrkin, Harlander, Kuhn, Steinhauser, 1997]
- **Small corrections, every contribution matter!**
- **Fully closed fermion loop contribution very helpful in checking our calculation** [Hoang, Teubner, NPB519 (1998) 285]

Comparison with Pade approximation

J. Gao, H.X.Z, PRD90.114022

$$\sigma_{\mathrm{NNLO},\gamma} = \sigma_{\mu^+\mu^-,\gamma}\left(R^{(0)} + \frac{\alpha_s(\mu^2)}{\pi}C_F R^{(1)} + \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2 R^{(2)}\right) \qquad r = 2m_t/\sqrt{S}$$



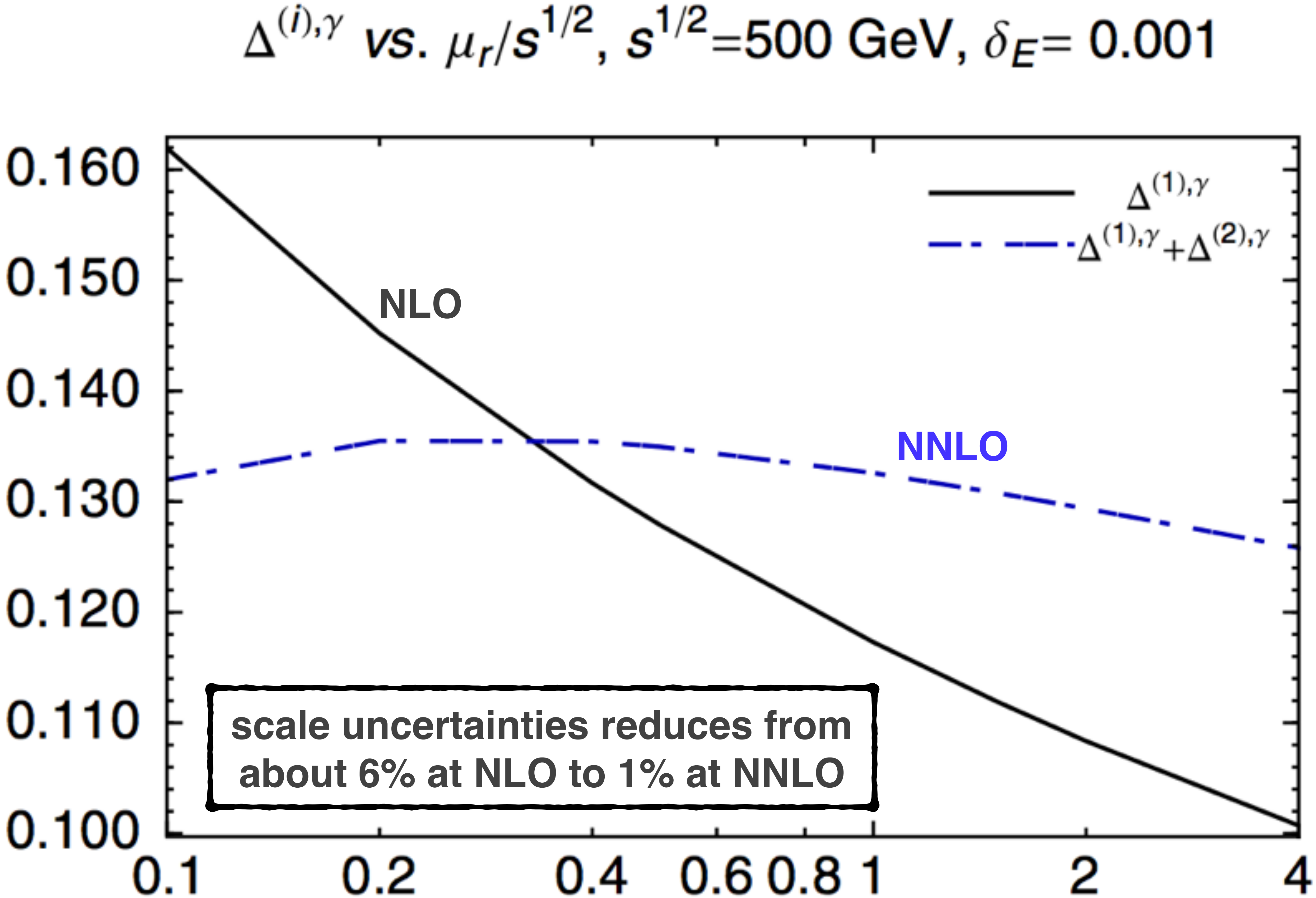


- **Two-loop Pade approximation:** [Chetyrkin, J. H. Kuhn, and M. Steinhauser, NPB482, 213(1996)]. **Three-loop also available** [Kiyo et al., NPB823(2009)269]
- **In general good agreement exact at very high energy S>(1000GeV)^2. Large power corrections proportional to δ_E*logδ_E*log^2(S/m_t^2) in the numerical calculation**

NNLO reduces scale uncertainties

J. Gao, H.X.Z, PRD90.114022

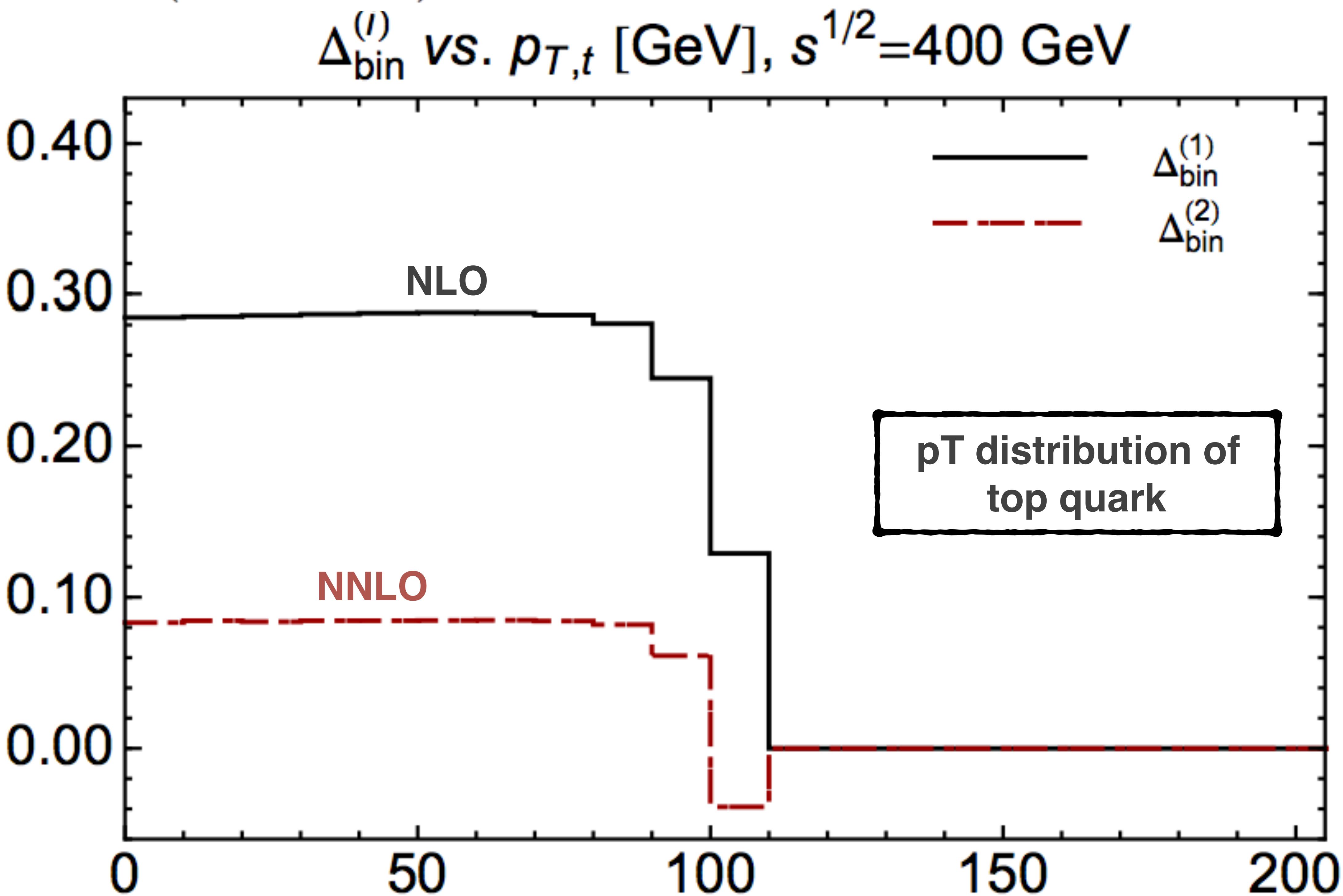
$$\sigma_{\mathrm{NNLO},\gamma} = \sigma_{\mathrm{LO},\gamma}(1 + \Delta^{(1),\gamma} + \Delta^{(2),\gamma})$$



Differential distribution at NNLO

$$\sigma_{NNLO} = \sigma_{LO}\left(1 + \Delta^{(1)} + \Delta^{(2)}\right)$$

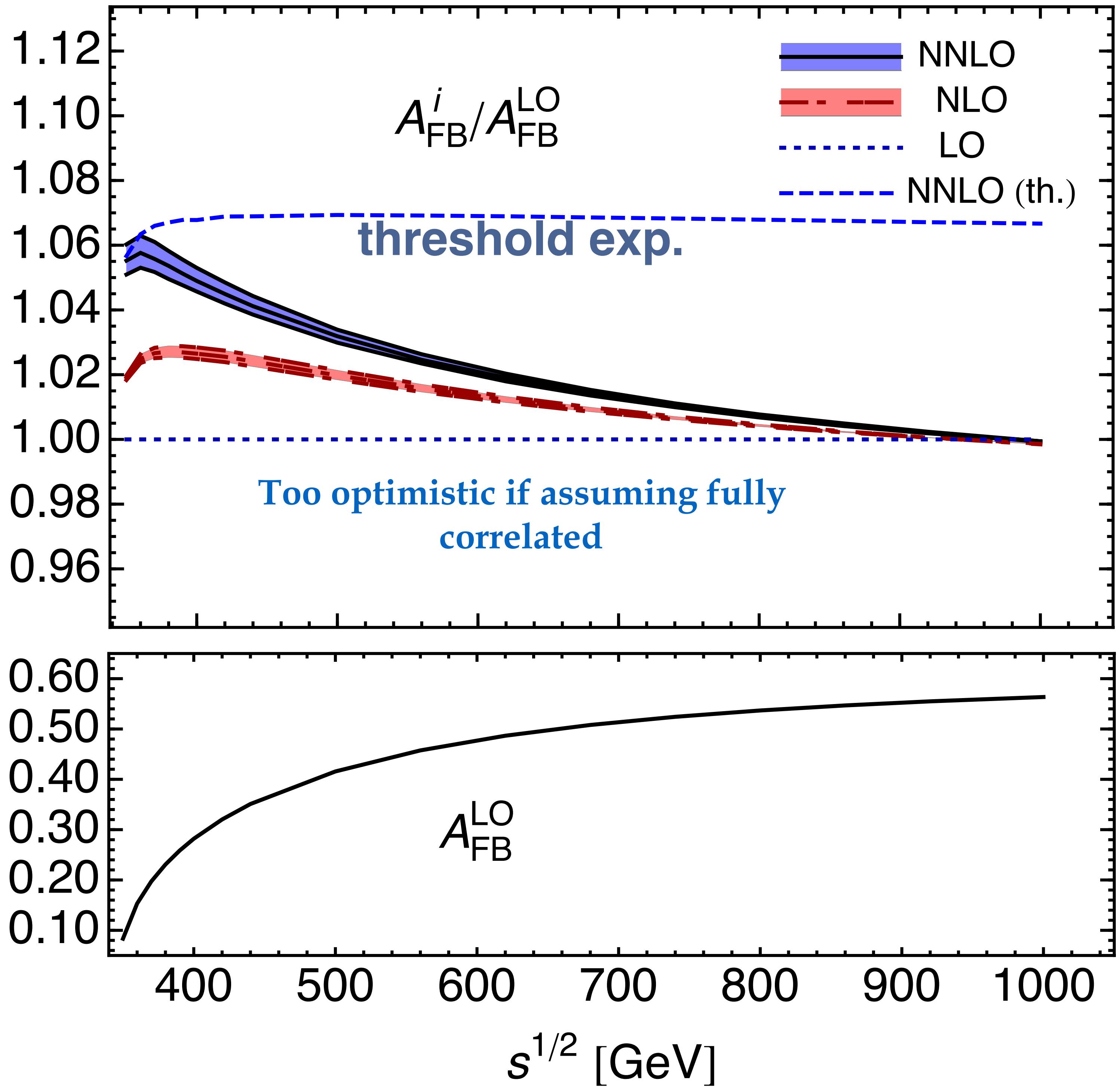
J. Gao, H.X.Z, PRL113(2014)262001



NNLO corrections to forward-backward asymmetry

J. Gao, H.X.Z, PRL113(2014)262001

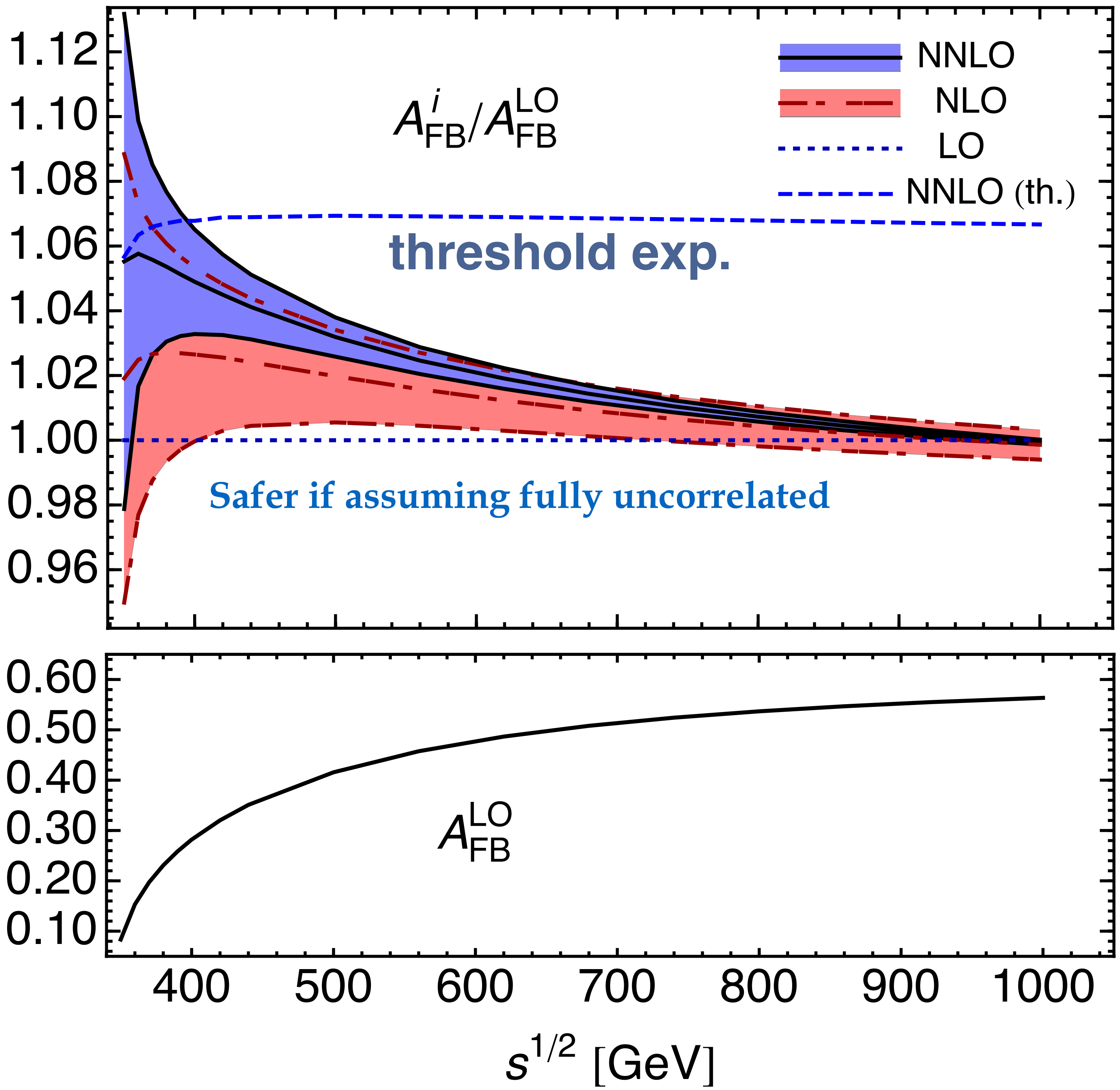
$$A_{FB} = \frac{\sigma_A}{\sigma_S} \equiv \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)}$$



- **Estimation of the theoretical uncertainties from scale variations requires assumption on correlations of the Forward and Backward bins**
- **Fully correlated scale setting leads to too small uncertainties**

NNLO corrections to forward-backward asymmetry

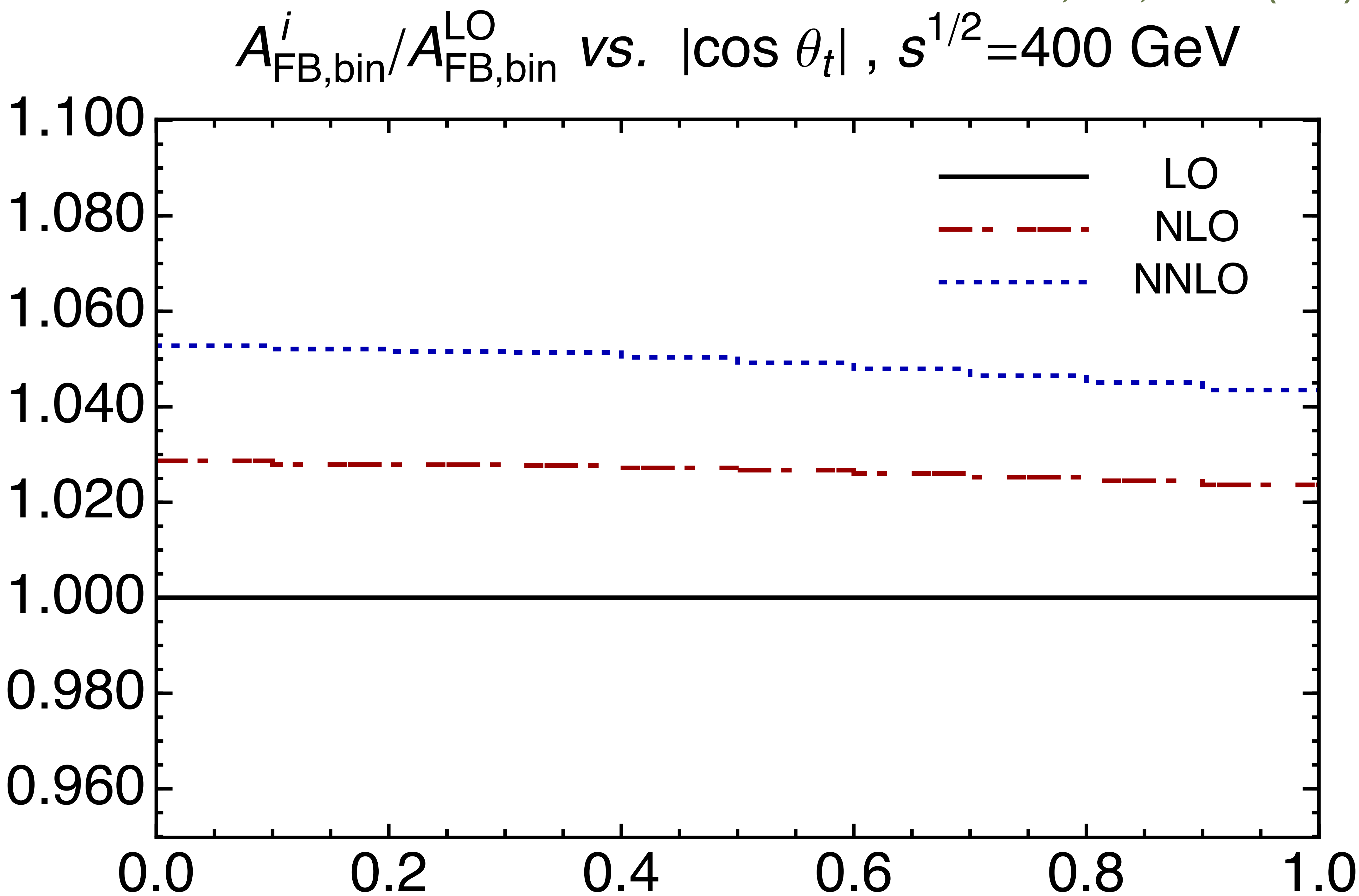
J. Gao, H.X.Z, PRL113(2014)262001



- **Uncorrelated scale setting leads to more realistic uncertainties estimate**
- **Large NNLO QCD corrections even at √S=500GeV (about half the size of NLO corrections)**
- **NLO EW corrections about 10% [Fleischer et al., EPJC31 (2003) 37]**
- **NNLO EW corrections highly desirable**

Forward-Backward asymmetry bin-by-bin

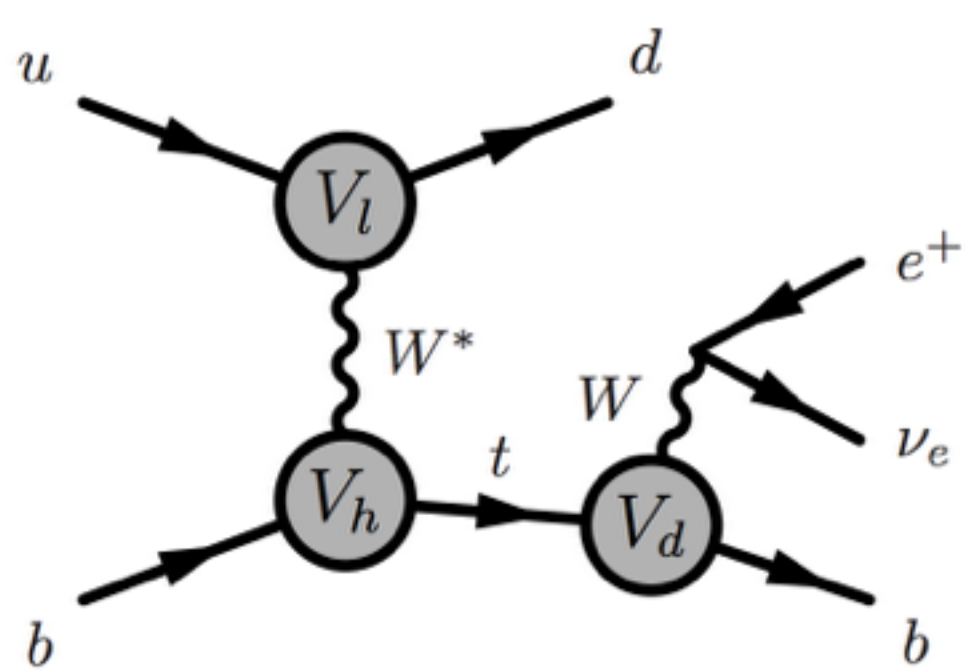
J. Gao, HXZ, PRL113(2014)262001



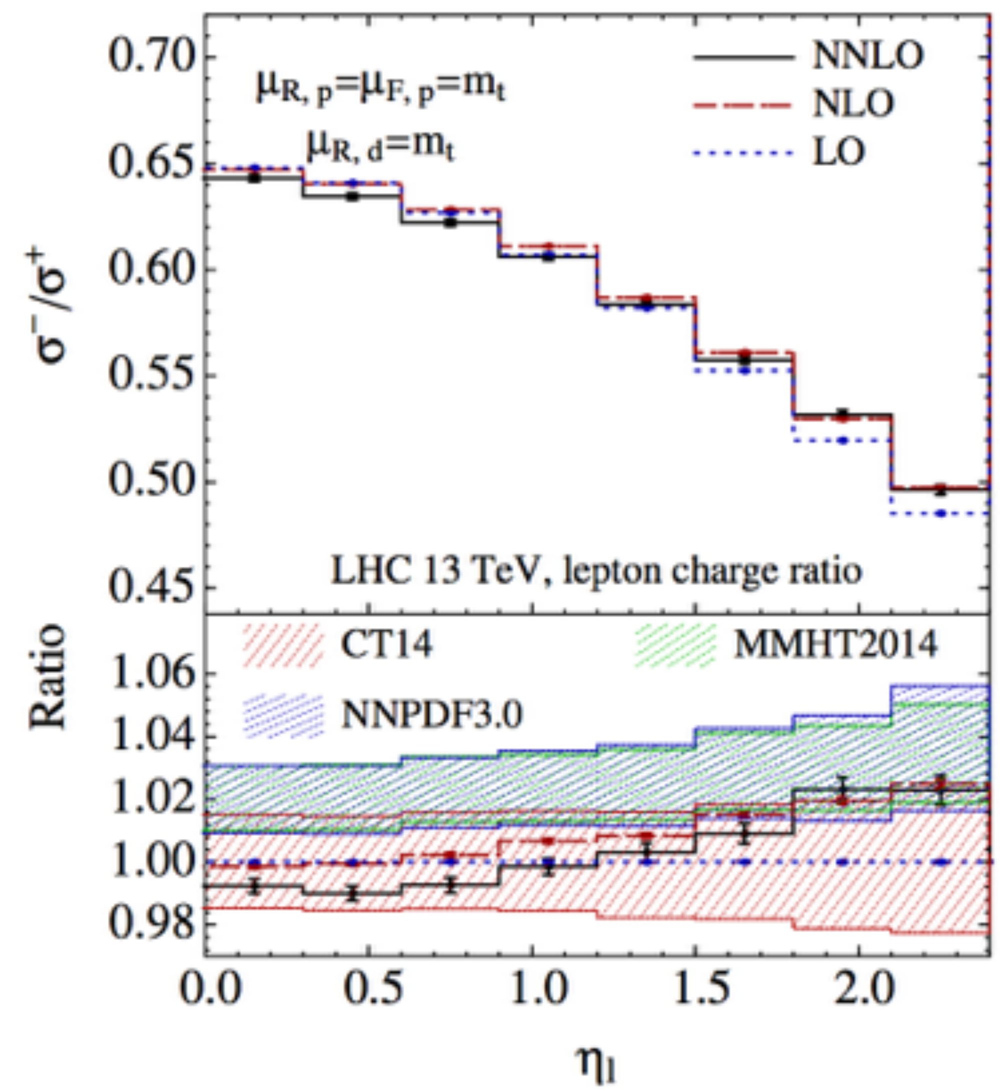
- Our results provide full kinematic dependence allowing for corrections of experimental acceptance

Going beyond stable top production

- **Now NNLO QCD corrections to tT production at LC, and NNLO QCD corrections to top decay are available**
- **It would be interesting to combine production and decay at NNLO to allow full spin correlation and parton level fiducial cross section**
- **Example: first t-channel single top production and decay at NNLO**



Structure function approximation



Berger, Gao, Yuan, HXZ, 1606.08463

Summary

- Precision calculation for top pair production at LC has long been a theoretical arena.
- Full NNLO QCD corrections for tT production in the continuum now available after many years of effort of different groups
- Large NNLO corrections to FB asymmetry (half the size of NLO corrections). Uncertainties reduce to 2% at $\sqrt{S}$=500GeV with conservative estimate
- With the current accuracy of QCD prediction, two-loop EW corrections become highly important and might become the driving force of further theoretical progress in the years to come

Thanks a lot for listening!

Backup slide

$$\sigma_{NNLO} = \sigma_{LO}\left(1 + \Delta^{(1)} + \Delta^{(2)}\right)$$

J. Gao, H.X.Z, PRL113(2014)262001