Effective field theory for top physics at LC

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(DESY)

EFT for $e^+e^- \to t\,\bar{t}$ under study
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Top physics at LC
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The EFT parametrization of NP

If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, [...] the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry.

[Weinberg 79]

– is quite general

– builds upon the SM, i.e., our knowledge lower-energy physics

– must be global to be consistent

– is useful only in a low-energy limit

– establishes a hierarchy between NP effects

– is a proper QFT, perturbatively improvable

The fermionic SM EFT

Use: – SM fields (fermion gauge eigenstates: q, u, d, l, e)
– SM symmetries (gauge and Lorentz)

dim-3 · no allowed fermion mass term: —

dim-4 · gauge: $\bar{\psi} \not{D} \psi$ and Yukawa: $\bar{\psi}\varphi\psi'$ operators

dim-5 · left-handed neutrino masses ($\Delta L = \pm 2$): $\overline{l^c}\varphi \; l\varphi$

dim-6 · four-fermion ($\Delta L = \Delta B = \pm 1$, or 0) [Grzadkowski et al 10']

basis reduction with Fierz and Schouten identities

· two-fermion:

D	φ		
3	0	—	
2	1	$\bar{\psi}\sigma^{\mu\nu}\psi'\varphi \quad X_{\mu\nu}$	Tensor
1	2	$\bar{\psi}\gamma^{\mu}\psi \quad \varphi^{\dagger}D_{\mu}\varphi$	Vector
0	3	$\bar{\psi}\psi'\varphi \quad \varphi^{\dagger}\varphi$	Scalar

basis reduction with EOMs

dim-7 · $\Delta L \neq 0$: ... [Lehman 14']

...

The up-sector neutral current operators

[Grzadkowski et al 10']

Two-quark operators:

Scalar: $O_{u\varphi} \equiv \bar{q} u\, \tilde{\varphi} \quad (\varphi^\dagger \varphi - v^2/2),$

Vector:

$$
\begin{aligned}
O^1_{\varphi q} &\equiv \bar{q}\gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi &&\equiv [O^+_{\varphi q} + O^-_{\varphi q}]/2,\\
O^3_{\varphi q} &\equiv \bar{q}\gamma^\mu \tau^I q \quad \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi, &&\equiv [O^+_{\varphi q} - O^-_{\varphi q}]/2 \quad \text{(charged current also)}\\
O_{\varphi u} &\equiv \bar{u}\gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi, &&\\
O_{\varphi \varphi} &\equiv \bar{u}\gamma^\mu d \quad \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi, && \text{(charged current only)}
\end{aligned}
$$

Tensor:

$$
\begin{aligned}
O_{uB} &\equiv \bar{q}\sigma^{\mu\nu} u\, \tilde{\varphi} \quad B_{\mu\nu},\\
O_{uW} &\equiv \bar{q}\sigma^{\mu\nu} \tau^I u\, \tilde{\varphi} \quad W^I_{\mu\nu},\\
O_{uG} &\equiv \bar{q}\sigma^{\mu\nu} T^A u\, \tilde{\varphi} \quad G^A_{\mu\nu}.
\end{aligned}
$$

Two-quark–two-lepton operators:

Scalar: $O^1_{lequ} \equiv \bar{l} e \;\varepsilon\; \bar{q} u,$

Vector:

$$
\begin{aligned}
O^1_{lq} &\equiv \bar{l}\gamma_\mu l \quad \bar{q}\gamma^\mu q &&\equiv [O^+_{lq} + O^-_{lq}]/2,\\
O^3_{lq} &\equiv \bar{l}\gamma_\mu \tau^I l \quad \bar{q}\gamma^\mu \tau^I q &&\equiv [O^+_{lq} - O^-_{lq}]/2,\\
O_{lu} &\equiv \bar{l}\gamma_\mu l \quad \bar{u}\gamma^\mu u, &&\\
O_{eq} &\equiv \bar{e}\gamma^\mu e \quad \bar{q}\gamma_\mu q, &&\\
O_{eu} &\equiv \bar{e}\gamma_\mu e \quad \bar{u}\gamma^\mu u, &&
\end{aligned}
$$

Tensor: $O^3_{lequ} \equiv \bar{l}\sigma_{\mu\nu} e \;\varepsilon\; \bar{q}\sigma^{\mu\nu} u.$

Four-quark operators: ...

$\overleftrightarrow{D}^{(I)}_\mu \equiv (\tau^I)\overrightarrow{D}_\mu - \overleftarrow{D}_\mu(\tau^I)$

Independent coefficients

Two-quark operators: 10 real degrees of freedom

Scalar: $C_{u\varphi}^{(33)}$,

Vector:

$$\begin{aligned} C_{\varphi q}^{+(33)} &= C_{\varphi q}^{+(3a)*}, && \text{(down-}Z\text{)} \\ C_{\varphi q}^{-(33)} &= C_{\varphi q}^{-(33)*}, && \text{(up-}Z\text{)} \\ C_{\varphi u}^{(33)} &= C_{\varphi u}^{(33)*}, \end{aligned}$$

Tensor: $C_{uB}^{(33)}$, $C_{uW}^{(33)}$, $C_{uG}^{(33)}$.

Two-quark–two-lepton operators: 9×3^2 real degrees of freedom

Scalar: $C_{lequ}^{1(33)}$,

Vector:

$$\begin{aligned} C_{lq}^{+(33)} &= C_{lq}^{+(33)*}, && (\text{up}–\nu,\ \text{down}–\ell) \\ C_{lq}^{-(33)} &= C_{lq}^{-(33)*}, && (\text{up}–\ell,\ \text{down}–\nu) \\ C_{lu}^{(33)} &= C_{lu}^{(33)*}, && (\text{up}–\ell,\ \text{up}–\nu) \\ C_{eq}^{(33)} &= C_{eq}^{(33)*}, && (\text{up}–\ell,\ \text{down}–\ell) \\ C_{eu}^{(33)} &= C_{eu}^{(33)*}, \end{aligned}$$

Tensor: $C_{lequ}^{3(33)}$.

Four-quark operators: ...

Gauge vs. physical basis

The CKM relates coefficients in the up$_L$ and down$_L$ sectors:

$$q \equiv (P_L u, \boldsymbol{V}_{\text{CKM}} P_L d)^T, \qquad u \equiv P_R u, \qquad d \equiv P_R d,$$
$$l \equiv (\boldsymbol{V}_{\text{PMNS}} P_L \nu, P_L e)^T, \qquad e \equiv P_R e$$

so that, *e.g.*:

$$\begin{aligned} C^-_{lq} O^-_{lq} = &[V^\dagger_{\text{PMNS}} V^\dagger_{\text{CKM}} C^-_{lq} V_{\text{CKM}} V_{\text{PMNS}}]^a{}_b{}^c{}_d \quad (\bar{\nu}_a \gamma^\mu P_L \nu^b \quad \bar{d}_c \gamma_\mu P_L d^d) \\ &+ [C^-_{lq}]^a{}_b{}^c{}_d \quad (\bar{e}_a \gamma^\mu P_L e^b \quad \bar{u}_c \gamma_\mu P_L u^d) \end{aligned}$$

with, e.g.:

$$[V^\dagger_{\text{CKM}} C V_{\text{CKM}}]^3{}_3 \simeq 0.998\, C^3{}_3 - 0.04\,(C^3{}_2 + C^2{}_3) + 0.02\,(C^3{}_1 + C^1{}_3) + \ldots$$

Anomalous vertices

$$t\bar{t}\gamma: \quad \gamma^\mu \overbrace{\left(F_{1,V}^{\gamma} + \gamma_5 F_{1,A}^{\gamma}\right)}^{\sim\emptyset} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2,V}^{\gamma} + i\gamma_5 F_{2,A}^{\gamma}\right)}^{\sim \mathrm{Re,Im}\{C_{uW}+C_{uB}\}}$$

$$t\bar{t}Z: \quad \gamma^\mu \underbrace{\left(F_{1,V}^{Z} + \gamma_5 F_{1,A}^{Z}\right)}_{\sim(C_{\phi u}\pm C_{\phi q}^{-})} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \underbrace{\left(F_{2,V}^{Z} + i\gamma_5 F_{2,A}^{Z}\right)}_{\sim \mathrm{Re,Im}\{c_W^2 C_{uW} - s_W^2 C_{uB}\}}$$

Insufficiencies:

- Conflict with gauge invariance
- Complex couplings where the tree-level EFT prescribes real ones
- Miss four-fermion operators
- Hide correlations induced by gauge invariance
  Preclude the combination of measurements in various sectors
- Do not allow for radiative corrections to be computed consistently

An EFT analysis

- Go global!
  - NP can generate several operators at a high scale
  - RG running down to low scales can mix them with others
  - Operators are re-express as combinations of others to form a basis

- Face interferences!

$$
\begin{aligned}
\sigma^{\sqrt{s}=500\ \mathrm{GeV}}_{e^+e^-\to t\bar{t}}\ [\mathrm{fb}] = +568 &+ \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2
\begin{pmatrix} C^A_{lq} \\ C^A_{eq} \\ C^A_{\phi q} \\ C^V_{lq} \\ C^V_{eq} \\ C^V_{\phi q} \\ C^R_{uZ} \\ C^R_{uA} \\ C^I_{uZ} \\ C^I_{uA} \end{pmatrix}^T
\begin{pmatrix} +221 \\ -194 \\ +7.01 \\ -1110 \\ -737 \\ -8.24 \\ +33.8 \\ +209 \\ \cdot \\ \cdot \end{pmatrix}
+ \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^4
\begin{pmatrix} C^A_{lq} \\ C^A_{eq} \\ C^A_{\phi q} \\ C^V_{lq} \\ C^V_{eq} \\ C^V_{\phi q} \\ C^R_{uZ} \\ C^R_{uA} \\ C^I_{uZ} \\ C^I_{uA} \end{pmatrix}^T
\begin{pmatrix}
+367 & \cdot & +13.2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & +367 & -11.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & +0.209 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & +868 & \cdot & +31.1 & -128 & -197 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & +868 & -27.3 & +112 & -197 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & +0.493 & -4.05 & -0.432 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +9.36 & +2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +25.2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +2.51 & +0.536 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +6.75
\end{pmatrix}
\begin{pmatrix} C^A_{lq} \\ C^A_{eq} \\ C^A_{\phi q} \\ C^V_{lq} \\ C^V_{eq} \\ C^V_{\phi q} \\ C^R_{uZ} \\ C^R_{uA} \\ C^I_{uZ} \\ C^I_{uA} \end{pmatrix} \\
&+ \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^4 \left\{ +1600 \left|C^S_{lequ}\right|^2 + 13900 \left|C^T_{lequ}\right|^2 \right\}
\end{aligned}
$$

- Combine observables!

- Offer yourself $\mathrm{N^nLO}$!

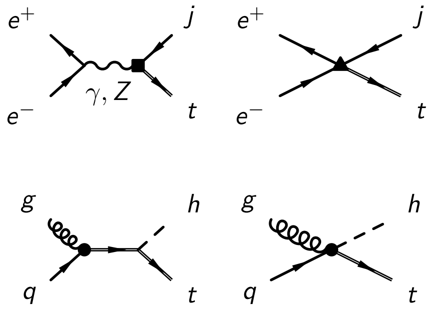
The top FCNC example

[GD,Maltoni,Zhang '14]

Top FCNC anomalous couplings

Schematically:

Scalar:	$\bar{t} q \quad h$
Vector:	$\bar{t}\gamma^\mu q \quad Z_\mu$
Tensor:	$\bar{t}\sigma^{\mu\nu} q \quad A_{\mu\nu}$
	$\bar{t}\sigma^{\mu\nu} q \quad Z_{\mu\nu}$
	$\bar{t}\sigma^{\mu\nu} T^A q \quad G^A_{\mu\nu}$



Issues:

1. Missing four-point interactions:
   - four-fermion operators
   - a *tqgh* vertex arising from $O_{uG} \equiv \bar{q}\sigma^{\mu\nu} T^A u\, \tilde{\varphi} \quad G^A_{\mu\nu}$
2. Operators of seemingly different dimensions
3. Hidden correlation:
   of '$v + h$' type
   of '$(t_L \quad [V_{\text{CKM}} d_L]^3)^T$' type

Top FCNC searches

			tqg, T	$tqgh$ T	$tq\gamma$ T	tqZ V,T	$tq\ell\ell$ S,V,T	$tqqq$ S,V,T	tqh S
The broken-phase effective Lagrangian:			✓	✗	✓	✓,✓	✗	✗	✓
production	• $e^+e^- \to t\, j$	OPAL, DELPHI, ALEPH, L3			✓	✓,✗	✗		
production	$e^- p \to e^- t$	**H1**, ZEUS			✓	✗	✗		
production	• $p\overset{(-)}{p} \to t$	CDF, **ATLAS**	✓						
production	$p\overset{(-)}{p} \to t\, j$	D0, **CMS**	✓		✗	✗		✗	
production	• $p\, p \to t\, \gamma$	CMS	✗		✓				
production	$p\, p \to t\, \ell^+\ell^-$	CMS	✓		✗	✗,✓	✗		
production	$p\, p \to t\, \gamma\gamma$	—	✗	✗	✗				✗
decay	$t \to j\gamma$	**CDF**, D0, ATLAS, CMS			✓				
decay	• $t \to j\ell^+\ell^-$	CDF, D0, ATLAS, **CMS**			✗	✓,✗	✗		
decay	• $t \to j\gamma\gamma$	**CMS**, ATLAS			✗				✓

Interferences and NLO

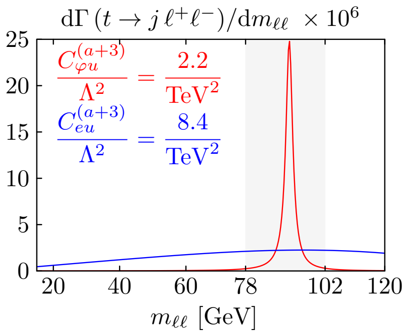
$$
\Gamma^{\text{on-peak + off-peak}}_{t\to j\ell^+\ell^-}/10^{-5}\,\text{GeV}\ \times(\Lambda/1\,\text{TeV})^4 =
$$

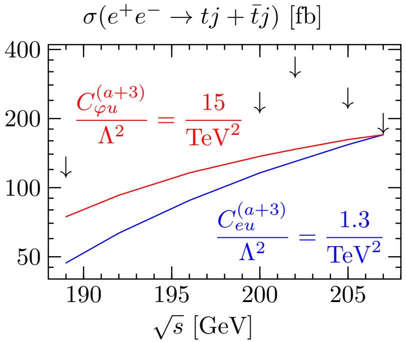
$$
\text{Re}\begin{pmatrix} C_{lq}^{-(a+3)} \\ C_{eq}^{(a+3)} \\ C_{\varphi q}^{-(a+3)} \\ C_{uB}^{(a3)} \\ C_{uW}^{(a3)} \\ C_{uG}^{(a3)} \end{pmatrix}^{\dagger}
\begin{pmatrix}
\underset{-8\%}{+0.29} & 0 & \underset{-12\%}{-0.035} - \underset{-8\%}{0.23i} & \underset{-7\%}{-0.19} - \underset{-8\%}{0.11i} & \underset{-7\%}{-0.33} + \underset{-8\%}{0.38i} & \underset{\text{—}}{+0.026} - \underset{\text{—}}{0.0025i} \\
 & \underset{-8\%}{+0.29} & \underset{-12\%}{+0.028} + \underset{-8\%}{0.18i} & \underset{-7\%}{-0.25} + \underset{-8\%}{0.087i} & \underset{-7\%}{-0.14} - \underset{-8\%}{0.30i} & \underset{\text{—}}{+0.00064} + \underset{\text{—}}{0.023i} \\
 & & \underset{-8\%}{+1.9} & \underset{-8\%}{+1.8} - \underset{-8\%}{0.016i} & \underset{-8\%}{-6.2} - \underset{-8\%}{0.016i} & \underset{\text{—}}{+0.29} + \underset{\text{—}}{0.22i} \\
 & & & \underset{-9\%}{+0.91} & \underset{-9\%}{-3.6} - \underset{-9\%}{0.049i} & \underset{\text{—}}{+0.14} + \underset{\text{—}}{0.12i} \\
 & & & & \underset{-9\%}{+7.6} & \underset{\text{—}}{-0.61} - \underset{\text{—}}{0.55i} \\
 & & & & & \underset{\text{—}}{+0.0068}
\end{pmatrix}
\begin{pmatrix} C_{lq}^{-(a+3)} \\ C_{eq}^{(a+3)} \\ C_{\varphi q}^{-(a+3)} \\ C_{uB}^{(a3)} \\ C_{uW}^{(a3)} \\ C_{uG}^{(a3)} \end{pmatrix}
$$

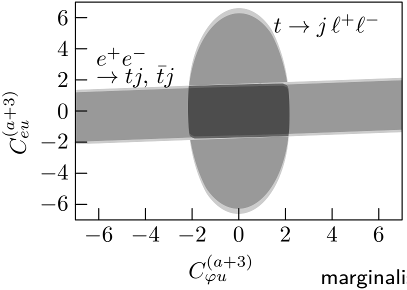
$$
+\,\text{Re}\begin{pmatrix} C_{lu}^{(a+3)} \\ C_{eu}^{(a+3)} \\ C_{\varphi u}^{(a+3)} \\ C_{uB}^{(3a)*} \\ C_{uW}^{(3a)*} \\ C_{uG}^{(3a)*} \end{pmatrix}^{\dagger}
\begin{pmatrix}
\underset{-8\%}{+0.29} & 0 & \underset{-12\%}{-0.035} - \underset{-8\%}{0.23i} & \underset{-7\%}{-0.19} - \underset{-8\%}{0.11i} & \underset{-7\%}{-0.33} + \underset{-8\%}{0.38i} & \underset{\text{—}}{+0.0068} + \underset{\text{—}}{0.021i} \\
 & \underset{-8\%}{+0.29} & \underset{-12\%}{+0.028} + \underset{-8\%}{0.18i} & \underset{-7\%}{-0.25} + \underset{-8\%}{0.087i} & \underset{-7\%}{-0.14} - \underset{-8\%}{0.30i} & \underset{\text{—}}{+0.016} + \underset{\text{—}}{0.0043i} \\
 & & \underset{-8\%}{+1.9} & \underset{-8\%}{+1.8} - \underset{-8\%}{0.016i} & \underset{-8\%}{-6.2} - \underset{-8\%}{0.016i} & \underset{\text{—}}{-0.18} - \underset{\text{—}}{0.092i} \\
 & & & \underset{-9\%}{+0.91} & \underset{-9\%}{-3.6} - \underset{-9\%}{0.049i} & \underset{\text{—}}{-0.13} - \underset{\text{—}}{0.096i} \\
 & & & & \underset{-9\%}{+7.6} & \underset{\text{—}}{+0.31} + \underset{\text{—}}{0.19i} \\
 & & & & & \underset{\text{—}}{+0.0053}
\end{pmatrix}
\begin{pmatrix} C_{lu}^{(a+3)} \\ C_{eu}^{(a+3)} \\ C_{\varphi u}^{(a+3)} \\ C_{uB}^{(3a)*} \\ C_{uW}^{(3a)*} \\ C_{uG}^{(3a)*} \end{pmatrix}
$$

$$
\underset{+1\%}{+0.082}\left(|C_{lequ}^{1(13)}|^2 + |C_{lequ}^{1(31)}|^2\right) + \underset{-8\%}{3.5}\left(|C_{lequ}^{3(13)}|^2 + |C_{lequ}^{3(31)}|^2\right)
$$

Combining observables





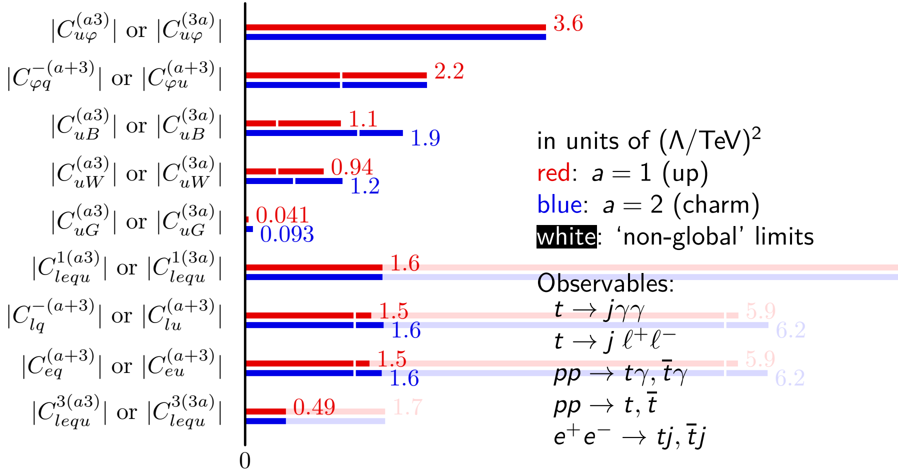


in units of $(\Lambda/\text{TeV})^2$

darker: $a = 1$, lighter: $a = 2$

marginalisation within final constraints

Global constraints



A partial top-Z analysis

[Berger, Cao, Low '09]

Operators considered

Top-Z:

$$\bar{t}\gamma^{\mu}\frac{\sqrt{g_1^2+g_2^2}}{2}\left[\frac{(C^3_{\varphi q}-C^1_{\varphi q})v^2}{\Lambda^2}P_L-\frac{C_{\varphi u}v^2}{\Lambda^2}P_R\right]tZ_{\mu}$$

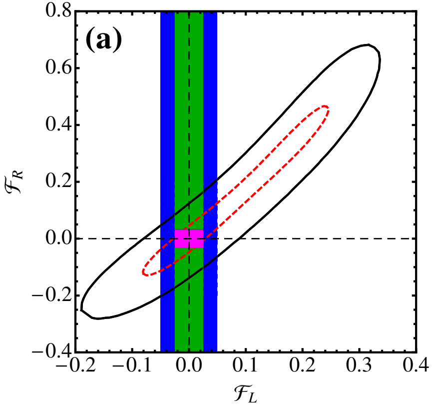
Bottom-Z:

$$-\bar{b}\gamma^{\mu}\frac{\sqrt{g_1^2+g_2^2}}{2}\left[\overbrace{\frac{(C^3_{\varphi q}+C^1_{\varphi q})v^2}{\Lambda^2}}^{Z\to b\bar{b}}P_L+\frac{C_{\varphi d}v^2}{\Lambda^2}P_R\right]bZ_{\mu}$$

Charged currents:

$$\bar{t}\gamma^{\mu}\frac{g_2}{\sqrt{2}}\left[\frac{C^3_{\varphi q}v^2}{\Lambda^2}P_L-\underbrace{\frac{C_{\varphi\varphi}v^2}{2\Lambda^2}}_{b\to s\gamma}P_R\right]bW^{+}_{\mu}\quad+\text{h.c.}$$

Impact on hadron and lepton colliders



$\mathcal{F}_R \equiv -C_{\varphi u} v^2/\Lambda^2$

$\mathcal{F}_L \equiv +C^3_{\varphi q} v^2/\Lambda^2$

- black, red: σ and $\Delta\phi_{\ell\ell_Z}$ in $t\bar{t}Z$ at the LHC, with 300/fb and 3/ab
- blue, green: σ(single top) at the LHC, 10% and 5% uncertainty
- violet: 100/fb at 500 GeV lepton collider

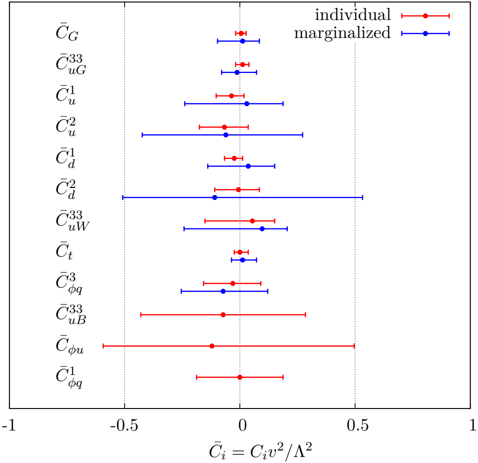
$\Longrightarrow$ LHC 3/ab already excludes many t' and 4th generation model.

Towards a global top EFT analysis

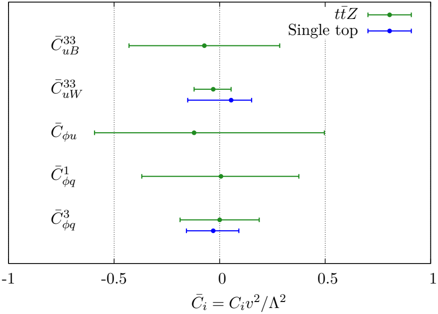
[Buckley *et al.* Jun. 15]
[Buckley *et al.* Dec. 15]

Towards a global top EFT analysis

- 195 observables (174 from differential distributions), 12 operators
- mainly from $t\bar{t}$, then single top, charge asymmetries, associated production, W helicity fraction in decay
- standard-model (N)NLO k-factors in each bin



individual 95% CL limits from $t\bar{t}Z$ and $t\bar{t}\gamma$:



MadGraph for NLO QCD
in the effective field theory

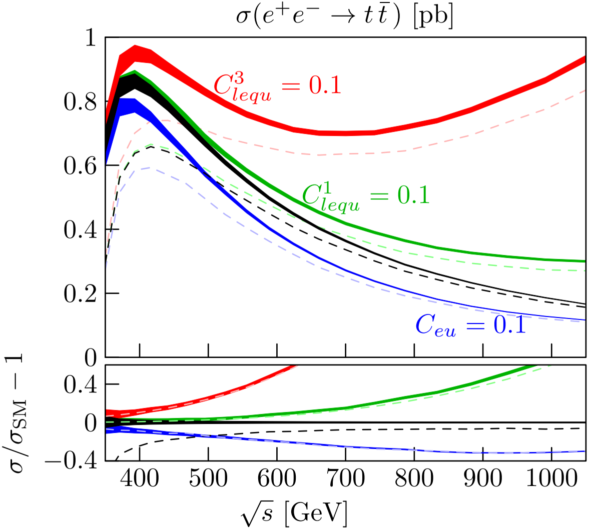
MadGraph for NLO QCD in the effective field theory

- The implementation of operators is required (UV and R2 CT's). Automation is in progress. [Degrande 14']
  - FCNCs [Degrande et al, 14']
  - top pair production [Franzosi et al. 15']
  - single top production [Zhang 16']
  - $t\bar{t}Z$, $t\bar{t}\gamma$ [Bylund et al. 16']
  - $t\bar{t}h$ [Maltoni et al. ...]
  - four-fermion operators [$\bar{l}l\bar{q}q$ OK, $\bar{q}q\bar{q}q$ ongoing]

- QCD NLO matched to parton shower is then automatic. Progresses are made towards EW NLO too. [Frixione et al. 15' ($t\bar{t}Z/W$)]

MadGraph for NLO QCD in the effective field theory

Example with four-fermion operators (NEW!):



$\Lambda = 1$ TeV, $\Lambda^{-2,-4}$ terms, $m_t = 172.5$ GeV, unpolarised beams, α_s scale variation only

Summary

The EFT is a well defined framework for constraining new physics in a model-independent way.

It allows for correlations between observables (arising from $SU(2)_L \times U(1)_Y$ gauge invariance) to be accounted for.

Constraints can then be set globally, and should be set globally.

Predictions are perturbatively improvable.

A lot of progresses have recently been made in both of these directions.